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THE CLUSTER SZ — MASS CORRELATION

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ABSTRACT

N-body + hydrodynamic simulations of galaxy clusters are used to demonstrate a correlation between galaxy cluster mass and the strength of the Sunyaev–Zel'dovich (SZ) effect induced by the cluster. The intrinsic scatter in the correlaton is larger than seen in the cluster mass — X–ray temperature correlation, but smaller than seen in the correlation between mass (or temperature) and X–ray luminosity, as expected. Using the convergence to self–similarity of cluster structure at larger radii, a simple area–averaged SZ value derived from mock SZ maps also correlates well with mass; the intrinsic scatter in this correlation is comparable to that seen in simulations for the mass — temperature correlation. Such a relation may prove a powerful tool for estimating cluster masses at higher redshifts.

Subject headings: Galaxies-clusters, cosmology-theory

1. MOTIVATION

Correlations between galaxy cluster mass and observables such as X–ray luminosity L_x or temperature T are used extensively to probe cosmology. For instance, the abundance of clusters as a function of temperature has been used to constrain the normalization of the power spectrum of primordial fluctuations, σ_8, while searches for evolution in the cluster X–ray luminosity function have been employed to argue for values of the cosmological density parameter Ω_0. Theoretical models such as the Press–Schechter (Press & Schechter 1974) formalism characteristically provide the abundance of clusters as a function of the cluster mass. The use of temperature or luminosity functions to constrain the parameters in the theoretical model therefore depends upon a relation between T or L_x and mass. Theoretical arguments and cluster simulations indicate that temperature and X–ray luminosity should have a power law dependence on mass, while observations of cluster abundances and correlations between observables are consistent with such simple power law relations holding true for galaxy clusters.

A comparatively new tool for investigating clusters is provided by observations of the Sunyaev–Zel'dovich (SZ) effect (Sunyaev & Zel'dovich 1972; Birkinshaw 1998) — the distortion in the cosmic microwave background spectrum produced by Compton upscattering of microwave background photons by electrons in the hot intracluster medium (ICM). The SZ effect has two properties that make it particularly interesting for examining high–redshift systems. First, the signal strength is not attenuated by redshift, in contrast with X–ray flux. Second, for clusters of fixed mass, self–similar scaling relations predict a strong positive evolution with redshift. Since observations of X–ray temperature, X–ray surface brightness, and the SZ effect depend upon the density and temperature structure of cluster gas in different ways, the SZ effect gives a complimentary probe of the state of the ICM.

An interesting question is whether the strength of the SZ effect correlates with cluster mass, as is expected of T or L_x. A simple argument reproduced below suggests that such a correlation should also be present; verification of this would provide a consistency check upon the simple theoretical models used to describe clusters. Furthermore, accurate X–ray temperature determinations become harder with increasing redshift. As the strength of the SZ signal is redshift–independent, and as clusters of a fixed mass produce a stronger effect at higher redshift, a correlation between SZ signal and mass yields a powerful method for estimating cluster masses at redshifts beyond those currently probed by X–ray temperature estimates.

In this *Letter*, we use simulated clusters to examine the relation between cluster mass and the SZ effect. We construct mock SZ images of the simulated clusters, extract SZ observables, and examine correlations with mass as well as scatter about the correlations. We find that the central SZ signal exhibits the expected self–similar scaling with mass in these simulations, with a scatter significantly smaller than is seen in simulated L_x–M relations or the observed cluster L_x–T relation. We will also note that a simple area–averaging of the SZ signal reduces the dispersion to a level near that expected from the cluster X–ray temperature. Some concerns about applying this result, as well as directions for future study, will be described at the end.

2. MODELS

The simulated clusters were produced using the N–body + hydrodynamic code P3MSPH; both the simulation code and analysis techniques are described in Evrard (1990). Each simulation run produced an individual cluster, with initial conditions drawn from the $\Omega = 1$, $\Omega_b = 0.1$, $h = 0.5$ (assumed throughout this paper), $\sigma_8 = 0.6$ cold dark matter model using the constrained–realization technique of Bertschinger (1987). The clusters simulated ranged in mass from 8×10^{13} to $1.1 \times 10^{15} M_\odot$, with ICM temperatures between 0.7 and 4.3 keV. A total of 65536 particles were used in each simulation, half in dark matter

and half in baryons. The fractional mass resolution was comparable amongst the simulated clusters, with around 3500 gas particles typically residing in the cluster at $z = 0$. A total of 73 simulated clusters were produced in this fashion. The final, $z = 0$ configuration of each simulated cluster was then "imaged" in the SZ by producing a pixel map of values of the Compton y–parameter, each pixel's value constructed by a line–of–sight pressure integral through the simulated cluster. For each run, an SZ map was produced in three orthogonal directions; this produced a total of 219 maps. In extracting a central value for the y–parameter, y_0, the cluster "center" is defined by the projected location of the bottom of the cluster potential well, which traces well the projected location of the maximum value of the y–parameter.

3. THE CLUSTER Y–M RELATION

The Compton y–parameter induced by upscattering of microwave background photons along a line of sight ℓ through the cluster is given by

$$y = \int n_e \left(\frac{\mathrm{kT}}{m_e c^2}\right) \sigma_T \mathrm{d}\ell. \tag{1}$$

This equation can be used to derive a self–similar scaling relation for clusters which are near isothermality when density–weighted. Writing the density in terms of the background density $\bar{\rho}$, expressing length scales in terms of the radius r_δ at some fixed overdensity δ, and factoring out the temperature, we have

$$y \propto \bar{\rho}\, r_\delta \mathrm{T} \int \left(\frac{\rho_{gas}}{\bar{\rho}}\right) \mathrm{d}\left(\frac{\ell}{\mathrm{r}_\delta}\right). \tag{2}$$

If clusters are self–similar, then the integral produces a number which is identical for clusters of any mass (Navarro, Frenk & White 1995; Metzler 1995; Metzler & Evrard 1998), and varies only with the choice of line of sight. With $M = \delta\bar{\rho}(4\pi/3)r_\delta^3$, $\bar{\rho} \propto (1+z)^3$, and the virial scaling $\mathrm{T} \propto M^{2/3}(1+z)$, we have

$$y \propto M(1+z)^3. \tag{3}$$

This yields the scaling, with mass and redshift, of the y–parameter induced along a line of sight with a 2–D radius equal to the 3–D radius of some fixed fiducial overdensity; the constant of proportionality depends upon the overdensity in question. In particular, the values of the SZ central temperature decrement should scale in this fashion.

Do the simulated clusters obey this relation? We have 219 pairs of y_0 and mass within an overdensity of 200 with which to test the relation; the results are plotted in Figure 1. The best–fit power law relation is plotted as a solid line; the best–fit power law slope is 0.98 ± 0.04, well in agreement with the theoretical expectation, Eq. (3). Demanding Eq. (3) be exactly correct yields a best–fit coefficient of proportionality which is not significantly different from that found in the general power–law fit.

While the success of the expected scaling law is quite striking, the small scatter about the mean relation is also of note. Modelling the distribution of possible values of the

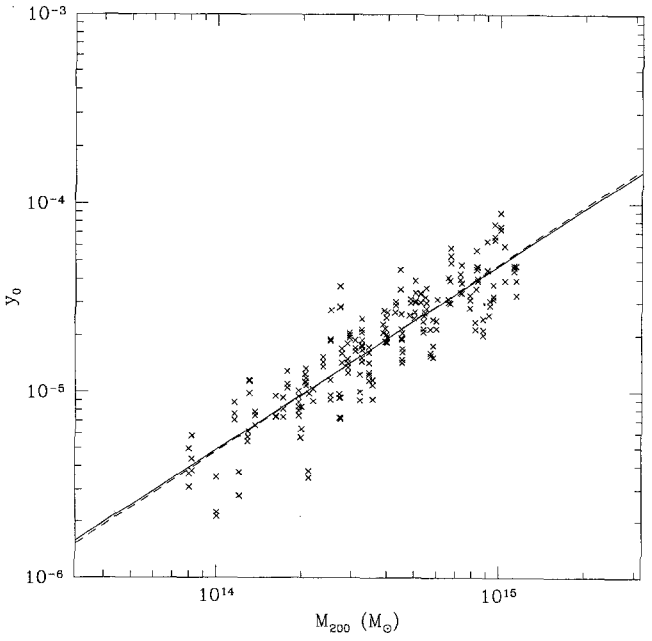

FIG. 1.— *Mass within an overdensity of 200 versus central y–distortion*, for the 219 simulated cluster observations in the dataset. Each point marks an observation. A solid line marks the least–squares best–fit to a power law, $y_0 = 10^{-4.4\pm0.7}\left(M/10^{15}\mathrm{M}_\odot\right)^{0.98\pm0.04}$. A linear relation, expected by theory, is not significantly different; forcing a fit to a linear relation produces the same coefficient, and the dashed line on the figure marking this relation cannot be distinguished from the solid line.

y–parameter for a cluster of a given mass as log–normal, the simulated data show a dispersion of $\sigma = 0.15$ in $\log y_0$. While larger than the scatter predicted by simulations for the T–M relation (Evrard, Metzler & Navarro 1996, hereafter EMN), this dispersion is significantly smaller than the scatter predicted by the luminosity–mass relation observed in simulations (Metzler 1995), and observed in the luminosity–temperature correlation for X–ray clusters (Arnaud & Evrard 1998).

The magnitude of the intrinsic scatter about the mean SZ–mass relation is important. Since the predicted abundance of clusters falls with increasing mass, the net effect of such scatter is to artificially populate the "bright" end of the abundance function. In other words, such an intrinsic scatter would make the real universe appear to have more high–mass clusters than are actually present. Since the high–mass end of the the cluster abundance function also contains most of the cosmological constraining power, a large intrinsic dispersion in the correlation of an observable with mass makes using the cluster abundance to constrain cosmological parameters extremely difficult. Most efforts to construct the cluster abundance include an attempt to correct for this. However, an accurate correction requires an understanding of the intrinsic dispersion in the correlation with mass, which has not been available.

Consequently, the comparatively small scatter suggests that the SZ central decrement is a much more robust observable to use in constructing an abundance function than X–ray luminosity. If this result holds at higher redshift as well, then along with the redshift–independence of the SZ

signal, and the expected self–similar scaling of the SZ signal strength with redshift at fixed mass, we can expect a large dataset of measurements of the SZ central temperature decrement to yield a much better estimate of the high–redshift cluster abundance.

4. THE CLUSTER $\langle Y \rangle_A$–M RELATION

Cluster entropy profiles drawn from simulations such as used here typically exhibit self–similar behavior outside of the cluster core, but a dispersion in scaled entropy values in the center of the cluster. Observations suggest such a dispersion in central entropy states, as we can demonstrate. First, note that the cluster luminosity–temperature relation is both theoretically and empirically well–described by a power law, $L_x \propto \mathrm{T}^{\alpha}$. Second, the fact that most emission originates from the cluster core allows us to approximate the X–ray luminosity by $L_x \propto n_0^2\, \mathrm{T}^{1/2}\, r_c^3$, where n_0 is the central gas density and r_c is the cluster X–ray core radius. Equating, we can solve for the quantity $s' = \mathrm{T}/n_0^{2/3}$, which is simply related to the entropy by $s = \log s'$; we obtain

$$s' \propto r_c\, \mathrm{T}^{(7-2\alpha)/6}. \tag{4}$$

If the relation between central entropy and cluster mass had little intrinsic scatter, then the tight mass—temperature relation would imply a small dispersion in observed values of cluster X–ray core radius at a given X–ray temperature. In other words, we should see a tight relation between X–ray core radius and X–ray temperature; this is contradicted by observations (Mohr 1998; Mushotzky 1998 1998). At fixed temperature, thus mass, there is a wide range of observed core radii, and thus by inference central entropy states.

Therefore, the result that the magnitude of the scatter in the cluster y—M relation lies between that of the T–M and L_x–M relations is not surprising. While the T–M relation is expected to be tight (EMN), as the gas specific energy is determined by the potential the gas falls through, the dispersion in central entropy noted above results in a dispersion in central gas density values. The y–parameter depends upon an integral of the gas density along the line of sight, so such an intrinsic dispersion introduces scatter in the correlation with mass. However, the X–ray emission depends upon an integral of the square of the density; the intrinsic scatter in the luminosity—mass relation is correspondingly larger.

That the gas density (and thus gas entropy) appears in simulations to converge to self–similar behavior at larger radii suggests that the scatter in correlations between mass and ICM–related properties could be reduced through the use of a statistic which increases the weight of data from outside the cluster core. For X–ray emission, this has been demonstrated through the prediction and observation of a tight correlation between cluster temperature and radius containing a fixed amount of X–ray emission (Mohr & Evrard 1997). It should be possible to construct an SZ observable which correlates well with mass and shows much smaller intrinsic disperson at fixed mass than the central y–distortion.

Defining our new statistic as $\langle y \rangle_A$, we can write $\langle y \rangle_A$ in terms of the y–distortion induced by a line–of–sight at angle θ from the cluster center by

$$\langle y \rangle_A = \int y\,(\theta)\, W\,(\theta)\, \mathrm{d}^2\theta, \tag{5}$$

where $W\,(\theta)$ is a window function that determines the relative weighting of different parts of the cluster. The simplest approach is to use a top–hat window function; $\langle y \rangle_A$ is then simply an average of the SZ map within some chosen radius.

It is important that the radius used to construct $\langle y \rangle_A$ be a *self–similar* radius, *i.e.* the radius out to some fiducial overdensity δ in 3-D, r_δ. This ensures that the same regions of different clusters are being examined in constructing $\langle y \rangle_A$. In practice, however, we would not know the radius r_δ for an observed cluster. It could be deduced from the X–ray temperature; but if we have a good measurement of the temperature, then we immediately have access to a mass estimator which is independent of the gas density, and thus presumably has smaller intrinsic error. The point is to develop an SZ–based mass estimator to be used at higher redshifts, where X–ray temperatures are difficult to obtain.

We solve this problem by *estimating* r_δ using the central value of the y–parameter alone. In the previous section, we demonstrated a correlation between the central y–distortion and cluster mass at r_{200}; we use this relation to estimate r_{200}, then use this value to set an outer limit for averaging an SZ map to construct $\langle y \rangle_A$. Here we will use $0.3\, r_{200}$, corresponding to $\delta \sim 2000$ for the NFW density profile. With our previous result of $y_0 = 3.98 \times 10^{-5} M_{15}^{0.98}$, where M_{15} is the mass within an overdensity of 200 in units of $10^{15}\mathrm{M}_\odot$, and with $M_{200} = 200\bar{\rho} \times 4\pi r_{200}^3/3$ by definition, we obtain $r_{200}^{est} \approx 1.61\, \Omega_0^{-1/3}\, (y_0/10^{-5})^{1/3}$ Mpc. For each mock image, we extract y_0, use this value to find r_{200}^{est}, and then construct the average $\langle y \rangle_A$ within $0.3\, r_{200}$; this radius would correspond to $\theta \approx 1.26'$ for $y_0 = 10^{-5}$ at $z = 0.5$.

The result is shown in Figure 2, with a best–fit power law shown as a solid line, while the best fit to Eq. (3) is shown as a dashed line. The two are barely distinguishable. The slope of the best fit relation is 0.97 ± 0.01, slightly shallower than the expected slope; the plot shows, however, that a linear scaling provides a good fit as well. More notably, the dispersion in the relation is much reduced; when modelled as log–normal, the scatter in $\log\langle y \rangle_A$ at fixed mass is only 0.06 — that is, 13–15% in $\langle y \rangle_A$ — much smaller than for the central decrement alone and comparable to that found by EMN for the mass–temperature relation at overdensities of ~ 500.

It may seem puzzling that the scatter in the $\log\langle y \rangle_A$ — M relation is less than that in the y_0 — M relation, given that we use y_0 to choose a radius within which to construct $\log\langle y \rangle_A$. However, note first that the dispersion in r_{200}^{est} will be only 1/3 the dispersion in mass inferred from the central signal. Second, the fact that the SZ signal should fall with cluster radius suppresses the error introduced by using an incorrect inferred r_{200}. For instance, imagine that a cluster with a given mass has an observed y_0 higher than

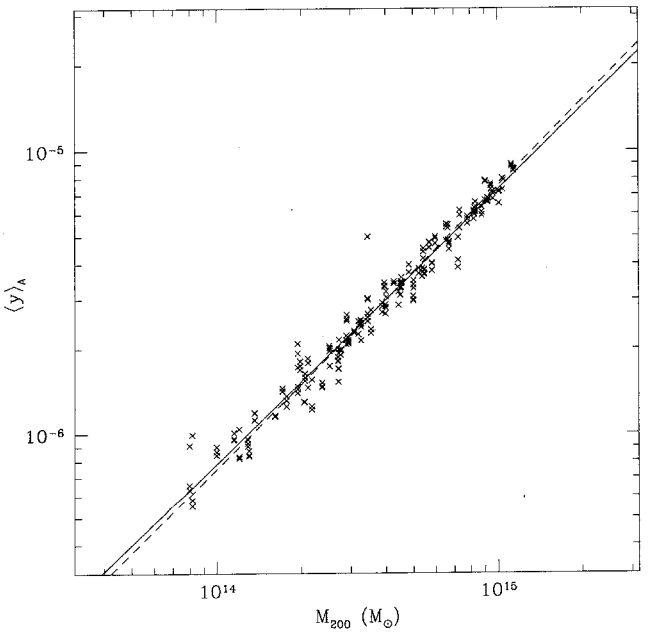

FIG. 2.— Mass within an overdensity of 200, M_{200}, versus average y–distortion over a circular area of radius $0.3 r_{200}^{est}$, $\langle y \rangle_A$. r_{200}^{est} is an estimate of the three–dimensional radius containing an overdensity of 200, derived from the central SZ signal y_0. Each point marks one of the 219 simulated cluster observations. A solid line marks the least–squares best–fit to a power law, $\langle y \rangle_A = 10^{-5.1 \pm 0.3} \left(M/10^{15} M_\odot\right)^{0.97 \pm 0.01}$. Forcing a fit to a linear relation produces the same coefficient; this curve is drawn in a dashed line on the figure.

expected from the mean relation; r_{200}^{est} will then be higher than the correct value. However, since the signal falls with radius, constructing an average within this incorrect radius depresses the result, counteracting the impact of the overlarge central value. This works to suppress the dispersion in $\langle y \rangle_A$ for clusters at a fixed mass that would be induced by the dispersion in y_0.

5. DISCUSSION

The simulation results discussed above evidence a correlation between the strength of SZ signal and collapsed mass, in agreement with theoretical expectation. The intrinsic scatter observed in the relation between the central value of the Compton y–parameter y_0 and mass is greater than seen in the mass—temperature relation, but considerably smaller than simulations show in the relation between X–ray luminosity and mass, as expected. The scatter in this relation can be reduced to a level near that of the mass — temperature relation through the use of statistics which depend less sensitively on central gas and more sensitively on gas at larger radii.

Ssome cautionary notes are in order. First of all, the *optimal* statistic to use will in all likelihood be experiment–dependent, since different experiments sample the sky differently. Also, such a correlation with mass should be affected by limits in observational resolution; when a cluster is not resolved, the y–parameter measured will scale with the angular–diameter distance to the cluster as $y \propto d_A^{-2}$.

There are also issues of concern about the simulations used here. Chief among these are numerical resolution limitations, which likely serve to reduce the dispersion in the SZ — mass correlation. Furthermore, the effect of missing physics which breaks self–similarity, such as cooling or the inclusion of supernova–driven galactic winds, must be considered. Next, the simulated clusters used in this investigation were taken at $z = 0$; numerical resolution issues are of greater concern as redshift increases. It would be worthwhile to investigate in detail, using higher–resolution simulations, the status of the SZ — mass correlation at redshifts of 0.5 or 1. Finally, the effects of the choice of background cosmology — particularly, the values chosen for Ω_0 and the baryon fraction $f_b = \Omega_b/\Omega_0$ — must be examined.

One hope for verifying or calibrating the SZ — mass correlation comes from comparing SZ–derived masses with those from X–ray observations or from gravitational lensing. Since low redshift clusters typically have better temperature estimates, the planned Viper Sunyaev–Zel'dovich Survey (Romer 1998) should provide a good opportunity to test this correlation.

Future work is planned to study these issues, as well as to examine the systematics of using SZ observations to constrain the cluster baryon fraction.

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