Neural Networks for Analysis of Top Quark Production

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Neural Networks for Analysis of Top Quark Production

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(July 15, 1999)

Abstract

Neural networks (NNs) provide a powerful and flexible tool for selecting a signal from a larger background. The DØ collaboration has used them extensively in studying $t\bar{t}$ decays. NNs were essential to the measurement of the $t\bar{t}$ production cross section in the all-jets channel ($t\bar{t} \rightarrow b\bar{b}qqq$), and were also used in the measurement of the mass of the top quark in the lepton+jets channel ($t\bar{t} \rightarrow b\bar{b}\ell\nuqq$). This paper will describe two new applications of neural networks to top quark analysis: the search for single top quark production, and an effort to increase the sensitivity in the dilepton channel $t\bar{t} \rightarrow b\bar{b}\ell\nu\ell\nu$ beyond that achieved in the published analysis.

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I. INTRODUCTION

Since the observation of the top quark in 1995 [1], much experimental effort has been invested in studying its properties [2]. Such analyses are difficult, owing to the small number of $t\bar{t}$ events available, the relatively large backgrounds, and the complex event geometries. Thus there has therefore been a great deal of interest in analysis techniques that could improve on the standard methods of selecting candidate events. One useful class of such techniques uses pattern classifiers based on feed-forward “neural networks.” [3]

The DØ experiment at the Fermilab Tevatron has made considerable use of neural network techniques in its analyses of top quark data. Both the cross section measurement in the all-jets channel [4] and the mass measurement in the lepton + jets channel [5] used neural networks; details of these analyses have already been published.

Here, we describe two more recent studies: a neural network analysis of single top quark production, and an effort to improve the efficiency for selecting $t\bar{t} \rightarrow e\mu$ events using neural networks. We shall start with a brief description of the kind of neural networks used in these analyses.

II. NEURAL NETWORKS

Figure 1 shows an example of the type of neural network used in these studies. It consists of a set of processing units, each of which has at least one input and one output. The output $y_i$ of a single unit $i$ is given in terms of its inputs $x_{ij}$ by

$$ y_i = g(\sum_j x_{ij} + \theta_i), $$

where $\theta_i$ is a threshold specific to the unit, and $g$ is a nonlinear squashing function, typically of the form

$$ g(x) = \frac{1}{1 + e^{-2x}}. $$

[Thus, the unit outputs are bounded in the range $(0, 1).$]

The units are arranged in layers, with the inputs of layer $n+1$ connected to the outputs of layer $n$ by a weight matrix:

$$ x_{ij}^{n+1} = w_{ij} y_j^n. $$

Typically, the last layer consists of only one unit, and is called the “output” layer; the other layers are called “hidden” layers. Often, the $y_i^0$ are said to be the outputs of a dummy “input” layer. No processing, however, is done in that “layer.” Such a network is quite flexible; in fact, it has been shown that a network with only one hidden layer can approximate any reasonable (Borel-measurable) function to any required degree of accuracy, provided that sufficient units are available in the hidden layer [6].

For pattern recognition, one wants to have the network output 1 if the input is most consistent with signal, and 0 if the input is most consistent with background. Typically, one has available a collection of $N$ inputs, some of which are known to be signal and some of which are known to be background. One defines an error function:

$$ \chi^2 = \frac{1}{N} \sum_{i=1}^{N} (O_i - t_i)^2, $$

where $O_i$ is the output of the network for input $i$, and $t_i$ is the desired output for that input. This quantity can be considered as a function of the weights $w$ and thresholds $\theta$; one then minimizes $\chi^2$ with respect to these variables to achieve an approximation to the desired function.

The minimization technique most often used is called “backpropagation,” which is a sort of stochastic gradient descent. Other minimization algorithms can also be used. This process is often referred to as “training” the network.

III. SINGLE TOP QUARK PRODUCTION

The first study we will examine is a search for single top quark production [7]. The processes relevant at the Tevatron are illustrated in Fig. 2; the total cross sections for these processes calculated at next-to-leading order (NLO) are [8]:

$$ \sigma_{\text{NLO}}(p\bar{p} \rightarrow t\bar{t}X + \text{c.c.}) = 0.724 \pm 0.043 \text{ pb}, $$

$$ \sigma_{\text{NLO}}(p\bar{p} \rightarrow tqX + \text{c.c.}) = 1.70 \pm 0.27 \text{ pb}. $$

Such processes are interesting because they directly probe the $W - t - b$ vertex. Assuming the Standard Model, measuring these cross sections gives a measurement of the $V_{tb}$ element of the Cabibbo-Kobayashi-Maskawa (CKM) matrix. Such measurements are also

![Figure 1. A feed-forward neural network.](image-url)
sensitive to any new physics in the weak interactions of the top quark [9].

After the decay of the top quark, the particles produced in these processes are $Wb\bar{b}$ and $Wb\bar{q}$, possibly with additional jets from QCD radiative effects. This study looks for leptonic decays of the $W$ boson, so the initial event selection requires a high-$p_T$ lepton, large missing transverse energy ($E_T$), and at least two jets. No $\delta$-tag is required in this study, in order to preserve signal efficiency (but if information about a tagging muon is present, it will be used).

The numbers of signal and background events expected to remain after this selection for D0’s Run 1 (109 pb$^{-1}$) are as follows:

<table>
<thead>
<tr>
<th>Process</th>
<th>$N_{\text{events}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$tb$</td>
<td>2.1</td>
</tr>
<tr>
<td>$tq\bar{b}$</td>
<td>5.1</td>
</tr>
<tr>
<td>QCD multijet</td>
<td>2411</td>
</tr>
<tr>
<td>$t\bar{t}$</td>
<td>22.3</td>
</tr>
<tr>
<td>$Wb\bar{b}$</td>
<td>11.4</td>
</tr>
<tr>
<td>$Wjj$ ($c, s$)</td>
<td>51.8</td>
</tr>
<tr>
<td>$Wjj$ ($g, u, d$)</td>
<td>1615.7</td>
</tr>
<tr>
<td>$WW$</td>
<td>36.9</td>
</tr>
<tr>
<td>$WZ$</td>
<td>5.3</td>
</tr>
</tbody>
</table>

As can be seen, the background is huge compared to the signal, with the dominant background sources being QCD multijet production (with a jet misidentified as a lepton) and the production of $W$ bosons with associated jets.

A crucial step in a neural network analysis is the selection of the variables used as input to the network. Adding more variables potentially increases the amount of information available to the network, but it also expands the space that must be searched during the minimization, making it more difficult to find a good minimum. In fact, with some procedures, adding variables of marginal utility can degrade the performance of a network. And while neural networks can in principle approximate any reasonable function, in practice complicated mappings may require too many hidden nodes for minimization to be practical.

A useful observation is that the rate for a scattering process is greatest in the regions of phase space near singularities in the corresponding matrix element [10]. If such singularities occur in different places for signal and background, then the dependence on the corresponding variables in which the singularities occur should differ strongly between signal and background. For example, the top quark production diagrams in Fig. 2 have a singularity at $M_W^2 = (p_t + p_W)^2 \rightarrow m_t^2$. In contrast, the dominant background diagrams, illustrated in Fig. 3, have singularity at

$$M_{g1,g2}^2 = (p_{g1} + p_{g2})^2 \rightarrow 0,$$

$$\hat{t}_{\phi,g1,g2} = (p_{g1} + p_{g2} - p_\phi)^2 \rightarrow 0,$$

$$\hat{t}_{\phi,g1} = (p_{g1} - p_\phi)^2 \rightarrow 0,$$

$$\hat{t}_{\phi,g2} = (p_{g2} - p_\phi)^2 \rightarrow 0.$$

These variables, however, are defined at the parton level, and cannot be directly measured, due to effects of QCD radiation, the unobserved neutrino, and unobserved momentum that escapes down the beam pipe. In such a situation, it is better to use other variables that are related to the singular variables, but can be derived directly from the observed final state. For example, the typical $t$-channel singular variable $\hat{t}_{ij,f}$ associated with the production of a light particle (or jet) $f$ can be written

$$\hat{t}_{ij,f} = (p_f - p_i)^2 = -\sqrt{s}Yp_{f}^{\perp}e^{-by_f},$$

where $\sqrt{s}$ is the total invariant mass of the produced system, $Y$ is its total rapidity, and $p_f^{\perp}$ and $y_f$ are the transverse momentum and rapidity of the produced $f$.

From these kinds of considerations, a nominal set of input variables can be defined as:

Set 1: $M_{j1,j2}$, $M_t$, $Y_{\text{tot}}$, $p_{T,j1}$, $y_{j1}$

$$p_{T,j2}$, $p_{T,j12}$, $y_{j12}$, $\sqrt{s}$

where $p_{T,j2}$ and $y_{j12}$ are the transverse momentum and rapidity of the system formed by the two highest $p_T$ jets, and $Y_{\text{tot}}$ is the total rapidity of the center of mass of the initial partons, as reconstructed from the final state. The $z$-component of the momentum of the $W$ boson is found by enforcing the $M_W$ mass constraint in the leptonic $W$ boson decay. Distributions of some of these variables are shown in Fig. 4.

Figure 5 compares this set with the simpler sets:

Set 2: $p_{T,j1}$, $p_{T,j2}$, $H_{\text{all}}$, $H_{T\text{all}}$

Set 3: $p_{T,j1}$, $p_{T,j2}$, $H_{\text{all}}$, $H_{T\text{all}}$, $M_t$
where $H_{\text{all}} = \sum E_{j}$ and $H_{T\text{all}} = \sum E_{Tj}$. The comparison is made by training a neural network for each of the sets on a sample of events consisting of top quark signal plus $Wjj$ background. It is seen that the neural network built using Set 1 performs better than those using Set 2 or Set 3.

Figure 5 also shows two other variations of the set of input variables. Set 4 is the same as Set 1, except that the variables $H_{\text{all}}$ and $H_{T\text{all}}$ are added. It is seen that this does worse than Set 1 — the additional variables do not add enough information to counteract the increase in the size of the minimization space. Set 5 adds to Set 1 the widths $w_{\text{jet}}$ of the two jets and the $p_{T}$ of a $b$-tagging muon (set to zero if there is no such tag). In this case, the added variables help: Set 5 has a lower $\chi^2$ than any of the others.

For the final analysis, a separate network is constructed for each of the major backgrounds, as shown in Fig. 6. The networks are trained using JETNET [11]; the results for each network are shown in Fig. 7. Figure 8 shows that the network output from Monte Carlo models agrees well with the data. Finally, individual cuts are made on each of the five network outputs. Figure 9 compares the results of this to a more conventional analysis. It is seen that for a given background level, the neural network analysis provides several times the signal efficiency of conventional cuts.
FIG. 7. Outputs of each of the neural networks for single top signal (dashed) and the indicated background (solid).

FIG. 8. A comparison of the combined output of the five networks for data (the open symbols) and a Monte Carlo model of signal and all backgrounds (the solid symbols). The individual network outputs are combined using $1/O_{\text{tot}} = (1/5) \sum_{i=1}^{5} 1/O_{\text{NN}i}$.

FIG. 9. Comparison of signal/background efficiencies for NN and conventional analyses. Each point represents one specific set of cuts.

FIG. 10. Distributions of input variables to the $\tau\tau$ neural network for signal (dashed) and $Z\to\tau\tau$ background (solid).

IV. $t\bar{t}$ DECAYS INTO $e\mu$

The "golden" channel for observing $t\bar{t}$ decays has long been the dilepton mode $t\bar{t} \rightarrow W^+W^-b\bar{b} \rightarrow e\nu, \mu\nu, b\bar{b}$. Due to the presence of two leptons with different flavors, this channel has a very low background. However, compared to the channels in which one of the $W$ bosons decays into jets, the $e\mu$ channel has a relatively small branching ratio — about 2.5%, versus about 15% for the $e+\text{jets}$ channel. Therefore, any new analysis techniques that can increase efficiency for identifying signal in this channel while maintaining the low background level are welcome.

This study starts from the published measurement of the $t\bar{t}$ production cross section [12], which selects $e\mu$ candidates as follows:

- An electron with $E_T > 15$ GeV and $|\eta| < 2.5$.
- A muon with $p_T > 15$ GeV/$c$ and $|\eta| < 1.2$.
- $E_T > 20$ GeV.
- At least two jets with $E_T > 20$ GeV and $|\eta| < 2.5$.
- $\Delta R_{b,\text{jet}} > 0.5$ and $\Delta R_{e,\mu} > 0.25$. ($\Delta R = \sqrt{(\Delta \phi)^2 + (\Delta \eta)^2}$)
- $H_T > 120$ GeV, where $H_T = E_T + \sum_{\text{jets}} E_{T,\text{jet}}$.

For the present study, this selection is relaxed by removing the cut on $H_T$ and reducing the $E_T$ and jet $E_T$ cuts to 15 GeV. This defines the sample used as input to the neural network.
There are three major backgrounds to contend with: QCD jet production with jets misidentified as leptons, \(Z\rightarrow\tau\tau\rightarrow e\mu\), and WW → eμ events. A separate network is trained to separate the signal from each of the three backgrounds. Six variables are used as inputs to each of the networks, these being \(E_T^e\), \(E_T^{jet}\), \(E_T\), \(H_T^{jets} = \sum_{jets} E_T^{jet}\), \(M_{e\mu}\), and \(\Delta\phi_{e\mu}\), except for the \(\tau\tau\) network, where \(E_T^{jet1}\) replaces \(E_T^{jet2}\). The input variables for the \(\tau\tau\) network are plotted in Fig. 10. Each network has seven hidden units. The networks are trained (using \textsc{jetnet}) on equal numbers of \(t\bar{t}\) signal and background events (2000 of each for the QCD network, and 1000 of each for the other two). The outputs of the three networks are combined, using usual

\[
O_{NN}^{comb} = \frac{1}{O_{NN1}^{comb}} + \frac{3}{O_{NN2}^{comb}} + \frac{1}{O_{NN3}^{comb}}. \tag{14}
\]

Distributions of this variable for signal and background are shown in Fig. 11. To define the candidate sample, a final cut of \(O_{NN}^{comb} > 0.88\) is imposed, which was determined by maximizing the expected relative significance, \(S/\sigma_B\). (\(\sigma_B\) is the uncertainty in the background estimate.)

The resulting signal efficiencies and estimated backgrounds for DØ’s Run 1 (108 pb\(^{-1}\)) are shown in Table I and Fig. 12. Compared to the standard (published) analysis, it is seen that the neural network analysis increases the signal efficiency by about 10%. In addition, the background is also slightly lower, although this is harder to evaluate due to the large statistical errors in the QCD background sample. Further comparison is made in Fig. 13.

<table>
<thead>
<tr>
<th>Background</th>
<th>(N_{expected})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z\rightarrow\tau\tau\rightarrow e\mu)</td>
<td>(0.10 \pm 0.10)</td>
</tr>
<tr>
<td>WW → eμ</td>
<td>(0.074 \pm 0.020)</td>
</tr>
<tr>
<td>(\gamma^*\rightarrow\tau\tau\rightarrow e\mu)</td>
<td>(0.006 \pm 0.005)</td>
</tr>
<tr>
<td>Fakes</td>
<td>(0.083 \pm 0.126)</td>
</tr>
<tr>
<td>Total</td>
<td>(0.26 \pm 0.16)</td>
</tr>
</tbody>
</table>

\[\text{FIG. 11. Distribution of } O_{NN}^{comb} \text{ for } t\bar{t} \text{ signal and background events.}\]

\[\text{FIG. 12. The neural network analysis compared to the standard analysis, for DØ’s Run 1. (a) Efficiency times branching ratio (%); (b) Ratio of NN analysis efficiency to standard analysis efficiency; (c) Expected number of signal events. Uncertainties displayed are statistical only; the systematic uncertainties (included in Table I) are highly correlated between the two analyses.}\]

\[\text{TABLE I. A comparison of the results of the conventional and neural network } t\bar{t} \rightarrow e\mu \text{ analyses. The numbers of background events are normalized for DØ’s Run 1 (108 pb\(^{-1}\)).}\]
FIG. 13. The neural network analysis compared to the standard analysis. Each point represents a different set of selection requirements.

V. CONCLUSIONS

In both the analyses considered here, neural networks provide a significant improvement over conventional analysis methods. We expect that such techniques will have a prominent place in the analysis of data from the upcoming Run 2 of the Tevatron.

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