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Covariance and a Decaying Cosmological Term

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Abstract

We investigate the physical consequences of a relativistic covariant evolution of the cosmological term Λ. While previous studies only consider a decaying Λ in a Robertson-Walker universe, we look into the evolution of Λ both in an isotropic and homogeneous universe and in a universe perturbed around the Robertson-Walker background. We show that when departures from homogeneity are taken into account an evolving Λ may be disfavored.

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I. INTRODUCTION

A cosmological constant Λ has been invoked over the last years to solve several cosmological issues [1,2]. Dynamical estimates of the total matter of the universe tend to indicate that the total matter density is below the critical value that makes the universe flat. A positive Λ fills the gap between this low total matter content and the energy density required to have a flat universe [3], the latter being predicted by many inflationary models of the very early universe. A positive Λ also solves a possible age crisis [4]. The recently measured luminosity distances of high redshift type Ia supernovae suggest the existence of a positive cosmological term [5]. On the other hand, gravitational lensing analyses set an upper bound on Λ [6].

The cosmologically preferred value for a present cosmological term is $\Lambda \sim H_0^2$, where H_0 is the present Hubble constant. In quantum field theory one can argue that the cosmological term should be related to the vacuum energy and therefore one would expect $\Lambda \sim GM^4$, where G is the Newton constant and M is the energy scale at which the cosmological term originates [7]. The most popular candidates for M are the Planck mass and the supersymmetry breaking scale. In both cases the value of Λ is many orders of magnitude larger than H_0^2. One way out of this problem is to assume that Λ decays in time from these large values at the epoch $t \sim G^{-1/2}M^{-2}$ to its small present value.

Several expressions for $\Lambda(t)$ have been proposed in the literature [8–20]. Most of them give $\Lambda(t)$ as a decreasing function of the scale factor $a(t)$ in a Robertson-Walker (RW) universe (or equivalently as a function of t or $H(t) \equiv \dot{a}/a$). However, according to general relativity, if Λ is evolving, one should be able to write down a covariant equation that describes the evolution of the cosmological term, which should depend on all the coordinates $\Lambda(x)$. Only for the background case, in which inhomogeneities are neglected, one would recover $\Lambda(t)$ as a function of $a(t)$. The purpose of our work is to investigate the consequences of including general covariance when studying a decaying cosmological term.

A reasonable expression for a covariant decay law of the cosmological term is

$$\Box\Lambda(x) = F\left(\Lambda(x), g_{\mu\nu}(x)\right), \tag{1}$$

where $\Box \equiv \nabla^\mu \nabla_\mu$, being ∇_μ the covariant derivative, and where F is a scalar function of Λ and the metric $g_{\mu\nu}$. This function is not completely arbitrary, certain physical requirements have to be obeyed by F. First of all, the solutions to Eq. (1) in the RW background have to decrease in time and always remain positive, as argued above. Another physical property that should be implemented in F is that the vacuum energy density must be much smaller than the radiation and matter density during most of the early evolution of the universe [9,21,22]. A vacuum energy density comparable to the radiation density at time ~ 1 sec would spoil the agreement between the observed cosmic abundances of light elements and the primordial abundances calculated with the standard big bang model. The vacuum energy density must be small compared with the matter density at matter-radiation equality and last scattering in order to leave gravitational growth of small matter density perturbations unscathed. Only until recent times the Λ energy density can begin to be significant. Therefore, expressions for F that give scaling solutions in a RW background (vacuum energy density proportional to radiation or matter density) are not allowed. On the other hand, F should not produce a Λ that decays very steeply, since we would like the cosmological term to be significant today and possibly to be the dominant energy contribution at present. Finally, if Λ is responsible for cosmic inflation in the very early universe, one should choose F so that Eq. (1) includes constant or nearly constant solutions for a vacuum dominated universe, in order to have a long enough inflationary epoch.

If the cosmological term evolves with time then the radiation plus matter energy-stress tensor is no longer conserved. From the Einstein equations

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} + \Lambda g_{\mu\nu} \tag{2}$$

one obtains

$$\nabla_\nu T^\nu_\mu = -\frac{1}{8\pi G}\partial_\mu \Lambda, \tag{3}$$

with $G_{\mu\nu}$ the Einstein tensor and T_μ^ν the energy-stress tensor of radiation plus matter. Equation (3) allows a nonconstant Λ to decay into radiation and/or matter.

A current subject of study is the possibility that an important contribution to the total energy density of the universe is given by a dynamical scalar field, sometimes referred as quintessence [23–26]. The effective pressure of this new component can be negative for certain scalar field potentials. An energy component with negative pressure seems to be favored by observational data [27]. In some models the late time behavior of this scalar field mimics that of a cosmological constant. We would like to emphasize the difference between these scalar field models and our work. For a scalar field ϕ one starts with a Lagrangian $\mathcal{L} = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$, and defines the energy-stress tensor for the scalar field component as

$$T^\phi_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \mathcal{L}, \tag{4}$$

which is conserved. We do not regard Λ as a scalar field in the usual sense. Although the decay law (1) could be derived from a Lagrangian $\tilde{\mathcal{L}}$ for Λ, we do not define its energy-stress tensor using this Lagrangian. The energy-stress tensor of Λ is

$$T^\Lambda_{\mu\nu} = \Lambda g_{\mu\nu}, \tag{5}$$

independently of the decay law (1) [28].

It turns out that the simplest form of F that obeys all the physical requirements previously listed is $F = 0$, so that a decay law for the cosmological term could be $\Box \Lambda = 0$. In the next section we shall solve this equation in a RW background and see that its behavior is physically correct. In Sec. III we consider a space-time that is slightly perturbed around the RW background. We shall show that a nonconstant cosmological term can alter significantly the standard scenario of structure formation. Finally, in Sec. IV we give the conclusions. We always assume negligible spatial curvature, as predicted by most inflationary models.

II. EVOLUTION OF Λ IN A RW BACKGROUND

For a flat homogeneous and isotropic universe Eq. (1) with $F = 0$ reduces to

$$\ddot{\Lambda} + 3H\dot{\Lambda} = 0. \tag{6}$$

During the radiation dominated era, the solution to this equation is

$$\Lambda(t) = c_1 + \frac{c_2}{a(t)}, \tag{7}$$

where c_i are constant factors; we can always choose these factors so that the vacuum energy density $\rho_\Lambda = \Lambda/8\pi G$ is much smaller that the radiation density ρ_r. Since at early times $\Lambda \propto a^{-1}$ while $\rho_r \propto a^{-4}$, the evolution of the cosmological term does neither modify big bang nucleosynthesis nor the standard evolution of the universe during the radiation dominated epoch.

At later times the universe becomes matter dominated. The solution to (6) is then

$$\Lambda(t) = c_1' + \frac{c_2'}{a(t)^{3/2}}. \tag{8}$$

However, since the matter density drops as $\rho_m \propto a^{-3}$, faster than ρ_Λ, it comes a time when the latter is comparable or larger than the matter density. Then expression (8) is no longer true. In order to solve Eq. (6) at late times we have to include Λ in the Friedmann-Lemaître (FL) equations. For a flat universe these equations can be written as

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \tag{9}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\rho + \frac{\Lambda}{3}, \tag{10}$$

where we have assumed for ease that the cosmological term decays only into matter (and we omit the label m in ρ). From the FL equations, or from Eq. (3) in a RW background, one obtains

$$\dot{\rho}_\Lambda = -\frac{1}{a^3}\frac{d\left(a^3\rho\right)}{dt}, \tag{11}$$

which explicitly shows that the cosmological term decays into matter.

Using observations of the cosmic gamma ray background it was shown [9] that a Λ that decays into baryons and that conserves baryon number is ruled out. However, it is not unnatural to assume that Λ is decaying mainly into some form of dark matter since, in fact, nonbaryonic dark matter seems to dominate the total matter content of the universe. Furthermore, if Λ is actually decaying, only unknown physics at high energy scales could account for it. Since at high energies violation of baryon number is needed to produce the cosmic matter-antimatter asymmetry, it does not seem unreasonable to suppose that the decay of Λ violates baryon number.

It is convenient to use the dimensionless variables $x \equiv \dot{\Lambda}/H^3$ and $y \equiv \Lambda/3H^2$ and recast Eqs. (6), (9), and (10) as a plane autonomous system

$$x' = -\frac{3}{2}x(1-3y), \tag{12}$$

$$y' = \frac{1}{3}x + 3y(1-y), \tag{13}$$

being $' \equiv d/d\ln a$. In addition, one has the flat universe constraint $\rho = \frac{3H^2}{8\pi G}(1-y)$. Once the present values of x and y (x_0, y_0) are specified, there exist a unique solution to this system of equations. In the particular case $x_0 = 0$ we recover the usual (nondecaying) cosmological constant scenario

$$x(a) = 0, \tag{14}$$

$$y(a) = \frac{1}{1 + \frac{\Omega_0}{1-\Omega_0}\left(\frac{a_0}{a}\right)^3}, \tag{15}$$

where Ω_0 and a_0 are the present matter density to critical density ratio and the present expansion factor, respectively.

Our autonomous system has three fixed points: the unstable nodes $(x, y) = (0, 0)$ and $(-2, 1/3)$, and the stable node $(0, 1)$. The fixed point $(0, 0)$ corresponds to the early universe, with a negligible Λ energy density. The unstable point $(-2, 1/3)$ corresponds to unaccelerated expansion $\ddot{a}(t) = 0$. The stable fixed point $(0, 1)$ represents a universe dominated by a cosmological constant, it acts as a late time attractor.

We plot in Fig. (1) numerical solutions to Eqs. (12) and (13) for several values of (x_0, y_0). The requirements $\dot{\Lambda} \leq 0$, $\rho > 0$ and $\Lambda > 0$ at any time, or equivalently, $x \leq 0$ and $0 < y < 1$, constrain the physical trajectories to lie inside a bounded region on the (x, y) plane. As a consequence, given a $y_0 = 1 - \Omega_0$ the present decay rate $\dot{\Lambda}_0$ is limited by $x_{0m} \leq x_0 \leq 0$, where x_{0m} can be calculated numerically. Figure (1) shows that all physical trajectories start with a matter density much larger than the Λ energy density. At low redshifts $z \sim 1 - 0.1$ the universe becomes Λ dominated and eventually all physical trajectories fall into the late time attractor.

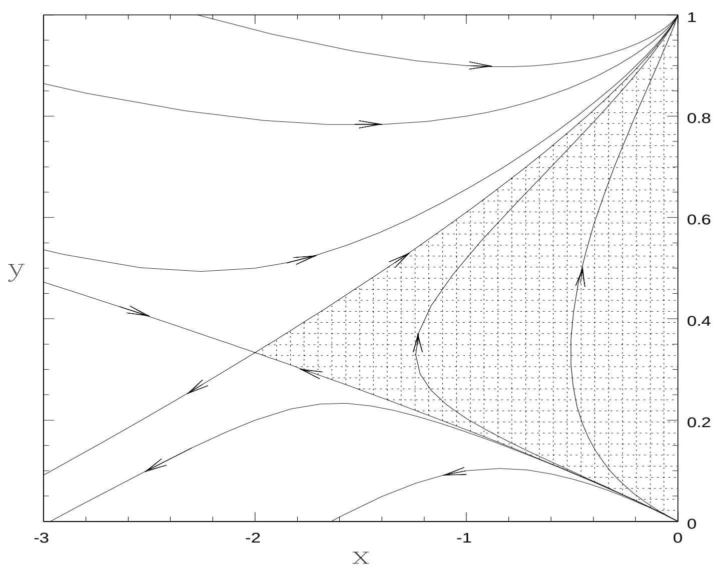

FIG. 1. The flow of trajectories on the (x, y) plane is depicted for the autonomous system given by Eqs. (12) and (13). The shaded region contains the physically meaningful trajectories. This region is bounded by two of the "asymptotes" of the unstable node $(-2, 1/3)$ and by $x = 0$.

Therefore, $F = 0$ in Eq. (1) gives a covariant decay law for the cosmological term that is physically acceptable in an unperturbed RW universe. Another possible choice for F, without introducing arbitrary mass scales, is $F = \beta(R + 4\Lambda)\Lambda$, where β is a dimensionless parameter and R is the Ricci scalar. It can be shown that unless β is small, $0 \leq \beta \leq 3/16$, the physical requirement $\dot{\Lambda} \leq 0$ is not obeyed.

On more speculative terms, if there is a nonvanishing Λ it should be somehow related to the vacuum state of a quantum theory, and if Λ is unstable its decay should be governed by quantum mechanics. Hence, one expects an exponential time decay for $t \leq \tau$, where τ is the lifetime of Λ. One can assume $\tau \sim M^{-1}$, being M the energy scale at which Λ originates. But for times $t \gg \tau$, the rules of quantum mechanics give slower decay rates, power-laws in time for the usual cases [29]. This is compatible with the decay rates (7) and (8) and some of the decay rates in a RW universe proposed in other papers [30]. For more involved expressions of F containing arbitrary mass scales, it may be harder to obtain power-law solutions in a RW background.

To end this section we would like to remark that, according to general relativity, any decay law for Λ in a RW background should be derived from a covariant decay law, independent of the coordinate frame, once one choses a coordinate frame in which the universe looks isotropic and homogeneous. It is not clear whether this can be achieved for all the decay laws in a RW universe proposed in the literature.

III. SMALL PERTURBATIONS

We have found that there are expressions for F in the covariant equation (1) that produce interesting decay laws of Λ in a RW background. We shall proceed to study the physical consequence of Eq. (1) in the realistic case that one has small departures from an isotropic and homogeneous universe in the early universe.

In an inhomogeneous universe an evolving cosmological term depends on all the spacetime coordinates, like the other energy components. Therefore, we consider small perturbations of both ρ and ρ_Λ respect to their background values

$$\rho(x) = \rho_b(t) + \delta\rho(x), \tag{16}$$

$$\rho_\Lambda(x) = \rho_{\Lambda b}(t) + \delta\rho_\Lambda(x). \tag{17}$$

The evolution of these perturbations is given by the continuity and Euler equations of a perfect fluid made of matter and an inhomogeneous Λ. Both equations can be derived from Eq. (3) only keeping terms that are first order in the perturbations

$$\dot{\delta} + \frac{1}{a}\partial_i v^i - 3\dot{\Phi} = \frac{\dot{\rho}_{\Lambda b}}{\rho_b}\delta - \frac{\delta\dot{\rho}_\Lambda}{\rho_b}, \tag{18}$$

$$\dot{v}^i + Hv^i + \frac{\partial_i \Phi}{a} + \frac{\partial_i \delta p}{a\rho_b} = \frac{\dot{\rho}_{\Lambda b}}{\rho_b}v^i + \frac{\partial_i \delta\rho_\Lambda}{a\rho_b}, \tag{19}$$

being v^i the peculiar velocity of matter, δp the matter pressure perturbation, and $\delta \equiv \delta\rho/\rho_b$. We always use the Newtonian gauge, Φ is the Newtonian potential [31]. The left-hand sides of Eq. (18) and (19) are the familiar expressions that one obtains when studying structure formation in a matter dominated universe. The nonvanishing right-hand side terms stem from the evolving Λ which acts as a source and sink of matter.

In addition, the evolution equation of Λ (1) gives, to first order in the perturbations,

$$\delta\ddot{\rho}_\Lambda + 3H\delta\dot{\rho}_\Lambda - \frac{\nabla^2\delta\rho_\Lambda}{a^2} - 4\dot{\rho}_{\Lambda b}\dot{\Phi} = 0, \tag{20}$$

where we have set $F = 0$.

At early times, when the universe is matter dominated, we can neglect the last term in Eq. (20). Hence, working in Fourier space the evolution equation for Λ can be written as

$$\delta\ddot{\rho}_\Lambda + 3H\delta\dot{\rho}_\Lambda + \frac{k^2\delta\rho_\Lambda}{a^2} = 0. \tag{21}$$

For modes outside the horizon $k < aH$ the solution to this equation is $\delta\rho_\Lambda = \mathit{constant} + \mathit{decaying\ term}$. For modes inside the horizon $k > aH$ we obtain

$$\delta\rho_\Lambda = \frac{1}{a}\left(\alpha_1 \exp ik\eta + \alpha_2 \exp -ik\eta\right), \tag{22}$$

where the parameters α_i are independent of time and η is the conformal time $dt = a\,d\eta$. Although in a RW background the equation $\Box\Lambda = 0$ renders a cosmological term that monotonously decays in time, when one considers departures form homogeneity one finds that, for subhorizon modes, this covariant equation allows $\rho \leftrightarrow \rho_\Lambda$ oscillations with a decreasing amplitude.

To study the impact of these oscillations on the growth of matter perturbations, we follow the standard procedure of combining the continuity (18) and Euler (19) equations in a single second order differential equation [32]. Neglecting matter pressure gradients (i.e. for modes smaller than the Jeans mode $k < k_J$), and for times when $\rho_b \gg \rho_{\Lambda b}$ we find

$$\ddot{\delta} + 2H\dot{\delta} - \frac{\nabla^2\Phi}{a^2} = -\frac{2}{\rho_b}\left(\frac{\nabla^2\delta\rho_\Lambda}{a^2} + H\delta\dot{\rho}_\Lambda\right). \tag{23}$$

Once again this equation is the same as that obtained in the standard case but with a source term on the right-hand side which accounts for the inhomogeneities of the cosmological term. For superhorizon scales the source term is approximately zero and one recovers the standard case. For scales smaller than the horizon, using Eq. (22) and the Poisson equation $\nabla^2\Phi = 4\pi G a^2 \rho_b \delta$, we obtain

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\rho_b\delta = \frac{2k^2}{a^3\rho_b}\left(\alpha_1 \exp ik\eta + \alpha_2 \exp -ik\eta\right). \tag{24}$$

The solution to this nonhomogeneous linear differential equation is

$$\delta = \delta_{stand} + \delta_{new}, \tag{25}$$

where we have split the solution into two terms

$$\delta_{stand} = \beta_1 H + \beta_2 a, \tag{26}$$

$$\delta_{new} = a^2\left(\gamma_1 \exp ik\eta + \gamma_2 \exp -ik\eta\right), \tag{27}$$

with β_i and $\gamma_i \equiv -2\alpha_i/(a^3\rho_b)$ independent of time. We have employed $a^3\rho_b \simeq constant$ at early times and used $k > aH$ or equivalently $k\eta > 1$, for scales smaller than the horizon. In addition to the standard decaying term $\propto H$ and the growing term $\propto a$, there is a new fast oscillating term with an amplitude growing as a^2. This new term can be written as

$$\delta_{new} = -2\,\frac{\rho_{\Lambda b}}{\rho_b}\,\delta_\Lambda, \tag{28}$$

where we have defined $\delta_\Lambda \equiv \delta\rho_\Lambda/\rho_{\Lambda b}$. In a matter dominated universe and for scales below the horizon we find $\delta_\Lambda \propto a^{1/2}$. Leaving out the decaying part and keeping only the amplitudes in the oscillatory terms we obtain

$$\frac{\delta_{new}}{\delta_{stand}} \sim \frac{\delta_\Lambda(t_k)}{\delta_{stand}(t_k)}\left(\frac{a_k}{a_0}\right)^{3/2}\frac{a}{a_k}, \tag{29}$$

being t_k and a_k the time and expansion factor when the scale k crosses the horizon, respectively (we concentrate on scales that cross the horizon after radiation matter equality). The standard scenario of structure formation is not altered provided $\delta_{new} \ll \delta_{stand}$. Therefore, $\delta_\Lambda(t_k)$ cannot be much larger than $\delta_{stand}(t_k)$. However, before the mode k enters the horizon the evolution of matter perturbations in the Newtonian gauge is essentially given by $\delta_{stand} = constant$ [31], while perturbations in Λ grow as $\delta_\Lambda \propto a^{3/2}$ in a matter dominated universe and $\delta_\Lambda \propto a$ in the radiation dominated epoch. Therefore, one has to start with vanishing or extremely small Λ perturbations in the very early universe compared to the ordinary matter density perturbations in order to have a δ_Λ that is not much larger than δ_{stand} at horizon crossing. In principle, there is no reason to expect perturbations in an evolving Λ to vanish initially. To avoid the disturbing consequences of the right-hand side in Eq. (23) one should assume that Λ is really a nonevolving term with initial perturbations set to zero, i.e. a truly cosmological constant.

IV. CONCLUSIONS

A decaying cosmological term has been considered in several papers to explain the discrepancy between the large value for Λ expected from quantum field theory and the small value suggested by observations. So far, only decay laws in a RW have been studied, and departures from homogeneity have never been taken into account. We notice that, according to general relativity, a proper description of a decaying Λ has to be addressed using a covariant equation governing its dynamics. Any possible decay law for Λ in a RW universe should be derived from a covariant equation once one selects a coordinate frame in which the universe looks homogeneous and isotropic. We have proposed a class of possible covariant decay laws (1) and studied the simplest case $\Box\Lambda = 0$ in detail. We have shown that it makes physical sense in a RW universe. We have gone a step further compared to previous work and we have investigated the consequences of our evolving Λ in a universe that is slightly perturbed around the homogeneous background. We have found that the inhomogeneities in Λ act as a source of matter perturbations and, unless the initial Λ perturbations are set to extremely small values, the growth of matter perturbations is boosted and the standard scenario of structure formation is destroyed. Similarly, an increase in radiation perturbations would be present if we assumed a cosmological term that decays into radiation. In a different approach one could consider a first order differential equation for the evolution of Λ. However, one should then write down an equation independent and compatible with Eq. (3), which should probably involve a new four-vector different from the four-velocity of the cosmic fluid. In any case, in this paper we have argued that the usual procedure of studying a decaying cosmological term only in a RW universe is oversimplified and that proper account of covariance and of departures from homogeneity should be included in any model of a decaying Λ.

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REFERENCES

[1] S. Carroll, W.H. Press, and E.L. Turner, Annu. Rev. Astron. Astrophys. **30**, 499 (1992).

[2] L.M. Krauss and M.S. Turner, Gen. Rel. Grav. 27, 1137 (1995).

[3] M.S. Turner, G. Steigman, and L. Krauss, Phys. Rev. Lett. **52**, 2090 (1984).

[4] W.L. Freedman, astro-ph/9706072; B. Chaboyer, P. Demarque, P.J. Kernan, and L.M. Krauss, astro-ph/9706128; L.M. Krauss, astro-ph/9706227.

[5] S. Perlmutter *et al.*, Nature **391**, 51 (1998); A.G. Riess *et al.*, astro-ph/9805201, to appear in Astron. J.; B.P. Schmidt *et al.*, astro-ph/9805200, to appear in Astrophys. J.

[6] E.L. Turner, Astrophys. J. **365**, L43 (1990); C.S. Kochanek, Astrophys. J. **466** 638 (1996).

[7] S. Weinberg, Rev. Mod. Phys. **61**, 1 (1989).

[8] M. Özer and M.O. Taha, Phys. Lett. B **171**, 363 (1986); Nucl. Phys. B **287**, 776 (1987); Mod. Phys. Lett. A **13**, 571 (1998).

[9] K. Freese, F.C. Adams, J.A. Frieman, and E. Mottola, Nucl. Phys. B **287**, 797 (1987).

[10] M. Gasperini, Phys. Lett. B **194**, 347 (1987).

[11] M. Reuter and C. Wetterich, Phys. Lett. B **188**, 38 (1987).

[12] W. Chen and Y.S. Wu, Phys. Rev. D **41**, 695 (1990).

[13] M.S. Berman, Phys. Rev. D **43**, 1075 (1991); A. Beesham, Phys. Rev. D **48**, 3539 (1993).

[14] A.M. Abdel-Rahman, Phys. Rev. D **45**, 3497 (1992).

[15] M.O. Calvao, H.P. de Oliveira, D. Pavón, and J.M. Salim, Phys. Rev. D **45**, 3869 (1992); J.C. Carvalho, J.A.S. Lima, and I. Waga, Phys. Rev. D **46**, 2402 (1992); I. Waga, Astrophys. J. **414**, 436 (1993); V. Silveira and I. Waga, Phys. Rev. D **50**, 4890

(1994); *ibid.* **56**, 4625 (1997); J.A.S. Lima and J.M.F. Maia, Phys. Rev. D **49**, 5597 (1994); J.A.S. Lima and M. Trodden, Phys. Rev. D **53**, 4280 (1996).

[16] Ph. Spindel and R. Brout, Phys. Lett. B **320**, 241 (1994).

[17] J. Matyjasek, Phys. Rev. D **51**, 4154 (1995).

[18] J.L. Lopez and D.V. Nanopoulos, Mod. Phys. Lett. A **11**, 1 (1996).

[19] M. van Putten, gr-qc/9802004, introduces a covariant equation for Λ but its cosmological implications are not studied.

[20] L.P. Chimento and D. Pavón, Gen. Rel. Grav. **30**, 643 (1998).

[21] M. Birkel and S. Sarkar, Astropart. Phys. **6**, 197 (1997).

[22] J.M. Overduin, P.S. Wesson, and S. Bowyer, Astrophys. J. **404**, 1 (1993).

[23] P.J.E. Peebles and B. Ratra, Astrophys. J. **325**, L17 (1988); B. Ratra and P.J.E. Peebles, Phys. Rev. D **37**, 3406 (1988).

[24] J.A. Frieman, C.T. Hill, A. Stebbins, and I. Waga, Phys. Rev. Lett. **75**, 2077 (1995); K. Coble, S. Dodelson, and J.A. Frieman, Phys. Rev. D **55**, 1851 (1997); M.S. Turner and M. White, Phys. Rev. D **56**, 4439 (1997).

[25] P.T.P. Viana and A.R. Liddle, Phys. Rev. D **57**, 674 (1998).

[26] R.R. Caldwell, R. Dave, and P.J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998); G. Huey, L. Wang, R. Dave, R.R. Caldwell, and P.J. Steinhardt, astro-ph/9804285.

[27] P.M. Garnavich *et al.*, astro-ph/9806396, to appear in Astrophys. J.

[28] Let us consider the case $\Box\Lambda = 0$. This equation can be derived from the Lagrangian $\tilde{\mathcal{L}} = \frac{1}{2m^2} g^{\mu\nu}\partial_\mu\Lambda\partial_\nu\Lambda$, with m a mass scale. Using this $\tilde{\mathcal{L}}$ to define the energy-stress tensor we find $m^2\tilde{T}_{\mu\nu} = \partial_\mu\Lambda\partial_\nu\Lambda - \frac{1}{2}g_{\mu\nu}g^{\sigma\rho}\partial_\sigma\Lambda\partial_\rho\Lambda$, which renders, in a RW universe, the equation of state $\tilde{p} = +\tilde{\rho}$. Our true energy-stress tensor is but $T_{\mu\nu} = \Lambda g_{\mu\nu}$, which gives

$p = -\rho$.

[29] L.S. Brown, *Quantum Field Theory* (Cambridge University Press, 1992); J.J. Sakurai, *Modern Quantum Mechanics* (Addison-Wesley, Reading MA, 1994).

[30] See J.M. Overduin and F.I. Cooperstock, astro-ph/9805260, to appear in Phys. Rev. D, for an extensive table of suggested decay laws for Λ in a RW universe.

[31] V.F. Mukhanov, H.A. Feldman, and R.H. Brandenberger, Phys. Rep. **215**, 203 (1992).

[32] P.J.E. Peebles, *Principles of Physical Cosmology* (Princeton University Press, 1993).