

First results from the asymmetric $\mathcal{O}(a)$ improved Fermilab action

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We present first results from calculations using an $\mathcal{O}(a)$ improved (FNAL) space-time asymmetric fermion action on a $12^3 \times 24$ quenched lattice at $\beta = 5.7$ and with $c_{SW} = 1.57$. The mass dependent asymmetry parameter ζ is determined non-perturbatively from the energy-momentum dispersion relation. Calculations have been made in the charm and bottom quark mass sectors in order to test the ζ dependence of the spectrum, since it is at these heavier masses that the asymmetry is expected to be most relevant.

1. The Fermilab improved action

For full details of this fermionic improvement scheme the reader is referred to [1]; here we merely note that it results in an action with mass dependent coefficients which is asymmetric in space and time.

The lattice dispersion relation may be written in the form

$$E^2(\mathbf{p}^2) = M_1^2 + \frac{M_1}{M_2} \mathbf{p}^2 + \mathcal{O}(\mathbf{p}^4), \quad (1)$$

defining the static mass $M_1 = E(\mathbf{0})$ and the kinetic mass $M_2 = \left(\frac{\partial^2 E}{\partial p_i^2} \right)^{-1}_{\mathbf{p}=\mathbf{0}}$.

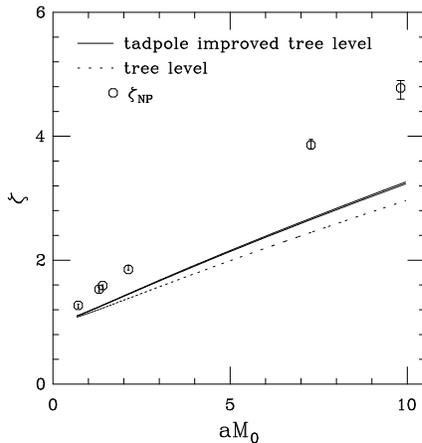


Figure 1. The non-perturbatively tuned ζ compared with tree-level perturbative predictions.

Lattice discretisation effects mean that in general $M_1 \neq M_2$ at $\mathcal{O}(am_{\text{quark}})$, and restoring the relativistic dispersion relation to this order is the first stage in the improvement program. This is achieved by introducing an asymmetry in the temporal and spatial quark propagation and adjusting it until $M_1 = M_2$, which then constitutes the first improvement condition of this scheme.

To this end we define the action

$$S_0 = \sum_x \left\{ \bar{\psi}_x \psi_x - \kappa_t [\bar{\psi}_x (1 - \gamma_0) U_{0x} \psi_{x+\hat{0}} + \bar{\psi}_x (1 + \gamma_0) U_{0x-\hat{0}}^\dagger \psi_{x-\hat{0}}] - \kappa_s \sum_i [\bar{\psi}_x (1 - \gamma_i) U_{ix} \psi_{x+\hat{i}} + \bar{\psi}_x (1 + \gamma_i) U_{ix-\hat{i}}^\dagger \psi_{x-\hat{i}}] \right\}. \quad (2)$$

It is helpful to parameterise this asymmetry by defining $\zeta = \kappa_s / \kappa_t$, in terms of which the quark mass is

$$M_0 = \frac{1}{2\kappa_t} - 3\zeta - 1 - \left(\frac{1}{2\kappa_{\text{crit}}} - 4 \right). \quad (3)$$

At some value $\zeta = \zeta_{\text{NP}}$, which we attempt here to find at various quark masses, $M_1 = M_2$.

In order to remove $\mathcal{O}(a)$ artifacts from the action the terms

$$S_E = i\kappa_s c_E \sum_{x,i} \bar{\psi}_x \sigma_{0i} F_{0i}(x) \psi_x \quad (4)$$

and

$$S_B = i\kappa_s c_B \sum_{x,i < j} \bar{\psi}_x \sigma_{ij} F_{ij}(x) \psi_x \quad (5)$$

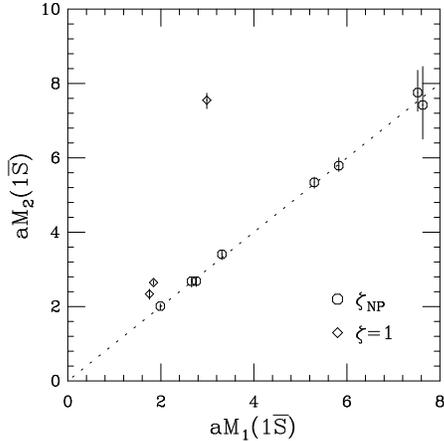


Figure 2. M_2 and M_1 at ζ_{NP} compared with the $M_1 = M_2$ line and the corresponding results from the symmetric action.

are added to S_0 . Here F_{0i} and F_{ij} are the standard clover representations of chromo-electric and chromo-magnetic parts of the field strength tensor.

2. Non-perturbative tuning of ζ

The strategy employed here was to tune ζ by requiring that $M_1 = M_2$ for the spin-averaged $1S$ quarkonium state. The results obtained from this non-perturbative tuning can be compared where appropriate with tree-level perturbation theory [1,2] and with results obtained using the symmetric ($\zeta = 1$) action [3].

These calculations were performed on 100 quenched $12^3 \times 24$ configurations at $\beta = 5.7$ with $c_E = c_B = 1.57$, the tree-level tadpole improved perturbative value on this lattice.

To find M_1 and M_2 , the ground state energy $E(\mathbf{p})$ was computed at five momenta using a two-state fit to a matrix of smeared correlators (as described in [4]). M_1 is simply $E(\mathbf{0})$ and M_2 was extracted from the coefficient a_1 obtained by fitting the dispersion relation to the function

$$E(\mathbf{p}^2) = a_0 + a_1 \mathbf{p}^2 + a_2 (\mathbf{p}^2)^2 + a_3 \sum_i p_i^4. \quad (6)$$

We obtain a graph (figure 1) of the non-perturbatively tuned ζ_{NP} as a function of M_0

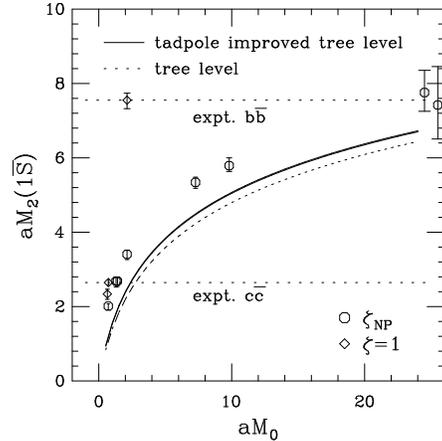


Figure 3. The variation of M_2 with M_0 for the tuned action, compared with the symmetric action and tree-level perturbation theory.

which we compare with tree level perturbation theory. The extent to which ζ_{NP} satisfies the improvement condition can be judged from figure 2, where for comparison the corresponding points previously obtained using the symmetric action on the same lattice are also plotted.

While figure 1 presents the dependence of ζ_{NP} upon M_0 , it is M_2 that emerges as the physically significant mass parameter in the heavy quark expansion. Therefore it is useful to know how M_2 depends on M_0 once ζ has been tuned, and this is shown in figure 3.

3. Quarkonium spectrum

The values of aM_2 that correspond on this lattice to the $c\bar{c}$ and $b\bar{b}$ mesons are known from the previous calculations with the symmetric action [3], and parameters of the asymmetric action yielding values of aM_2 comparable to these were found (see figures 3 and 1). To verify that the asymmetric action reproduces the same physics a spectral calculation was performed at these parameters on 300 configurations. $2S$ states were obtained using a three-state fit to the full correlator matrix. The scale was set from the spin-averaged $1P - 1S$ splitting. Figure 4 shows the masses of charmonium and bottomonium states expressed as splittings from the spin-averaged $1S$

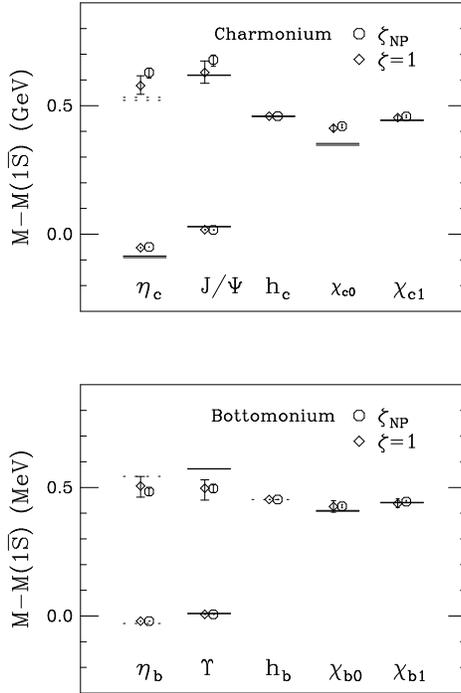


Figure 4. The quarkonium spectra compared with results obtained with the symmetric action.

mass. The hyperfine splitting is shown in figure 5 as a function of M_2 for all the data sets examined (*i.e.* with untuned asymmetric actions as well as at ζ_{NP} and $\zeta = 1$).

4. Conclusions

We have demonstrated the feasibility of a non-perturbative tuning of the first parameter of the Fermilab improved action, and find this value to be rather higher than the tree level prediction, although the mass dependence is already qualitatively predicted at tree level. The bare quark mass required to reach a particular physical régime is found to be much greater than for the symmetric action (again this behaviour is qualitatively predicted at tree level) which can have algorithmic implications in the quark propagator computation.

The quarkonium spectra (and a similar analy-

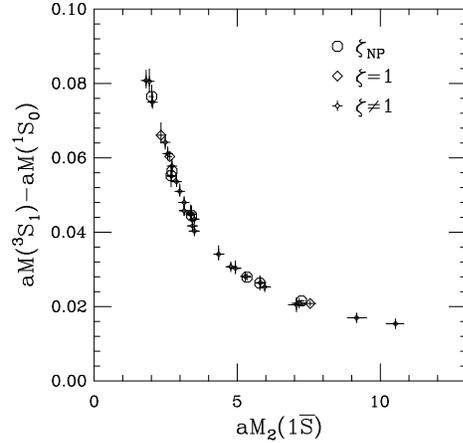


Figure 5. The hyperfine splitting as a function of M_2 computed using the symmetric, the tuned and the asymmetric untuned actions.

sis of the hyperfine, fine structure and the $2S-1S$ splittings) show that the tuned action produces the expected physics, and supports the use of M_2 as the physically relevant mass scale in computations where ζ is not tuned. Further confirmation comes from figure 5 where it can be seen that the hyperfine splittings lie on the same curve regardless of whether the action is tuned or not, indicating that it is dependent on M_2 only.

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