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# The $0^{++}$ and $0^{-+}$ mass of light-quark hybrid in QCD sum rules

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**Abstract**

We calculate the masses of the light-quark hybrid mesons with the quantum number $0^{++}$ and $0^{-+}$ by using QCD sum rules. Two kinds of the interpolated currents with the same quantum number are employed. We find that the approximately equal mass is predicted for $0^{-+}$ hybrid state from the different current and the different mass value is obtained for the $0^{++}$ hybrid state from the different current. The prediction depends on the interaction between the gluon and quarks in the low-lying hybrid mesons. The mixing effect on the mass value of the light-quark hybrid meson through Low-energy theorem has been examined too, it is found that this mixing shifts the mass of hybrid meson and glueball a little.

## 1 Introduction

One believes that the gluon degrees of freedom play an important role in hadrons. QCD theory predicts the existence of glueball and hybrid states. Searching for glueballs and hybrid mesons on experiments has been carried for a long time since 1980s, so far there is no conclusive evidence on them. Glueballs and hybrids are particularly difficult to identify on experiments since their mass spectroscopy overlaps with the ordinary $\bar{q}q$ meson spectroscopy and they can mix with each other. From theoretical point of view, we have no effective non-perturbative theory in QCD to predict their mass precisely.

The QCD sum rule[1] calculation of hybrid mesons was given by J. Govaerts *et. al*[2]. They presented the masses of hybrid mesons with various $J^{pc}$. They analyzed sets of coupled sum rules for heavy-quark hybrids by using the different interpolated currents and have found that the mass predictions for the same $J^{pc}$ from totally different sum rules essentially agree within the errors of their procedure. These states with the same $J^{pc}$ were considered as the same state.

In this paper, we extend their approach to the light-quark case for $0^{++}$ and $0^{-+}$ hybrids by using two kinds of the interpolated currents: $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^aq(x)$ and $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^aq(x)$. Although Govaerts *et. al* also calculated the light-quark hybrid mesons for the $0^{++}$, $1^{-+}$, $0^{--}$ and $1^{+-}$ states by using the vector and axial-vector current, respectively, they didn't do the corresponding calculation from the current $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$. The $\bar{q}q$ combination in the current $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$ has the quantum number $J^{pc} = 1^{+-}$. Obviously, the $\bar{q}q$ combination in the current $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x)$ has $J^{pc} = 1^{--}$. The interaction between quarks and gluon in these two different currents is different. Thus one can't expect the same prediction from these two different currents in the light-quark hybrid mesons. It is similar to the situation of hybrid mesons in the MIT bag model[3]. For instance, the $\bar{q}q$ combination of $0^{++}$ hybrid meson $\bar{q}qg$ may have $J^{pc} = 1^{--}$ with the gluon in TE($1^{--}$) mode[4] or $J^{pc} = 1^{+-}$ with the gluon in TM($1^{+-}$) mode. These two $0^{++}$ states have different intrinsic structure and energy. Therefore the hybrid mesons with the same $J^{pc}$ can be obtained from totally different sum rules by using different interpolated currents. We calculate the masses of the light-quark hybrid mesons, $0^{++}$ and $0^{-+}$ states, by using two different currents. Our result shows that the prediction depends on the interaction between the quarks and gluon in the low-lying hybrid meson. The approximately equal mass is predicted for the $0^{-+}$ hybrid mesons from the different currents and the different mass value is obtained for the $0^{++}$ hybrid mesons from two different currents.

We also consider the mixing effect on the mass determination of hybrid meson between the low-lying $0^{++}$ glueball and hybrid meson $\bar{q}qg$ . By using the low-energy theorem[5], we can construct a sum rule for the mixing correlation function(one gluonic current and one hybrid current). Through these relationship and based on the assumption of two states (lowest-lying states of glueball and hybrid meson $\bar{q}qg$) dominance, we find the mass for $0^{++}$ glueball is around: 1.8 GeV, which is a little higher than the pure resonance prediction and the mass for the $0^{++}$ $\bar{q}qg$ hybrid meson is around: 2.6 GeV, which is a little higher than the pure resonance prediction too.

The paper is organized as follows. The analytic formalism of QCD sum rules for hybrid is given In Sec. 2. In Sec. 3 we give the numerical results for the mass of $0^{++}$ and $0^{-+}$ light-quark hybrid mesons and discuss them with those in the bag model with the same $J^{pc}$. The mixing effect of the glueball with the hybrid meson state is studied in Sec. 4. The summary is given in the last section .

# 2 QCD sum rules for light-quark hybrid mesons

To construct the sum rules for light-quark hybrid mesons $\bar{q}qg$ , we use the composite operators with the same quantum numbers as these states to build the correlation functions. In order to obtain the $0^{++}$ hybrid meson sum rules, we define two different currents

$$j(x) = g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \tag{1}$$
$$j_\mu(x) = g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x), \tag{2}$$

where $q(x)$ and $G^a_{\alpha\mu}(x)$ are the light-quark field and gluon field strength tensor, respectively. $T^a$ are the color matrices.

To get the OPE, we expand the correlation function of $j(x)$ in the background field gauge[6] only in the leading order, which includes the perturbative part(a), the two-quark condensate(b), the two-gluon condensate(c) and the four-quark condensate(d). The result can be obtained from Feynman diagrams in Figs. (1a-1d)

$$\begin{aligned} \Pi(q^2) &= i\int e^{iqx}\langle 0|T\{j(x),j(0)\}|0\rangle dx \qquad (3)\\ &= -A(q^2)^3 ln(-q^2/\Lambda^2) - Bq^2 ln(-q^2/\Lambda^2) - C ln(-q^2/\Lambda^2) - D\frac{1}{q^2} + const \end{aligned}$$

where

$$A = \frac{\alpha_s}{24\pi^3} \quad , \quad B = \frac{4\alpha_s}{\pi}\langle m\bar{q}q\rangle,$$
$$C = -\frac{m^2}{\pi}\langle\alpha_s G^2\rangle \quad , \quad D = \frac{8\pi\alpha_s}{3}\langle m\bar{q}q\rangle^2.$$

when the u and d quarks are taken to be massless, the coefficient C vanishes.

In order to relate the calculation on QCD with the hadron physics, the standard dispersion relation is used

$$\Pi(q^2) = \frac{1}{\pi}\int\frac{Im\Pi(s)}{s-q^2}ds. \qquad (4)$$

We saturate $Im\Pi(s)$(so-called spectral density) by one narrow resonance and a continuum in form of a $\theta$-function. Thus the $Im\Pi(s)$ is given by

$$Im\Pi(s) = \pi g_R^2(m_R^2)^4\delta(s-m_R^2) + \pi(As^3 + Bs + C)\theta(s-s_0). \qquad (5)$$

where $g_R$ is the coupling of the current to the hybrid meson state and $m_R$ refers to mass of the hybrid meson.

In practice, it is more convenient to define the moments $R_k$[8] to proceed the sum rule calculation instead, which is expressed by

$$\begin{aligned} R_k(\tau,s_0) &= \frac{1}{\tau}\hat{L}[(q^2)^k\{\Pi(Q^2)-\Pi(0)\}] - \frac{1}{\pi}\int_{s_0}^{+\infty} s^k e^{-s\tau} Im\Pi^{\{pert\}}(s)ds \qquad (6)\\ &= \frac{1}{\pi}\int_0^{s_0} s^k e^{-s\tau} Im\Pi(s)ds, \end{aligned}$$

where $\hat{L}$ is the Borel transformation and $\tau$ is the Borel transformation parameter, $s_0$ is the starting point of the continuum threshold.

Substituting Eq. (3) into Eq. (6), the $R_0(\tau,s_0)$ behaves as

$$R_0(\tau,s_0) = \frac{1}{\tau^4}\{6A[1-\rho_3(s_0\tau)] + B\tau^2[1-\rho_1(s_0\tau)] + C\tau^3[1-\rho_0(s_0\tau)] + D\tau^4\}, \qquad (7)$$

where

$$\rho_k(x) = e^{-x}\sum_{j=0}^{k}\frac{x^j}{j!}. \qquad (8)$$

and higher moments $R_k$ can be related to the $R_0$

$$R_k(\tau,s_0) = (-\frac{\partial}{\partial\tau})^k R_0(\tau,s_0). \qquad (9)$$

Similar to Eq. (3), we can calculate the correlator $\Pi_{\mu\nu}(q^2)$ from the current $j_\mu(x)$

$$\begin{aligned}\Pi_{\mu\nu}(q^2) &= i\int e^{iqx}\langle 0|T\{j_\mu(x), j_\nu(0)\}|0\rangle dx \\ &= (\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu})\Pi_v(q^2) + \frac{q_\mu q_\nu}{q^2}\Pi_s(q^2)\end{aligned} \tag{10}$$

and

$$\Pi_s(q^2) = -A'(q^2)^3 ln(-q^2/\Lambda^2) - B'q^2 ln(-q^2/\Lambda^2) - C' ln(-q^2/\Lambda^2) - D'\frac{1}{q^2} + const \tag{11}$$

where

$$\begin{aligned} A' &= \frac{\alpha_s}{480\pi^3} \quad , \quad B' = -(\frac{\alpha_s}{3\pi}\langle m\bar{q}q\rangle + \frac{\langle\alpha_s G^2\rangle}{24\pi}), \\ C' &= -\frac{m^2}{8\pi}\langle\alpha_s G^2\rangle \quad , \quad D' = -\frac{2\pi\alpha_s}{3}\langle m\bar{q}q\rangle^2. \end{aligned}$$

the coefficient of the two-quark condensate in $B'$ is a little different with reference[2]. The $\Pi_v(q^2)$ is the same as this reference[2].

Replacing $G^a_{\alpha\mu}(x)$ in the Eq. (1) and Eq. (2) by

$$\tilde{G}^a_{\alpha\mu}(x) = \frac{1}{2}\epsilon_{\alpha\mu\rho\sigma}G^a_{\rho\sigma}(x), \tag{12}$$

we can get the sum rules for the resonance states with opposite parity $(0^{-+})$, the results of the correlation functions and moments are almost the same as before except that the sign of the gluon condensate is changed.

# 3 Numerical results and $J^{pc}$ analysis

From Eq. (6), the mass of the hybrid meson is given by (with $k \geq 1$)

$$m_R^2 = \frac{R_{k+1}}{R_k}, \tag{13}$$

the moments $\frac{R_1}{R_0}$ and $\frac{R_2}{R_1}$ are both suitable for the mass determination according to the ordinary QCD sum rules criteria. They are employed in the following calculation.

To get the numerical results, the parameters are chosen as

$$\begin{aligned} \Lambda = 0.2GeV \quad &, \quad m_s = 0.15GeV, \\ \langle\bar{q}q\rangle = -(0.25GeV)^3 \quad &, \quad \langle m\bar{q}q\rangle = -(0.1GeV)^4, \\ \langle m\bar{s}s\rangle = -0.15 * 0.8 * 0.25^3 GeV^4 \quad &, \quad \frac{\langle\alpha_s G^2\rangle}{\pi} = 0.33^4 GeV^4, \\ \alpha_s(\tau) = -\frac{4\pi}{9ln(\tau\Lambda^2)} \quad &. \end{aligned}$$

where q refers to u or d quark field.

Corresponding to the current $j(x)$ in Eq. (1), the mass of $0^{++}$ $\bar{s}sg$ hybrid meson determined from $\frac{R_1}{R_0}$ is shown as Fig. 2, it reads 2.35 GeV. If we use $\frac{R_2}{R_1}$, the result is almost the same $\sim$ 2.30 GeV. When the quark mass vanishes, which corresponds to $q = u, d$, the results change a little (see Fig. 3).

Corresponding to the current $j_\mu(x)$ in Eq. (2), $\frac{R_1}{R_0}$ gives the mass of $0^{++}$ $\bar{s}sg$ hybrid meson around 3.4 GeV (Fig. 4), the higher moment shifts the mass a little lower. When the quark mass goes to zero, the result is shown as Fig. 5.

There is no platform in the $0^{-+}$ hybrid meson case, we deal with it as reference[2], and the masses of the $0^{-+}$ hybrid mesons corresponding to currents $j(x)$ and $j_\mu(x)$ have an approximately equal value: 2.3 GeV. They are shown in Fig. 6 and Fig. 7.

All these results are obtained at suitable $s_0$, which account for the ordinary QCD sum rules criteria for threshold choosing. The results change slightly with the $s_0$.

It is apparent that the mass of the light-quark hybrid meson depends on what interpolated current we choose: the mass value of the $0^{++}$ hybrid from current $j_\mu(x)$ is about 1.0 GeV higher than that from current $j(x)$ while the mass value of the $0^{-+}$ hybrid from the two different currents is approximately the same.

As we know, the hybrid meson is a three body system and the valence quark, anti-quark and gluon may have different internal $J^{pc}$ combination. $J^{pc}$ of the combination $\bar{q}q$ in the current $j(x)$ and $j_\mu(x)$ are $1^{+-}$ and $1^{--}$ respectively. Only the state with the same overall and 'local' quantum number can dominate the corresponding correlation function, where we refer the quantum number of intrinsic $\bar{q}q$ combination or gluon, such as $J^{pc}$ of them, to 'local' quantum number. Therefore the correlation function which consists of the current $j(x)$ is dominated by the $0^{++}$ state with the gluon in TE($1^{+-}$) mode and the correlation function which consists of the current $j_\mu(x)$ is dominated by the $0^{++}$ state with the gluon in TM($1^{--}$) mode. The predicted $0^{++}$ mass from the different current may be different since the interaction between quarks and gluon is different. In fact, the interaction between quarks and gluon in the current $j(x)$ is in magnetic form, while the interaction in the current $j_\mu(x)$ is in electric form.

From the identity for free light quark

$$2m\bar{q}\gamma_\mu q = (k' + k)_\mu \bar{q}q + \bar{q}i\sigma_{\mu\nu}(k - k')_\nu q, \tag{14}$$

we can expect $\bar{q}\sigma_{\mu\nu}(k - k')_\nu q \sim m\bar{q}\gamma_\mu q$. So when $m$ goes to zero, the interaction energy between quarks and gluon in $j(x)$ is much smaller than the one in $j_\mu(x)$. It also can be seen from the contribution of the two-gluon condensate to the correlation function ( Eq. (3) and Eq. (11)). This kind of interaction makes a large contribution to the energy of the hybrid states and the $0^{++}$ hybrid mass with magnetic interaction has a lower mass than that with electric interaction. However, in the heavy hybrid system, we have $m^2 \sim Q^2$. This is the reason that the author in [2] got the almost same masses from two different sum rules for the heavy hybrid mesons.

It is helpful to note that the $J^{pc}$ of the $\bar{q}q$ combination in the bag model has the same structure as that in the current $j_\mu(x)$ and $j(x)$, thus we can compare our picture with that in the bag model. In the bag model, the energy of the hybrid meson consist of the volume energy, the zero-point energy, the mode energy and the $O(\alpha_s)$ quantum corrections. The valence quarks and gluon in the $\bar{q}qg$ hybrid mesons may have different excited mode and each mode has different energy. Besides, the $O(\alpha_s)$ quantum corrections are spin dependent.

so they have different energy corresponding to different internal $J^{pc}$ combination of the $\bar{q}q$ with the gluon. The quarks and gluon in $0^{++}$ hybrid mesons may be in $s_{\frac{1}{2}}s_{\frac{1}{2}}TM$ mode with internal $J^{pc}$: $1^{--}\otimes 1^{--}$ or in $s_{\frac{1}{2}}p_{\frac{1}{2}}TE$ mode with internal $J^{pc}$: $1^{+-}\otimes 1^{+-}$, so the same overall $J^{pc}$ states may have different energy.

# 4 Mixing of the $0^{++}$ hybrid meson with the glueball

In this section, we discuss the mixing effect[7] on the mass of the $0^{++}$ hybrid meson. Since the mass of the $0^{++}$ hybrid(3.4 GeV) from current $j_\mu$ is much larger than the pure $0^{++}$ glueball(1.7 GeV) in sum rules, so we do not discuss this situation. We consider only the mixing between the $0^{++}$ hybrid(2.3 GeV) from current $j(x)$ and the $0^{++}$ glueball. We choose the scalar gluonic current

$$j_1(x) = \alpha_s G^a_{\mu\nu} G^a_{\mu\nu}(x), \tag{15}$$

for the $0^{++}$ glueball and the current

$$j_2(x) = g\bar{q}\sigma_{\mu\alpha} G^a_{\alpha\mu} T^a q(x), \tag{16}$$

for the $0^{++}$ hybrid meson.

The correlation function of the current in Eq. (15) was given in [8]

$$\begin{aligned}\Pi_1(q^2) &= a_0(Q^2)^2\ln(Q^2/\nu^2) + b_0\langle\alpha_s G^2\rangle \\ &+ c_0\frac{\langle gG^3\rangle}{Q^2} + d_0\frac{\langle\alpha_s^2 G^4\rangle}{(Q^2)^2}\end{aligned} \tag{17}$$

where $Q^2 = -q^2 > 0$, and

$$\begin{aligned}a_0 &= -2(\tfrac{\alpha_s}{\pi})^2(1+\tfrac{51}{4}\tfrac{\alpha_s}{\pi}),\\ c_0 &= 8\alpha_s^2,\\ b_0 &= 4\alpha_s(1+\tfrac{49}{12}\tfrac{\alpha_s}{\pi}),\\ d_0 &= 8\pi\alpha_s,\\ \langle\alpha_s G^2\rangle &= \langle\alpha_s G^a_{\mu\nu}G^a_{\mu\nu}\rangle,\\ \langle gG^3\rangle &= \langle g f_{abc} G^a_{\mu\nu} G^b_{\nu\rho} G^c_{\rho\mu}\rangle,\\ \langle\alpha_s^2 G^4\rangle &= 14\langle(\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\rho\nu})^2\rangle - \langle(\alpha_s f_{abc} G^a_{\mu\rho} G^b_{\lambda\nu})^2\rangle\end{aligned}$$

From (6),(17) and (3) ,we have the following expressions:

$$\begin{aligned}R_0(\tau,s_0) &= -\tfrac{2a_0}{\tau^3}\lfloor 1-\rho_2(s_0\tau)\rfloor + c_0\langle gG^3\rangle + d_0\langle\alpha_s^2 G^4\rangle\tau,\\ R_1(\tau,s_0) &= -\tfrac{6a_0}{\tau^4}\lfloor 1-\rho_3(s_0\tau)\rfloor - d_0\langle\alpha_s^2 G^4\rangle,\\ R_2(\tau,s_0) &= -\tfrac{24a_0}{\tau^5}\lfloor 1-\rho_4(s_0\tau)\rfloor,\\ R_0'(\tau,s_0) &= \tfrac{1}{\tau^4}\{6A[1-\rho_3(s_0\tau)] + B\tau^2[1-\rho_1(s_0\tau)] + C\tau^3[1-\rho_0(s_0\tau)] + D\tau^4\},\\ R_1'(\tau,s_0) &= \tfrac{1}{\tau^5}\{24A[1-\rho_4(s_0\tau)] + 2B\tau^2[1-\rho_2(s_0\tau)] + C\tau^3[1-\rho_1(s_0\tau)]\},\end{aligned} \tag{18}$$

where $R_k$ and $R_k'$ in (18) are the moments corresponding to current $j_1(x)$ and $j_2(x)$ respectively.

By using the Low-energy theorem [5], we can construct another correlator with one $j_1(x)$ and one $j_2(x)$

$$\lim_{q\to 0} i\int dxe^{iqx}\langle 0|T[g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \alpha_s G^2(0)]|0\rangle = \frac{40\pi}{9}\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle. \quad (19)$$

for the light quark, $\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle$ can be expressed in terms of $\langle 0|\bar{q}q|0\rangle$ as[9]

$$\langle 0|g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q|0\rangle = -m_0^2\langle 0|\bar{q}q|0\rangle \quad (20)$$

where $m_0^2 \approx 0.8$ GeV$^2$.

In order to factorize the spectral density, we define the couplings of the currents to the physical states in the following way

$$\begin{aligned}\langle 0|j_1|H\rangle &= f_{12}m_2 \quad , \quad \langle 0|j_1|G\rangle = f_{11}m_1, \\ \langle 0|j_2|H\rangle &= f_{22}m_2 \quad , \quad \langle 0|j_2|G\rangle = f_{21}m_1,\end{aligned} \quad (21)$$

where $m_1$ and $m_2$ refer to the glueball(including few part of quark component) mass and the $\bar{q}qg$ hybrid meson(including few part of pure gluon component) mass. $|H\rangle$ and $|G\rangle$ refer to the $\bar{q}qg$ hybrid meson state and the glueball state respectively. After choosing the two resonances plus continuum state approximation, the spectral density of the currents of $j_1(x)$ and $j_2(x)$ read, respectively

$$ImΠ_1(s) = m_2^2 f_{12}^2\delta(s-m_2^2) + m_1^2 f_{11}^2\delta(s-m_1^2) + \frac{2}{\pi}s^2\alpha_s^2\theta(s-s_0), \quad (22)$$

$$ImΠ_2(s) = m_2^2 f_{22}^2\delta(s-m_2^2) + m_1^2 f_{21}^2\delta(s-m_1^2) + \pi(As^3 + Bs + C)\theta(s-s_0). \quad (23)$$

Then it is straightforward to get the moments

$$R_0 = \frac{1}{\pi}\{m_2^2 e^{-m_2^2\tau} f_{12}^2 + m_1^2 e^{-m_1^2\tau} f_{11}^2\}, \quad (24)$$

$$R_1 = \frac{1}{\pi}\{m_2^4 e^{-m_2^2\tau} f_{12}^2 + m_1^4 e^{-m_1^2\tau} f_{11}^2\}, \quad (25)$$

$$R_2 = \frac{1}{\pi}\{m_2^6 e^{-m_2^2\tau} f_{12}^2 + m_1^6 e^{-m_1^2\tau} f_{11}^2\}, \quad (26)$$

$$R_0' = \frac{1}{\pi}\{m_2^2 e^{-m_2^2\tau} f_{22}^2 + m_1^2 e^{-m_1^2\tau} f_{21}^2\}, \quad (27)$$

$$R_1' = \frac{1}{\pi}\{m_2^4 e^{-m_2^2\tau} f_{22}^2 + m_1^4 e^{-m_1^2\tau} f_{21}^2\}. \quad (28)$$

In the meantime, assuming the states $|G\rangle$ and $|H\rangle$ saturate the l.h.s of Eq. (19), one can obtain

$$\lim_{q\to 0} i\int dxe^{iqx}\langle 0|T[g\bar{q}\sigma_{\mu\alpha}G^a_{\alpha\mu}T^a q(x), \alpha_s G^2(0)]|0\rangle = f_{22}f_{12} + f_{21}f_{11}. \quad (29)$$

To get the numerical result, the following additional parameters are chosen

$$\begin{aligned}\langle gG^3\rangle &= (0.27GeV^2)\langle\alpha_s G^2\rangle, \\ \langle\alpha_s^2 G^4\rangle &= \frac{9}{16}\langle\alpha_s G^2\rangle^2.\end{aligned}$$

The next step is to equate the QCD side with the hadron side one by one, and we get a set of equations. After giving various of reasonable parameters $s_0$ and $\tau$, we can get a series of mass value of the two states through solving this set of equations. Our result is illustrated in Fig. 8. In this figure, the doted line corresponds to the hybrid meson and the solid line corresponds to the glueball. It is found that $s_0 = 8.0$ GeV$^2$ is the best favorable value, then from the figure follows the masses prediction: hybrid meson with mass around 2.6 GeV and glueball with mass around 1.8 GeV. It concludes that the masses of the glueball and the hybrid meson are both a little higher than their pure states.

# 5 Summary

In this paper, we calculate the $0^{++}$ and $0^{-+}$ masses of the light-quark hybrid meson by using QCD sum rules with two different kinds of interpolated currents: $g\bar{q}\sigma_{\mu\nu}G^a_{\nu\mu}T^a q(x)$ and $g\bar{q}\gamma_\alpha G^a_{\alpha\mu}T^a q(x)$. Numerical result is found: the $0^{++}$ hybrid meson mass from current $j_\mu(x)$ is around 3.4 GeV, which is 1.0 GeV higher than that(which is around 2.35 GeV) from current $j(x)$; the masses of the $0^{-+}$ hybrid meson from these two current are approximatively equal: 2.3 GeV.

The $J^{pc}$ of the $\bar{q}q$ combination in these two current are $1^{+-}$ and $1^{--}$ respectively, so the interaction between the quarks and gluon is different and the two different kind of $0^{++}$ or $0^{-+}$ hybrid meson dominating the spectral density of these two different current are different states correspondingly. For the light-quark hybrids, the interaction between the quarks and gluon makes a large contribution to the energy of the states, their mass thus may be different, while for heavy-quark hybrid, $m^2 \sim Q^2$, so different currents result in the approximately equal mass prediction. Our picture can be compared to MIT bag model and confirm the reasonness of the mode analysis in the bag model.

For the $0^{++}$ hybrid, the contribution of the two-gluon condensate to the correlation function( Eq. (11)) from the current $j_\mu(x)$ is large, while the contribution of the two-gluon condensate to the correlation function( Eq. (3)) from the current $j(x)$ is small because of the factor $m^2$ in the coefficient C, these two $0^{++}$ states have different mass value. When the sign of two-gluon condensate terms changes which corresponds to the $0^{-+}$ hybrid case, it results in a large mass difference compared to the $0^{++}$ case for current $j_\mu(x)$ but makes a small mass difference compared to the $0^{++}$ situation for the current $j(x)$, so these two $0^{-+}$ light hybrid from the two different currents have approximately equal mass.

The mixing effect on the mass determination of $0^{++}$ hybrid meson is considered too. We find that the mixing of the $0^{++}$ hybrid meson with the glueball shifts the masses of both the hybrid and the glueball a little higher compared to their pure states.

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[9] L. J. Reinders, H. Rubinstein and Yazaki, Phys. Rep. **127**, 1 (1985).

# Figure caption

Figure 1: Feynman diagrams of the leading order contributing to the correlation function.

Figure 2: $0^{++}$ $\bar{s}sg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 8.0$ GeV$^2$ corresponding to current $j(x)$.

Figure 3: $0^{++}$ $\bar{q}qg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 8.0$ GeV$^2$ corresponding to current $j(x)$.

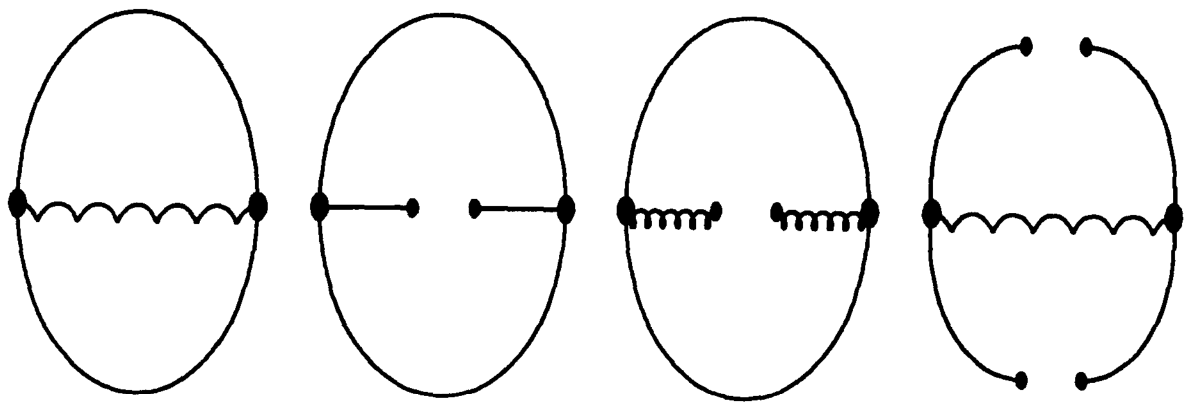
Figure 4: $0^{++}$ $\bar{s}sg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 13.0$ GeV$^2$ corresponding to current $j_\mu(x)$.

Figure 5: $0^{++}$ $\bar{q}qg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 13.0$ GeV$^2$ corresponding to current $j_\mu(x)$.

Figure 6: $0^{-+}$ $\bar{s}sg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 8.0$ GeV$^2$ corresponding to current $j(x)$.

Figure 7: $0^{-+}$ $\bar{s}sg$ mass from $\frac{R_1}{R_0}$ versus $\tau$ at $s_0 = 8.0$ GeV$^2$ corresponding to current $j_\mu(x)$.

Figure 8: $0^{++}$ $\bar{s}sg$ mass versus $\tau$ at $s_0 = 8.0$ GeV$^2$ corresponding to the mixing figure.



(a) (b) (c) (d)

*Fig.1*

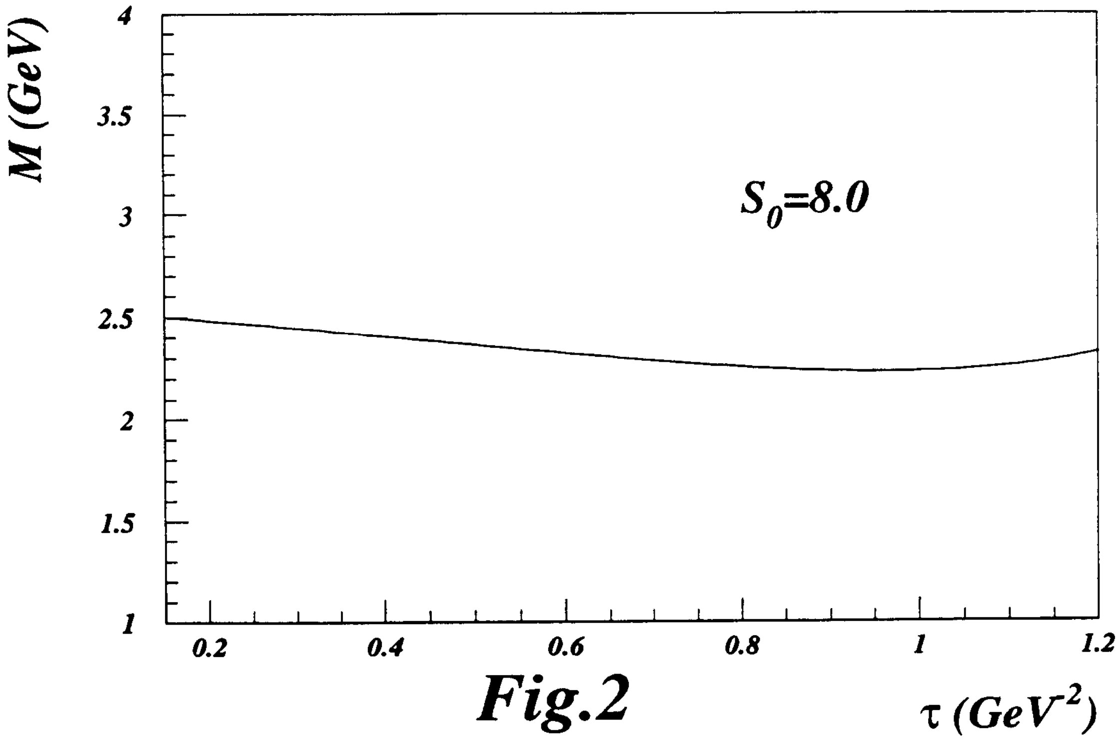



**Fig.2**

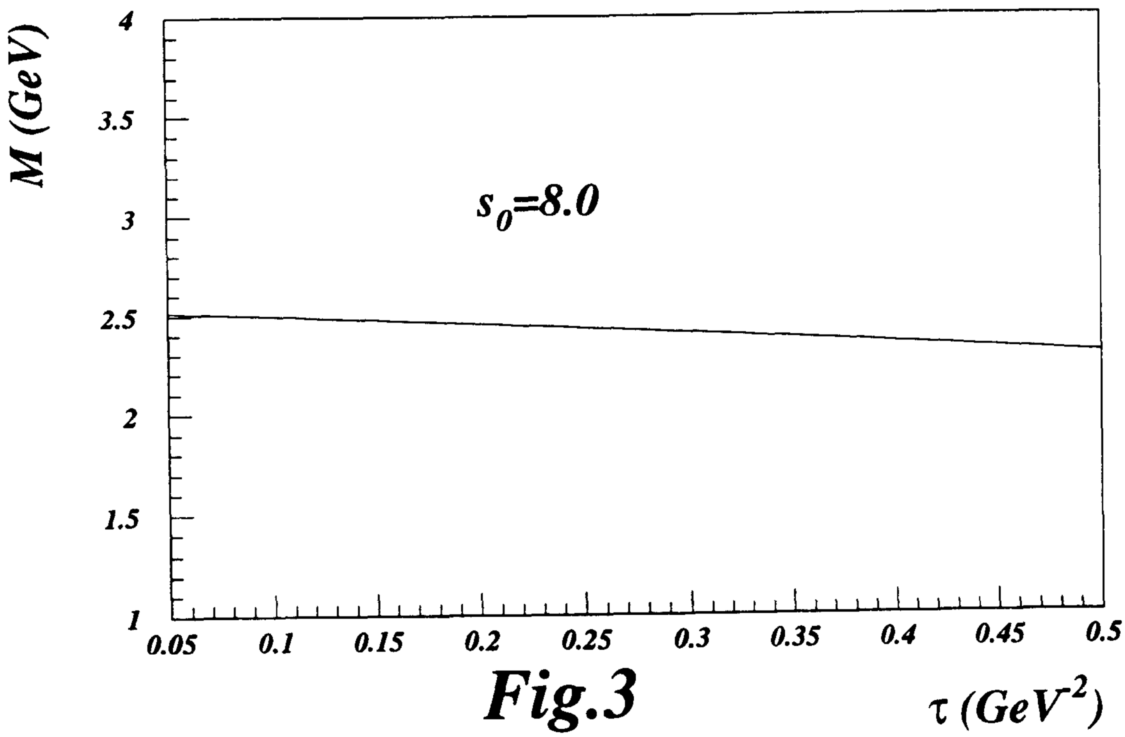



**Fig.3**

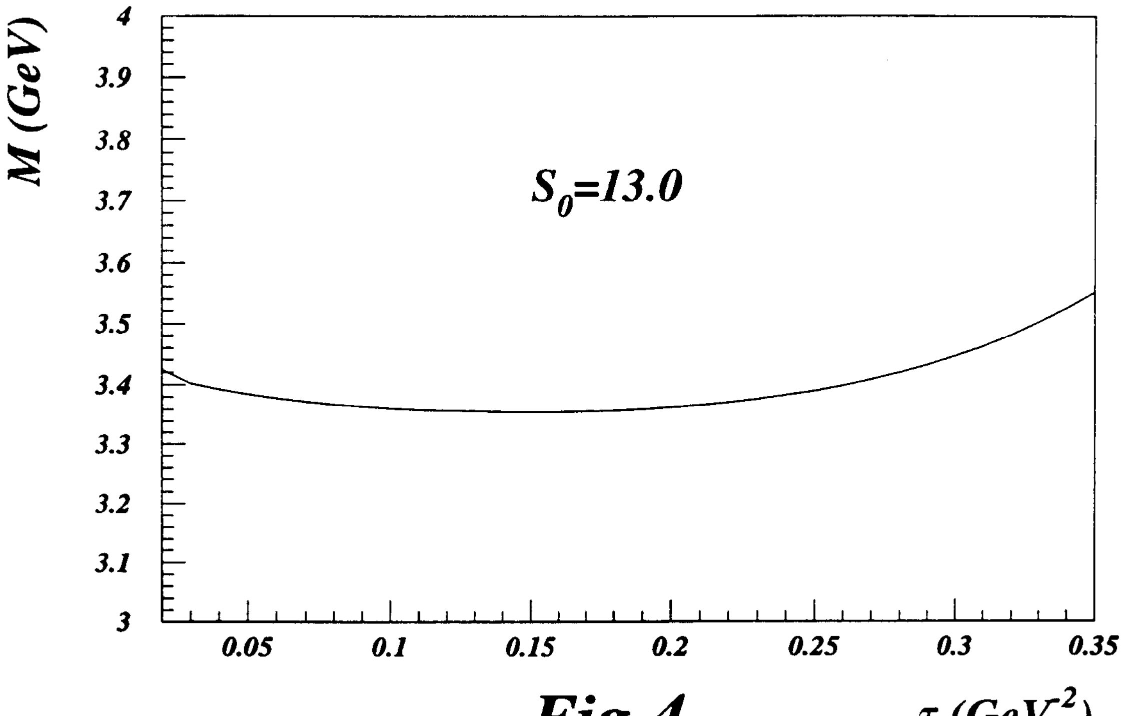
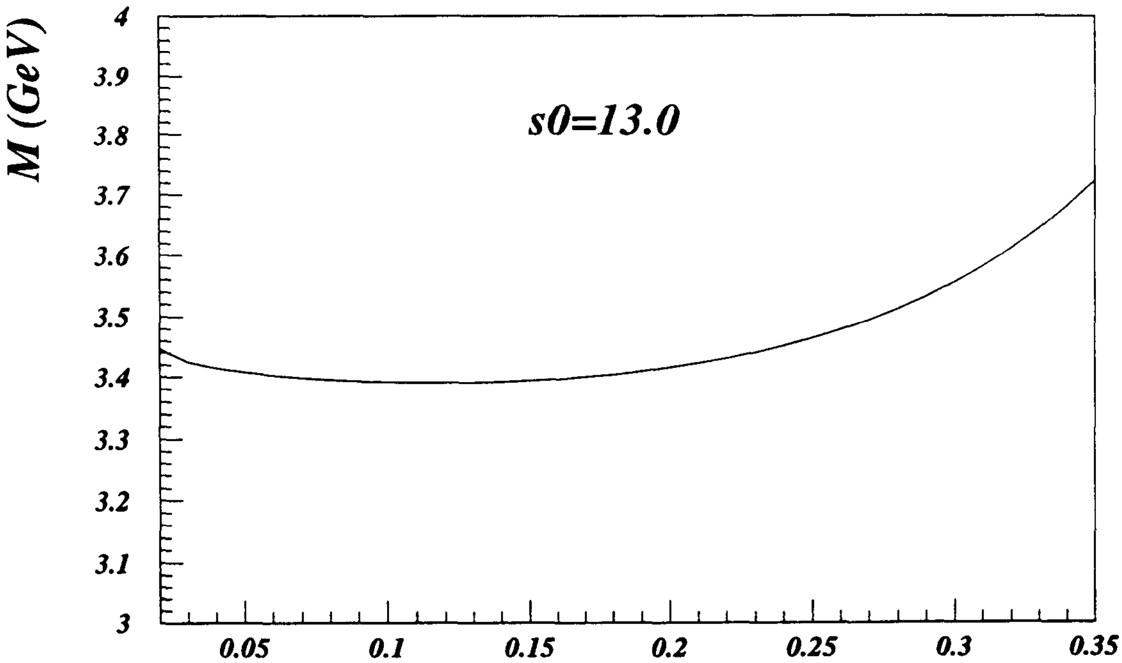
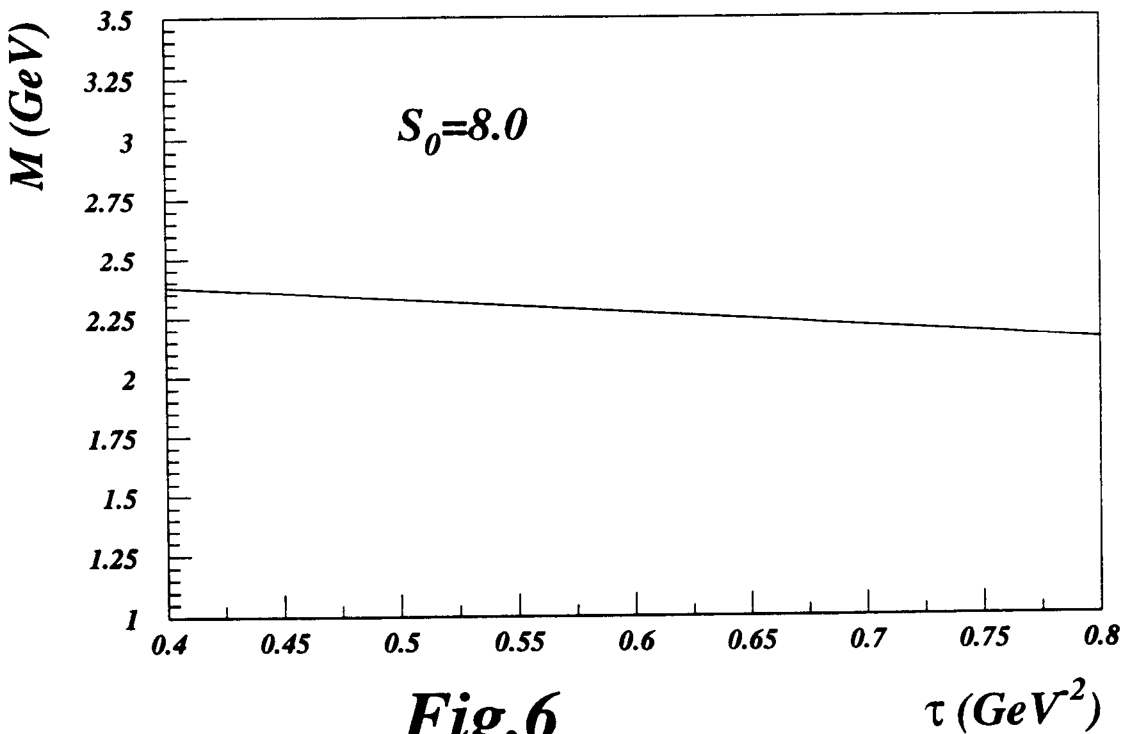

M (GeV)
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3
$S_0$=13.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
τ ($GeV^{-2}$)

**Fig.4**



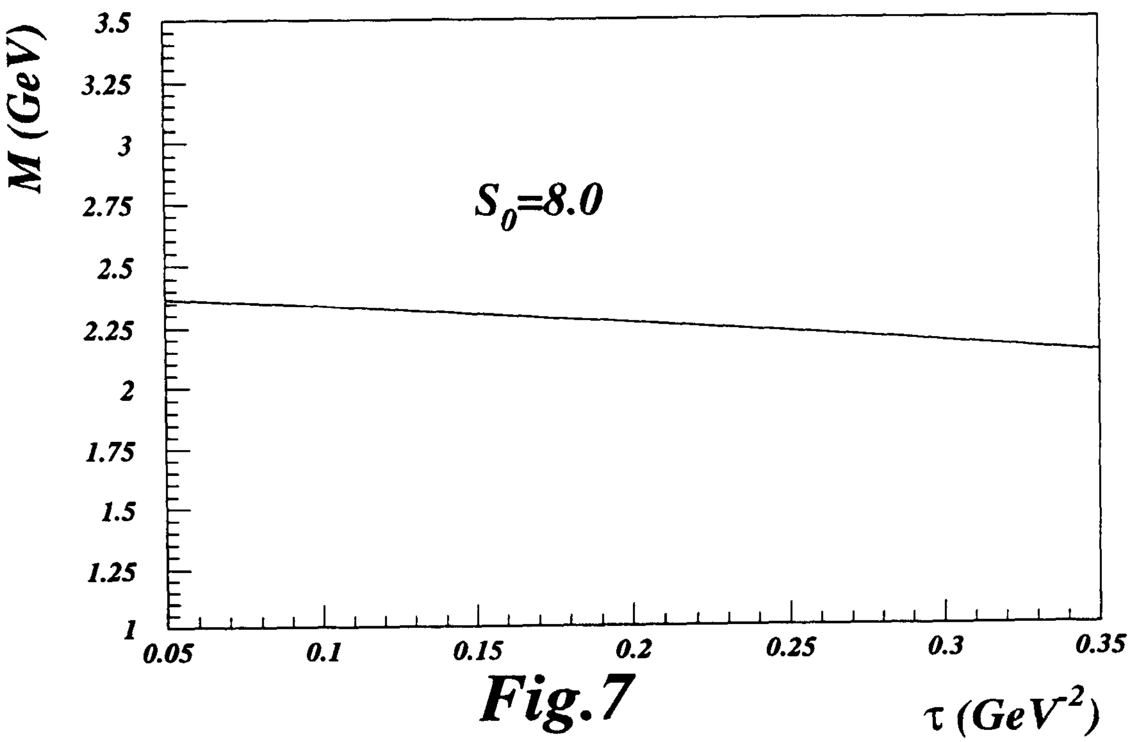
M (GeV)
4
3.9
3.8
3.7
3.6
3.5
3.4
3.3
3.2
3.1
3
s0=13.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
τ (GeV^-2)


*Fig.5*



M (GeV)
3.5
3.25
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
$S_0$=8.0
0.4
0.45
0.5
0.55
0.6
0.65
0.7
0.75
0.8
τ ($GeV^{-2}$)


**Fig.6**



M (GeV)
3.5
3.25
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
$S_0$=8.0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
τ ($GeV^{-2}$)


**Fig.7**

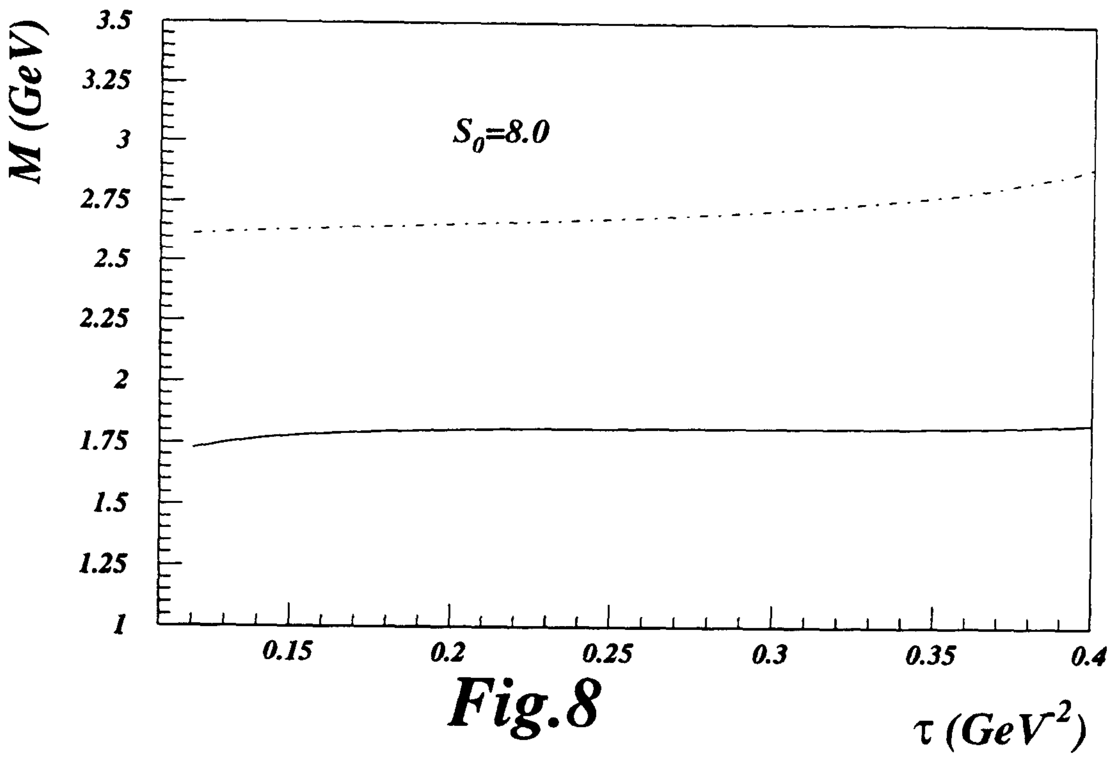

M (GeV)
3.5
3.25
3
2.75
2.5
2.25
2
1.75
1.5
1.25
1
$S_0$=8.0
0.15
0.2
0.25
0.3
0.35
0.4
τ (GeV$^2$)

Fig.8