Fermi National Accelerator Laboratory

FERMILAB-Pub-99/248-T

Focus Points and Naturalness in Supersymmetry

Jonathan L. Feng and Takeo Moroi

Institute for Advanced Study
Princeton, New Jersey 08540

Konstantin T. Matchev

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

September 1999

Submitted to *Physical Review D*

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Disclaimer

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, expressed or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise, does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

Distribution

Approved for public release; further dissemination unlimited.

Copyright Notification

This manuscript has been authored by Universities Research Association, Inc. under contract No. DE-AC02-76CH03000 with the U.S. Department of Energy. The United States Government and the publisher, by accepting the article for publication, acknowledges that the United States Government retains a nonexclusive, paid-up, irrevocable, worldwide license to publish or reproduce the published form of this manuscript, or allow others to do so, for United States Government Purposes.

IASSNS–HEP–99–81
FERMILAB–PUB–99/248–T
hep-ph/9909334
September 1999

Focus Points and Naturalness in Supersymmetry

Jonathan L. Feng[a], Konstantin T. Matchev[b], and Takeo Moroi[a]

[a] *School of Natural Sciences, Institute for Advanced Study*
Princeton, NJ 08540, U.S.A.

[b] *Theoretical Physics Department, Fermi National Accelerator Laboratory*
Batavia, IL 60510, U.S.A.

Abstract

We analyze focus points in supersymmetric theories, where a parameter's renormalization group trajectories meet for a family of ultraviolet boundary conditions. We show that in a class of models including minimal supergravity, the up-type Higgs mass has a focus point at the weak scale, where its value is highly insensitive to the universal scalar mass. As a result, scalar masses as large as 2 to 3 TeV are consistent with naturalness, and *all* squarks, sleptons and heavy Higgs scalars may be beyond the discovery reaches of the Large Hadron Collider and proposed linear colliders. Gaugino and Higgsino masses are, however, still constrained to be near the weak scale. The focus point behavior is remarkably robust, holding for both moderate and large $\tan\beta$, any weak scale gaugino masses and A parameters, variations in the top quark mass within experimental bounds, and for large variations in the boundary condition scale.

I. INTRODUCTION

An understanding of electroweak symmetry breaking is currently one of the most important objectives in high energy particle physics. Renormalizability requires that electroweak symmetry be spontaneously broken; in the minimal standard model, this is realized by the condensation of the elementary Higgs field. In such a theory, however, the squared Higgs mass receives quadratically divergent radiative corrections. The Higgs mass, and with it the weak scale, is therefore expected to be of order the cut-off scale, which is typically identified with the grand unified theory (GUT) or Planck scale. The fact that the weak scale is much smaller than the cut-off scale requires a large fine-tuning and is therefore considered unnatural in the minimal standard model [1].

Supersymmetry removes quadratic divergences and therefore provides a framework for naturally explaining the stability of the weak scale with respect to radiative corrections [2]. However, the requirement of naturalness constrains supersymmetric models, as, in these models, the weak scale is generated when electroweak symmetry breaking is induced by (negative) squared mass parameters for the Higgs scalars. The weak scale is therefore related to these supersymmetry breaking parameters. It is hoped that an understanding of the mechanism of supersymmetry breaking will shed light on the origin of the weak scale. Even without this knowledge, though, it is clear that naturalness in supersymmetric theories requires that the supersymmetry breaking parameters in the Higgs potential be not too far above the weak scale.

As naturalness implies *upper* bounds on supersymmetry breaking parameters and superpartner masses, its implications are obviously of great importance for supersymmetry searches. These implications depend on the assumed structure of supersymmetry breaking. With respect to the scalar masses, broadly speaking, three possibilities exist. In the first, all supersymmetry breaking scalar mass parameters are roughly of the same order; for example, they may be of the same order when generated at some high scale and then remain of the same order when evolved through renormalization group (RG) equations to the weak scale. Naturalness then demands that scalar Higgs, squark, and slepton masses all be near the weak scale. Such light particles are within the discovery reach of the Large Hadron Collider (LHC) and future lepton colliders.

Another possibility is that a hierarchy exists between the various scalar masses. This hierarchy may be present at the scale at which supersymmetry breaking parameters are generated [3] or may be generated dynamically through RG evolution [4]. In either case, one finds that naturalness bounds on the first and second generation squarks and sleptons are much weaker than those for the third generation [5]. First and second generation sfermions may then be much heavier than 1 TeV and far beyond the reach of near future colliders. However, top and bottom squarks, for example, are still constrained to have masses of order the weak scale, and should be discovered by the LHC.

A third possibility, however, is that the RG trajectories of the Higgs mass parameters may meet at a "focus point" [6,7], where their values are independent of their ultraviolet

boundary values.[1] If this focus point is near the weak scale, the Higgs potential at the weak scale may be insensitive to the ultraviolet values of certain supersymmetry breaking parameters, including the scalar masses. In this case, naturalness, while constraining (unphysical) Higgs mass parameters, may lead to very weak upper bounds on the squark, slepton, and heavy Higgs boson masses, and these scalars may *all* be beyond the reach of near future colliders.

The last possibility is the subject of this study. We will show that it is realized in a class of models that includes minimal supergravity. Minimal supergravity is at present probably the single most widely-studied framework for evaluating the potential of new experiments to probe physics beyond the standard model. We therefore concentrate on this model and explore the implications of focus points in minimal supergravity for naturalness and the superpartner spectrum.

The organization of this paper is as follows. In Sec. II, we analyze the focus point behavior of the RG evolution of supersymmetry breaking parameters. In Sec. III, we discuss the implications of the up-type Higgs focus point for naturalness in minimal supergravity. In particular, we will see that multi-TeV scalar masses are consistent with naturalness. The implications of these results for superpartner searches are considered in Sec. IV. Finally, we conclude in Sec. V with a summary of our results and some philosophical comments concerning the concept of naturalness.

II. FOCUS POINTS

In this section, we explore the phenomenon of focus points in the RG evolution of supersymmetry breaking parameters. We will see that, in certain circumstances, the supersymmetry breaking up-type Higgs mass has such a focus point at the weak scale, where it becomes insensitive to its boundary value at the high scale (for example, the GUT scale). Since the supersymmetry breaking masses for the Higgses are related to the weak scale, this fact has implications for naturalness, as we will discuss in Sec. III.

We start by considering the RG behavior of supersymmetry breaking scalar masses. Denoting the mass of the i-th scalar field by m_i, the one-loop RG evolution of scalar masses is given schematically[2] by the inhomogeneous equations

$$\frac{dm_i^2}{d\ln Q} \sim \frac{1}{16\pi^2}\left[-g_a^2 M_a^2 + \sum_j y_j^2 m_j^2 + \sum_j y_j^2 A_j^2\right] , \tag{1}$$

[1]Focus points are not be confused with the well-known phenomena of fixed and quasi-fixed points [8]. As we will see, when RG trajectories have a focus point behavior, they do not asymptotically approach a limit curve, but rather meet and then disperse.

[2]In Eqs. (1)–(3) and (5), we neglect positive $\mathcal{O}(1)$ coefficients for each term.

where g_a and y_j are gauge and Yukawa coupling constants, respectively, Q is the renormalization scale, and the summation is over all chiral superfields coupled to the i-th chiral superfield through Yukawa interactions. The gaugino masses M_a and supersymmetry breaking trilinear scalar couplings A_j have RG equations

$$\frac{dM_a}{d\ln Q} \sim \frac{1}{16\pi^2} g_a^2 M_a \ , \tag{2}$$

$$\frac{dA_i}{d\ln Q} \sim \frac{1}{16\pi^2}\left[g_a^2 M_a + \sum_j y_j^2 A_j\right] \ . \tag{3}$$

As one can see, the evolution of the m_i^2 parameters depends on the gaugino masses and A parameters, as well as on the scalar masses themselves. On the other hand, the gaugino masses and A parameters evolve independently of the scalar masses.[3] This structure implies that, if $m_i^2|_{\rm p}$ is a particular solution to Eq. (1) with fixed values of the gaugino masses and A parameters, then for arbitrary constant ξ,

$$m_i^2(Q) = m_i^2|_{\rm p}(Q) + \xi\Delta_i^2(Q) \tag{4}$$

is also a solution if the Δ_i^2 obey the following linear and homogeneous equation:

$$\frac{d\Delta_i^2}{d\ln Q} \sim \frac{1}{16\pi^2}\sum_j y_j^2 \Delta_j^2 \ . \tag{5}$$

The evolution of the Δ_i^2 depends only on the Δ_i^2, and the Δ_i^2 are themselves solutions to the RG equations in the limit $M_a, A_j \to 0$.

With a given boundary condition, Δ_i^2 may vanish for some i at some renormalization scale $Q_{\rm F}^{(i)}$. At this scale, m_i^2 is given by $m_i^2|_{\rm p}$ irrespective of ξ, and the family of boundary conditions parameterized by $m_i^2(Q_0) = m_i^2|_{\rm p}(Q_0) + \xi\Delta_i^2(Q_0)$, with various ξ, all yield the same value of m_i^2 at the scale $Q_{\rm F}^{(i)}$. (Here, Q_0 is the scale where the boundary condition is given.) We call $Q_{\rm F}^{(i)}$ the focus point scale or "focus point" for m_i^2. For large ξ, $m_i^2(Q_0) \gg m_i^2|_{\rm p}(Q_0)$, and if $M_a^2(Q_0), A^2(Q_0) \sim m_i^2|_{\rm p}(Q_0)$, a large hierarchy $m_i^2(Q_0) \gg m_i^2(Q_{\rm F}^{(i)})$ is generated.

This concludes our general discussion of focus points. We now consider the existence of focus points in the minimal supersymmetric standard model. Before showing the results of detailed numerical calculations, we first analyze the focus point behavior by using one-loop RG equations. In the minimal supersymmetric standard model, the possibly large Yukawa couplings are those for the third generation quarks, y_t and y_b.[4] The system of RG equations for the Δ_i^2 is then

---

[3]Note that this implies that if a hierarchy $M_a, A_j \ll m_i$ is generated at some high scale, for example, by an approximate R-symmetry, it will not be destabilized by RG evolution.

[4]For large $\tan\beta$, the τ Yukawa coupling y_τ may be sizable, too. Here, for our simplified discussion, we neglect y_τ, although its effects are included in our numerical calculations.

$$\frac{d}{d\ln Q}\begin{bmatrix}\Delta^2_{H_u}\\ \Delta^2_{U_3}\\ \Delta^2_{Q_3}\\ \Delta^2_{D_3}\\ \Delta^2_{H_d}\end{bmatrix} = \frac{y_t^2}{8\pi^2}\begin{bmatrix}3&3&3&0&0\\2&2&2&0&0\\1&1&1&0&0\\0&0&0&0&0\\0&0&0&0&0\end{bmatrix}\begin{bmatrix}\Delta^2_{H_u}\\ \Delta^2_{U_3}\\ \Delta^2_{Q_3}\\ \Delta^2_{D_3}\\ \Delta^2_{H_d}\end{bmatrix} + \frac{y_b^2}{8\pi^2}\begin{bmatrix}0&0&0&0&0\\0&0&0&0&0\\0&0&1&1&1\\0&0&2&2&2\\0&0&3&3&3\end{bmatrix}\begin{bmatrix}\Delta^2_{H_u}\\ \Delta^2_{U_3}\\ \Delta^2_{Q_3}\\ \Delta^2_{D_3}\\ \Delta^2_{H_d}\end{bmatrix} , \tag{6}$$

where Q_3, U_3, and D_3 represent the third generation $\mathrm{SU}(2)_L$ doublet, singlet up-type, and singlet down-type squarks, and H_u and H_d are the up- and down-type Higgses, respectively. All other Δ_i^2's are not coupled to large Yukawa coupling constants and hence are (almost) scale independent.

To find the focus point, it is simplest to begin by considering small or moderate values of $\tan\beta$, for which y_b is negligible. In this case, $\Delta^2_{D_3}$ and $\Delta^2_{H_d}$ remain constant, but the RG equations for $\Delta^2_{H_u}$, $\Delta^2_{U_3}$, and $\Delta^2_{Q_3}$ are solved by

$$\begin{bmatrix}\Delta^2_{H_u}(Q)\\ \Delta^2_{U_3}(Q)\\ \Delta^2_{Q_3}(Q)\end{bmatrix} = \kappa_6\begin{bmatrix}3\\2\\1\end{bmatrix}e^{6I(Q)} + \kappa_0\begin{bmatrix}1\\0\\-1\end{bmatrix} + \kappa_0'\begin{bmatrix}0\\1\\-1\end{bmatrix} , \tag{7}$$

where

$$I(Q) \equiv \int_{\ln Q_0}^{\ln Q}\frac{y_t^2(Q')}{8\pi^2}d\ln Q' . \tag{8}$$

The κ's are constants determined by the boundary conditions at the scale Q_0 and are independent of the renormalization scale Q. For given κ's, the focus point for $m^2_{H_u}$ is given by

$$3\kappa_6 e^{6I(Q_{\rm F}^{(H_u)})} + \kappa_0 = 0 . \tag{9}$$

For large $\tan\beta$, we cannot neglect y_b, and the above arguments do not apply. For a general large $\tan\beta$, the evolution of the parameters is complicated and will be studied numerically below. However, for the specific case $\tan\beta \simeq m_t/m_b$, we can assume $y_b = y_t$ and follow an analysis similar to the one above.[5] In this case, the Δ_i^2's evolve according to

$$\begin{bmatrix}\Delta^2_{H_u}(Q)\\ \Delta^2_{U_3}(Q)\\ \Delta^2_{Q_3}(Q)\\ \Delta^2_{D_3}(Q)\\ \Delta^2_{H_d}(Q)\end{bmatrix} = \kappa_7\begin{bmatrix}3\\2\\2\\2\\3\end{bmatrix}e^{7I(Q)} + \kappa_5\begin{bmatrix}3\\2\\0\\-2\\-3\end{bmatrix}e^{5I(Q)} + \kappa_0\begin{bmatrix}1\\-1\\0\\0\\0\end{bmatrix} + \kappa_0'\begin{bmatrix}0\\1\\-1\\1\\0\end{bmatrix} + \kappa_0''\begin{bmatrix}0\\0\\0\\-1\\1\end{bmatrix} . \tag{10}$$

[5] In fact, y_t and y_b, even if initially identical, will be slightly split in their RG evolution by y_τ and the $\mathrm{U}(1)_Y$ gauge interaction. In this discussion, we neglect this difference. This approximation is justified by the numerical calculations to follow.

The focus point for $m_{H_u}^2$ is then given by

$$3\kappa_7 e^{7I(Q_{\rm F}^{(H_u)})} + 3\kappa_5 e^{5I(Q_{\rm F}^{(H_u)})} + \kappa_0 = 0 \ . \tag{11}$$

The actual focus point depends on the boundary condition. Here, we first consider the well-studied case of minimal supergravity. In this framework, the supersymmetric Lagrangian is specified by five new fundamental parameters: the universal scalar mass m_0, the unified gaugino mass $M_{1/2}$, the supersymmetric Higgs mass μ_0, the universal trilinear coupling A_0, and the bilinear Higgs scalar coupling B_0. These parameters are given at the GUT scale $M_{\rm GUT}$, which, in our analysis, is defined as the scale where the $\mathrm{SU}(2)_L$ and $\mathrm{U}(1)_Y$ gauge couplings meet. (Numerically, $M_{\rm GUT} \simeq 2 \times 10^{16}$ GeV.) All the supersymmetry breaking scalar masses are universal at $M_{\rm GUT}$, and we may take[6]

$$m_i^2|_{\rm p}(M_{\rm GUT}) = 0 \ , \tag{12}$$

$$\xi \Delta_i^2(M_{\rm GUT}) = m_0^2 \ . \tag{13}$$

With these boundary conditions, the coefficients κ are

$$(\kappa_6, \kappa_0, \kappa_0') = m_0^2 \left(\frac{1}{2}, -\frac{1}{2}, 0\right) \ , \quad y_b \ll y_t \ , \tag{14}$$

$$(\kappa_7, \kappa_5, \kappa_0, \kappa_0', \kappa_0'') = m_0^2 \left(\frac{3}{7}, 0, -\frac{2}{7}, -\frac{1}{7}, -\frac{2}{7}\right) \ , \quad y_b = y_t \ , \tag{15}$$

and the focus point scale is determined by

$$e^{6I(Q)} = 1/3 \ , \ \text{for } y_b \ll y_t \ , \tag{16}$$

$$e^{7I(Q)} = 2/9 \ , \ \text{for } y_b = y_t \ . \tag{17}$$

Note that the $I(Q)$ in Eqs. (16) and (17) are not identical, as y_t runs differently for small and large $\tan\beta$.

Eqs. (16) and (17) determine the focus point scale in terms of the gauge couplings and the top quark Yukawa coupling, or equivalently, the top quark mass. Remarkably, for the physical gauge couplings and top quark mass $m_t \approx 174$ GeV, both conditions yield focus points that are very close to the weak scale! Thus, in minimal supergravity, the weak scale value of $m_{H_u}^2$ is highly insensitive to the universal scalar mass m_0.

We now show numerically that the focus point is near the weak scale for $m_t \approx 174$ GeV. (For an analytical discussion, see the Appendix.) To study the focus point more carefully, we have evolved the supersymmetry breaking parameters with the full two-loop RG equations [9]. The one-loop threshold corrections from supersymmetric particles

[6] We choose $m_i^2|_{\rm p}(M_{\rm GUT}) = 0$ so that $m_i^2|_{\rm p}$ is independent of m_0 and is always $\mathcal{O}(M_{1/2}^2)$ (or $\mathcal{O}(A_0^2)$) or smaller.

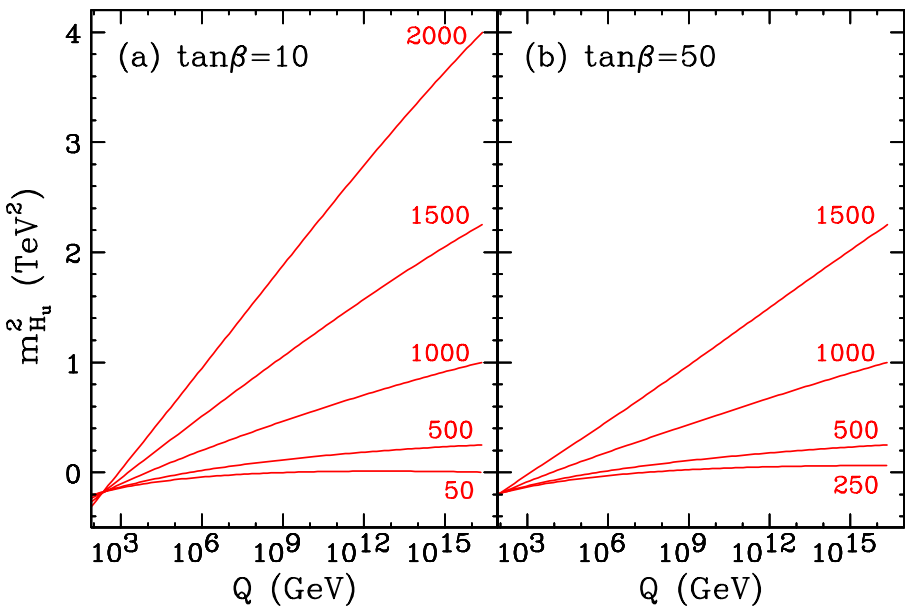

FIG. 1. The RG evolution of $m^2_{H_u}$ for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, several values of m_0 (shown, in GeV), $M_{1/2} = 300$ GeV, $A_0 = 0$, and $m_t = 174$ GeV. For both values of $\tan\beta$, $m^2_{H_u}$ exhibits an RG focus point near the weak scale, where $Q_{\rm F}^{(H_u)} \sim \mathcal{O}(100\ {\rm GeV})$, irrespective of m_0.

to the gauge and Yukawa coupling constants are also included [10,11]. We take as inputs $\alpha_{\rm em}^{-1} = 137.0359895$, $G_F = 1.16639 \times 10^{-5}$, $\alpha_s(m_Z) = 0.117$, $m_Z = 91.187$ GeV, $m_\tau^{DR}(m_Z) = 1.7463$ GeV, bottom quark pole mass $m_b = 4.9$ GeV, and, unless otherwise noted, top quark pole mass $m_t = 174$ GeV.

The scale dependence of $m^2_{H_u}$ for various values of m_0 in minimal supergravity is shown in Fig. 1. To high accuracy, all of the RG trajectories meet at $Q \sim \mathcal{O}(100\ {\rm GeV})$. In fact, in this case, the weak value of $m^2_{H_u}$ is determined by the other fundamental parameters $M_{1/2}$ and A_0, and hence at least one of these parameters is required to be $\mathcal{O}(100\ {\rm GeV})$.

In Fig. 1, two values of $\tan\beta$ were presented. In Fig. 2, we show the focus point scale of $m^2_{H_u}$ as a function of $\tan\beta$. The focus point is defined here as the scale where $\partial m^2_{H_u}/\partial m_0 = 0$. As noted above, we have included the low-energy threshold corrections to the gauge and Yukawa coupling constants, which depend on the soft supersymmetry breaking parameters. As a result, the RG trajectories do not all meet at one scale, and the focus point given in Fig. 2 has a slight dependence on m_0. For small values of $\tan\beta$, say, $\tan\beta \sim 2-3$, the focus point is at very large scales. However, the important point is that, for all values of $\tan\beta \gtrsim 5$, including both moderate values of $\tan\beta$ and large values where y_b and y_τ are not negligible, $Q_{\rm F}^{(H_u)} \sim \mathcal{O}(100\ {\rm GeV})$, and the weak scale value of $m^2_{H_u}$ is insensitive to m_0.

So far, we have considered only the case of a universal scalar mass. However, the $m^2_{H_u}$ focus point remains at the weak scale for a much wider class of boundary conditions. For example, for small $\tan\beta$, Eq. (7) shows that the parameter κ_0' does not affect the evolution of $m^2_{H_u}$. As a result, the focus point of $m^2_{H_u}$ does not change even if we vary κ_0', and the

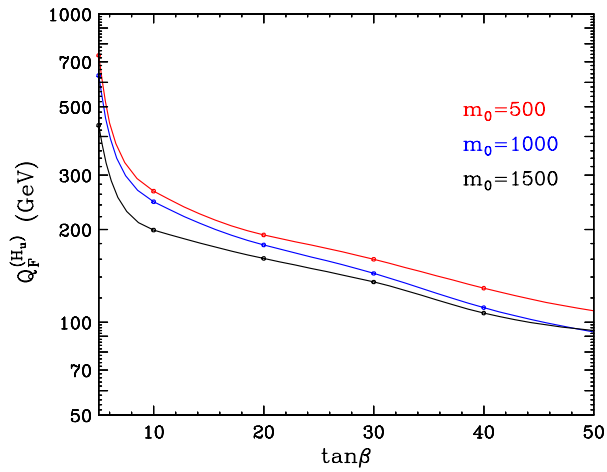

FIG. 2. The focus point renormalization scale $Q_{\rm F}^{(H_u)}$ as a function of $\tan\beta$ for $m_0 = 500$, 1000, and 1500 GeV (from above), $M_{1/2} = 300$ GeV, $A_0 = 0$, and $m_t = 174$ GeV. The focus point scale is defined as the scale at which $\partial m_{H_u}^2/\partial m_0 = 0$.

weak scale focus point is realized with any boundary condition of the form

$$(m_{H_u}^2, m_{U_3}^2, m_{Q_3}^2) \propto (1, 1+x, 1-x) \ , \tag{18}$$

with x an arbitrary constant. Similarly, for the case of $y_b = y_t$, the possible variation is

$$(m_{H_u}^2, m_{U_3}^2, m_{Q_3}^2, m_{D_3}^2, m_{H_d}^2) \propto (1, 1+x, 1-x, 1+x-x', 1+x') \ , \tag{19}$$

with both x and x' arbitrary.

Another possible modification of the boundary conditions may be seen by viewing the $m_i^2|_{\rm p}(Q_0)$ as perturbations. Since it was never necessary to specify the particular solution in the general focus point analysis, we may consider arbitrary $m_i^2|_{\rm p}(Q_0)$ without changing the focus point scale. The only constraint on $m_i^2|_{\rm p}$ is from naturalness. As will be discussed in the next section, $m_{H_u}^2$ is required to be $\mathcal{O}((100\ {\rm GeV})^2)$ at the weak scale for natural electroweak symmetry breaking. As a result, $m_{H_u}^2|_{\rm p}$, $m_{U_3}^2|_{\rm p}$, and $m_{Q_3}^2|_{\rm p}$ (and also $m_{H_d}^2|_{\rm p}$ and $m_{D_3}^2|_{\rm p}$ for large $\tan\beta$) are required to be of the order of the weak scale. Therefore, deviations from the boundary conditions of Eqs. (18) and (19) of order $\delta m^2 \sim \mathcal{O}((100\ {\rm GeV})^2)$ are acceptable and do not lead to fine-tuning problems. Similar arguments show that deviations $M_a, A_j \sim \mathcal{O}(100\ {\rm GeV})$ are allowed. In particular, the focus point is independent of gaugino mass or A parameter universality, and may therefore be found in many other frameworks. For example, focus points also exist in anomaly-mediated supersymmetry breaking models with additional universal scalar masses [6].

Before closing this section, we discuss another way of formulating the focus point, which was originally used in Ref. [12] for the specific case of minimal supergravity. By dimensional analysis, the evolution of $m_{H_u}^2$ may be parameterized as

$$m_{H_u}^2(Q) = \eta_{m_0^2}(Q) m_0^2 + \eta_{M_{1/2}^2}(Q) M_{1/2}^2 + \eta_{M_{1/2}A_0}(Q) M_{1/2} A_0 + \eta_{A_0^2}(Q) A_0^2 \ , \tag{20}$$

where the coefficients η are determined by the (dimensionless) gauge and Yukawa coupling constants, and are independent of the (dimensionful) supersymmetry breaking parameters. In this formulation, the focus point is given by the scale where $\eta_{m_0^2} = 0$, since at that scale, the value of $m_{H_u}^2$ is insensitive to m_0.[7] Based on this observation, it was noted in Ref. [12] that, for minimal supergravity, and neglecting y_b, $\eta_{m_0^2} = 0$ at the weak scale, and the weak scale becomes very insensitive to the universal scalar mass m_0, for $m_t \simeq 160 - 170$ GeV. As may be seen from the general analysis of focus points above, however, this conclusion holds much more generally: it is valid even when y_b is not negligible, holds for the more general boundary conditions given in Eqs. (18) and (19), and is independent of all other scalar masses. In addition, as noted above and as is evident from Eq. (20), the conclusion is valid also for non-universal gaugino masses and A parameters, as long as they are not too much larger than the weak scale.

III. NATURALNESS

In the previous section, we saw that the weak scale value of $m_{H_u}^2$ is highly insensitive to the high scale scalar mass boundary conditions in a class of models that includes minimal supergravity. This fact has important implications for the naturalness of the gauge hierarchy, since $m_{H_u}^2$ determines, to a large extent, the shape of the Higgs potential.

In minimal supergravity, $m_{H_u}^2$ and other sfermion masses have the same origin, the universal scalar mass m_0, and it has typically been believed that naturalness constraints on $m_{H_u}^2$ also give similar bounds on the sfermion masses. Such beliefs have led to great optimism in the search for scalar superpartners at future colliders in the framework of minimal supergravity [13]. However, as we have seen, the relation between $m_{H_u}^2$ and other sfermion masses is not as trivial as typically assumed. In the following, we therefore reconsider the naturalness bounds on the sfermion masses in the minimal supergravity model.

To begin, it is instructive to start with the tree-level expression for the weak scale. The Z boson mass is determined by minimizing the tree-level Higgs potential to be

$$\frac{1}{2} m_Z^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \mu^2 \ . \tag{21}$$

For all $\tan\beta$, $m_{H_u}^2 \gg m_Z^2$ is disfavored by the naturalness criterion, as in that case, a large cancellation between $m_{H_u}^2$ and μ^2 is needed to arrive at the physical value of the weak scale. For moderate and large values of $\tan\beta$, however, $m_{H_d}^2 \gg m_Z^2$ does not necessarily lead to fine-tuning.

---

[7] Notice that in this formulation, it is clear that all RG trajectories meet at a focus point to all orders in the RG equations. Of course, as noted above, threshold effects smear out the focus point slightly — see Fig. 2.

For more detailed discussions of naturalness, it is convenient to define a quantitative measure of fine-tuning [12,14–17]. Following previous work, we use the sensitivity of the weak scale (i.e., m_Z) to fractional variations in the fundamental parameters as such a measure.

In any discussion of naturalness, several subjective choices must be made. The choice of supersymmetry breaking framework is crucial. For example, in GUT models, the gaugino masses are all governed by one parameter, whereas in general, all three gaugino masses may be varied independently, and the sensitivity of the weak scale to each of them must be considered. In the following, we will specialize to minimal supergravity. As noted previously, minimal supergravity introduces five new fundamental parameters: m_0, $M_{1/2}$, μ_0, A_0, and B_0. All quantities at the weak scale are fixed by these parameters. In particular, the vacuum expectation values of the Higgs bosons depend on these quantities. Therefore, some combination of them is constrained to yield the correct Z boson mass. (At tree level, this constraint is that of Eq. (21). At one-loop, the Higgs masses squared are shifted by the corresponding tadpole contributions [18], as will be discussed below.) From the low energy point of view, it is therefore more convenient to consider $\tan\beta$ and $\mathrm{sign}(\mu)$ as free parameters, instead of μ_0 and B_0.

We adopt the following procedure to calculate the magnitude of fine-tuning at all physically viable parameter points:
(i) We consider the minimal supergravity framework with its $4+1$ input parameters

$$\{P_{\rm input}\} = \{m_0, M_{1/2}, A_0, \tan\beta, \mathrm{sign}(\mu)\} \ . \tag{22}$$

Any point in the parameter space of minimal supergravity is specified by these parameters.
(ii) The naturalness of each point is then calculated by first determining all the parameters of the theory (Yukawa couplings, soft supersymmetry breaking masses, etc.), consistent with low energy constraints. RG equations are used to relate high and low energy boundary conditions. In particular, using the relevant radiative breaking condition, $|\mu_0|$ and B_0 are determined consistent with the low energy constraints.
(iii) We choose to consider the following set of (GUT scale) parameters to be free, continuously valued, independent, and fundamental:

$$\{a_i\} = \{m_0, M_{1/2}, \mu_0, A_0, B_0\} \ . \tag{23}$$

(iv) All observables, including the Z boson mass, are then reinterpreted as functions of the fundamental parameters a_i, and the sensitivity of the weak scale to small fractional variations in these parameters is measured by the sensitivity coefficients

$$c_a \equiv \left| \frac{\partial \ln m_Z^2}{\partial \ln a} \right| \ , \tag{24}$$

where all other fundamental (not input) parameters are held fixed in the partial derivative.[8]

[8]The sensitivity of $v^2 = v_u^2 + v_d^2$, where v_u and v_d are the vacuum expectation values of the up-

(v) Finally, we form the fine-tuning parameter

$$c \equiv \max\{c_{m_0}, c_{M_{1/2}}, c_{\mu_0}, c_{A_0}, c_{B_0}\} \ , \tag{25}$$

which is taken as a measure of the naturalness of point $\{P_{\rm input}\}$, with large c corresponding to large fine-tuning.

Among the choices made in the prescription above, the choice of fundamental parameters a_i is of particular importance. This choice varies throughout the literature [12,14–17]. Since we are interested in the naturalness of the supersymmetric solution to the gauge hierarchy problem, we find it most reasonable to include only supersymmetry breaking parameters (and μ, as its origin is likely to be tied to supersymmetry breaking) and not standard model parameters, such as y_t or the strong coupling. We will return to this issue in Sec. V.

Given the prescription for defining fine-tuning described above, we may now present the numerical results. In minimizing the Higgs potential, we use the one-loop corrected Higgs potential, calculated with parameters evaluated with two-loop RG equations. Denoting the physical stop masses by $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$, we choose to minimize the potential at the scale

$$Q_{\tilde{t}} = (m_{\tilde{t}_1} m_{\tilde{t}_2})^{1/2} \ , \tag{26}$$

where the one-loop corrections to the Higgs potential tend to be smallest [15]. In terms of the fundamental parameters, $Q_{\tilde{t}} \simeq 0.5(m_0^2 + 4M_{1/2}^2)^{1/2}$.

In Figs. 3–5, we give contours of constant c_{m_0}, $c_{M_{1/2}}$, and c_{μ_0} in the $(m_0, M_{1/2})$ plane for $\tan\beta = 10$ and 50. (The parameters c_{A_0} and c_{B_0} are typically negligible relative to these, and are so for the parameter ranges displayed.)

Several features of these figures are noteworthy. First, as is evident from Fig. 3, the fine-tuning coefficient c_{m_0} is small even for scalar masses as large as $m_0 \sim 2$ TeV. This is a consequence of the focus point behavior of $m_{H_u}^2$: for moderate and large $\tan\beta$, $m_{H_u}^2$ is insensitive to m_0, and, therefore, so is m_Z^2. The deviation of c_{m_0} from zero for very large m_0 is a consequence of the fact that the focus point does not coincide exactly with the electroweak scale, or, more precisely, with $Q_{\tilde{t}}$. We will explain this statement in more detail below.

On the other hand, large gaugino masses lead to large $m_{H_u}^2$ through RG evolution, and hence $c_{M_{1/2}}$ increases as $M_{1/2}$ increases, as shown in Fig. 4. Multi-TeV values of $M_{1/2}$ are therefore inconsistent with naturalness.

The behavior of c_{μ_0}, presented in Fig. 5, is also interesting. Since $\mu \propto \mu_0$, Eq. (21) implies $c_{\mu_0} \approx 4\mu^2/m_Z^2$. In particular, c_{μ_0} is small in the region bordering the excluded

---

and down-type Higgs scalars, respectively, may be a more accurate choice, especially if variations of the gauge coupling constants are considered. In this paper, however, we follow the literature and define sensitivity coefficients as in Eq. (24).

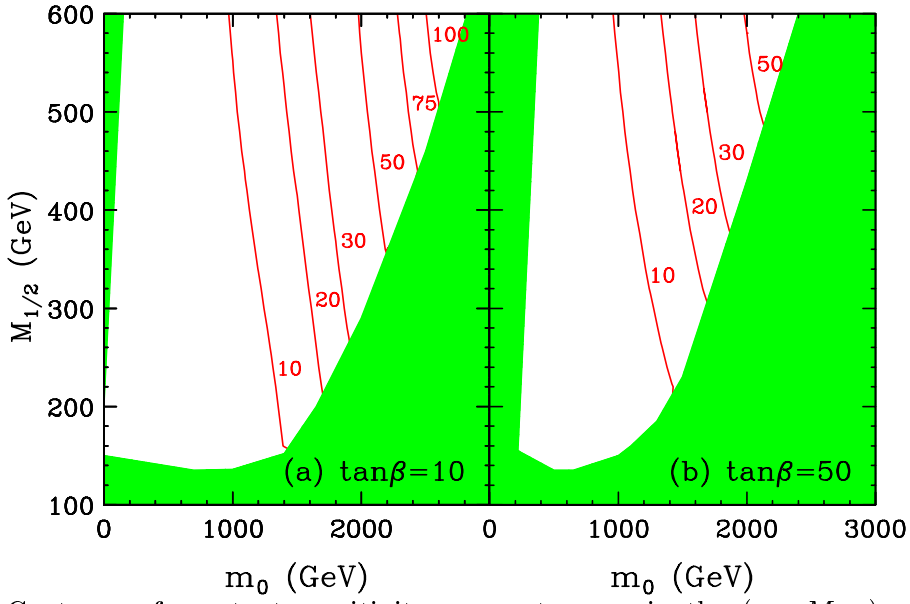

FIG. 3. Contours of constant sensitivity parameter c_{m_0} in the $(m_0, M_{1/2})$ plane for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, $A_0 = 0$, $\mu > 0$, and $m_t = 174$ GeV. The bottom and right shaded region is excluded by the chargino mass limit of 90 GeV. The top left region is also excluded if a neutral LSP is required.

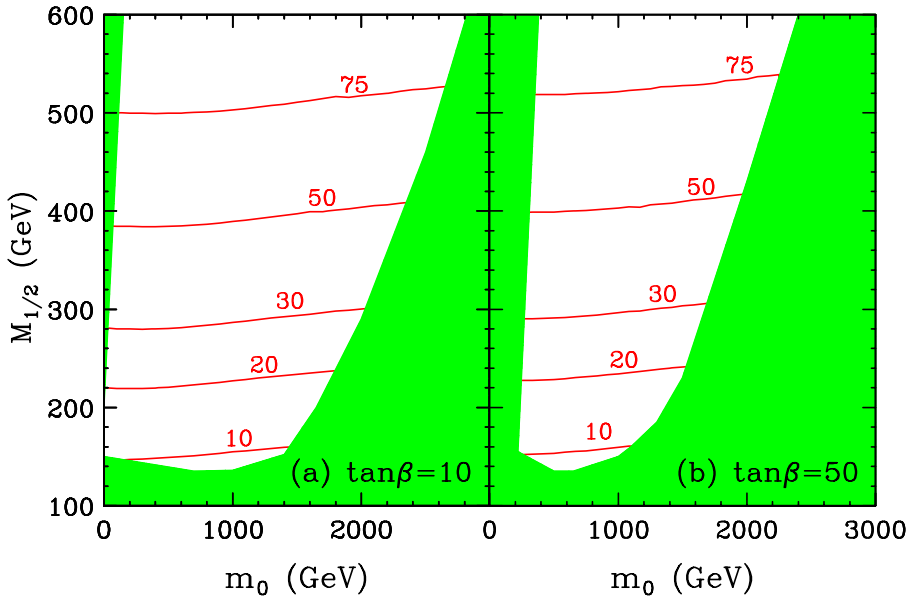

FIG. 4. As in Fig. 3, but for the sensitivity parameter $c_{M_{1/2}}$.

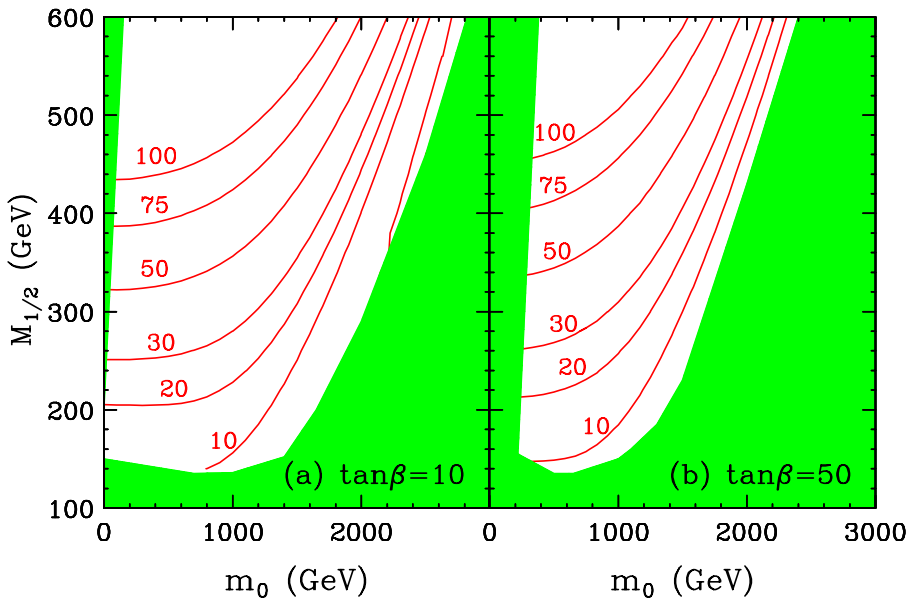

FIG. 5. As in Fig. 3, but for the sensitivity parameter c_{μ_0}.

right-hand region, as there μ is suppressed by a cancellation between the $\eta_{m_0^2}$ and $\eta_{M_{1/2}^2}$ terms in Eq. (20). However, in this region, c_{m_0} and $c_{M_{1/2}}$ are large and this region is fine-tuned; the simple criterion of requiring low μ for naturalness [19] fails here.

In Fig. 6, we show the overall fine-tuning parameter c, the maximum of c_a, in the $(m_0, M_{1/2})$ plane. The fine-tuning c is determined by c_{μ_0}, $c_{M_{1/2}}$, and c_{m_0}. For small m_0, c_{μ_0} is the largest. As m_0 increases, however, $c_{M_{1/2}}$ becomes dominant, and c is therefore almost independent of m_0 in this region. Finally, for extremely large m_0, c_{m_0} becomes important. (For large $\tan\beta$, large m_0 is excluded by the chargino mass limit before c_{m_0} becomes dominant. As a result, we do not see the c_{m_0} segment in Fig. 6b.) Note that, for fixed $M_{1/2}$ in Fig. 6, values of $m_0 \gtrsim 1$ TeV are actually *more* natural than small m_0: for large m_0, the parameter μ, and therefore c_{μ_0}, is reduced. Of course, eventually as m_0 increases to extremely large values, either μ becomes so small that the chargino mass bound is violated or c_{m_0} becomes large, and so very large m_0 is either excluded or disfavored.

As one can see, regions of parameter space with $m_0 \sim 2-3$ TeV are as natural as regions with $m_0 \lesssim 1$ TeV. As will be discussed more fully in Sec. IV, in the region of parameter space with $m_0 \sim 2-3$ TeV, all squarks and sleptons have multi-TeV masses, and discovery of these scalars will be extremely challenging at near future colliders. On the other hand, the gaugino mass $M_{1/2}$ cannot be multi-TeV since it generates unnaturally large $m_{H_u}^2$. Although the focus point mechanism allows multi-TeV scalars consistent with naturalness, the same conclusion does not apply to gauginos and Higgsinos.

As discussed in Sec. III, we expect also that the A parameters should be bounded by naturalness to be near the weak scale. In Fig. 7, we present contours of constant c in the (m_0, A_0) plane. As expected, large A terms lead to large $m_{H_u}^2$, and A_0 is also required to

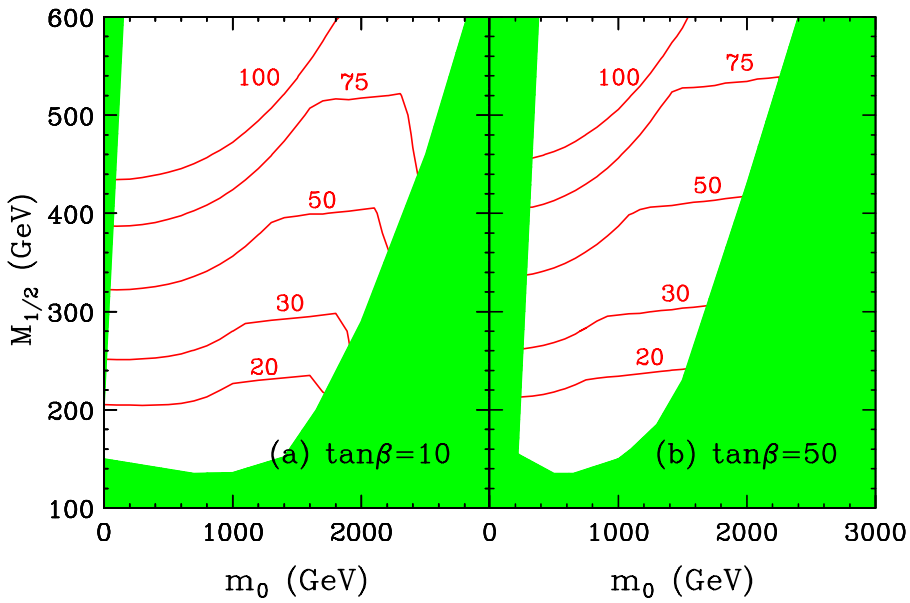

FIG. 6. As in Fig. 3, but for the fine-tuning parameter c.

be $\mathcal{O}(100$ GeV). In Fig. 7, for increasing m_0, c is determined successively by $c_{\mu 0}$, c_{A_0} and c_{m_0}. (The c_{m_0} segments are missing for the $A_0 > 0$ contours in Fig. 7b.)

The dependence of the fine-tuning parameter c on $\tan\beta$ is illustrated in Fig. 8. From this figure, for a given $\tan\beta$ and maximal allowed c, we can determine the upper bound on m_0. The exact range of c required for a natural model is, of course, subjective. However, taking as an example the requirement $c \leq 50$, corresponding to $\mu \lesssim 300$ GeV at parameter points where $c = c_{\mu 0}$, we find $m_0 \lesssim 2$ TeV for $\tan\beta = 10$.

So far, we have assumed $m_t = 174$ GeV in our calculations. However, given the experimentally allowed range $m_t = 173.8 \pm 5.2$ GeV [20], we now consider the top quark mass dependence of the naturalness constraint on m_0. This can be understood only after accounting for the one-loop corrections to the Higgs effective potential.[9] At one-loop, the relation between m_Z and the Higgs mass parameters, Eq. (21), is modified to [11,18]

$$\begin{aligned} \frac{1}{2}m_Z^2 &= \frac{(m_{H_d}^2 - T_d/v_d) - (m_{H_u}^2 - T_u/v_u)\tan^2\beta}{\tan^2\beta - 1} - \mu^2 - \mathrm{Re}\,\Pi_{ZZ}^T(M_Z) \\ &\simeq -\, m_{H_u}^2 \,+\, T_u/v_u \,-\, \mu^2, \;\; \text{for} \;\; \tan\beta \gg 1 \;, \end{aligned} \tag{27}$$

where T_u and T_d are the tadpole contributions to the effective potential and $\Pi_{ZZ}^T(p)$, the transverse part of the Z boson self-energy, is negligible. In minimal supergravity, the dominant terms in T_u and T_d are from third generation squark loops and have the generic form

[9] In fact, the sensitivity coefficient c_{m_0} can only be reliably calculated, and, formally, is only meaningful, at one-loop, since at tree-level there is no preferred scale at which to enforce Eq. (21).

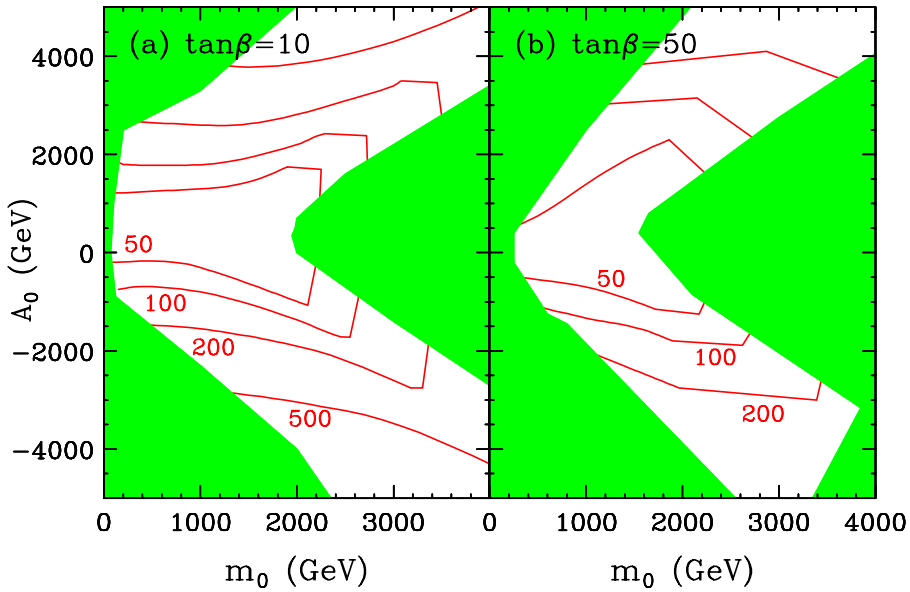

FIG. 7. Contours of constant fine-tuning c in the (m_0, A_0) plane for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, $M_{1/2} = 300$ GeV, $\mu > 0$, and $m_t = 174$ GeV. The shaded regions in the upper and lower left corners are excluded by top squark mass bounds. The shaded region on the right is excluded by the chargino mass limit, while the region in the lower right corner of panel (b) is excluded by Higgs searches, in particular, searches for the CP-odd Higgs boson. The thin strip on the left in panel (a) is excluded if a neutral LSP is required.

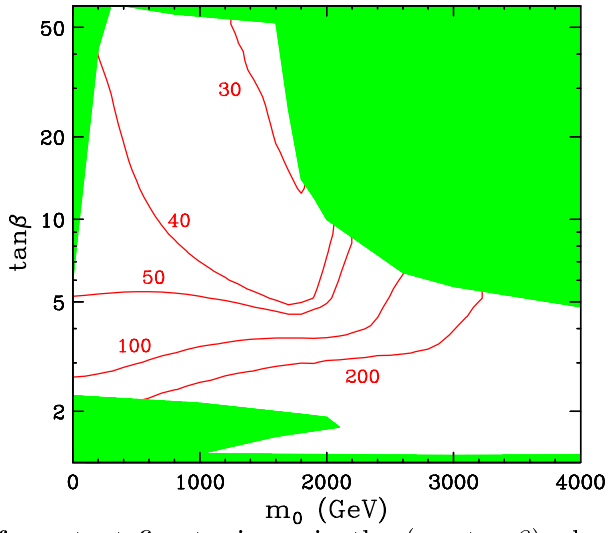

FIG. 8. Contours of constant fine-tuning c in the $(m_0, \tan\beta)$ plane for $M_{1/2} = 300$ GeV, $A_0 = 0$, $\mu > 0$, and $m_t = 174$ GeV. The thin shaded strip on the bottom is excluded by the requirement that y_t remain perturbative up to the GUT scale ($y_t(M_{\rm GUT}) < 3.5$). The region in the lower left corner is excluded by the LEP Higgs mass limit $m_h > 95$ GeV, the region in the upper right corner is excluded by chargino searches, and the region at very large $\tan\beta$ on the top is again ruled out by Higgs searches, as the CP-odd Higgs becomes too light. Finally, the upper left region is also excluded if a neutral LSP is required.

$$\frac{T_{u,d}}{v_{u,d}} \sim \frac{3y_{t,b}^2}{16\pi^2}\, m_{\tilde{t},\tilde{b}}^2 \left[\frac{1}{2} - \ln\left(\frac{m_{\tilde{t},\tilde{b}}}{Q}\right)\right] . \tag{28}$$

Using Eqs. (27), (20) and (28), we find

$$\begin{aligned} \frac{1}{2}\, m_Z^2 &\simeq - \left\{ \eta_{m_0^2}(Q)\, m_0^2 + \ldots \right\} + \frac{3y_t^2}{16\pi^2}\, m_{\tilde{t}}^2 \left[\frac{1}{2} - \ln\left(\frac{m_{\tilde{t}}}{Q}\right)\right] + \ldots - \mu^2 \\ &\equiv - \left[\eta_{m_0^2}(Q) + \eta'_{m_0^2}(Q)\right] m_0^2 - \mu^2 + \ldots , \end{aligned} \tag{29}$$

where $\eta'_{m_0^2}(Q)$ encodes the dependence on m_0 arising at one-loop through the tadpole.

Recall that the focus point $Q_{\rm F}^{(H_u)}$ is defined by

$$\eta_{m_0^2}(Q = Q_{\rm F}^{(H_u)}) \;=\; 0 , \tag{30}$$

while empirically we find

$$\eta'_{m_0^2}(Q = Q_{\tilde{t}}) \;\approx\; 0 , \tag{31}$$

which may also be understood to a good approximation from Eq. (29). We have already seen from Fig. 2 that for $m_t = 174$ GeV, $Q_{\rm F}^{(H_u)} \sim \mathcal{O}(100$ GeV), well below the typical stop mass scale $Q_{\tilde{t}}$. The sensitivity of $m_{H_u}^2$ to m_0 may then be understood in two ways: either we may minimize the potential at $Q_{\tilde{t}}$ where the tadpole contributions are negligible, but $\eta_{m_0^2}$ is non-vanishing, or we may choose to minimize the potential at $Q_{\rm F}^{(H_u)}$, in which case $\eta_{m_0^2} = 0$, but m_0 dependence arises from non-vanishing tadpole contributions. Either way, there is some residual dependence on m_0, as can be seen in Fig. 3.

If $Q_{\rm F}^{(H_u)}$ and $Q_{\tilde{t}}$ are identical, however, $\eta_{m_0^2}(Q) + \eta'_{m_0^2}(Q)$ vanishes at $Q_{\tilde{t}}$. This can happen in two ways: for a fixed top quark mass, $Q_{\tilde{t}}$ can be lowered to $Q_{\rm F}^{(H_u)}$ by lowering m_0, or, for fixed large m_0, $Q_{\rm F}^{(H_u)}$ may be raised to $Q_{\tilde{t}}$ by increasing m_t (and thereby increasing the top Yukawa renormalization effect on $m_{H_u}^2$).

In Fig. 9, we present contours of fine-tuning c in the (m_0, m_t) plane. On the dotted contour, the focus point and $Q_{\tilde{t}}$ coincide, and $c_{m_0} = 0$. This occurs for m_t above 174 GeV, in accord with the discussion above. More generally, the sensitivity c_{m_0} is indeed reduced for larger m_t. As a result, the upper bound on m_0 is increased for larger m_t, and for $m_t \simeq 180$ GeV, near the 1σ upper bound, $m_0 \sim 3.4$ TeV is allowed for $c \leq 50$. For smaller top quark mass, the naturalness bound on m_0 becomes more stringent. Thus, while the focus point behavior persists for all m_t within current experimental bounds, future improvements in top mass measurements [21] may provide important information about the extent to which multi-TeV scalars are allowed in minimal supergravity.

We may also consider variations in the high scale Q_0 where the supersymmetry breaking parameters are assumed to be generated. So far we have assumed $Q_0 = M_{\rm GUT}$. It is interesting to consider the effects of assuming that the boundary conditions are specified at a different scale, e.g., the string or Planck scale. To investigate this, we have taken a simple

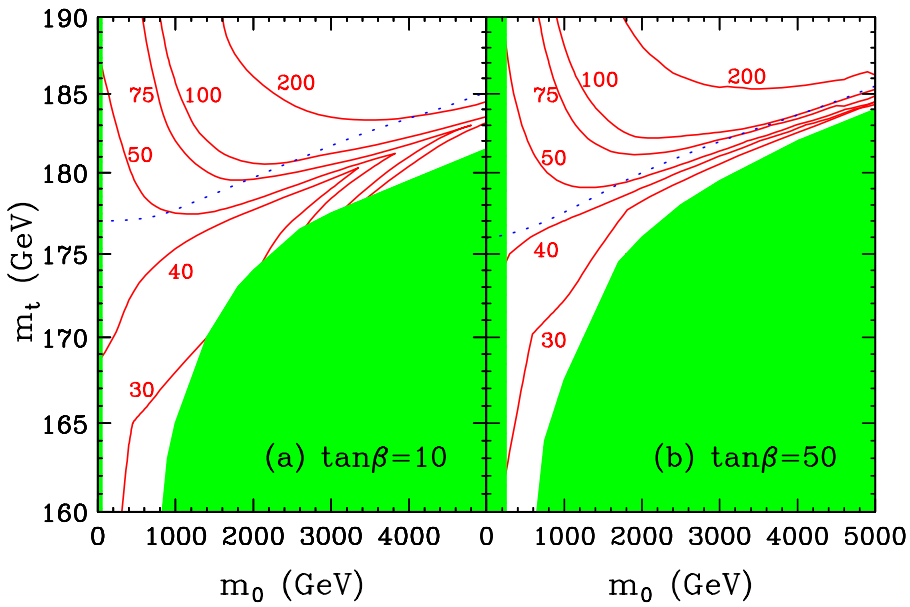

FIG. 9. Contours of constant fine-tuning c in the (m_0, m_t) plane for $M_{1/2} = 300$ GeV, $A_0 = 0$, $\tan\beta = 10$, and $\mu > 0$. The bottom and right shaded region is excluded by the chargino mass limit of 90 GeV. The left region is excluded if a neutral LSP is required. The sensitivity coefficient c_{m_0} vanishes on the dotted contour (see text).

approach, and evolved the gauge couplings up to some fixed Q_0, set the supersymmetry breaking parameters at that scale, and then evolved them down to the weak scale. The minimal field content is assumed throughout the RG evolution; in particular, no additional GUT particle content is assumed for $Q_0 > M_{\rm GUT}$, and the unification of gauge couplings at $M_{\rm GUT}$ is unexplained. In Fig. 10 we show contours of constant c in the (m_0, Q_0) plane. Just as in Fig. 6, the fine-tuning parameter c is determined successively, for increasing m_0, by c_{μ_0}, $c_{M_{1/2}}$ and c_{m_0}. We see that increasing the scale Q_0 also allows even larger scalar masses. For example the requirements $c \leq 50$ and $Q_0 < M_{\rm Pl}$ allow m_0 as large as 2.9 TeV.

In Fig. 11 we show contours of constant c in the (m_t, Q_0) plane. As expected, smaller values of m_t can be compensated for by larger evolution intervals, and vice versa. Notice, however, that varying m_t within its current experimental uncertainty leads to changes in c that are as large as those caused by varying Q_0 by several orders of magnitude.

IV. IMPLICATIONS FOR SUPERSYMMETRY SEARCHES

We have seen that the naturalness bound on m_0 (i.e., the typical sfermion mass) may be as large as a few TeV. In this section, we discuss the implication of these results for the superpartner spectrum and, in particular, the discovery prospects for scalar superpartners at future colliders. (Of course, it is clear that such heavy scalars also drastically reduce the size of supersymmetric effects in low energy experiments, but we will not address this

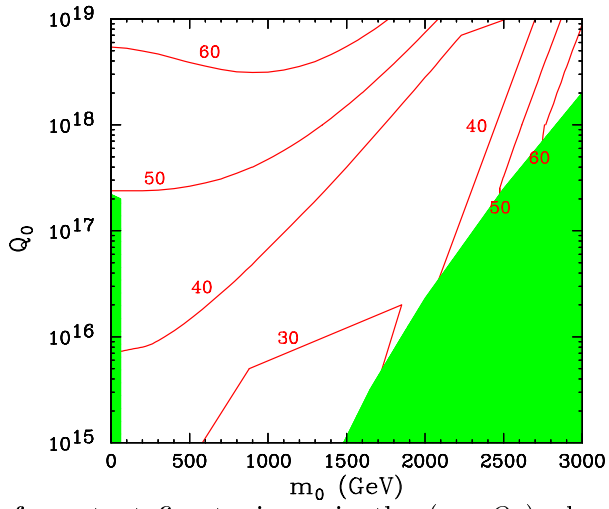

FIG. 10. Contours of constant fine-tuning c in the (m_0, Q_0) plane for $M_{1/2} = 300$ GeV, $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, and $m_t = 174$ GeV. The shaded region on the right is excluded by the chargino search, while the shaded region on the left is excluded if a neutral LSP is required.

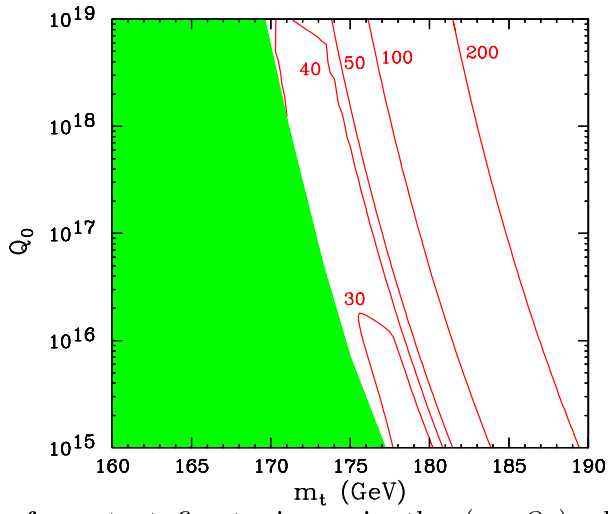

FIG. 11. Contours of constant fine-tuning c in the (m_t, Q_0) plane for $m_0 = 2$ TeV, $M_{1/2} = 300$ GeV, $A_0 = 0$, $\tan\beta = 10$, and $\mu > 0$. The shaded region is excluded from the chargino mass limit.

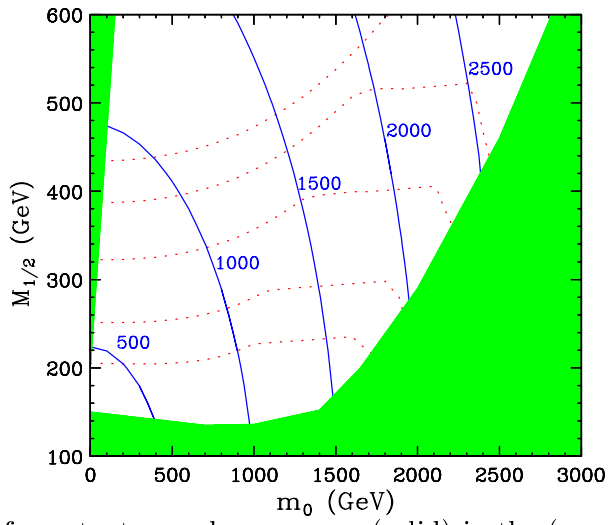

FIG. 12. Contours of constant squark mass $m_{\tilde{u}_L}$ (solid) in the $(m_0, M_{1/2})$ plane for $A_0 = 0$, $\tan\beta = 10$, $\mu > 0$, and $m_t = 174$ GeV. Fine-tuning contours (dotted) are also presented for $c = 20$, 30, 50, 75, and 100 (from below). The bottom and right shaded region is excluded by the chargino mass limit of 90 GeV. The top left region is also excluded if a neutral LSP is required.

further.)

The implications for sleptons are fairly straightforward. Sleptons have small Yukawa couplings (with the possible exception of staus for large $\tan\beta$), and so their masses are virtually RG-invariant, with $m_{\tilde{l}} \approx m_0$ in this scenario. Multi-TeV sleptons are beyond the kinematic limit $m_{\tilde{l}} < \sqrt{s}/2$ of all proposed linear colliders. They will also escape detection at hadron colliders, as they are not strongly produced, and will not be produced in large numbers in the cascade decays of strongly interacting superparticles.

We now turn to squarks. Multi-TeV squarks will, of course, also evade proposed linear colliders. Traditionally, however, it has been expected that future hadron colliders, particularly the LHC, will discover all squarks in the natural region of minimal supergravity parameter space [13]. This conclusion is based on the expectation that all squarks have masses $\lesssim 1 - 2$ TeV. In Fig. 12, we present contours for $m_{\tilde{u}_L}$.[10] (All first and second generation squarks are nearly degenerate.) In the same figure, we have also included contours of the fine-tuning parameter c. For $c \leq 50$, we see that squark masses of $m_{\tilde{u}_L} =$ 2.2 TeV are allowed, and more generally, the parameter space with multi-TeV squarks is as natural as parameter space with $m_{\tilde{u}_L} \lesssim 1$ TeV. Recall also that these mass limits may be extended to as large as ~ 3 TeV for variations of m_t within its current bounds. Squarks of mass ~ 2 TeV may be detected at the LHC with large integrated luminosities of several 100 fb^{-1}. However, squarks with masses significantly beyond 2 TeV are likely to escape detection altogether [22].

[10] One-loop corrections [11] are included in all superpartner masses.

Since the top squarks and left-handed bottom squark interact strongly through y_t, they are lighter than the other squarks. For small $\tan\beta$, and small $M_{1/2}$ and A_0 parameters, their masses at the focus point of $m_{H_u}^2$ are given by

$$m_{U_3}^2 \simeq \frac{1}{3}m_0^2, \quad m_{Q_3}^2 \simeq \frac{2}{3}m_0^2 \ . \tag{32}$$

In general, these squark masses, particularly the stop masses, may also be influenced by left-right mixing. However, because naturalness constrains the A and μ parameters to be of order the weak scale, left-right mixing effects are sub-leading for multi-TeV m_0. As a result, the lighter (heavier) stop is mostly right-handed (left-handed), and the lighter (heavier) sbottom is mostly left-handed (right-handed). For large $\tan\beta$, y_b also suppresses third generation squark masses. For example, for $y_b = y_t$, we obtain

$$m_{U_3}^2 \simeq m_{D_3}^2 \simeq m_{Q_3}^2 \simeq \frac{1}{3}m_0^2 \ . \tag{33}$$

In Figs. 13 and 14 we present the masses of the stops and sbottoms, respectively. By comparing with Fig. 12, we see that the stops are always lighter than the first and second generation squarks. The lighter sbottom and heavier stop are nearly degenerate, since they are (approximately) in the same $\mathrm{SU}(2)_L$ doublet. The heavier sbottom, which is mostly the right-handed sbottom, may also become significantly lighter than the first two generation squarks for large $\tan\beta$. Therefore, in the multi-TeV m_0 scenario, stop and sbottom production will be the most promising modes for squark discovery at the LHC.

In contrast to the sfermions, gauginos cannot be very heavy in this scenario. This fact is explicit in Fig. 4: for large $M_{1/2}$, the fine-tuning coefficient $c_{M_{1/2}}$ becomes unacceptably large, irrespective of $\tan\beta$. As a result, naturalness requires fairly light gauginos. For example, the constraint $c \leq 50$ implies $M_{1/2} \lesssim 400$ GeV, corresponding to $M_1 \lesssim 160$ GeV, $M_2 \lesssim 320$ GeV, and $M_3 \lesssim 1.2$ TeV. Such gauginos will be produced in large numbers at the LHC, and will be discovered in typical scenarios.[11]

It is also interesting to consider the implications of the focus point for Higgs masses. In supersymmetric models with minimal field content, the lightest Higgs boson mass m_h is bounded by m_Z at tree level. However, this upper bound may be significantly violated by radiative corrections [24]. In particular, top-stop loop contributions, approximately proportional to $\ln(m_{\tilde{t}}/m_t)$, may be important. Since the focus point allows heavy stops, one may wonder if this affects the upper bound on the lightest Higgs mass. To answer this question, we have calculated the one-loop radiative corrections to the lightest Higgs mass. The result is shown in Fig. 15; we emphasize the A_0 dependence, as the radiative

[11] In some scenarios, however, the detection of all gauginos may be challenging. This is particularly true in scenarios with degeneracies, such as the Wino LSP scenario [23]. To our knowledge, the detectability of all gauginos (not including the invisible LSP) in such scenarios at the LHC remains an open question.

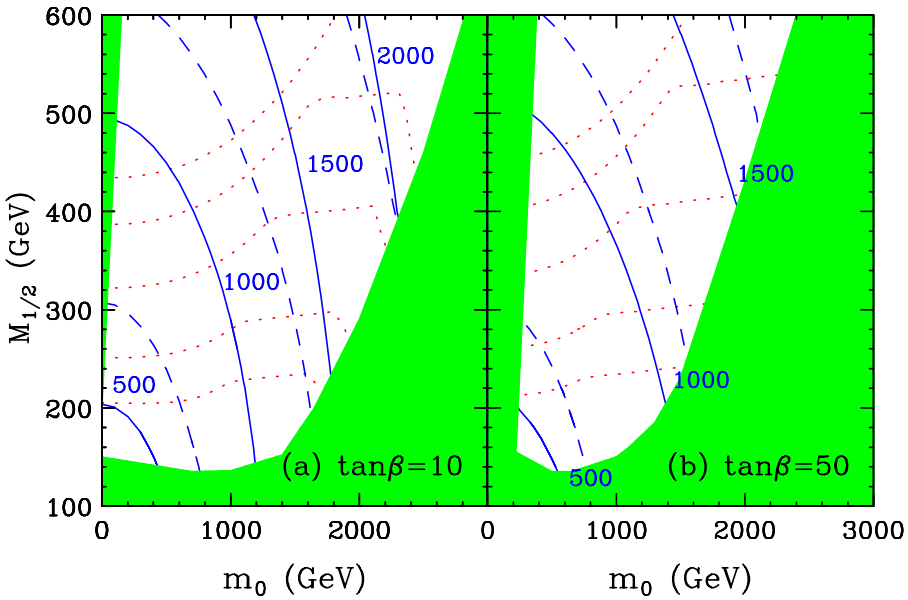

FIG. 13. Contours of constant $m_{\tilde{t}_1}$ (dashed) and $m_{\tilde{t}_2}$ (solid) in the $(m_0, M_{1/2})$ plane for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, $A_0 = 0$, $\mu > 0$, and $m_t = 174$ GeV. Fine-tuning contours (dotted) are also presented for $c = 20$, 30, 50, 75, and 100 (from below). The bottom and right shaded region is excluded by the chargino mass limit of 90 GeV. The top left region is also excluded if a neutral LSP is required.

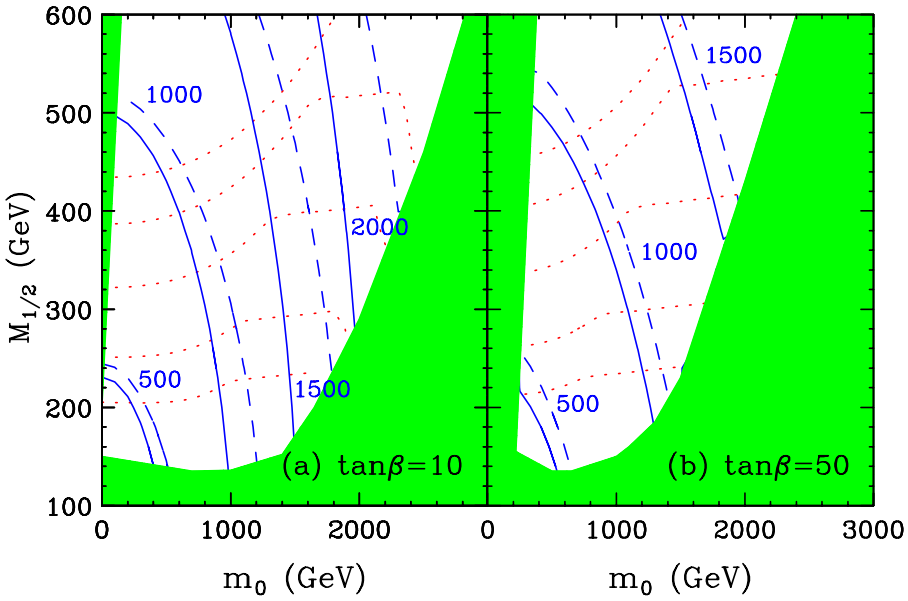

FIG. 14. As in Fig. 13, but for $m_{\tilde{b}_1}$ (dashed) and $m_{\tilde{b}_2}$ (solid).

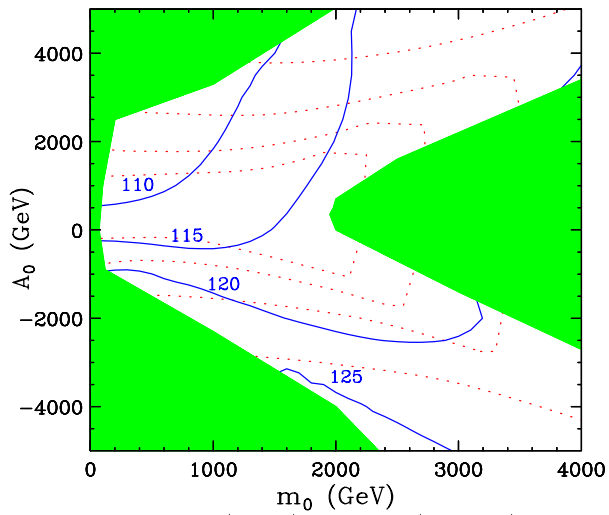

FIG. 15. Contours of constant m_h (solid) in the (m_0, A_0) plane for $M_{1/2} = 300$ GeV, $\tan\beta = 10$, $\mu > 0$, and $m_t = 174$ GeV. Fine-tuning contours (dotted) are also presented for $c = 50$, 100, 200, and 500 (see Fig. 7a).

correction is sensitive to left-right mixing through the A parameter. For the region with small fine-tuning parameter, say, $c \leq 50$, we find $m_h < 118$ GeV. It is important that a large A parameter is forbidden by naturalness, as this suppresses left-right stop mixing, which usually significantly increases m_h. Therefore, even in the focus point scenario with multi-TeV squarks, Run II of the Tevatron will probe much of parameter space in its search for the lightest Higgs boson [25].

Discovery of the heavy Higgs scalars is more challenging. At tree-level, the masses of the heavy Higgs scalars are approximately given by

$$m_A \simeq m_H \simeq m_{H^\pm} \simeq \sqrt{m_{H_u}^2 + m_{H_d}^2 - 2\mu^2} \ . \tag{34}$$

Although $m_{H_u}^2$ and μ^2 are always bounded by naturalness, for moderate $\tan\beta$, $m_{H_d}^2$ is only weakly bounded by naturalness. For negligible y_b, $m_{H_d}^2$ does not participate in the focus point behavior and is roughly RG-invariant. For larger $\tan\beta$, on the other hand, $m_{H_d}^2$ may be significantly suppressed by y_b during RG evolution. In particular, in the case of $\tan\beta \simeq m_t/m_b$, there is an approximate symmetry under interchanging $H_u \leftrightarrow H_d$ and $U_3 \leftrightarrow D_3$, and so $m_{H_d}^2$ also has a focus point near the weak scale. In Fig. 16, we present the pseudoscalar Higgs mass m_A in the $(m_0, M_{1/2})$ plane. For $\tan\beta = 10$, $m_{H_d}^2 \simeq m_0^2$, $m_A \sim m_0$, and detection of the heavy Higgses at the LHC or proposed linear colliders becomes impossible for large m_0. However, for large $\tan\beta$, $m_{H_d}^2$ is suppressed by y_b, and $m_A \sim \mathcal{O}(100 \text{ GeV})$. For large $\tan\beta$, heavy Higgses with masses of several hundred GeV may be found at the LHC through the decays $H, A \to \tau\bar{\tau}$ [26].

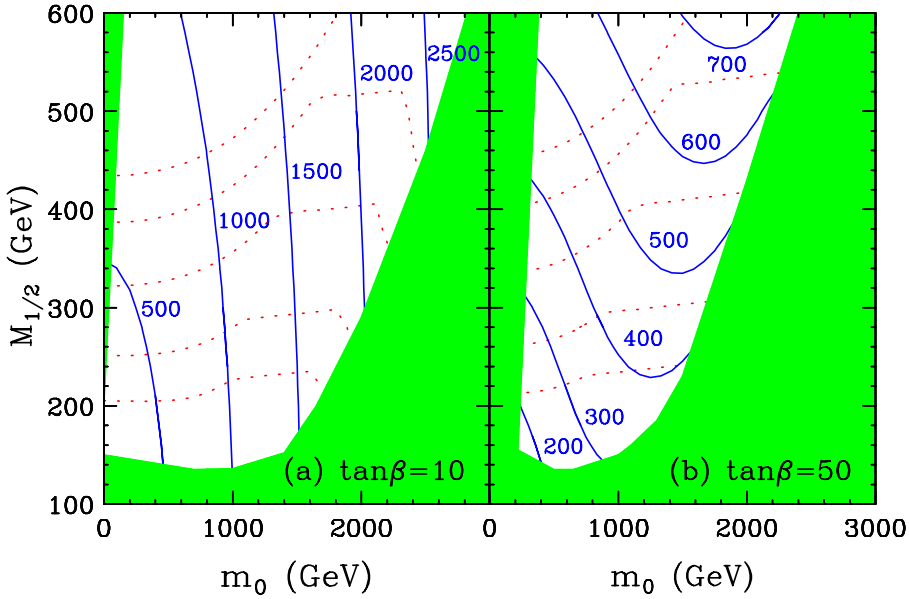

FIG. 16. Contours of constant heavy Higgs mass $m_A \approx m_H \approx m_{H^\pm}$ (solid) in the $(m_0, M_{1/2})$ plane for (a) $\tan\beta = 10$ and (b) $\tan\beta = 50$, $A_0 = 0$, $\mu > 0$, and $m_t = 174$ GeV. Fine-tuning contours (dotted) are also presented for $c = 20$, 30, 50, 75, and 100 (from below).

V. SUMMARY AND DISCUSSION

In this paper, we have explored the existence of focus points in the RG behavior of supersymmetry breaking parameters and their implications for naturalness and experimental searches for supersymmetry.

For the experimentally measured top quark mass, the supersymmetry breaking up-type Higgs mass parameter $m^2_{H_u}$ has a focus point at the scale $Q \sim \mathcal{O}(100\text{ GeV})$ in a class of models which includes minimal supergravity. The value of $m^2_{H_u}$ at the weak scale is therefore highly insensitive to the universal scalar mass m_0 at the GUT scale. We have also seen that this focus point behavior exists for all values of $\tan\beta \gtrsim 5$.

Since $m^2_{H_u}$ plays an important role in the determination of the weak scale, this focus point behavior affects the naturalness of electroweak symmetry breaking in minimal supergravity. In particular, because a large m_0 can result in a reasonably small $m^2_{H_u}$, naturalness constraints on m_0 are not as severe as typically expected. To discuss this issue quantitatively, we have calculated the fine-tuning parameter c, which is determined by the sensitivity of the weak scale to fractional variations of the fundamental parameters. As we have seen, in regions of parameter space with $m_0 \sim 2-3$ TeV this fine-tuning parameter may be as small as in regions with $m_0 \lesssim 1$ TeV. As a result, multi-TeV sfermions are as natural as sfermions lighter than 1 TeV. We note that the region of multi-TeV scalars and light gauginos *and* Higgsinos is also somewhat preferred by gauge coupling unification in minimal SU(5) [10,27,28], as well as b-τ Yukawa unification at moderate to large $\tan\beta$ [29,30]. The discovery of squarks and sleptons at the LHC and proposed linear colliders may therefore be extremely challenging, and may require some even more energetic

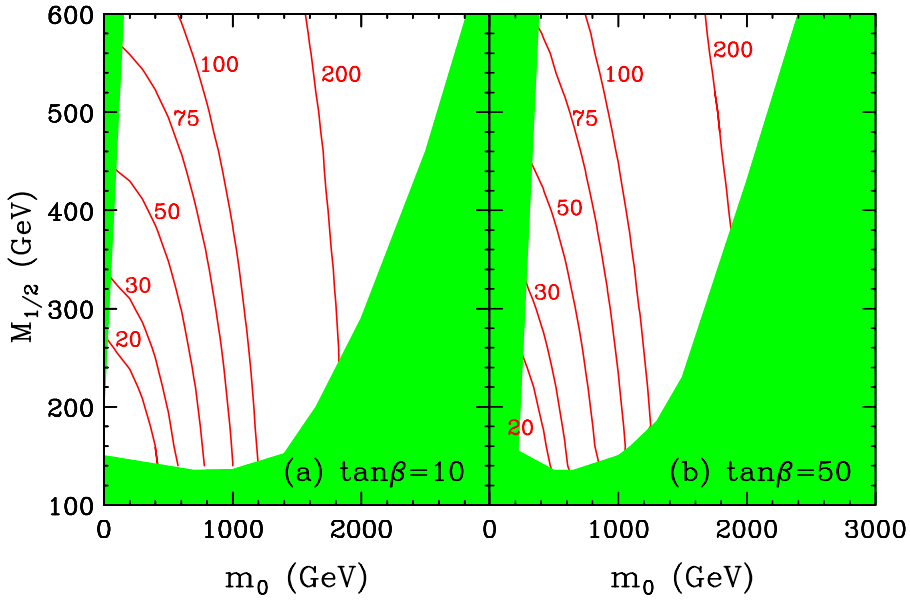

FIG. 17. As in Fig. 3, but for the sensitivity parameter c_{y_t}.

machines, such as muon or very large hadron colliders.

In our analysis, we did not include the Yukawa couplings, notably y_t, and gauge couplings in the calculation of the the fine-tuning parameter c. In Fig. 17, we present the sensitivity coefficient c_{y_t} for the (GUT scale) top Yukawa coupling. For $m_0 \geq 1$ TeV, c_{y_t} is always larger than 70, and so if c_{y_t} is included in the calculation of the fine-tuning parameter c, the naturalness bound on m_0 becomes much more stringent and $m_0 \gtrsim 1$ TeV is disfavored. The inclusion of y_t and other standard model parameters in the fine-tuning calculation would thus lead to significantly different conclusions.

A definitive resolution to this question of whether or not to include variations of standard model parameters in fine-tuning calculations cannot, we believe, be achieved without a more complete understanding of the fundamental theories of flavor and supersymmetry breaking. Without such knowledge, any discussion necessarily becomes somewhat philosophical. Nevertheless, several remarks are in order.

First, it is sometimes argued that one should not consider variations with respect to parameters that have been measured or are highly correlated with measured quantities. According to this view, standard model parameters such as the strong coupling constant, and possibly also y_t, should not be included among the a_i. We do not subscribe to this view.[12] If in the future the Higgsino mass μ is measured to be 10 TeV, given our current notions of naturalness, we believe this should be considered fine-tuned, irrespective of the

[12]Note that the exclusion of standard model parameters from the fundamental parameters a_i does not imply that current experimental data are ignored in the calculation of fine-tuning. All experimental data are used in step (*ii*) of the fine-tuning prescription to specify the physical

accuracy with which the Higgsino mass is measured. Of course, if this were the case, the fact that a 10 TeV μ parameter is realized in nature would be a strong motivation to consider alternative, and perhaps more fundamental, theoretical frameworks in which a 10 TeV μ parameter is not unnatural.

There are, however, other considerations which favor the exclusion of standard model parameters from the list of a_i. As noted in Sec. III, we are interested in the naturalness of the supersymmetric explanation of the gauge hierarchy. We should not require that supersymmetry also solve the problem of flavor. In fact, in many supergravity frameworks, the supersymmetry breaking parameters and the Yukawa couplings are expected to be determined independently. For example, in hidden sector scenarios, the supersymmetry breaking parameters are determined in one sector, while the Yukawa couplings are fixed in some other sector and by a completely independent mechanism. In this case, it seems reasonable to assume that y_t is fixed to its observed value in some sector not connected to supersymmetry breaking, and we therefore should not consider variations with respect to it.

Finally, it is worth noting that there are several possible scenarios in which it is clear that c_{y_t} should not be included in c or is at least negligibly small. One possibility is that y_t may evolve from some higher scale, such as the Planck scale, to a fixed or focus point at the GUT scale. The weak scale is then highly insensitive to variations in the truly fundamental parameter, i.e., y_t at the Planck scale. Alternatively, the top Yukawa coupling may arise as a renormalizable operator with coefficient determined by a correlation function of string vertex operators. The coupling y_t would then be fixed to its current value (or possibly one of a discrete set of values), and it is again inappropriate to consider continuous variations with respect to y_t. Note that in both of these scenarios, y_t may receive additional contributions from non-renormalizable operators of the form $\delta y_t \sim g\epsilon$, where g is a coupling constant, and ϵ is some small expansion parameter, such as v/M_{Pl}, where v is some vacuum expectation value. In this case, c_g and c_ϵ should be included in the definition of fine-tuning, but they will be negligible for small ϵ.

ACKNOWLEDGMENTS

We thank J. Bagger, G. Giudice, and C. Wagner for discussions. This work was supported in part by the Department of Energy under contracts DE–FG02–90ER40542 and DE–AC02–76CH03000, by the National Science Foundation under grant PHY–9513835, through the generosity of Frank and Peggy Taplin (JLF), and by a Marvin L. Goldberger Membership (TM).

---

hypersurface of parameter space.

APPENDIX: DETERMINATION OF FOCUS POINT SCALE FOR $y_b \ll y_t$ AND $y_b = y_t$

In this appendix, we discuss the focus point analytically at one-loop for the two cases $y_b \ll y_t$ and $y_b = y_t$. Solutions to the RG equations are well-known for these two cases [31]. For both cases, we derive a closed form expression involving the gauge couplings and m_t that must be satisfied if the focus point scale $Q_{\rm F}$ is to be at the weak scale. We also show that if the focus point is at the weak scale for $y_b \ll y_t$, it is also at the weak scale for $y_b = y_t$.

In the analysis of the $y_b = y_t$ case, we neglect the effects of y_τ and the hypercharge differences in the y_t and y_b RG equations, so y_t and y_b remain degenerate throughout their RG evolution. The validity of these approximations is verified only by the numerical results in Sec. II. In addition, the intermediate case, where $y_b \neq y_t$ but y_b may not be neglected, is not considered. However, the following analysis may be helpful in understanding the numerical results, and in particular, the behavior of Figs. 1 and 2.

Let us define

$$\tilde{\alpha}_a \equiv \frac{g_a^2}{16\pi^2}, \quad a = 1, 2, 3, \tag{A1}$$

$$\tilde{\alpha}_y \equiv \frac{y_t^2}{16\pi^2} \ . \tag{A2}$$

These quantities obey the following RG equations:

$$\frac{d\tilde{\alpha}_a}{d\ln Q} = 2b_a \tilde{\alpha}_a^2 \ , \tag{A3}$$

$$\frac{d\tilde{\alpha}_y}{d\ln Q} = 2\Big(s\tilde{\alpha}_y - \sum_a r_a \tilde{\alpha}_a\Big)\tilde{\alpha}_y \ . \tag{A4}$$

In the minimal supersymmetric standard model, $(b_1, b_2, b_3) = (11, 1, -3)$, $(r_1, r_2, r_3) = (13/9, 3, 16/3)$, and $s = 6$ and 7 for $y_b \ll y_t$ and $y_b = y_t$, respectively. The solution for $\tilde{\alpha}_y$ is

$$\tilde{\alpha}_y(Q) = \frac{\tilde{\alpha}_y(Q_0)E(Q)}{1 - 2s\tilde{\alpha}_y(Q_0)F(Q)} \ , \tag{A5}$$

where

$$E(Q) = \prod_a \left[1 - 2b_a\tilde{\alpha}_a(Q_0)\ln(Q/Q_0)\right]^{r_a/b_a} \ , \tag{A6}$$

$$F(Q) = \int_{\ln Q_0}^{\ln Q} E(Q')d\ln Q' \ . \tag{A7}$$

We find then that

$$e^{sI(Q)} \equiv \exp\left(2s\int_{\ln Q_0}^{\ln Q} \tilde{\alpha}_y(Q')d\ln Q'\right) = \frac{1}{1 - 2s\tilde{\alpha}_y(Q_0)F(Q)} = 1 + \frac{2s\tilde{\alpha}_y(Q)F(Q)}{E(Q)} \ . \tag{A8}$$

For a universal scalar mass, the conditions for the focus point (see Sec. II) are

$$e^{sI} = \begin{cases} 1/3 \ , \ \text{for } y_b \ll y_t \\ 2/9 \ , \ \text{for } y_b = y_t \end{cases} \ . \tag{A9}$$

We see that these are simultaneously satisfied at $Q = m_t$ if

$$\frac{\tilde{\alpha}_y(m_t) F(m_t)}{E(m_t)} = -\frac{1}{18} \ . \tag{A10}$$

In terms of the top quark mass, this requirement corresponds to

$$\frac{1}{16\pi^2} \left[\frac{m_t(m_t)}{v \sin\beta} \right]^2 \frac{F(m_t)}{E(m_t)} = -\frac{1}{18} \ , \tag{A11}$$

where $m_t(m_t)$ is the running top quark mass, and $v = 174$ GeV.

Given fixed gauge coupling constants, Eq. (A11) specifies which top quark mass will place the focus point at the weak scale. For $Q_0 = M_{\rm GUT} = 2 \times 10^{16}$ GeV and $\alpha(M_{\rm GUT}) = 1/24$, we obtain $F \simeq -130$ and $E \simeq 13$ at $Q = 174$ GeV. For $\sin\beta \simeq 1$, the requirement is then $m_t(m_t) \simeq 160$ GeV, which is very close to the running mass corresponding to the physical pole mass $m_t \approx 174$ GeV. Therefore, for the experimentally measured top quark mass, the focus point of $m_{H_u}^2$ is close to the weak scale for $y_b = y_t$ and also for $y_b \ll y_t$.

REFERENCES

[1] K. Wilson, unpublished;
L. Susskind, Phys. Rev. **D20**, 2619 (1979);
G. 'tHooft, in *Recent Developments in Gauge Theories*, ed. G. 'tHooft et al., (Plenum Press, New York, 1980), p. 135.

[2] L. Maiani, in *Proc. Gif-sur-Yvette Summer School* (Paris, 1980), p. 3;
E. Witten, Nucl. Phys. **B188**, 513 (1981);
M. Veltman, Acta. Phys. Pol. **12**, 437 (1981);
R. Kaul, Phys. Lett. **B109**, 19 (1982).

[3] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. **B388**, 588 (1996);
G. Dvali and A. Pomarol, Phys. Rev. Lett. **77**, 3728 (1996); Nucl. Phys. **B522**, 3 (1998);
R. N. Mohapatra and A. Riotto, Phys. Rev. **D55**, 1 (1997);
R.-J. Zhang, Phys. Lett. **B402**, 101 (1997);
A. E. Nelson and D. Wright, Phys. Rev. **D56**, 1598 (1997);
H. P. Nilles and N. Polonsky, Phys. Lett. **B412**, 69 (1997);
D. E. Kaplan, F. Lepeintre, A. Masiero, A. E. Nelson, and A. Riotto, Phys. Rev. **D60**, 055003 (1999);
J. Hisano, K. Kurosawa, and Y. Nomura, Phys. Lett. **B445**, 316 (1999).

[4] J. L. Feng, C. Kolda, and N. Polonsky, Nucl. Phys. **B546**, 3 (1999);
J. Bagger, J. L. Feng, and N. Polonsky, to appear in Nucl. Phys. **B**, hep-ph/9905292.

[5] M. Drees, Phys. Rev. **D33**, 1468 (1986);
M. Dine, R. Leigh, and A. Kagan, Phys. Rev. **D48**, 4269 (1993);
S. Dimopoulos and G. F. Giudice, Phys. Lett. **B357**, 573 (1995);
A. Pomarol and D. Tommasini, Nucl. Phys. **B466**, 3 (1996).

[6] J. L. Feng and T. Moroi, hep-ph/9907319.

[7] J. L. Feng, K. T. Matchev, and T. Moroi, hep-ph/9908309.

[8] B. Pendleton and G. G. Ross, Phys. Lett. **98B**, 291 (1981);
C. T. Hill, Phys. Rev. **D24**, 691 (1981).

[9] S. P. Martin and M. T. Vaughn, Phys. Rev. **D50**, 2282 (1994);
I. Jack and D. R. T. Jones, Phys. Lett. **B333**, 372 (1994);
Y. Yamada, Phys. Rev. **D50**, 3537 (1994);
I. Jack, D. R. T. Jones, S. P. Martin, M. T. Vaughn, and Y. Yamada, Phys. Rev. **D50**, 5481 (1994).

[10] J. Bagger, K. Matchev, and D. Pierce, Phys. Lett. **B348**, 443 (1995).

[11] D. Pierce, J. Bagger, K. Matchev, and R.-J. Zhang, Nucl. Phys. **B491**, 3 (1997).

[12] R. Barbieri and G. F. Giudice, Nucl. Phys. **B306**, 63 (1988).

[13] See, e.g., D. G. Cassel, L. T. Gennari, and R. H. Siemann (ed.), New directions for high-energy physics (Snowmass, CO, 1996).

[14] J. Ellis, K. Enqvist, D.V. Nanopoulos, and F. Zwirner, Mod. Phys. Lett. **1**, 57 (1986);
G. G. Ross and R. G. Roberts, Nucl. Phys. **B377**, 571 (1992).

[15] B. de Carlos and J. A. Casas, Phys. Lett. **B309**, 320 (1993).

[16] G. W. Anderson and D. J. Castano, Phys. Lett. **B347**, 300 (1995); Phys. Rev. **D52**, 1693 (1995); Phys. Rev. **D53**, 2403 (1996).
[17] P. Ciafaloni and A. Strumia, Nucl. Phys. **B494**, 41 (1997);
G. Bhattacharyya and A. Romanino, Phys. Rev. **D55**, 7015 (1997);
P. H. Chankowski, J. Ellis, and S. Pokorski, Phys. Lett. **B423**, 327 (1998);
K. L. Chan, U. Chattopadhyay, and P. Nath, Phys. Rev. **D58**, 096004 (1998);
D. Wright, hep-ph/9801449;
P. H. Chankowski, J. Ellis, M. Olechowski, and S. Pokorski, Nucl. Phys. **B544**, 39 (1999);
R. Barbieri and A. Strumia, Phys. Lett. **B433**, 63 (1999);
L. Giusti, A. Romanino, and A. Strumia, Nucl. Phys. **B550**, 3 (1999);
G. L. Kane and S. F. King, Phys. Lett. **B451**, 113 (1999).
[18] D. M. Pierce, A. Papadopoulos, and S. Johnson, Phys. Rev. Lett. **68**, 3678 (1992);
V. Barger, M.S. Berger, and P. Ohmann, Phys. Rev. **D49**, 4908 (1994).
[19] See, e.g., K. L. Chan et al., in Ref. [17].
[20] C. Caso et al., Eur. Phys. J. **C3**, 1 (1998).
[21] See, e.g., "Thinkshop on Top Quark Physics for Run II," Fermilab, 16–18 October 1998, http://lepton.bu.edu/∼heintz/pm.html.
[22] I. Hinchliffe, talk given at the Circle Line Tours seminar series, Fermilab, 9 September 1999, http://sbhep1.physics.sunysb.edu/∼grannis/circleline.html.
[23] C.-H. Chen, M. Drees, J. F. Gunion, Phys. Rev. Lett. **76**, 2002 (1996); Phys. Rev. **D55**, 330 (1997); Erratum, hep-ph/9902309;
J. L. Feng, T. Moroi, L. Randall, M. Strassler, and S. Su, Phys. Rev. Lett. **83**, 1731 (1999);
J. F. Gunion and S. Mrenna, hep-ph/9906270;
G. Landsberg, talk presented at SUSY '99, Fermilab, 14–19 June 1999.
[24] Y. Okada, M. Yamaguchi, and T. Yanagida, Prog. Theor. Phys. **85**, 1 (1991);
H. E. Haber and R. Hempfling, Phys. Rev. Lett. **66**, 1815 (1991);
J. Ellis, G. Ridolfi, and F. Zwirner, Phys. Lett. **B257**, 83 (1991); **B262**, 477 (1991).
[25] Final Report of the Higgs Working Group, http://fnth37.fnal.gov/higgs/draft.html.
[26] CMS Collaboration, Technical Proposal, CERN/LHCC/94-38 (1994);
ATLAS Collaboration, Technical Proposal, CERN/LHCC/94-43 (1994).
[27] P.H. Chankowski, Z. Pluciennik, and S. Pokorski, Nucl. Phys. **B439**, 23 (1995).
[28] J.A. Bagger, K.T. Matchev, D.M. Pierce and R. Zhang, Phys. Rev. Lett. **78**, 1002 (1997); Erratum, ibid., **78**, 2497 (1997).
[29] K. Matchev and D. Pierce, Phys. Lett. **B445**, 331 (1999).
[30] T. Blazek, M. Carena, S. Raby, and C. E. M. Wagner, Phys. Rev. **D56**, 6919 (1997).
[31] See, e.g., L. E. Ibanez and C. Lopez, Nucl. Phys. **B233**, 511 (1984);
L. E. Ibanez, C. Lopez, and C. Munoz, Nucl. Phys. **B256**, 218 (1985);
M. Carena, M. Olechowski, S. Pokorski, and C. E. M. Wagner, Nucl. Phys. **B419**, 213 (1994); ibid., **B426**, 269 (1994);
M. Olechowski and S. Pokorski, Phys. Lett. **B344**, 201 (1995);

M. Carena, P. Chankowski, M. Olechowski, S. Pokorski, and C.E. M. Wagner, Nucl. Phys. **B491**, 103 (1997).