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 $B_{u,d} \rightarrow J/\psi K^*$  and  $B_s \rightarrow J/\psi \phi$  Decays**

Amol S. Dighe

*The Abdus Salam International Centre for Theoretical Physics  
34100 Trieste, Italy*

Isard Dunietz

*Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois 60510*

Robert Fleischer

Theory Division, CERN  
CH-1211 Geneva 23, Switzerland

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# Resolving a Discrete Ambiguity in the CKM Angle $\beta$ through $B_{u,d} \rightarrow J/\psi K^*$ and $B_s \rightarrow J/\psi \phi$ Decays

Amol S. Dighe

*The Abdus Salam International Centre for Theoretical Physics  
34100 Trieste, Italy*

Isard Dunietz

*Theoretical Physics Division, Fermi National Accelerator Laboratory  
Batavia, IL 60510, USA*

Robert Fleischer

*Theory Division, CERN, CH-1211 Geneva 23, Switzerland*

## Abstract

It is well known that  $\sin(2\beta)$ , where  $\beta$  is one of the angles of the unitarity triangle of the CKM matrix, can be determined in a theoretically clean way by measuring mixing-induced CP violation in the decay  $B_d \rightarrow J/\psi K_S$ . Another clean extraction of this CKM angle is provided by the time-dependent angular distribution for the decay products of  $B_d \rightarrow J/\psi(\rightarrow l^+l^-) K^{*0}(\rightarrow \pi^0 K_S)$ , where we have more observables at our disposal than in the case of  $B_d \rightarrow J/\psi K_S$ , so that in addition to  $\sin(2\beta)$  also  $\cos(2\beta)$  can be probed in a direct way. Unfortunately a sign ambiguity remains in  $\cos(2\beta)$ . If it could be resolved, a discrete ambiguity in the extraction of the CKM angle  $\beta$  could be resolved as well, which would allow a more incisive test of the CKM model of CP violation. This note shows that detailed time-dependent studies of  $B_{u,d} \rightarrow J/\psi K^*$  and  $B_s \rightarrow J/\psi \phi$  decay processes can determine the sign of  $\cos(2\beta)$ , thereby removing the corresponding ambiguity in the extraction of the CKM angle  $\beta$ .



The conventional methods for determining the angles  $\alpha$ ,  $\beta$  and  $\gamma$  of the usual unitarity triangle [1] of the Cabibbo–Kobayashi–Maskawa matrix (CKM matrix) [2] leave several discrete ambiguities [3]. This is also the case for the “gold-plated” mode  $B_d \rightarrow J/\psi K_S$ . The mixing-induced CP asymmetry arising in this channel allows only a theoretically clean determination of  $\sin(2\beta)$ , so that a discrete four-fold ambiguity for the extracted value of  $\beta \in [0^\circ, 360^\circ]$  remains. In the recent literature, several strategies were proposed to resolve ambiguities of this kind [4].

Another clean probe of the CKM angle  $\beta$  is provided by the observables of the angular distributions for the decay products of  $B_d \rightarrow J/\psi(\rightarrow l^+l^-) K^{*0}(\rightarrow \pi^0 K_S)$  modes [5]–[7]. Such observables can in general be expressed in terms of decay amplitudes as

$$|A_f(t)|^2, \quad \text{Re}[A_{\tilde{f}}^*(t) A_f(t)], \quad \text{Im}[A_{\tilde{f}}^*(t) A_f(t)], \quad (1)$$

where  $f$  and  $\tilde{f}$  are labels for specific final-state configurations. The full three-angle distributions for tagged  $B_d(t) \rightarrow J/\psi K^{*0}(\rightarrow \pi^0 K_S)$  decays are given in [6]–[8]. Throughout this note, by tagging we mean making the distinction of initially, i.e. at  $t = 0$ , present unmixed  $B_{d,s}^0$  and  $\overline{B}_{d,s}^0$  mesons. Weighting functions have been derived to extract the corresponding observables in an efficient way from experimental data [6, 7]. The time evolution of the interference terms in (1), i.e. of the real and imaginary parts of bilinear combinations of certain decay amplitudes, allows the determination [7] of  $\sin(\delta_{1,2})$  and  $\cos(\delta_1 - \delta_2)$ , where  $\delta_1$  and  $\delta_2$  are CP-conserving strong phases, and of  $\sin(2\beta)$  and

$$\cos(\delta_{1,2}) \cos(2\beta). \quad (2)$$

The CP-conserving observables  $|A_f(t=0)|$ ,  $\sin(\delta_{1,2})$  and  $\cos(\delta_1 - \delta_2)$  can be determined to a higher accuracy from the much larger data samples arising for  $B_s^\pm \rightarrow J/\psi K^{*\pm}$  transitions, and untagged  $B_d$  decays into  $J/\psi K^{*0}(\rightarrow K^+\pi^-)$  and  $J/\psi \overline{K}^{*0}(\rightarrow K^-\pi^+)$  states [6, 7]. At first sight, one may think that  $\sin(\delta_{1,2})$  and  $\cos(\delta_1 - \delta_2)$  extracted this way from the  $B_{u,d} \rightarrow J/\psi K^*$  angular distributions will allow the determination of  $\cos(2\beta)$  with the help of the terms given in (2). A closer look shows, however, that this is unfortunately not the case, since we do not have sufficient information to fix the *signs* of  $\cos(\delta_{1,2})$ , thereby leaving a sign ambiguity for  $\cos(2\beta)$ .

The purpose of this letter is to point out that this ambiguity can be resolved with the help of tagged, time-dependent studies of  $B_s \rightarrow J/\psi \phi$  decays. The angular distributions are given in [7, 8], and weighting functions to extract the observables from experimental data can be found in [7]. An important feature of these observables is that they allow the determination of a CP-violating weak phase  $\phi$  [7, 9], which takes a very small value, of  $\mathcal{O}(0.03)$ , within the Standard Model, and represents a sensitive probe for new-physics contributions to  $B_s^0\text{--}\overline{B}_s^0$  mixing. Provided there is a sizeable mass difference between the mass eigenstates  $B_s^H$  and  $B_s^L$ , this phase can even be extracted from *untagged*  $B_s$  data samples [10], where the rapid  $\Delta m_s t$  oscillations cancel [11].

Another important feature is the fact that the tagged, time-dependent  $B_s^{(-)}(t) \rightarrow J/\psi(\rightarrow l^+l^-)\phi(\rightarrow K^+K^-)$  observables corresponding to the “Im” terms in (1) provide

sufficient information to determine  $\widehat{\delta}_{1,2}$  *unambiguously*, where the strong phases  $\widehat{\delta}_{1,2}$  are the flavour  $SU(3)$  counterparts of  $\delta_{1,2}$ . In the strict  $SU(3)$  limit, we have  $\widehat{\delta}_{1,2} = \delta_{1,2}$ . The time evolution of these observables takes the following form [7]:

$$e^{-\overline{\Gamma}t} \sin(\widehat{\delta}_k - \Delta m_s t) + \frac{(-)}{2} \left( e^{-\Gamma_H t} - e^{-\Gamma_L t} \right) \cos(\widehat{\delta}_k) \phi, \quad (3)$$

where terms of  $\mathcal{O}(\phi^2)$  have been neglected,  $\overline{\Gamma}, \Gamma_H, \Gamma_L$  denote the decay widths of the  $B_s$  mass eigenstates,  $\overline{\Gamma} \equiv (\Gamma_H + \Gamma_L)/2$ , and  $k = 1, 2$ . Consequently, the strong phases  $\widehat{\delta}_{1,2}$  can be determined *unambiguously* by resolving the rapid  $\Delta m_s t$  oscillations in (3). Comparing the resulting values for  $\sin(\widehat{\delta}_{1,2})$  and  $\cos(\widehat{\delta}_1 - \widehat{\delta}_2)$  with their unhatted analogues, which can be determined from the  $B_{u,d} \rightarrow J/\psi K^*$  observables, we obtain valuable information on  $SU(3)$  breaking. In order to fix the sign of  $\cos(2\beta)$  with the help of (2), we just need the sign of  $\cos(\delta_1)$  or  $\cos(\delta_2)$ , which is provided by the sign of  $\cos(\widehat{\delta}_k)$  determined from the  $B_s(t) \rightarrow J/\psi \phi$  observables.

The  $SU(3)$  flavour symmetry should work reasonably well to determine this sign, unless  $|\sin(\delta_k)|$  is close to 1, implying  $\delta_k$  close to  $90^\circ$  or  $270^\circ$ , where  $\cos(\delta_k)$  flips its sign. However, for such values of  $\delta_k$ , the  $\cos(\delta_k)\cos(2\beta)$  terms (2) appearing in the  $B_d \rightarrow J/\psi(\rightarrow l^+l^-)K^{*0}(\rightarrow \pi^0 K_S)$  angular distribution – which are essential for our strategy – will anyway be highly suppressed, so that it is doubtful that the sign ambiguity can be resolved in this case. It is of course not yet clear whether future experiments will encounter such an unfortunate situation. Within the framework of “factorization”, we have  $\delta_{1,2} \in \{0^\circ, 180^\circ\}$ , i.e.  $|\cos(\delta_{1,2})| = 1$ . Since  $B_{u,d} \rightarrow J/\psi K^*$  decays and their  $B_s$  counterpart  $B_s \rightarrow J/\psi \phi$  are colour-suppressed modes, “factorization” is not expected to be a good approximation in this case. Consequently, the actual values of  $\delta_{1,2}$  may deviate significantly from these trivial values.

Angular distribution measurements for  $B \rightarrow J/\psi K^*$  modes have already been reported [12], and others may soon be made public [13]. The modes considered here are very appealing, because of the ability to trigger on the  $J/\psi$  meson. The prospects are bright for resolving the rapid  $\Delta m_s t$  oscillations in  $B_s(t) \rightarrow J/\psi \phi$  decays at planned experiments at the Tevatron and the LHC. Thus, in a not too distant future, the determination of  $\widehat{\delta}_k$  and the resolution of the ambiguity (related to the sign of  $\cos(2\beta)$ ) in the extraction of the CKM angle  $\beta$  from  $B_d \rightarrow J/\psi K^{*0}(\rightarrow \pi^0 K_S)$  decays may become feasible. Let us note that there remains a two-fold ambiguity for  $\beta$  in this approach, since we cannot decide whether  $\beta$  lies within the intervals  $[0^\circ, 180^\circ]$  or  $[180^\circ, 360^\circ]$ . In each interval,  $\beta$  is, however, fixed unambiguously. Consequently, the original four-fold ambiguity arising in the extraction of  $\beta$  from  $\sin(2\beta)$  can be reduced to just a two-fold ambiguity. Usually it is argued that  $\varepsilon_K$ , which measures indirect CP violation in the kaon system, implies the former range [4].

While the  $B_d \rightarrow J/\psi K^{*0}(\rightarrow \pi^0 K_S)$  mode is very accessible at  $B$  factories operating at the  $\Upsilon(4S)$  resonance, detectors at hadron accelerators should study the feasibility of the  $\pi^0$  reconstruction. The  $B_d \rightarrow J/\psi \rho^0, J/\psi \omega$  modes could be added to  $B_d \rightarrow J/\psi K^{*0}(\rightarrow \pi^0 K_S)$  in order to resolve the  $\beta$  ambiguity. If penguin amplitudes are neglected, the time evolution of these decay modes also depends on the CKM angle  $\beta$  and, in the limit

of the  $SU(3)$  flavour symmetry, their strong phases are equal to those of their  $SU(3)$  counterparts.

In summary, traditional methods allow tests of the CKM picture of CP violation only up to discrete ambiguities. The resolution of these ambiguities would make such CKM tests significantly more powerful. In this letter, making use of the many observables that are available from angular correlations, we have proposed an approach to resolve a discrete ambiguity in the determination of the CKM angle  $\beta$  that may be simpler than strategies advocated earlier [4]. More generally, angular-correlation methods can also be formulated to remove discrete CKM ambiguities in  $\beta$ ,  $2\beta + \gamma = \pi + \beta - \alpha$  and  $\gamma$  from colour-allowed processes [14].

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