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Steady State and Transient Current Lead Analysis.

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Abstract—A mathematical model of the gas-cooled, resistive portion of a binary current lead has been developed. An analytical solution of the time-dependent differential equations for the resistive portion of the forced flow cooled current lead is presented which allows one to calculate the evolution of the temperature profile and voltage drop. A comparison of analytical with numerical calculations and a comparison of the calculations with experimental data are given.

I. INTRODUCTION

Fermilab's superconducting accelerator, the Tevatron, contains about 50 pairs of conventional current leads, which Fermilab is considering replacing with current leads which utilize HTS technology. The basic idea is to use leads which have two sections: (a) an HTS section from 4.3 - 80 K cooled with helium liquid and vapor, and (b) a copper section from 80 - 300 K cooled with liquid nitrogen, nitrogen vapor, and helium vapor (Fig.1). American Superconductor Corporation (ASC) and Intermagnetics General Corporation (IGC) have developed R&D HTS current leads for Fermilab which have been successfully tested at rated current of 5 kA. Although the emphasis of these tests was on demonstrating the operational stability of the HTS material, the copper section stability of these leads had to be tested as well.

A mathematical model of current lead behavior can be a useful tool to help understand and predict the transient behavior of a lead under various conditions. Mathematical modeling enables one either to find an analytical solution or to perform the calculation numerically. Both analytical and numerical solutions have been used for the study of the copper portion of these leads. The analytical solution was obtained for a constant thermal conductivity and with electrical resistivity calculated as a linear function of temperature. These assumptions are very reasonable for an upper lead section between 80 K and 300 K which is made from copper or copper alloys. Numerical calculation was done with temperature-dependent thermal conductivity. A comparison of analytical with numerical calculations and a comparison of the calculations with experimental data are given.

II. THE CURRENT LEAD MODEL

Figure 1 shows two types of gas cooling schemes: forced flow and boil-off cooling. With the forced flow cooling scheme, one can reduce N₂ cooling flow down to zero, but the boundary conditions are assumed to remain the same. In particular, the junction temperature is assumed to remain at 80 K. With the boil-off cooling scheme, one cannot reduce the N₂ cooling flow to less than the minimum one; otherwise, the lower boundary condition will not be satisfied: the junction temperature will start increasing. If the LN₂ flow is less than the minimum, all liquid will be evaporated and the temperature of the junction between copper and HTS sections will begin growing; with LN₂ flow a bit less than minimum a new equilibrium could be reached at higher temperature. Although the current leads which have been tested at Fermilab more nearly match the boil-off cooling concept, as a first step in analyzing the leads an analytical model for forced flow cooling of the copper section has been developed.

The current lead was modeled by applying a transient energy balance to the solid conductor and to the coolant, which can be helium and/or nitrogen for the resistive section of a current lead. A differential equation describing the transient thermal behavior of the lead is derived from an energy balance on the lead considering heat stored, thermal conduction, convection, and electrical heating. A second differential equation is derived from an energy balance on the gas coolant considering energy stored, mass flow, and convection. The equations, both of which neglect transverse temperature gradients, are [1],[2],[3],[4]:

$$Ac\sigma\frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left[Ak(T)\frac{\partial T}{\partial x} \right] - \frac{hS(T - T_g)}{L} + J^2\frac{\rho(T)}{A}$$
$$A_g c_p \sigma_g \frac{\partial T_g}{\partial t} = \dot{m} c_p \frac{\partial T_g}{\partial x} - \frac{hS(T - T_g)}{L} \quad (1)$$

where: T and T_g are the lead and the coolant temperature at the coordinate x , A is the cross-sectional area of the conductor, S is the heat exchange surface area, x is the axial coordinate on the lead, L is the length of conductor, $k(T)$ is the copper thermal conductivity, $\rho(T)$ is the electrical resistivity, J is the current through lead, \dot{m} is the cooling gas mass flow, c_p is the specific heat capacity of cooling gas, h is the heat exchange coefficient between lead and gas, where: c is the specific heat capacity of copper, A_g is the cross-sectional area of gas passage, σ is the

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density of copper, σ_g is the density of cooling gas, t is the time.

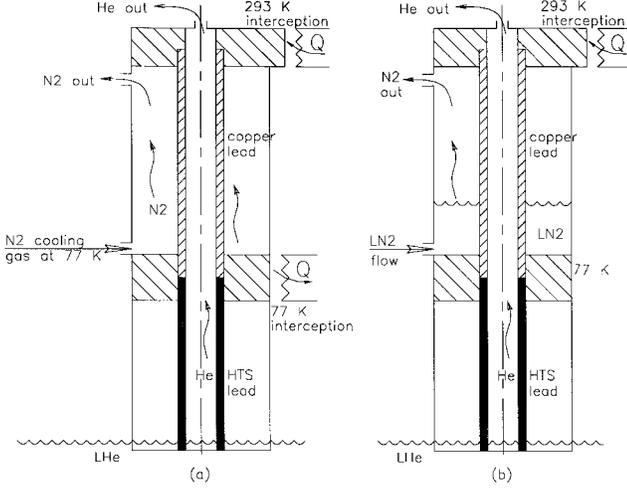


Fig. 1. Forced flow (a) and boil-off (b) cooled leads.

In order to reduce these equations to a form that can be integrated analytically, the following assumptions are made. First, that the temperature difference between cooling gas and conductor is constant along the lead. It means that heat exchange coefficient h is constant, that with good heat exchange is a reasonable assumption. Secondly that the electrical resistivity is considered to be a linear function of temperature,

$$\rho(T) = aT + b, \text{ where } a > 0 \quad (2)$$

where a and b are constant coefficients. A, k, S, c_p, c, h, A_g are assumed constant.

III. STEADY STATE ANALYSIS

For the steady state case $t = 0$ and with the simplifying assumptions above, the system of the two differential equations (1) results in the following differential equation of the second order with the constant coefficients.

$$\frac{\partial^2 T}{\partial x^2} - \frac{\dot{m}c_p}{Ak} \frac{\partial T}{\partial x} + \frac{J^2(aT + b)}{A^2k} = 0 \quad (3)$$

The solution form of the differential equation depends on the roots of the characteristic equation of (3). For the real lead operating conditions the roots are complex. The solution for temperature along the lead is of the form:

$$T(x) = -\frac{b}{a} + e^{-Bx} \cdot \left(T_c + \frac{b}{a} \right) \cos [Dx] + \frac{\left\{ T_h + \frac{b}{a} - e^{-BL} \cdot \left(T_c + \frac{b}{a} \right) \cos [DL] \right\} \sin [Dx] \cdot e^{B(L-x)}}{\sin [DL]}, \quad (4)$$

$$B = -\frac{\dot{m}c_p}{2Ak}; D = \sqrt{\frac{J^2 a}{A^2 k} - \left(\frac{\dot{m}c_p}{2Ak} \right)^2}$$

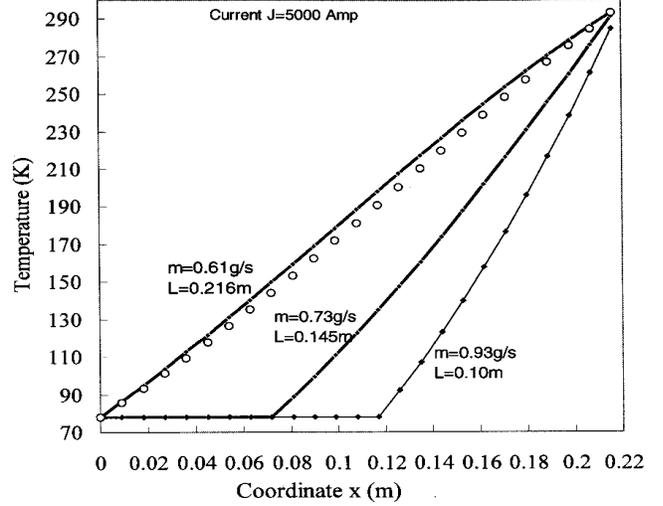


Fig. 2. Temperature profiles of boil-off cooled lead for LN₂ flows 0.61, 0.73 and 0.93 g/sec. L is corresponding exposed lead length. Line and circles are respectively, numerical and analytical calculations.

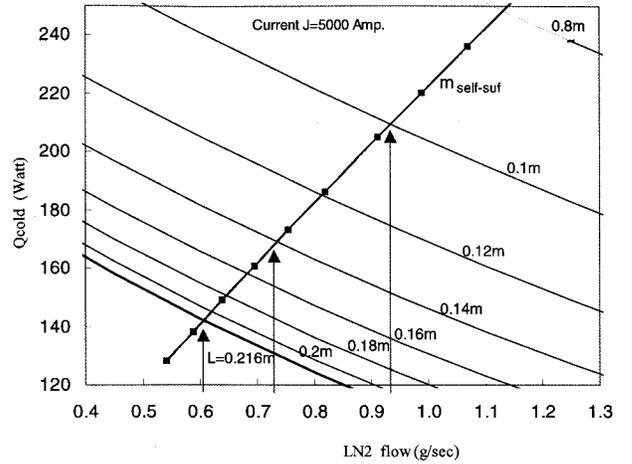


Fig. 3. Heat flow to lead cold end vs. N₂ cooling flow for different exposed lead lengths.

where T_h and T_c are lead temperature at the hot and cold ends, respectively.

In Fig. 2, the calculated temperature profiles of the copper section based on equation(4) are plotted using as an example the current lead developed by IGC. The three different temperature profiles correspond to three different cooling conditions. At an applied current value the exposed length of the lead depends on the LN₂ flow rate. Since the heat flow is proportional to the temperature gradient, knowing the temperature distribution along the lead, one can calculate the heat flows to the warm or to the cold lead ends as shown in Fig. 3. The higher the cooling flow rate at a certain exposed length, the lower the heat flow to the junction (*curves* Q_{cold}). On the other hand for the IGC boil-off cooled lead, with lower heat flow less cooling vapor is generated. The equilibrium is attained at the exposed length corresponding to the self

sufficient evaporation rate (line $\dot{m}_{self-suff}$). The minimum LN₂ flow \dot{m}_{min} is such a flow when the exposed length is maximum (0.216 m for IGC lead).

Evaporated LHe could be used to cool the HTS section, junction and copper section. The total heat flow to the junction which governs the minimum LN₂ flow is the difference between heat flows to the junction from the copper and HTS sections: $Q_{junction} = Q_{cold}^{Cu} - Q_{hot}^{HTS}$. Thus, taking into account the contribution of He gas cooling, the minimum LN₂ flow to maintain the junction at 80 K can be reduced by Q_{hot}^{HTS}/r , where r is the latent heat of vaporization of liquid nitrogen. For the IGC lead this \dot{m}_{min} is equal to 0.61 g/s at 5 kA and 0.41 g/s at zero current without any He cooling contribution and, respectively, 0.534 g/s and 0.34 g/s with the He cooling contribution taken into account. This is an average of a 13% reduction of the total heat load to LN₂.

IV. TRANSIENT ANALYSIS

The above analysis considers the steady state condition. This section reviews the dynamic behavior of the forced flow cooled lead. Let us assume that one of the system parameters such as current or LN₂ flow is abruptly changed. Then the former equilibrium is disturbed and temperature begins to change. Such a parameter to jump could be the current or coolant flow. The evolution equation for the transient process $\tau \neq 0$ follows from (1):

$$\frac{\partial \tilde{T}}{\partial \tau} = \frac{\partial^2 \tilde{T}}{\partial x^2} - \frac{\dot{m}c_p}{Ak} \frac{\partial \tilde{T}}{\partial x} + \frac{J^2 a}{A^2 k} \tilde{T} \quad (5)$$

$\tilde{T}(\tau, x) = T_2(x) - T(\tau, x)$ is the temperature deviation from the final stable temperature profile $T_2(x)$ and τ is the new time variable:

$$\tau = \frac{Ak}{(A_g c_p \sigma_g + A c \sigma)} \cdot t$$

Since the temperatures at the lead ends are fixed, the temperature deviation $\tilde{T}(\tau, x)$ satisfies the zero boundary condition:

$$\tilde{T}(\tau, 0) = \tilde{T}(\tau, L) = 0. \quad (6)$$

Initial conditions follow from the assumption of an equilibrium before the change of a parameter. When $\tau = 0$:

$$\tilde{T}(0, x) = T_2(x) - T_1(x), \quad (7)$$

where $T_1(x)$ is the initial stable temperature profile before the change. Applying the Fourier transformation to (5), the solution $\tilde{T}(\tau, x)$ is obtained:

$$\tilde{T}(\tau, x) = n = 1 \sum_{n=1}^{\infty} C_n \cdot \left[\exp(-\Lambda_n \cdot \tau) \cdot \sin\left(\frac{\pi n x}{L}\right) \cdot \exp(Bx) \right], \quad (8)$$

where: n goes through all positive integers; $n = 1, 2, \dots, \infty$. The constants C_n are found from the initial con-

ditions (7):

$$C_n = \frac{2}{L} \int_{x=0}^L \left\{ [T_2(x) - T_1(x)] \sin\left(\frac{\pi n x}{L}\right) \exp(-Bx) \right\} dx, \quad (9)$$

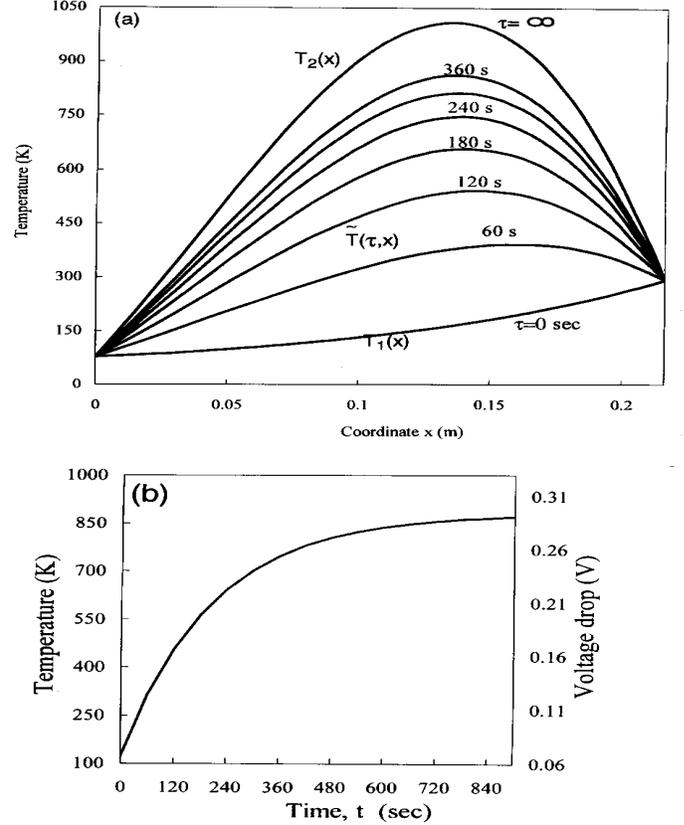


Fig. 4. Time evolution of temperature profile (a), lead temperature at coordinate $x=0.095$ m and voltage drop (b) after current change from 0 to 11.6 kA with LN₂ flow 1 g/sec.

where $T_2(x)$ and $T_1(x)$ are computed by (4) and the rates Λ_n are found from (5):

$$\Lambda_n = \left(\frac{\pi \cdot n}{L}\right)^2 + \left(\frac{\dot{m}_2 \cdot c_p}{2Ak}\right)^2 - \frac{J_2^2 a}{A^2 k}. \quad (10)$$

Here \dot{m}_2 and J_2 are new values of mass flow rate and current after these parameters change. The rates Λ_n characterize the dynamic behavior of the current lead. For the system to be stable all the rates Λ_n must be positive.

If all Λ_n are positive, $\Lambda_n > 0$, then $\Delta T(\tau, x)$ tends to zero at $\tau \rightarrow \infty$. That is, the temperature profile after the parameter change tends from the initial stable state to the final one. The most dangerous for the stability case is $n=1$, therefore the stability condition can be presented in the following form:

$$\left(\frac{\pi}{L}\right)^2 + \left(\frac{\dot{m}_2 \cdot c_p}{2Ak}\right)^2 - \frac{J_2^2 a}{A^2 k} > 0 \quad (11)$$

There are two mathematical cases where the lead can be damaged. The first is the burn-out situation, when the

stability condition is mathematically satisfied and a new steady state could be reached, but at so high temperature (for example 1000 K, Fig. 4) that irreversible damage will occur before the steady state will be reached. The second is the run-away situation when the stability condition (11) is not satisfied. This run-away is caused by the positive feedback between the temperature and electrical resistivity, assumed at (2).

TABLE I
Comparison of analytical and numerical calculations with experiment.

5 kA current lead Test		Analytic. Model	Experiment	Anal.-Exper. Discrepancy	
S t e a d y S t a t e L e a d	No current, J=0 Amp. Exposed Length L= 0.12 m.				
	LN2 cooling flow, (g/sec)	0.73	0.66	9.6%	
	No current, J=0 Amp. Exposed Length L= 0.216 m.				
	LN2 cooling flow, (g/sec)	0.34	0.24	29 %	
	Temperature of a point x=0.095 m, (K)	161	177	10 %	
	Current J=5000 Amp. Exposed Length L= 0.216 m.				
	LN2 cooling flow, (g/sec)	0.534	0.53	1 %	
	Temperature of a point x=0.095 m, (K)	183	200	10 %	
	Voltage Drop of copper lead (mV)	35	36	3 %	
	T r a n s i e n t L e a d	No current, J=0 Amp. Exposed Length L= 0.216 m. Initial LN2 flow: m1=0.3 g/sec. Final LN2 flow: m2=0.24 g/sec			
Temperature of coordinate x=0.095 m, (K)		Initial T1 Final T2	158 161	159 177	0.5 % 10 %
Stabilization time, (min)			3	16	5 times
Current J=5000 Amp. Exposed Length L= 0.216 m. Initial LN2 flow: m1=0.59 g/sec Final LN2 flow: m2=0.53 g/sec					
Temperature of coordinate x=0.095 m, (K)		Initial T1 Final T2	179.7 183.3	182 202	0.5 % 10 %
Voltage Drop of copper part, (mV)		Initial V1 Final V2	34.9 35.4	34.5 37	1.5 % 5 %
Stabilization time, (min)			5	16.8	3.4 times

V. EXPERIMENTAL RESULTS

Experimental results were obtained during tests of 5 kA binary copper-HTS leads. During these tests, the LN₂ level in the upper section varied depending on the applied current and LN₂ mass flow rate, so the exposed length was variable. To estimate the LN₂ level, one temperature sensor was embedded in the top of the copper-HTS junction and one in the middle of the upper section

(0.095 m above the junction) corresponding to exposed lengths above the liquid of 0.216 m and 0.12 m, respectively. Comparison of the experimental results with ones calculated analytically and numerically are given in table 1. One can see that the difference between analytical results and experimental results for the steady state flows and temperatures are within 30 percent, but for the transient lead the stabilization time discrepancy between analysis and experiment is a factor of 3 to 5. One likely reason for such a big discrepancy may be that the model assumes that the exposed length is not changed from the moment of disturbance until a steady state is attained¹. In reality, a significant additional time is required for a new LN₂ level to be fixed. An important refinement of the model would be to include this effect of LN₂ mass storage in the lead, thus modelling more closely the self-cooling or boil-off situation in the copper section of the lead.

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¹ Detailed derivation of analytical solutions for steady state and transient current lead is described in [5].