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Distribution

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Abstract

We study the time evolution of the quantum fluctuations of the axion field for both the QCD axion as well as axions arising in the context of supergravity and string theories. We explicitly keep track not only of the coherently oscillating zero momentum mode of the axion but also of the higher non-zero momentum modes using the full axion potential. The full axion potential makes possible two kinds of instabilities: spinodal instabilities and parametric resonance instabilities. The presence of either of these instabilities can lead to a quasi-exponential increase in the occupation of non-zero momentum modes and the build-up of the quantum fluctuations of the axions. If either of these becomes a significant effect then axions would no longer be a suitable cold dark matter candidate. First, we check that in Minkowski space the axion fluctuations build up on a timescale of order $10^2 m_a^{-1}$ (in units where $\hbar = 1 = c$). This timescale is in fact much shorter than the age of the universe. Our results confirm the conventional wisdom that these effects are not significant in the setting of an expanding FRW universe and hence axions are indeed cold dark matter candidates.

I. INTRODUCTION AND MOTIVATION

Over the years there has been growing evidence that nonbaryonic, cold dark matter plays an important role in the structure and evolution of the universe (see, e.g. [1, 2, 3]). Axions are among the most promising candidates for the non-baryonic cold dark matter of the universe[4, 5, 6]. The concept of the axion was originally introduced and developed to solve the strong CP problem of QCD in an appealing and phenomenologically acceptable way[7]. In these models, axions are pseudo-Goldstone bosons of a U(1) symmetry. After the advent of supergravity and string theories, it became clear that particles with the properties of axions were in fact more generic, and that in these theories there could be additional axion fields which may play a significant role in the history of the universe (see e.g. [8]).

In this paper, we examine the time evolution of the quantum fluctuations of the axion field as it oscillates about the minimum of its potential. The standard picture is that the axion oscillates coherently in its potential; only the zero-momentum mode is important. The original papers presenting the axion as a dark matter candidate[4, 5, 6] considered simple estimates of instabilities in the axion field that could result in energy being pumped from the zero mode into higher-momentum modes. If such an effect were significant, then it is possible that the energy stored in the axion field would be largely converted to kinetic energy and subsequently redshifted away. According to the original estimates, these effects are not significant.

However in recent years we have come to realise that there are two kinds of instabilities occuring in the time evolution of generic mode functions that have the potential of changing this situation. Thus, either spinodal instabilities[9, 10, 11, 12] or parametric resonance instabilities[13, 14, 15], if they last for a sufficiently long time, can lead to an explosive growth of quantum fluctuations through the exponential growth of non-zero momemtum modes. Thus in the light of the recent understanding of the role of these instabilities in the growth of non-zero momentum modes, it is worthwhile to re-examine the role of the quantum fluctuations of the axion.

In what follows we carefully and quantitatively study the time evolution of both the zero and non-zero momentum modes of the axion. We are thus able to ascertain the magnitude of the quantum fluctuations of the axion and compare it with the value of the coherently oscillating axion field. We do this both for the QCD axion as well as other axions that arise in the context of supergravity and string theories. We show that if the axions were born, lived and died in Minkowski space then there would in fact have been an explosive growth of quantum fluctuations resulting from the quasi-exponential growth of some non-zero momentum modes of the axion field. However, the energy density of axions is diluted by the expansion of the universe, which implies a decrease in the amplitude of oscillation of the coherently oscillating zero mode. Since it is this oscillation that drives the instabilities and the explosive exponential growth of the non-zero modes, it is clear that when the amplitude of the zero mode falls below some critical value, the instabilities will be shut off and there will be no further growth of the fluctuations. The issue thus becomes one of the initial amplitudes and timescales involved.

We now turn to a quantitative analysis of the problem to determine whether there is enough time to build up the fluctuations significantly. In the next section we will describe and layout the equations that determine the time evolutions of the axion field and its fluctuations.

In section 3 we will study and analyze the solution to these equations for (i) the QCD axion and (ii) other axions arising from supergravity and string theories. In both cases we will compare the evolution in Minkowski space with the behaviour in an expanding FRW universe to gain insight into the relative roles of instabilities and the expansion of the universe. Finally we will conclude by stating the implications of our results and place things in perspective.

II. EVOLUTION EQUATIONS

The derivation of the appropriate evolution equations[27, 28] has been intensively studied during the past few years by a number of groups [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26]. Here we will summarise the key formulae along the lines presented by Boyanovsky, de Vega and Holman[29].

In a spatially flat FRW cosmology, the metric is

$$ds^2 = dt^2 - a^2(t) d\vec{x}^2 \ . \tag{2.1}$$

The action and Lagrangian density for the axion field Φ are given by

$$S = \int d^4x \mathcal{L} \tag{2.2}$$

$$\mathcal{L} = a^3(t) \left[\frac{1}{2} \dot{\Phi}^2(\vec{x},t) - \frac{1}{2} \frac{(\vec{\nabla}\Phi(\vec{x},t))^2}{a(t)^2} - V(\Phi(\vec{x},t)) \right] \tag{2.3}$$

$$V(\Phi) = \Lambda^4 \left[1 - \cos\left(\frac{\Phi(\vec{x},t)}{f_a} \right) \right] \ , \tag{2.4}$$

where f_a is the axion decay constant and Λ is related to f_a and the axion mass m_a via $\Lambda = f_a m_a$. We will be interested in two kinds of axions: the standard QCD axion for which $\Lambda = \Lambda_{QCD} \sim 200\,\mathrm{MeV}$ and $f_a \sim 10^{12}\,\mathrm{GeV}$, and axions that can arise in the context of supergravity and string theories, for which we will take $\Lambda \sim 10^{16}\,\mathrm{GeV}$ and $f_a \sim M_{Pl} \sim 10^{19}\,\mathrm{GeV}$ (although the parameters are much less constrained in this case). Axions with these parameters have been considered earlier in the context of natural inflation[30]. Here we will concentrate on computing the magnitude of the fluctuations of the axion field compared to the amplitude of the coherently oscillating zero momentum mode of the axion for general values of Λ and f_a.

The canonical momentum conjugate to Φ is

$$\Pi(\vec{x},t) = a^3(t) \dot{\Phi}(\vec{x},t) \ , \tag{2.5}$$

and the Hamiltonian becomes

$$H(t) = \int d^3x \left\{ \frac{\Pi^2}{2a^3(t)} + \frac{a(t)}{2} (\vec{\nabla}\Phi)^2 + a^3(t) V(\Phi) \right\} \ . \tag{2.6}$$

In the Schrödinger representation (at an arbitrary fixed time t_o), the canonical momentum is represented as

$$\Pi(\vec{x}) = -i \frac{\delta}{\delta \Phi(\vec{x})} \ .$$

Wave functionals obey the time dependent functional Schrödinger equation

$$i\frac{\partial\Psi[\Phi,t]}{\partial t} = H\Psi[\Phi,t] \ . \tag{2.7}$$

For the systems we'll be interested in it is convenient to work with a functional density matrix $\hat{\rho}$ with matrix elements in the Schrödinger representation $\rho[\Phi(\vec{\cdot}),\tilde{\Phi}(\vec{\cdot});t]$. Normalizing the density matrix such that $Tr\hat{\rho}=1$, the "order parameter" is defined as

$$\phi(t) = \frac{1}{\Omega}\int d^3x\langle\Phi(\vec{x},t)\rangle = \frac{1}{\Omega}\int d^3x Tr\hat{\rho}(t)\Phi(\vec{x}) \ , \tag{2.8}$$

where Ω is the comoving volume, and the scale factors cancel between the numerator (in the integral) and the denominator. Note that we have used the fact that the field operator does not evolve in time in this picture. In this paper we will use the terms "order parameter", "mean value of the field" and "zero momentum mode of the field" interchangably to refer to quantity defined above. The evolution equations for the order parameter are

$$\frac{d\phi(t)}{dt} = \frac{1}{a^3(t)\Omega}\int d^3x\,\langle\Pi(\vec{x},t)\rangle = \frac{1}{a^3(t)\Omega}\int d^3x Tr\hat{\rho}(t)\Pi(\vec{x}) = \frac{\pi(t)}{a^3(t)} \tag{2.9}$$

$$\frac{d\pi(t)}{dt} = -\frac{1}{\Omega}\int d^3x a^3(t)\left\langle\frac{\delta V(\Phi)}{\delta\Phi(\vec{x})}\right\rangle \ . \tag{2.10}$$

It is now convenient to write the field in the Schrödinger picture as

$$\Phi(\vec{x}) = \phi(t)+\eta(\vec{x},t) \tag{2.11}$$

$$\langle\eta(\vec{x},t)\rangle = 0 \ . \tag{2.12}$$

Expanding the right hand side of (2.10) in powers of $\eta(\vec{x},t)$ we find the effective equation of motion for the order parameter:

$$\frac{d^2\phi(t)}{dt^2}+3\frac{\dot{a}(t)}{a(t)}\frac{d\phi(t)}{dt}+V'(\phi(t))+\frac{V'''(\phi(t))}{2\Omega}\int d^3x\langle\eta^2(\vec{x},t)\rangle+O(\eta^4)=0 \ . \tag{2.13}$$

where primes stand for derivatives with respect to ϕ. For our case, with $V(\Phi)$ given by (2.4), we find

$$\frac{d^2\phi(t)}{dt^2}+3\frac{\dot{a}(t)}{a(t)}\frac{d\phi(t)}{dt}+\frac{\Lambda^4}{f_a}\sin\left(\frac{\phi}{f_a}\right)\left[1-\frac{\langle\eta^2\rangle}{2f_a^2}\right]+O(\eta^4)=0 \ . \tag{2.14}$$

It is legitimate to neglect the $O(\eta^4)$ and higher terms in this equation as long as $\langle\eta^{2n}\rangle/f_a^{2n}$ remains small. We will assume that this is the case provided $\langle\eta^2\rangle/f_a^2$ remains small. Of course, if we were to find $\langle\eta^2\rangle/f_a^2$ of order 1 at some time, then beyond that time it would not be legitimate to neglect the higher order terms, and we would need to keep track of them through some non-perturbative technique such as the Hartree approximation. The first issue to address is if and when $\langle\eta^2\rangle/f_a^2$ ever does become of order 1, and this we can do while neglecting the $O(\eta^4)$ terms. To do this it is sufficient to consider only the lowest order terms in $\langle\eta^2\rangle/f_a^2$.

In order to follow the time evolution of the fluctuations $\langle\eta^2\rangle$ it is convenient to introduce mode functions $\varphi_k(t)$ which obey a simple evolution equations:

$$\frac{d^2\varphi_k}{dt^2}+3\frac{\dot{a}}{a}\frac{d\varphi_k}{dt}+\left[\frac{\vec{k}^2}{a^2}+\frac{\Lambda^4}{f_a}\cos\left(\frac{\phi}{f_a}\right)\left[1-\frac{\langle\eta^2\rangle}{2f_a^2}\right]\right]\varphi_k=0\ , \tag{2.15}$$

where we have dropped the $O(\eta^4)$ and higher terms. In terms of the functions $\varphi_k(t)$ the initial conditions are taken to be

$$\varphi_k(t_o)=[a^3(t_o)W_k(t_o)]^{-1/2} \tag{2.16}$$

$$\dot{\varphi}_k(t)\mid_{t_o}\ =i[a^{-3}(t_o)W_k(t_o)]^{1/2} \tag{2.17}$$

where $W_k(t_o)=[a^{-2}(t_o)k^2+m_0^2]^{1/2}$. This initial condition corresponds to taking an initial gaussian wave-packet for the field Φ with a width determined by the parameter m_0 which has the dimensions of mass. The smaller the parameter m_0 the more sharply peaked the initial distribution is around $k=0$. Since we are interested in studying the process of building up the occupation of higher non-zero momentum modes starting off with essentially only the zero momentum mode we are interested in the case where m_0 is much less than m_a. We have also varied m_0 and checked that the rate of the build-up of fluctuations is insensitive to the choice of m_0. This is actually so because when the non-zero momentum modes do grow they do so at an exponential rate and hence any differences in the initial occupation of modes quickly becomes insignificant.

The equal time two-point function for the fluctuations which we have been denoting by $\langle\eta^2\rangle$ can then be expressed as:

$$\langle\eta^2\rangle=\frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\frac{\mid\varphi_k(t)\mid^2}{2}. \tag{2.18}$$

Having described the formalism and arrived at the evolution equations for the axion fields of interest for us we are now in a position to study the solutions to these evolution equations. We turn to this in the following section. At this point we point out that there are some aspects of a similar problem specifically in the context of natural inflation that are currently being investigated[31].

III. SOLUTIONS TO THE EVOLUTION EQUATIONS AND ANALYSIS.

To analyze the solutions to the evolution equations we have written down in the previous section it is convenient to rescale variables into dimensionless ones in the following way:

$$\xi=\frac{\phi}{f_a}\ ,\quad m_a=\frac{\Lambda^2}{f_a}\ ,\quad \tau=m_a t\ ,\quad \langle\chi^2\rangle=\frac{\langle\eta^2\rangle}{2f_a^2}\ ,\quad \eta_k=\phi_k\sqrt{m_a}\ ,\quad q=\frac{k}{m_a}\ . \tag{3.1}$$

In the numerical work that follows we will take two sets of Λ and f_a. For the QCD axion we will take $\Lambda\simeq 200\,\mathrm{MeV}$ and $f_a\simeq 10^{12}\mathrm{GeV}$, wheras for axions arising in the context of supergarvity and string theories we will take $\Lambda\simeq 10^{16}\mathrm{GeV}$ and $f_a\simeq 10^{19}\mathrm{GeV}$. The rescaled equations in terms of the dimensionless variables introduced in this section are:

$$\frac{d^2\xi(\tau)}{d\tau^2} + 3\frac{\dot{a}(\tau)}{a(\tau)}\frac{d\xi(\tau)}{d\tau} + \sin\xi\left[1 - \langle\chi^2\rangle\right] = 0 \tag{3.2}$$

$$\left[\frac{d^2}{d\tau^2} + 3\frac{\dot{a}(\tau)}{a(\tau)}\frac{d}{d\tau} + \frac{q^2}{a^2(\tau)} + \cos\xi\left[1 - \langle\chi^2\rangle\right]\right]\eta_k(\tau) = 0. \tag{3.3}$$

$$\langle\chi^2\rangle = \left(\frac{m_a}{f_a}\right)^2 \frac{1}{8\pi^2}\int q^2 dq \frac{\mid \eta_k(t) \mid^2}{2}. \tag{3.4}$$

We further introduce the dimensionless quantity:

$$g = \left(\frac{m_a}{f_a}\right)^2 \frac{1}{8\pi^2}. \tag{3.5}$$

By doing this we completely encode all the various dimensional parameters for the two different particle physics settings for the two kinds of axions we are interested in into two different values of the dimensionless parameter g. Thus we now have,

$$\langle\chi^2\rangle = g\int q^2 dq \frac{\mid \eta_k(t) \mid^2}{2}. \tag{3.6}$$

We will study axions and their fluctuations in two different space-time settings, in order to compare the behaviour and gain insight into the physics of the build-up of fluctuations. Thus we will study axions in a radiation dominated expanding universe setting and a static Minkowski space-time. Both of these situations can be captured by parametrizing the time evolution of the scale factor as:

$$a(\tau) = a_o\tau^n. \tag{3.7}$$

We will take $a_o = 1$ and consider the two cases $n = 1/2$ and $n = 0$ corresponding to a radiation dominated (RD) Universe and Minkowski space-time respectively. In addition to the two different space-time settings, for each space-time setting we study two different kinds of axions: the QCD axion and axions that arise in the context of supergravity and string theories. These two different kinds of axions have very different values of Λ and f_a which enter into the quantity g which arose in the way we rescaled variables into a dimensionless form. Thus for the QCD axion we take $g = 2 \times 10^{-53}$ and for other axions arising from supergravity and string theories we take $g = 10^{-14}$. Then the solutions to the evolution equations can be displayed in terms of plots of quantities as a function of time. We have organised these into four figures: Figure 1 has plots representative of the QCD axion in a RD universe ($n = 1/2, g = 2 \times 10^{-53}$); Figure 2 has plots representative of the QCD axion in a Minkowski space-time ($n = 0, g = 2 \times 10^{-53}$); Figure 3 has plots representative of supergravity and string theory axions in a RD universe ($n = 1/2, g = 10^{-14}$); Figure 4 has plots representative of supergravity and string theory axions in a Minkowski space-time ($n = 0, g = 10^{-14}$).

For each of these cases above we plot the time evolution of the axion field zero momentum mode (ξ); and the logarithm of the axion fluctuations ($\log[\langle\chi^2\rangle]$).

By examining the plots we note the following facts:

1. Figures 1a, 1b, 2a and 2b show that axion fluctuations are not likely to be important for the QCD axion.

2. Figures 1b and 2b show that there is however an important qualitative difference between the behaviour of axion fluctuations in an expanding FRW universe compared with that in Minkowski space.

3. Finally, figures 3 and 4 highlight the difference between the Minkowski space and the FRW universe behaviour and importance of axion fluctuations. In particular, it shows that for axions arising from string theory and supergravity theories that axion fluctuations would quickly become significant in Minkowski space whereas they would not be significant in an expanding universe setting.

IV. CONCLUSIONS

In this paper we examined the time evolution of the axion field and its quantum fluctuations. This was done by keeping track not only of the zero momentum mode of the axion field but also of the higher non-zero momentum modes of the axion. We studied different kinds of axions: the axion required to solve the strong CP problem in the context of QCD, as well as other axions that would arise in the context of supergravity and string theories. Further, we studied the dynamics in two different space-time settings: in Minkowski space as well as in an expanding radiation dominated FRW universe.

This study was motivated by the recent progress in understanding the important role of phenomena such as spinodal instabilities and parametric resonance instabilities that can lead to a rapid build-up of fluctuations. Indeed, we found that in Minkowski space axion fluctuations rapidly build up in a quasi-exponential manner. In the expanding universe there are a few additional factors which we understand through our quantitative analysis. First, we note that because the axion is a pseudo-Goldstone boson of a compact $U(1)$ symmetry, the potential for the axion is periodic and hence the displacement from the minimum is bounded. This limits the initial amplitude of coherent oscillations of the axion. Further, since the instabilities in the higher momentum modes are driven by the coherent oscillations of the zero momentum mode it is clear that if the amplitude of oscillations drop below some threshold the instabilities will not get a chance to really take off. The result of our analysis is that in fact in an expanding universe quantum fluctuations of the axion do not become significant.

Indeed this result preserves a central piece of axion lore. If the higher non-zero momentum modes had been significantly occupied and quantum fluctuations had not been negligible then it would not have been accurate to think of axions as oscillating coherently and axions would not have been a suitable cold dark matter candidate.

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REFERENCES

[1] Particle Physics and Cosmology: Dark Matter, M. Srednicki (ed.), Elselvier Science Pub., 1990.

[2] The Early Universe, E.W. Kolb and M.S. Turner, Addison - Wesley, 1990.

[3] G. Jungman, M. Kamionkowski and K. Griest, Phys.Rept.267: 195, 1996.

[4] J. Preskill, M. Wise and F. Wilczek, Phys. Lett. **B 120**, 127 (1983).

[5] L. Abbott and P. Sikivie, Phys. Lett. **B 120**, 133 (1983).

[6] M. Dine and W. Fischler, Phys. Lett. **B 120**, 137 (1983).

[7] R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977) ; S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978) ; F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978) ; M. Dine, W. Fischler and M. Srednicki, Phys. Lett. **B 104**, 199 (1981).

[8] T. Banks, M. Dine, "The cosmology of string theoretic axions", hep-th/9609046 ; K. Choi, E. Chun and J. Kim, "Cosmological implications of radiatively generated axion scale", hep-ph/9608222.

[9] A. Guth and S.-Y. Pi, Phys. Rev. **D 32**, 1899 (1985).

[10] E. Weinberg and A. Wu, Phys. Rev. **D 36**, 2474 (1987).

[11] D. Boyanovsky and H. J. de Vega, Phys. Rev. **D47**, 2343 (1993).

[12] D. Boyanovsky, D-S Lee and A. Singh, Phys. Rev. **D 48**, 800 (1993).

[13] J. Traschen and R. Brandenberger, Phys Rev **D42**, 2491 (1990);
Y. Shtanov, J. Traschen and R. Brandenberger, Phys. Rev. **D51**, 5438 (1995).

[14] L. A. Kofman, A. Linde, and A. Starobinksy, Phys. Rev. Lett. **73** 3195 (1994).

[15] D. Boyanovsky, H. J. de Vega, R. Holman, D.-S. Lee and A. Singh,
Phys. Rev. **D51**, 4419 (1995);
D. Boyanovsky, M. D'Attanasio, H. J. de Vega, R. Holman and D. S. Lee,
Phys. Rev. **D52**, 6805 (1995);

[16] D. Boyanovsky, H.J. de Vega, R. Holman, J.F.J. Salgado, Phys. Rev. **D54**, 7570 (1996); astro-ph/9609007.

[17] F. Cooper and E. Mottola, Mod. Phys. Lett. **A2**, 635 (1987); F. Cooper, S. Habib, Y. Kluger, E. Mottola, J. P. Paz, P. R. Anderson, Phys. Rev. **D50**, 2848 (1994); F. Cooper, S. Habib, Y. Kluger and E. Mottola, "Non-Equilibrium Dynamics of Symmetry Breaking in Φ^4 Field Theory", (hep-ph/9610345).

[18] O. Eboli, R. Jackiw and S.-Y. Pi, Phys. Rev. **D37**, 3557 (1988);
M.Samiullah, O. Eboli and S-Y. Pi, Phys. Rev. **D44**, 2335 (1991).

[19] L. A. Kofman, A. D. Linde and A. A. Starobinsky, Phys. Rev. Lett. **76** 1011 (1996).

[20] E.W. Kolb, A.D. Linde and A. Riotto, Phys.Rev.Lett. **77** 4290 (1996).

[21] G. W. Anderson, A. D. Linde and A. Riotto, Phys. Rev. Lett. **77** 3716 (1996).

[22] E.W. Kolb and A. Riotto, astro-ph/9602095 (1996); A. Riotto and I.I. Tkachev, Phys. Lett. **B385**, 57 (1996); S. Yu. Khlebnikov and I.I. Tkachev, Phys. Rev. Lett. **77**, 219 (1996); S. Yu. Khlebnikov and I.I. Tkachev, hep-ph/9608458 (1996).

[23] D.T. Son, Phys. Rev. **D54**, 3745 (1996); hep-ph/9601377, (1996).

[24] D.I. Kaiser, Phys. Rev **D53**, 1776 (1996).

[25] H. Fujisaki, K. Kumekawa, M. Yamaguchi and M.Yoshimura, Phys. Rev. **D53**, 6805 (1996); M. Yoshimura, Progr. Theor. Phys. **94**, 873 (1995); hep-ph/9605246 (1996).

[26] D. Boyanovsky, D. Cormier, H. J. de Vega, R. Holman, A. Singh and M. Srednicki, hep-ph/9609527; hep-ph/9703327.

[27] J. Schwinger, J. Math. Phys. **2**, 407 (1961); P. M. Bakshi and K. T. Mahanthappa, J. Math. Phys. **4**, 1 (1963); *ibid*, 12; L. V. Keldysh, Sov. Phys. JETP **20**, 1018 (1965); A. Niemi and G. Semenoff, Ann. of Phys. (N.Y.) **152**, 105 (1984); Nucl. Phys. B [FS10], 181 (1984); E. Calzetta, Ann. of Phys. (N.Y.) **190**, 32 (1989); R. D. Jordan, Phys. Rev. **D33**, 444 (1986); N. P. Landsman and C. G. van Weert, Phys. Rep. **145**, 141 (1987); R. L. Kobes and K. L. Kowalski, Phys. Rev. **D34**, 513 (1986); R. L. Kobes, G. W. Semenoff and N. Weiss, Z. Phys. **C29**, 371 (1985).

[28] For a thorough exposition of non-equilibrium methods in cosmology see, for example: E. Calzetta and B.-L. Hu, Phys. Rev. **D35**, 495 (1988); *ibid* **D37**, 2838 (1988); J. P. Paz, Phys. Rev. **D41**, 1054 (1990); *ibid* **D42**, 529 (1990); B.-L. Hu in *Bannf/Cap Workshop on Thermal Field Theories: proceedings*, edited by F.C. Khanna, R. Kobes, G. Kunstatter, H. Umezawa (World Scientific, Singapore, 1994), p.309 and in the *Proceedings of the Second Paris Cosmology Colloquium, Observatoire de Paris*, edited by H. J. de Vega and N. Sánchez (World Scientific, Singapore, 1995), p.111 and references therein.

[29] D. Boyanovsky, H. J. de Vega, and R. Holman, Phys. Rev. **D49**, 2769 (1994).

[30] K. Freese, J. Frieman and A. Olinto, Phys. Rev. Lett. **65**, 3233 (1990); F. Adams, J. R. Bond, K. Freese, J. Frieman and A. Olinto, Phys. Rev. **D47**, 426 (1993).

[31] D. Cormier, R. Holman and S. P. Kumar, Work in progress: Private communication.

FIGURES

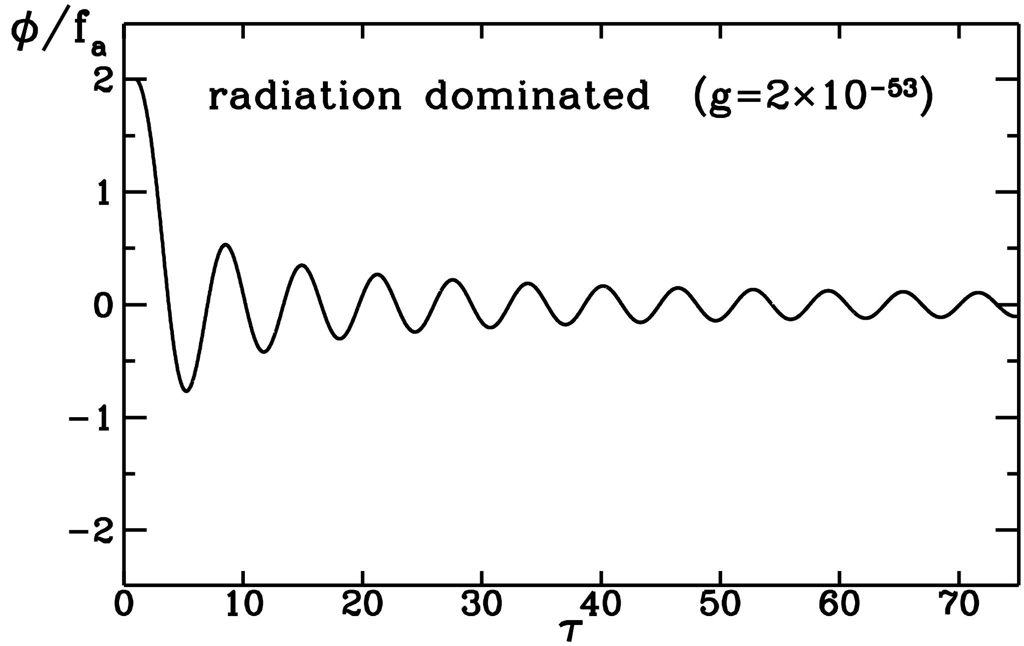

Fig. 1a: Time evolution of the QCD axion zero momentum mode in a radiation-dominated universe.

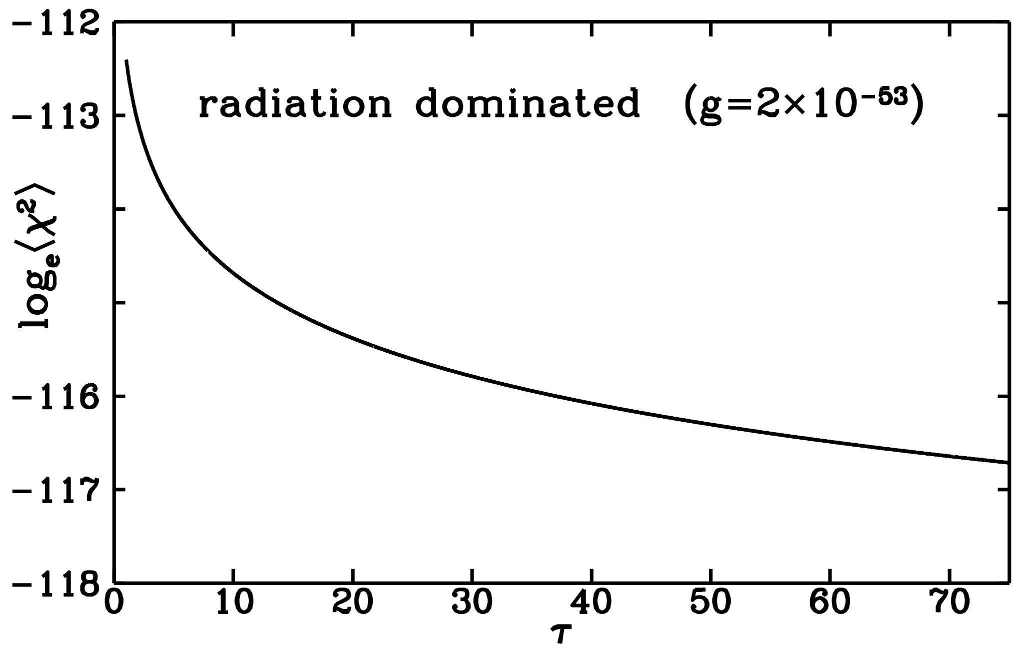

Fig. 1b: Time evolution of the QCD axion fluctuations in a radiation-dominated universe.

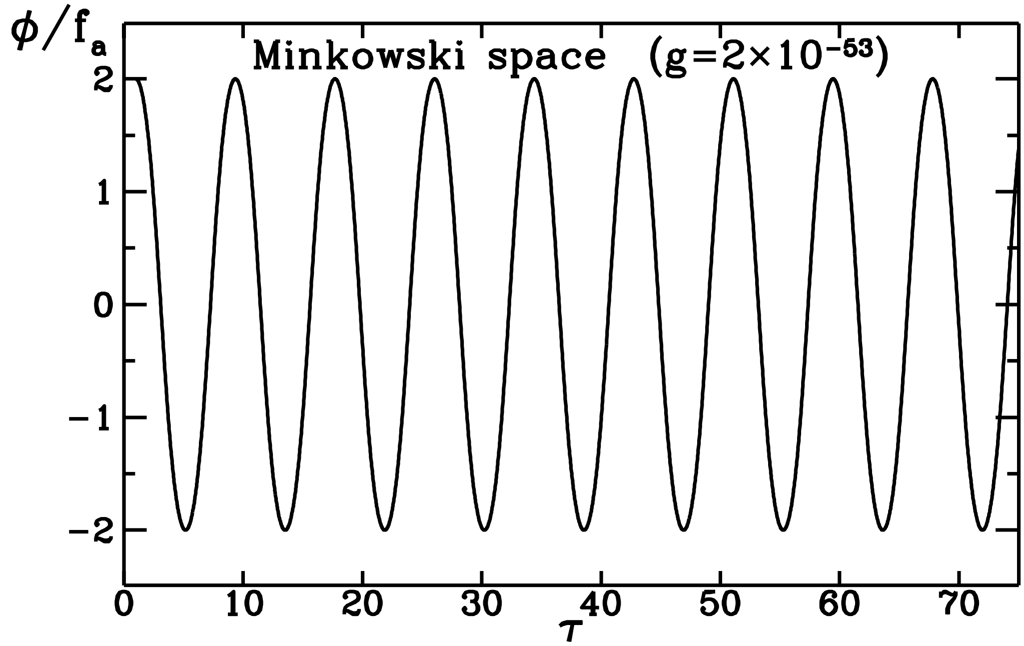

Fig. 2a: Time evolution of the QCD axion zero momentum mode in Minkowski space.

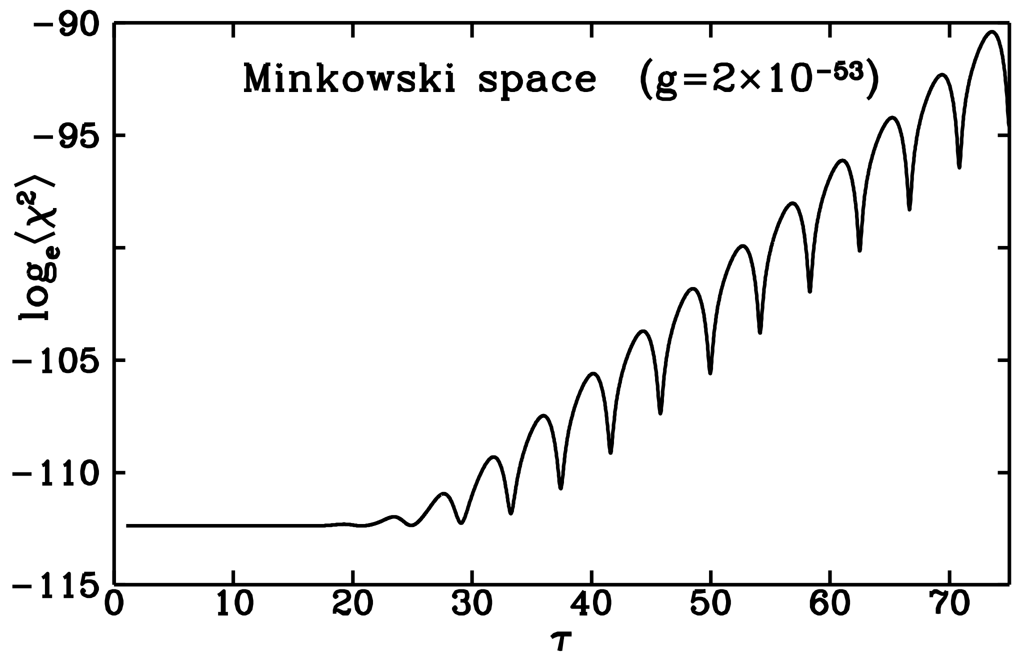

Fig. 2b: Time evolution of the QCD axion fluctuations in Minkowski space.

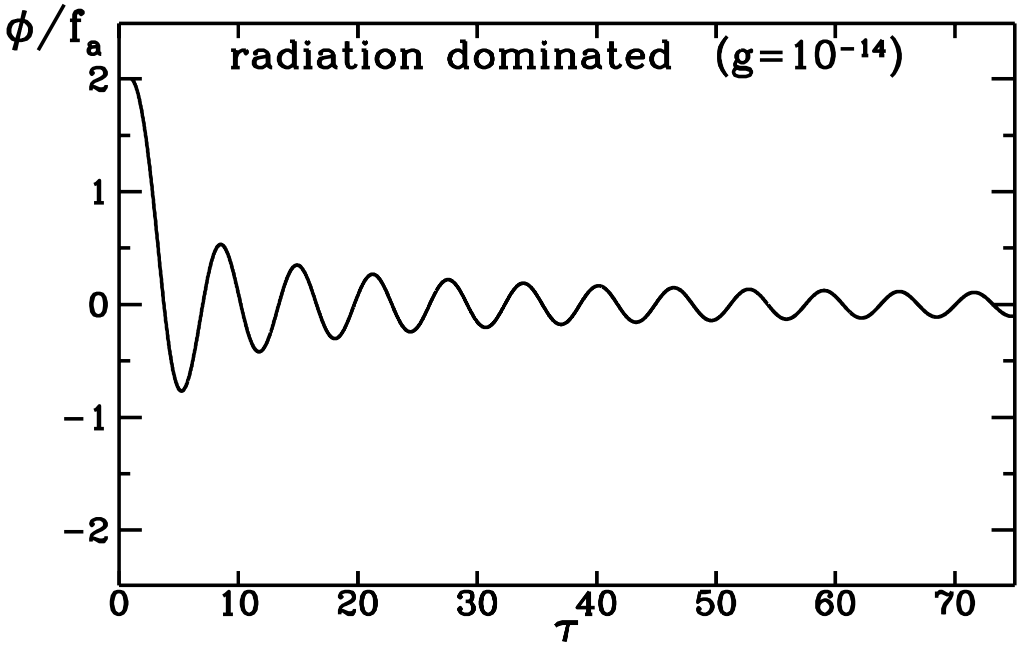

Fig. 3a. Time evolution of the supergravity and string theory axion zero momentum mode in a radiation-dominated universe.

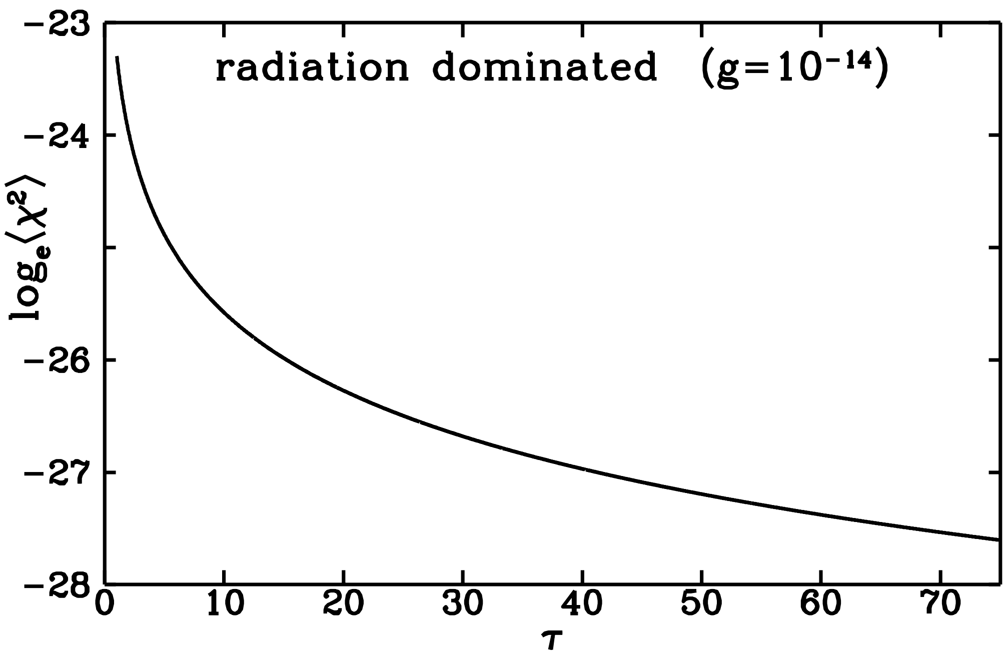

Fig. 3b: Time evolution of the supergravity and string theory axion fluctuations in a radiation-dominated universe.

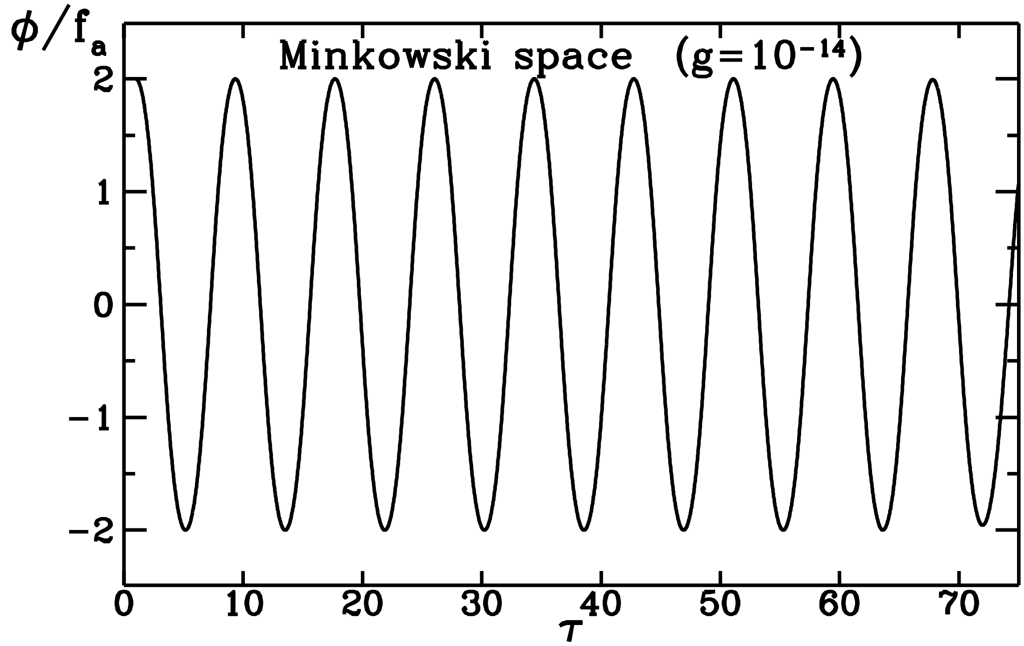

Fig. 4a: Time evolution of the supergravity and string theory axion zero momentum mode in Minkowski space.

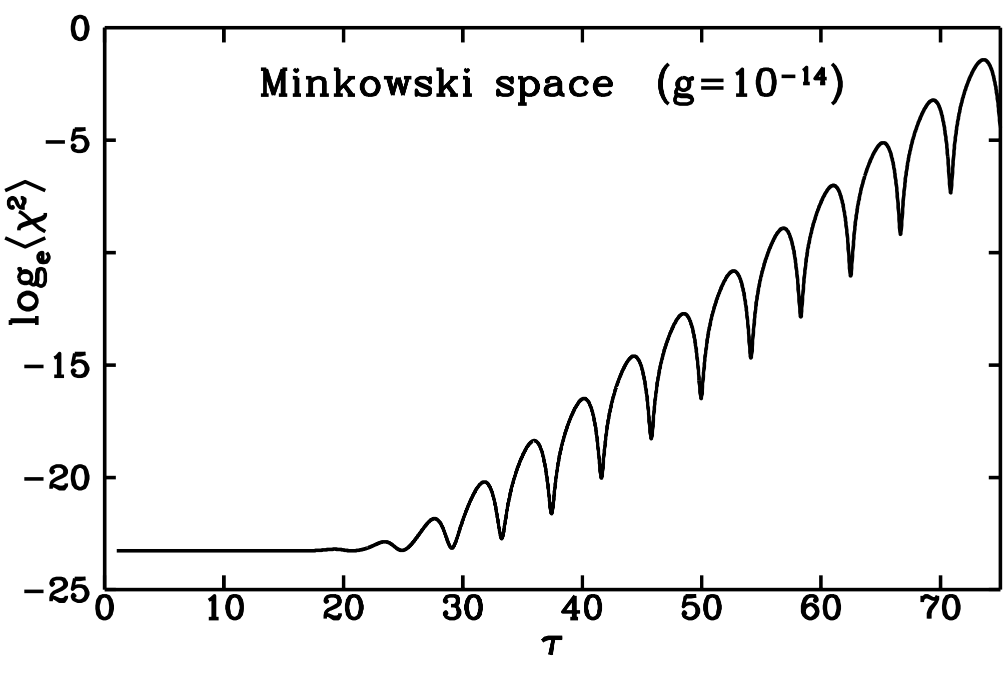

Fig. 4b: Time evolution of the supergravity and string theory axion fluctuations in Minkowski space.