



$\Lambda - \pi$ Phase Shifts in Chiral Perturbation Theory

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Abstract

We calculate the S and P wave phase shifts in $\Lambda - \pi$ scattering at the Ξ mass using the full relativistic $SU(3)_L \times SU(3)_R$ chiral perturbation theory. We get small phase shifts similar to previous calculations using $SU(2)_L \times SU(2)_R$ chiral perturbation theory in the heavy baryon limit. We also consider possible off-shell effects in the coupling of the Rarita-Schwinger particle $\Sigma^*(1385)$. Using $SU(3)$ we estimate the off-shell coupling of the Σ^* to $\Lambda\pi$ from the off-shell coupling of the Δ to $N\pi$ which is obtained from a fit to the pion-nucleon data. We find that the contributions from the off-shell coupling can be of the same size as the other terms in the $\Lambda\pi$ scattering amplitude.

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1 Introduction

The amplitudes for non-leptonic decays of hyperons are modulated by the final state strong scattering[1]. These final state phase shifts are necessary in calculating the various CP violating asymmetries in hyperon decays [2]. Some of the asymmetries depend on $\sin \delta$ where δ is some combination of the final state scattering phase shifts and a knowledge of δ is necessary to make predictions for CP violations in hyperon decays. Calculations of $\Lambda - \pi$ phase shifts are relevant to the measurement of CP violation in the hyperon decay $\Xi \rightarrow \Lambda \pi$ [2]. An experiment to measure the combined asymmetry $\Delta\alpha = \Delta\alpha_\Lambda + \Delta\alpha_\Xi$ is being carried out at Fermilab [3]. Here, for example, $\Delta\alpha_\Xi = \alpha_\Xi + \bar{\alpha}_\Xi$ where α_Ξ and $\bar{\alpha}_\Xi$ are the up-down asymmetries in the decay $\Xi \rightarrow \Lambda \pi$ and its charge conjugate process.

The CP violating asymmetry $\Delta\alpha_\Xi$ is proportional to $\tan(\delta_S - \delta_P)$ where δ_S and δ_P are the S and P wave phase shifts in $\Lambda - \pi$ scattering. There have been calculations of $\Lambda - \pi$ scattering phase shifts in the framework of $SU(2)_L \times SU(2)_R$ baryon chiral perturbation theory (HBCHPT) [4], with much smaller S wave phase shift than some earlier dispersive estimates [5]. The P wave phase shift in [4] was approximately of the same sign and magnitude as in [5]. The phase shifts in [4] were calculated using tree level exchanges of low lying positive and negative parity Σ states of spin $\frac{1}{2}$ and $\frac{3}{2}$. Recently the $\Lambda - \pi$ phase shifts were calculated in $SU(3)$ baryon chiral perturbation theory [6] and a large S wave phase shift was obtained while the P wave phase shift ~ -3 degrees was of the same order as before [4]. In particular, the S wave phase shift was calculated to be $\delta_S = -17.6$ degrees compared to a maximum S wave phase shift of 0.5 degrees calculated in Ref [4]. It has been claimed [6] that the large S wave phase shift is due to corrections to the heavy baryon limit. This is difficult to understand as corrections to the heavy baryon limit should be suppressed with respect to the leading order contribution by factors of $\frac{p_\pi}{M_\Lambda}$ or $\frac{M_\pi}{M_\Lambda}$ where p_π is the magnitude of the pion three momentum. So the subleading effects are expected to be at about the 14

% level . Moreover calculations of phase shifts in the pion-nucleon system using HBCHPT give a fairly good fit to the data suggesting the approximate validity of the HBCHPT [7].

In this paper we calculate the S and P wave phase shifts in the framework of the fully relativistic $SU(3)_L \times SU(3)_R$ chiral Lagrangian and find small phase shifts for both S and P wave in disagreement with the results in Ref [6]. We also consider possible off-shell coupling of the spin $\frac{3}{2}(1385)$ Σ resonance, denoted as Σ^* , to $\Lambda \pi$. This off-shell coupling cannot be determined from the decay $\Sigma^* \rightarrow \Lambda \pi$ since it vanishes for the on-shell Σ^* . In the $SU(3)$ limit the magnitude of this off-shell coupling can be inferred from the off-shell $\pi N \Delta$ coupling.

In Section 2 we describe our formalism and in Section 3 we present our results and conclusions.

2 Chiral Lagrangian for Baryons

The lowest order $SU(3)_L \times SU(3)_R$ chiral involving the involving the 0^- mesons, M and the $\frac{1}{2}^+$ baryon octet B can be written as[8]

$$\begin{aligned}
L_1 = & \frac{f_\pi^2}{8} Tr(\partial_\mu \Sigma \partial^\mu \Sigma^\dagger) + iT r(\bar{B} \gamma^\mu \partial_\mu B + \bar{B} \gamma^\mu [V_\mu, B]) - m Tr(\bar{B} B) \\
& + D Tr \bar{B} \gamma_\mu \gamma_5 \{A_\mu, B\} + F Tr \bar{B} \gamma_\mu \gamma_5 [A_\mu, B],
\end{aligned} \tag{1}$$

where

$$\begin{aligned}
V_\mu &= \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) \\
A_\mu &= \frac{i}{2}(\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi) \\
\xi &= \exp(i \frac{M}{f_\pi}) \\
\Sigma &= \xi^2
\end{aligned} \tag{2}$$

M and B are the meson and the baryon matrices given by

$$M = \begin{bmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \bar{\Xi}^0 & -\sqrt{\frac{2}{3}}\Lambda \end{bmatrix}$$

The transformations of the various fields under $SU(3)_L \times SU(3)_R$ are

$$\begin{aligned} \Sigma &\rightarrow L\Sigma R^\dagger \\ \xi &\rightarrow L\xi U^\dagger = U\xi R^\dagger \\ B &\rightarrow UBU^\dagger \end{aligned} \tag{3}$$

The constants $D=0.8\pm 0.14$ and $F=0.5\pm 0.12$ are obtained from a fit to hyperon semileptonic decays [9] and $f_\pi = 131$ GeV is the pion decay constant.

The $\frac{3}{2}$ baryon $\Sigma^*(1385)$ belongs to the decouplet representation in SU(3). The interaction Lagrangian for the decouplet field D_μ has the general form

$$L_{int} = g\bar{D}_\mu(g^{\mu\nu} - z\gamma^\mu\gamma^\nu)A_\mu B + h.c., \tag{4}$$

where we have suppressed the SU(3) indices and we have retained only terms relevant to our calculation. Expanding A_μ in terms of the meson fields, the lowest order term describing the interaction of $\Sigma^{\frac{3}{2}+}(1385)$ is

$$L_{int} \rightarrow L_2 = \frac{g}{f_\pi}\bar{\Sigma}_\mu^*[g^{\mu\nu} - z\gamma^\mu\gamma^\nu]\partial_\nu\pi\Lambda + h.c., \tag{5}$$

where Σ_μ^* is the Rarita-Schwinger field describing the $\Sigma(1385)$ baryon. The coupling g may be obtained from the branching ratio of $\Sigma^* \rightarrow \Lambda\pi$. The coupling z remains undetermined because $\gamma_\mu u^\mu = 0$ for a free Rarita-Schwinger spinor u^μ and so it does not contribute to the

decay width of $\Sigma^* \rightarrow \Lambda\pi$. However in the SU(3) limit we can infer this off-shell coupling from the $\pi N\Delta$ system. The $\frac{3}{2}$ propagator is usually taken as [10]

$$S_{\mu\nu}^1 = \frac{i}{P^2 - M^2}(\gamma \cdot P + M)\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2P_\mu P_\nu}{3M^2} + \frac{\gamma_\mu P_\nu - \gamma_\nu P_\mu}{3M}\right]. \quad (6)$$

On-shell, $S_{\mu\nu}^1$ satisfies the following conditions

$$\begin{aligned} P^\mu S_{\mu\nu}^1 &= 0, \\ \gamma^\mu S_{\mu\nu}^1 &= 0. \end{aligned} \quad (7)$$

In the study of the pion-nucleon system it has been suggested that the above conditions for $S_{\mu\nu}$ should also be satisfied off-shell[11]. This leads to a unique form for the $\frac{3}{2}$ propagator

$$S_{\mu\nu}^2 = \frac{i}{P^2 - M^2}(\gamma \cdot P + M)\left[-g_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{\gamma \cdot P(\gamma_\mu P_\nu) + (\gamma_\nu P_\mu)\gamma \cdot P}{3P^2}\right]. \quad (8)$$

The advantage of this form is that the off-shell coupling does not contribute because of the above conditions (Eq. 7) which, in this case, are also true off-shell and there is no dependence on the arbitrary parameter z in the amplitude. In the heavy baryon limit the Lagrangian L_2 , in standard HQET notation, reduces to

$$L_2 = \frac{g}{f_\pi} \bar{h}_\mu^*(v)[g^{\mu\nu} - z(v^\mu v^\nu - 4S^\mu S^\nu)]\partial_\nu \pi h(v) + h.c., \quad (9)$$

where v_μ is the baryon four-velocity and $S_\mu = i\gamma_5\sigma_{\mu\nu}v^\nu$ is the spin operator. Both the propagators $S_{\mu\nu}^1$ and $S_{\mu\nu}^2$ reduce in the heavy baryon limit to

$$S_{HB}^{\mu\nu}(\Sigma^*) = -\frac{i}{v \cdot k} \left[g^{\mu\nu} - v^\mu v^\nu + \frac{4}{3} S^\mu S^\nu \right], \quad (10)$$

where

$$P = Mv + k$$

and the momentum $k \ll M$ represents the amount by which P is off the mass shell. In the heavy baryon limit the off-shell coupling in L_2 does not contribute since

$$v_\mu S_{HB}^{\mu\nu} = S_\mu S_{HB}^{\mu\nu} = 0.$$

So the off-shell coupling in HBCHPT would correspond to higher order $\frac{1}{M}$ effects. However such effects can be important if these higher order terms are associated with large coefficients and if z is not too small.

The invariant transition amplitude for $\Lambda - \pi$ scattering has the general form [12]

$$T = \bar{u}_f[A(k, \theta) + \frac{1}{2}\gamma \cdot (k_1 + k_2)B(k, \theta)]u_i, \quad (11)$$

where θ is the scattering angle, k is the centre of mass momentum and k_1 and k_2 are the incoming and outgoing pion four-momenta. The scattering amplitude is then given by

$$\begin{aligned} f(\theta) &= \chi_f^\dagger [f_1 + f_2 \frac{\sigma \cdot \mathbf{k}_2 \sigma \cdot \mathbf{k}_1}{k^2}] \chi_i \\ &= \chi_f^\dagger [h + ig \frac{\sigma \cdot (\mathbf{k}_2 \times \mathbf{k}_1)}{k^2}] \chi_i \end{aligned} \quad (12)$$

with

$$\begin{aligned} f_1 &= \frac{E + M}{8\pi E_{cm}} \{A + (E_{cm} - M)B\} \\ f_2 &= \frac{E - M}{8\pi E_{cm}} \{-A + (E_{cm} + M)B\} \\ h &= f_1 + f_2 \cos\theta \\ g &= f_2 \end{aligned} \quad (13)$$

where χ_f, χ_i are the two component spinor representing the final and initial state Λ with mass M and energy E . The functions h and g represent the non spin-flip and the spin-flip amplitude.

The partial waves $f_{L\pm}$ can now be projected out as

$$f_{L\pm} = \frac{1}{2} \int_{-1}^1 [P_L(x)f_1 + P_{L\pm 1}(x)f_2] dx \quad (14)$$

where $x = \cos\theta$.

Since we are interested in the strong scattering of the $\Lambda - \pi$, system which is the decay product in the weak decay of Ξ , the total angular momentum of the $\Lambda - \pi$ system is $J = L \pm \frac{1}{2} = \frac{1}{2}$ and hence the relevant partial waves are $f_{J=0+\frac{1}{2}} = f_{0+}$ and $f_{J=1-\frac{1}{2}} = f_{1-}$. The phase shifts can then be calculated from

$$f_{S,P} = f_{0+,1-} = \frac{1}{k} e^{i\delta_{S,P}} \sin \delta_{S,P}. \quad (15)$$

For small phase shifts we have

$$\tan \delta_{S,P} \approx k f_{S,P}. \quad (16)$$

Since we are calculating only the tree level amplitude we are unable to satisfy partial wave unitarity.

In our calculation, the $\Lambda - \pi$ scattering takes place through the exchange of Σ and Σ^* (1385). In the former case both s and u channel amplitudes contribute while only the u channel contributes for the latter. The contributions to A and B for Σ exchange are

$$\begin{aligned} A_\Sigma &= \left(\frac{2D}{\sqrt{6}f_\pi}\right)^2 [(M_\Sigma + M_\Lambda) \{2 + (M_\Sigma^2 - M_\Lambda^2) \left(\frac{1}{s - M_\Sigma^2} + \frac{1}{u - M_\Sigma^2}\right)\}] \\ B_\Sigma &= \left(\frac{2D}{\sqrt{6}f_\pi}\right)^2 [(M_\Sigma + M_\Lambda)^2 \left\{\frac{1}{u - M_\Sigma^2} - \frac{1}{s - M_\Sigma^2}\right\}] \end{aligned} \quad (17)$$

where

$$s = (p_1 + k_1)^2 = (p_2 + k_2)^2$$

and

$$u = (p_2 - k_1)^2 = (p_1 - k_2)^2$$

with p_1, p_2 being the initial, final baryon momenta and k_1, k_2 the initial, final pion momenta.

We note that in the lowest order the F term in the Lagrangian does not contribute.

The contributions from Σ^* exchange using the propagator $S_{\mu\nu}^1$ are

$$A_{\Sigma^*} = \frac{g^2 [(M + M_\Lambda)(-M_\Lambda^2 + u - 3k_1 \cdot k_2)M^2 + P \cdot k_1(-M_\pi^2 - M_\Lambda^2 + u)M + 2(P \cdot k_1)^2(M + M_\Lambda)]}{f_\pi^2 3M^2(M^2 - u)}$$

$$\begin{aligned}
& - \frac{g^2}{f_\pi^2} \frac{2z(u - M_\Lambda^2)}{3M} \\
& + \frac{g^2}{f_\pi^2} z^2 \left[\frac{2M_\Lambda(u - M_\Lambda^2)}{3M^2} + \frac{4(u - M_\Lambda^2)}{3M} \right], \\
B_{\Sigma^*} & = \frac{g^2}{f_\pi^2} \frac{[(-M_\pi^2 + 2M_\Lambda^2 + 3k_1 \cdot k_2 + 2MM_\Lambda)M^2 + 2P \cdot k_1 M_\Lambda M - 2(P \cdot k_1)^2]}{3M^2(M^2 - u)} \\
& + \frac{g^2}{f_\pi^2} z \left[\frac{4P \cdot k_1}{3M^2} - \frac{4M_\Lambda}{3M} \right] \\
& + \frac{g^2}{f_\pi^2} z^2 \left[-\frac{2M_\pi^2}{3M^2} + \frac{4M_\Lambda^2}{3M^2} - \frac{4P \cdot k_1}{3M^2} + \frac{8M_\Lambda}{3M} \right]. \tag{18}
\end{aligned}$$

We note that the off-shell terms do not have a pole since the numerator in the amplitude of the off-shell terms is proportional to $(P^2 - M^2)$ which cancels the pole term in the denominator.

If the propagator $S_{\mu\nu}^2$ is used then we have

$$\begin{aligned}
A_{\Sigma^*} & = \frac{g^2}{f_\pi^2} \frac{(M + M_\Lambda) \{u(-M_\Lambda^2 + u - 3k_1 \cdot k_2) - 2P \cdot k_1 M_\pi^2\}}{3(M^2 - u)u} \\
B_{\Sigma^*} & = \frac{g^2}{f_\pi^2} \frac{-2(P \cdot k_1)^2 - uM_\pi^2 + 2(u + P \cdot k_1)(M_\Lambda^2 + MM_\Lambda) + 3uk_1 \cdot k_2}{3(M^2 - u)u} \tag{19}
\end{aligned}$$

As noted before, there is no z dependence in this case since the off-shell terms do not contribute.

3 Results and Discussions

In this section we present and discuss our results. The relevant masses and widths for our calculation are taken from Ref[13]. The phase shifts from the Σ exchange are $\delta_S = -0.13$ degrees and $\delta_P = -2.84$ degrees for $D=0.8$. If we vary D between its limits 0.66-0.94 then we obtain $\delta_S = -0.09$ degrees to -0.18 degrees and $\delta_P = -1.9$ degrees to -3.9 degrees. For the Σ^* exchange we determine the coupling constant g from the width of the $\Sigma^* \rightarrow \Lambda\pi$,

$$\frac{g^2}{f_\pi^2} = \frac{12\pi\Gamma}{\left(\frac{M_{\Sigma^*}}{M_\Lambda}\right)\left(\frac{E_\Lambda}{M_\Lambda} + 1\right)p_\pi^3}.$$

From the above we obtain $g \approx 1.13$. The contribution of the Σ^* exchange using the propagator $S_{\mu\nu}^1$ can be written as

$$\begin{aligned} f_S &= -0.127 + 0.059z + 0.089z^2, \\ f_P &= 0.119 - 0.012z + 0.054z^2. \end{aligned} \tag{20}$$

We observe that the contribution from the offshell terms can be almost of the same order as the other terms if $z \sim 1$. If z is large with $z > 1$, say $z \sim 2 - 3$, then the offshell term can even dominate the other terms. However this is unlikely as it would correspond to anomalously large SU(3) breaking since the magnitude [14] of the off-shell coupling in the $\pi N\Delta$ system is < 1 . In the heavy baryon framework Eq. (20) indicates that HBCHPT with baryons is slowly converging as in the case of the $SU(2)_L \times SU(2)_R$ HBCHPT describing the pion-nucleon system [7]. In the figure we plot δ_S and δ_P from the Σ^* contribution as a function of z . Studies in the $\pi N\Delta$ system obtain a range of z to be between -0.3 to 0.8 from fit a to the pion-nucleon data [14]. In the SU(3) limit we can use this range for our calculations. Assuming reasonable SU(3) breaking effects we take the range of z in our calculation between -1 and 1. From the figure we see that δ_P is always positive and δ_S is mostly negative for most values of z within the range of -1 to 1. The total phase shifts in $\Lambda - \pi$ scattering is to a good approximation the sum of the phase shifts from the Σ and Σ^* exchange. So the net δ_S and δ_P can have values between ~ -1.3 to 0.1 degrees and between ~ -3 degrees to -0.4 degrees, respectively.

If we use the propagator $S_{\mu\nu}^2$ then we get $\delta_S = 0.62$ degrees and $\delta_P = 1.12$ degrees from the Σ^* exchange. This leads to total phase shifts of $\delta_S = 0.53$ to 0.44 degrees and $\delta_P = -0.8$ to -2.8 degrees. The inclusion of the next low lying negative parity Σ and Σ^* resonances in our calculations is not expected to make a significant contribution to the phase shifts [4].

We now compare our results with those obtained in [6]. As noted in the introduction a large S wave phase shift but a small P wave phase shift was obtained in Ref[6]. We should

recover the results of [6] if we neglect the off-shell coupling of the Σ^* and set the pion mass⁴ $M_\pi = 0$. Now if one performs an expansion of the amplitude in the small parameter $x = p_\pi/M_\Lambda$ then the S-wave phase shift is always suppressed with respect to the P-wave phase shift by $x \sim 0.14$ and so a large S-wave phase shift of -17.6 degrees is not expected when the P wave phase shift is only ~ -3 degrees.

In summary we have calculated the S and P phase shifts for $\Lambda - \pi$ scattering in the fully relativistic $SU(3)_L \times SU(3)_R$ invariant chiral Lagrangian. We also included possible off-shell couplings of the Σ^* baryon to $\Lambda\pi$. Assuming reasonable SU(3) breaking this off-shell coupling is taken to be of the same order as in the $\pi N\Delta$ system. We find small phase shifts for both the S and the P waves which are of the same order as those calculated using $SU(2)_L \times SU(2)_R$ invariant chiral Lagrangian in the heavy baryon limit.

Note Added: When this manuscript was under preparation we became aware of a new version of [6] that gets much smaller phase shifts than previous versions. The new phase shifts in [6] are similar to our results.

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⁴ We point out that it is inconsistent to neglect the pion mass as p_π is of the same order as the pion mass.

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3.1 Figure Caption

- **Figure:** S and P wave phase shifts δ_S and δ_P from Σ^* exchange versus the offshell coupling z .