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Dynamical Supersymmetry Breaking

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Abstract

Dynamical supersymmetry breaking is a fascinating theoretical problem. It is also of phenomenological significance. A better understanding of this phenomenon can help in model building, which in turn is useful in guiding the search for supersymmetry. In this article, we review the recent developments in the field. We discuss a few examples, which allow us to illustrate the main ideas in the subject. In the process, we also show how the techniques of holomorphy and duality come into play. Towards the end we indicate how these developments have helped in the study of gauge mediated supersymmetry breaking. The review is intended for someone with a prior knowledge of supersymmetry who wants to find out about the recent progress in this field.

1 Introduction.

Supersymmetry is a beautiful idea in theoretical physics. Unlike any conventional symmetry, it relates bosons and fermions. It has proved important in many of the major theoretical developments in recent times. For example, it plays a vital role in string theory.

There are phenomenological reasons that make supersymmetry attractive as well. The standard model presents us with a puzzle: why is the electroweak scale so much smaller than the Planck scale? This puzzle is called the hierarchy problem. Supersymmetric theories promise to solve this problem. The Higgs particle can be naturally incorporated as a light elementary scalar in these theories. Quadratically divergent contributions to its mass are then automatically canceled by equal and opposite contributions arising from fermions. Moreover, in supersymmetric extensions of the standard model, the large top Yukawa coupling, together with radiative effects, provides a mechanism to break electroweak symmetry.

But these positive features come at a price: by pairing fermions with bosons, supersymmetry doubles the number of known particles. The extra particles must clearly be heavy, leading to the conclusion that supersymmetry must be broken in nature.

We do not have a good understanding of how this breaking of supersymmetry might happen. Theoretically, as we will see, this is a fascinating and challenging question. It is of phenomenological importance as well. The phenomenology of supersymmetric extensions of the standard model depends in an important way on the masses of the superpartners and the other soft parameters, which are all ultimately determined by how supersymmetry breaks. In the absence of a better understanding of supersymmetry breaking, the soft parameters are taken to be arbitrary, resulting in a huge parameter space. This makes a thorough exploration of the resulting phenomenology daunting. A better understanding of the mechanisms of supersymmetry breaking can, in turn, help in exploring scenarios with restricted choices of the soft parameters. Such explorations are useful in guiding the experimental search for supersymmetry.

In this review, we will discuss various mechanisms for dynamical supersymmetry breaking. In the supersymmetric context, the electroweak scale is ultimately related to the supersymmetry breaking scale. Thus, the hierarchy problem can be recast in the form: why is the supersymmetry breaking scale so much smaller than the Planck scale? Dynamical supersymmetry breaking provides the most attractive answer to this question [1]. The idea is that non-perturbative effects in a gauge theory are responsible for supersymmetry breaking. For these effects to be important, the gauge coupling must be large. Moreover, asymptotic freedom tells us that this can happen at a scale much lower than the Planck scale. Thus we have an appealing answer to the hierarchy problem: the electroweak scale is so much lower than the Planck scale because gauge couplings only run logarithmically.

Recently, there has been phenomenal progress in our understanding of the dynamical behavior of supersymmetric gauge theories [2]-[7]. This progress in turn has led to a better understanding of dynamical supersymmetry breaking [8]-[28]. Our main aim is to review some of these developments. Specifically, we will study theories with $N = 1$ global supersymmetry in 4 dimensions. The restriction to $N = 1$ supersymmetry arises because of the phenomenological requirement of chiral matter content. Considering only globally supersymmetric theories is less well motivated. We do so here, first, because the recent progress has mostly been confined to such theories, and, second, because it allows for constructing models where the supersymmetry-breaking dynamics takes place at scales low enough to be observed in the foreseeable future. The study of such models carries a certain phenomenological appeal.

A very large number of models exhibiting dynamical supersymmetry breaking have been constructed, using the newly developed techniques, in the recent past [13]-[28]. Clearly, it would be pointless to try and describe them all. Instead, we will attempt to build up an understanding of supersymmetry breaking by studying a few illustrative examples. The general progression will be from simpler to more complicated theories. As the reader will see, many of the main ideas will recur throughout this study in different contexts. Wherever possible, we will also attempt to make contact with other examples studied in the literature (for short reviews on the subject, see [29]-[32]).

The review is structured as follows. In Section 2, we first provide a very quick overview of some of the recent developments in supersymmetric gauge theories. The discussion is by no means complete and is intended more to remind the reader about some salient features, which will be important in the discussion of supersymmetry breaking. Section 3 is a brief digression, in which we study a supersymmetric quantum mechanics problem with supersymmetry breaking. This provides a convenient setting in which to introduce some of the important ideas. Thereafter we turn to field theories. First, in Section 4, some general features of supersymmetry breaking as well as some simple examples of tree level supersymmetry breaking are discussed. Section 5.1 then deals with calculable models of dynamical supersymmetry breaking. In these models the low-energy effective theory in which supersymmetry breaking occurs can be completely controlled. This allows a great deal to be learned about the resulting supersymmetry breaking ground state. Section 5.2 deals with more complicated theories. In some of these (Sections 5.2.1-5.2.3), we will be able to definitely establish supersymmetry breaking without being able to calculate in detail where the resulting vacuum lies. In other instances (Section 5.2.4), we rely on the global symmetries and the Witten index to plausibly argue that supersymmetry breaking occurs. Finally, in Section 6, we describe how some of these theories of dynamical supersymmetry breaking

might apply to nature in the gauge-mediated supersymmetry breaking scenario.

2 Key ideas in the study of nonperturbative supersymmetric gauge dynamics.

As was mentioned in the Introduction, recently there has been a great deal of progress in our understanding of the non-perturbative dynamics of supersymmetric gauge theories. In this section, we briefly discuss some of the key ideas that have played an important role in these developments. We then also review the case of supersymmetric QCD to illustrate the different kinds of non-perturbative effects that can occur in a gauge theory. For an in-depth discussion of supersymmetric gauge theory dynamics, we refer the reader to the reviews [5], [6], [7].

The recent progress in our understanding of four dimensional $N = 1$ supersymmetric gauge theories was initiated by the work of Seiberg [2], [3], [4] (for a review of important work on the subject in the 1980's, see [33]). Two central ideas have played a particularly important role in these developments:

1. *Holomorphy*. The key realization is that the superpotential of the Wilsonian effective action of supersymmetric theories is a holomorphic function of the chiral superfields.¹ In addition, one can regard the couplings of the theory (the strong coupling scale, Λ , or the various superpotential couplings) as expectation values of nondynamical chiral superfields [2]. The couplings can be assigned charges under various symmetries (which are broken by their expectation values). This leads to certain “selection rules,” restricting how the couplings can appear in the effective action. Now the Wilsonian superpotential has to be a holomorphic function of the chiral superfields as well as the couplings of the theory. Holomorphy in the fields and couplings, together with the requirement of consistency with various limits and the above mentioned “selection rules” allow one to exactly determine the Wilsonian superpotential of the theory in many cases [2].
2. *Duality* is the second crucial ingredient in our understanding of $N = 1$ supersymmetric theories. Generalizing the notion of electric-magnetic duality in Maxwell electrodynamics, Seiberg suggested that, in many cases, the infrared limit of a supersymmetric gauge theory (the “electric” theory) is equivalent to the infrared limit of another supersymmetric gauge theory (the “magnetic” theory) [4]. In some cases, both theories

¹Holomorphy can, in fact, be used to prove [2] the “old” nonrenormalization theorem, that the superpotential is not renormalized at any order of perturbation theory [34].

flow to a nontrivial infrared fixed point and a description of the fixed point in terms of either theory is appropriate (it is said then that both theories are in the “conformal window”), although only one of the descriptions may be weakly coupled. In other cases, there is only one description of the infrared theory, in terms of the either the electric or magnetic degrees of freedom (in these cases the relevant theory is often infrared free). It was also shown that often the exact superpotential in an electric confining theory can be calculated by doing an instanton calculation in a weakly coupled and completely higgsed dual theory. While there is no proof of duality in $N = 1$ theories, Seiberg’s conjecture has survived many nontrivial tests [4], [5], [35], most recently from brane dynamics [36].

Let us illustrate what these insights teach us by studying an $N = 1$ $SU(N_c)$ gauge theory with N_f flavors of quarks (supersymmetric “QCD”). By this we mean N_f chiral superfields, which we denote as Q_i^α , $i = 1, \dots, N_f$ in the \square representation and N_f fields \bar{Q}_α^i , $i = 1, \dots, N_f$ in the $\bar{\square}$ representation. It is useful to study the behavior of this theory as N_f is varied. We start by first considering the case $N_f = N_c - 1$. Classically, the theory has a D-term potential which is set to its minimum when the fields satisfy the conditions:

$$Q^\dagger \cdot T^a Q - \bar{Q}^\dagger (T^a)^* \bar{Q} = 0 \quad (1)$$

for each group generator T^a . These conditions do not select a unique vacuum. Instead, the potential has a set of flat directions. A general result [37] says that the flat directions can be parametrized by gauge invariant chiral superfields. In the present case, there are N_f^2 flat directions. These correspond to the “meson” gauge invariants $M_j^i \equiv \bar{Q}^i \cdot Q_j$. Along these flat directions the $SU(N)$ gauge symmetry is, generically, completely broken. The $SU(N)$ vector multiplets are heavy and the low-energy dynamics can be described in an effective theory containing only the mesons M_j^i .

We now turn to the quantum theory. A non-renormalization theorem [34] states that the flat directions are not lifted at any order in perturbation theory. But they can be lifted non-perturbatively. In fact, in the present case with $N_f = N_c - 1$, such a superpotential is induced by instanton effects in the Wilsonian effective theory for the mesons, M_j^i . It has the form:

$$W_{NP} = C \frac{\Lambda^{b_0}}{\det M} \quad (2)$$

Here Λ is the strong coupling scale of the gauge theory, and $b_0 = 3N_c - N_f$ is the coefficient of the one loop beta function. We note that holomorphy and the various symmetries dictate that a superpotential can only have this form. An explicit constrained instanton calculation then shows that its coefficient C is indeed non-zero [8]. Note that the superpotential results

in a potential energy that goes to zero as some of mesons go to infinity. This is often referred to as “runaway” behavior and results in an unstable ground state.

Now let us vary N_f . When $N_f < N_c - 1$, the moduli are still described by the meson fields M_j^i for the appropriate number of flavors. Generically in moduli space a $SU(N_c - N_f)$ group is left unbroken. This group confines, giving rise at low energies to an effective theory involving only the mesons. The non-perturbative superpotential in this effective theory arises due to gaugino condensation and is given by:

$$W_{NP} = C_{N_c, N_f} \left(\frac{\Lambda^{b_0}}{\det M} \right)^{\frac{1}{N_c - N_f}} . \quad (3)$$

For $N_f > N_c - 1$ things get more interesting. For example, for $N_f = N_c$ one finds that the flat directions include, besides the meson chiral superfields, additional “baryons,” $B \sim Q^{N_c}$ and $\bar{B} \sim \bar{Q}^{N_c}$. These are not all independent. Correspondingly, the quantum theory has a constraint relating them, which is implemented by adding a term in the superpotential:

$$W_{NP} = A \left(\det M - B \bar{B} - \Lambda^{b_0} \right) . \quad (4)$$

Here A is the Lagrange multiplier whose F term implements the constraint. In the quantum theory this moduli space is smooth and the low-energy effective theory in terms of the mesons and baryons is valid everywhere in moduli space. In particular, there is no submanifold of the moduli space where extra degrees of freedom became massless. The theory with $N_f = N_c + 1$ also has a smooth quantum moduli space. The main difference is that the “origin” is part of the moduli space in the quantum theory too. All the global symmetries are unbroken at the origin and the mesons and baryons satisfy the ’t Hooft anomaly matching there as well.

Finally, for $N_f > N_c + 1$ one finds that the theory has a dual “magnetic” description in terms of an $SU(N_f - N_c)$ gauge theory with N_f flavors of quarks, q^i and \bar{q}_i , $i = 1, \dots, N_f$. In addition, there are N_f^2 chiral superfields M_j^i which can be identified with the mesons of the electric theory. Note that in the dual description the mesons are elementary fields present in the microscopic “magnetic” theory. The dual theory also has a tree level superpotential of the form:

$$W = \bar{q}_i M_j^i q^j . \quad (5)$$

This brings us to the end of our lightning review of the recent developments. To summarize, holomorphy and duality help determine the appropriate low-energy degrees of freedom—the ones pertinent to the physics below the strong coupling scale of the gauge theory—and determine the exact superpotential of the low-energy effective theory.

How do these insights help study supersymmetry breaking? We note that the ideas described above are useful in determining the low-energy dynamics of supersymmetric gauge

theories. We will encounter many instances, during the study of supersymmetry breaking, in which, by adjusting an appropriate coupling, the scale of supersymmetry breaking can be made lower than the scale of strong dynamics in the gauge theory. In these cases, the breaking of supersymmetry can be conveniently studied in a low-energy supersymmetric effective theory. The ideas described above will prove very useful then in determining this effective theory and studying its behavior.

The effective theory in these cases will usually only involve a set of chiral superfields, Φ^i . The corresponding Wilsonian effective lagrangian is then given by a supersymmetric nonlinear sigma model:

$$L_{eff} = \int d^4\theta K(\Phi^\dagger, \Phi) + \left(\int d^2\theta W(\Phi) + \text{h.c.} \right) . \quad (6)$$

Here K is the Kähler potential of the low-energy theory (a real function of the chiral superfields Φ^i), and W is the superpotential of the theory—a holomorphic function of the chiral superfields (and couplings). The complete component expansion of (6) can be found in [38]. Here we will only give the expression for the scalar potential of the sigma model (6):

$$V = W_{i^*}^* K^{-1}{}^{i^*j} W_j , \quad (7)$$

where $W_i = \partial W / \partial \Phi^i$, $W_{j^*}^* = \partial W^* / \partial \Phi^{*j}$, and $K^{-1}{}^{i^*j}$ is the matrix inverse to the Kähler metric $K_{ij^*} = \partial^2 K / \partial \Phi^i \partial \Phi^{*j}$; in (7) all functions are understood to depend on the scalar components of the superfields only.

As we will see in Sections 3 and 4, supersymmetry is broken if and only if the vacuum energy is nonvanishing. Since the Kähler metric K_{ij^*} (and its inverse) is a positive definite matrix—so that L_{eff} makes sense as a physical theory—the potential² V from (7) is positive semi-definite. It vanishes only if the F term conditions, $W_i = \partial W / \partial \Phi^i = 0$ are met for all the fields ϕ_i . If, on the other hand, these F term conditions cannot all be met, the vacuum energy must necessarily not vanish and supersymmetry is broken. Once we have found, by holomorphy and duality, the correct degrees of freedom and the exact superpotential of the low-energy effective theory, we can say with certainty whether a given theory breaks supersymmetry.

Upon inspection of the effective Lagrangian (6), one sees that other important physical properties of the low-energy theory, such as the expectation values of the fields, the vacuum energy, the masses and interactions of the light fields, depend on the Kähler potential. In some cases, we will be able to explicitly determine it, while in other cases we will be at least able to establish that the corresponding Kähler metric is non-singular.

²Strictly speaking, to find the ground state energy one has to use the 1PI rather than the Wilsonian effective action. The superpotentials of the 1PI and Wilsonian effective actions, however, are identical because of the nonrenormalization theorem (for discussions see [5], [7], [39]).

3 A toy model.

In this section, we digress from the study of field theories to explain some key ideas of supersymmetry breaking in a quantum-mechanical setting. We will consider in some detail Witten's supersymmetric quantum mechanics. This example will be used to introduce several important concepts: the order parameter for supersymmetry breaking and the Witten index. The hope is, that by encountering them in a simpler context the reader will gain a better appreciation for these ideas.

3.1 Supersymmetry breaking in quantum mechanics.

The quantum mechanical system is that of a spin-1/2 particle moving on the line [1]. The state of the spin-1/2 particle is described by a two-component wave function $\Psi(x)$; the two components of Ψ are the wave functions of the particle with spin projections $+1/2$ and $-1/2$, respectively. The supersymmetric Hamiltonian is:

$$H = \frac{1}{2} p^2 + \frac{1}{2} W'(x)^2 + \frac{1}{2} \sigma_3 W''(x) . \quad (8)$$

Here and below $\sigma_{1,2,3}$ denote the Pauli matrices, $W'(x) = dW/dx$, etc. The supersymmetry generators are:

$$Q_1 = \frac{1}{2} \sigma_1 p + \frac{1}{2} \sigma_2 W'(x) , \quad Q_2 = \frac{1}{2} \sigma_2 p - \frac{1}{2} \sigma_1 W'(x) . \quad (9)$$

They obey the supersymmetry algebra:

$$\{ Q_i , Q_j \} = \delta_{ij} H , \quad i, j = 1, 2, \quad (10)$$

with H given by (8). The function $W(x)$ is called the superpotential; it completely determines the interactions (in order to fully underline the analogy with quantum field theory, we have slightly changed notations from [1]). Note that the Hamiltonian (8) is similar to the one obtained in $3 + 1$ dimensional renormalizable supersymmetric field theory with spin-0 and spin-1/2 fields only: all interactions are derived by the derivatives of a single function, the superpotential $W(x)$. In the field theory case, the "spin-orbit" term corresponds to the Yukawa interaction between the bosons and fermions in the supermultiplet. Also, we see from (10) that the supersymmetry generators transform the $+1/2$ eigenstate of σ_3 to the one with eigenvalue $-1/2$. Thus, these two eigenstates are the analogue of bosons and fermions in this quantum mechanics problem. Note, in particular, that the Hamiltonian, eq. (8) commutes with σ_3 and does not change "fermion number".

The first issue we want to discuss is the order parameter for supersymmetry breaking. The spontaneous breaking of supersymmetry means that even though the dynamics is invariant

under supersymmetry, the ground state is not. The noninvariance of the ground state $|0\rangle$ under supersymmetry transformations implies that the supersymmetry generators Q_i do not annihilate the ground state, $Q_i|0\rangle \neq 0$. Consider now the ground state energy of the system, E_0 , and the following chain of equalities:

$$E_0 \equiv \langle 0 | H | 0 \rangle = 2 \langle 0 | Q_i Q_i | 0 \rangle = 2 \| Q_i | 0 \rangle \|^2 > 0, \text{ iff } Q_i | 0 \rangle \neq 0, \quad (11)$$

where we used the fact that the supersymmetry algebra (10) relates the supersymmetric Hamiltonian to the square of the supersymmetry generators (there is no sum over i in eq. (11)). The inequality in (11) holds whenever supersymmetry is broken, i.e. $Q_i|0\rangle \neq 0$. We thus see that the ground state energy of a supersymmetric system is positive if and only if supersymmetry is broken and zero if and only if supersymmetry is unbroken. The ground state energy is therefore the order parameter for supersymmetry breaking. We note that this conclusion trivially generalizes to quantum field theory: the relativistic supersymmetry algebra reduces to (10) in the rest frame of the system.

At the classical level—ignoring the spin-orbit interaction and the zero-point energies—it is easy to see whether supersymmetry is broken or not. We only have to look at the graph of the potential energy $V \sim W^2$. We have shown three possibilities on Fig. 1. Fig. 1a shows a potential which is everywhere positive. Thus, classically, the ground state energy is positive and supersymmetry is broken. The potentials on Fig. 1b,c both allow for classical states of zero energy, hence, classically, supersymmetry is unbroken.

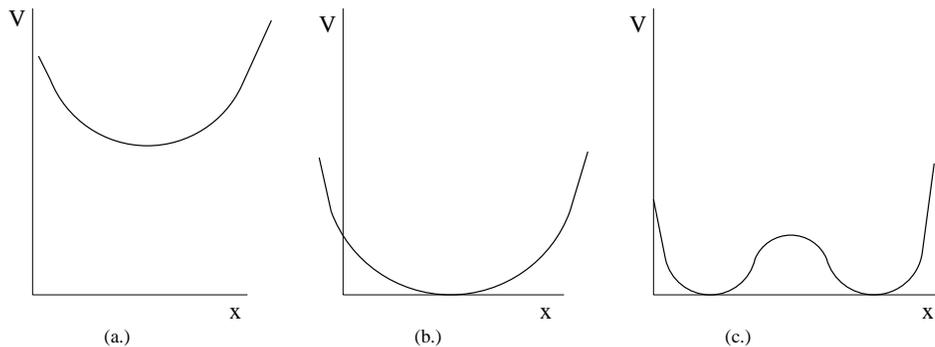


Figure 1: The three potentials discussed in the text: (a.) supersymmetry broken at tree level, (b.) supersymmetry unbroken, (c.) supersymmetry unbroken at tree level, but broken due to instantons (tunneling between the wells).

The classical approximation is, of course, not the whole story. It is natural to ask whether quantum corrections can change the classical answer. In supersymmetric systems, as we will see throughout this article, it is often possible to give exact answers to questions about the ground state. As discussed above, supersymmetry is unbroken if and only if there is a normalizable zero energy state (we assume here that the system has a discrete spectrum).

Finding the zero-energy state implies solving the second order Schrödinger equation $H|0\rangle = 0$. Now eq. (11) shows that $E_0 = 0$ if and only if $Q_i|0\rangle = 0$; hence, it suffices, instead, to solve the first order equation $Q_i|0\rangle = 0$ (see the definition of Q_i , eq. (9)). While a general second order equation can only be solved numerically, the corresponding first order equation can be solved for an arbitrary superpotential $W(x)$. Using simple Pauli matrix algebra, it is easy to check that

$$\Psi_0(x) = e^{\sigma_3 W(x)} \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} e^{W(x)} c_1 \\ e^{-W(x)} c_2 \end{pmatrix} \quad (12)$$

is the general solution for the zero-eigenvalue wavefunction. Ψ_0 depends on two integration constants c_1, c_2 and is normalizable only in two cases: either $c_1 = 0$ and $W(x) \rightarrow +\infty$ as $x \rightarrow \pm\infty$, or $c_2 = 0$, while $W(x) \rightarrow -\infty$ as $x \rightarrow \pm\infty$. Thus a normalizable ground state of zero energy exists only if the superpotential $W(x)$ is “even at infinity,” i.e. has the same limit (+ or $-\infty$) at both $x = +\infty$ and $x = -\infty$. A smooth function $W(x)$ with this property will necessarily have an odd number of extrema (and its derivative W' —an odd number of zeros). Equivalently, since $V(x) = (W')^2/2$, we find that the criterion for unbroken supersymmetry is that the potential has an odd number of zeros.

We can now revisit the three potentials on Fig. 1 and find whether supersymmetry is broken or not in the exact ground state. The potential on Fig. 1a has no zeros, hence according to our criterion, supersymmetry, being broken at the tree level, remains broken once quantum corrections are included. The potential on Fig. 1b has one minimum, hence supersymmetry remains unbroken in the quantum theory. Finally, in the case of Fig. 1c, the potential has an even number of zeros. Therefore, even though supersymmetry is unbroken at the classical level, it is broken by quantum effects. We will see that all three cases have counterparts in quantum field theory.

It is the case depicted on Fig. 1c that will be of most interest for us. The reason is that the breaking of supersymmetry in the supersymmetric system with a double-well potential is due to nonperturbative effects—it occurs because of tunneling between the two wells. We found earlier that in the classical approximation (and, even though we did not show this, also in perturbation theory, including the zero-point energy and the spin-orbit interaction) the ground state energy vanishes and supersymmetry is unbroken. The effect of tunneling can be evaluated in the semiclassical approximation [40]. The WKB formula for the ground state energy splitting gives $E_0^{WKB} = \langle 0|\hat{H}|0\rangle \sim \hbar\omega \exp\left(-\frac{1}{\hbar} \int dx \sqrt{2V(x)}\right) \ll \hbar\omega$ where ω is the frequency of classical motion near the bottom of the well, and the integral is over the classically forbidden region of x . Since, for appropriate parameters of the potential (or, in the semiclassical $\hbar \rightarrow 0$ limit), the tunneling probability is exponentially suppressed, the scale of supersymmetry breaking—the ground state energy—is much smaller than the characteristic frequency of motion inside the wells.

The generation of small scales, described above, also occurs naturally in many field theory models of dynamical supersymmetry breaking and is the key property that makes them interesting candidates for explaining the hierarchy of scales in nature.

3.2 The Witten index.

In the remainder of this section we will introduce another important concept in the study of supersymmetric theories: the Witten index [41]. As we saw in our discussion of the supersymmetric quantum mechanical model, whether supersymmetry is broken or not depends only on the behavior of the superpotential $W(x)$ at large x . Consequently, any continuous change of $W(x)$ that does not change its asymptotics at infinity will have no effect on whether the model breaks supersymmetry. This is an indication that the issue of supersymmetry breaking has topological nature: it depends only on asymptotics and global properties of the theory. As a measure whether supersymmetry can break or not in a given model, Witten introduced the index, $\text{Tr}(-1)^F$, with F —the fermion number:

$$\text{Tr} (-1)^F \equiv \sum_E n_B(E) - n_F(E) = n_B(0) - n_F(0) . \quad (13)$$

Here $n_{B(F)}(E)$ denotes the number of bosonic (fermionic) states of energy E . The reason for the second equality is that in a supersymmetric system every bosonic state of nonvanishing energy is degenerate with a fermionic state (its superpartner), hence $n_B(E) = n_F(E)$ for $E \neq 0$, and only the zero energy states contribute to the index.³ Note that since supersymmetry is unbroken if the theory has a zero energy state, $\text{Tr}(-1)^F \neq 0$ implies that the vacuum is supersymmetric.

The main utility of the index (13) is that it is invariant under changes of the Hamiltonian that do not change the asymptotics of the potential (i.e. changing the Hamiltonian such that the added terms do not grow faster at infinity than the ones already present). This is because under continuous changes of the parameters states can leave or descend to the zero energy level, but can do so only in pairs (because of the doubling of all $E \neq 0$ levels), and hence do not affect the index. Because of this invariance, a calculation of the index at weak coupling (i.e. in perturbation theory) allows one to deduce information about the ground state even at strong coupling.

We note that if a calculation of the index yields $\text{Tr}(-1)^F = 0$, without separate knowledge of whether zero energy states exist ($n_B(0) = n_F(0) \neq 0$) one can not decide whether supersymmetry is broken. In the case of vanishing index, under continuous deformations of

³The pairing of nonzero energy states is true whether or not supersymmetry is broken—in the case of broken supersymmetry, every state is degenerate with the state obtained from it by adding a zero-momentum goldstino (Sect. 4.2).

the parameters of the Hamiltonian, all states can leave zero energy, so it is possible that supersymmetry is broken for some values of the parameters and not for others. In this case, more dynamical information is required to find whether the ground state is supersymmetric.

As an example of the application of the Witten index, we can quickly calculate it in our quantum mechanical model (we define $F = 1$ for spin projection $|+1/2\rangle$ and $F = 0$ for $|-1/2\rangle$), for the three potentials of Fig. 1. In perturbation theory, the potential of Fig. 1a does not allow for any zero-energy states, hence the index vanishes (and, as eq. (12) shows, there are no states of zero energy in the exact solution, so supersymmetry is broken). The potential of Fig. 1b allows for a single zero energy state (in the harmonic approximation near the minimum, depending on the sign of W'' , it is either bosonic or fermionic), hence $\text{Tr}(-1)^F = \pm 1$ (say), and supersymmetry is unbroken, even when all quantum effects are taken into account. Finally, Fig. 1c has two perturbative zero-energy states—in the harmonic approximation to eq. (8) near each of the minima, one of them has spin $+1/2$, and the other $-1/2$ —so $\text{Tr}(-1)^F = 0$ and supersymmetry can be broken (and, as the exact solution, eq. (12) shows, indeed is).

We should also note that by continuously changing the parameters we can not interpolate between the theories of Fig. 1a and Fig. 1b (or Fig. 1b and Fig. 1c) without changing the Witten index. In order to deform, say, the potential of Fig. 1b to that of Fig. 1c, we would have to change the asymptotic behavior of the superpotential $W(x)$ from being even to being odd at infinity. This change of asymptotic behavior causes vacua to “appear” (or “disappear”) from infinity (i.e. the second minimum of Fig. 1c). We will see examples [19] of such behavior when we consider field theory models; see Section 5.2.2.

Finally, we add some comments on the Witten index, $\text{Tr}(-1)^F$, in field theory. Witten [41], [42] calculated the index in pure supersymmetric Yang-Mills theory (i.e. without matter), and found it nonvanishing. Thus, pure SYM theory does not break supersymmetry. A corollary from Witten’s result is that vectorlike gauge theories without classical flat directions (or, which is the same, with added mass terms for all matter fields) also do not break supersymmetry. This is because at low energies the vectorlike theories with massive matter flow to pure SYM, for which the index calculation gives a nonzero result. This argument can fail in vectorlike theories with classical flat directions (since such theories have a moduli space which renders the index ill-defined at the classical level) [41, 19]. Even so, we will see that most known theories exhibiting dynamical supersymmetry breaking are chiral.

4 Supersymmetry breaking in field theory.

4.1 The order parameter.

We start this section by first reviewing some general features associated with supersymmetry breaking in quantum field theory. The order parameter for (global) supersymmetry breaking is simply the vacuum energy. To see this we note that the $N = 1$ four-dimensional supersymmetry algebra [38]

$$\{Q_\alpha, \bar{Q}_{\dot{\alpha}}\} = -2 \sigma_{\alpha\dot{\alpha}}^\mu P_\mu \quad (14)$$

reduces, in the rest frame of the system, $P_0 = H, \vec{P} = 0$, after appropriate rescaling, to the nonrelativistic algebra (10). Thus, the arguments following eq. (10) can be repeated in the field theory case, showing that the order parameter for supersymmetry breaking is the vacuum energy.

4.2 Goldstone fermions.

A straightforward generalization of Goldstone's theorem implies, very generally, that if global supersymmetry is broken there must be a massless fermion in the spectrum, coupling to the supercurrent. It is called a goldstino. The basic idea is to consider a Green's function, $G_{\beta\dot{\alpha}}^\mu(x) = \langle 0|T S_\beta^\mu(x) \bar{\psi}_{\dot{\alpha}}(0)|0\rangle$, involving the supercurrent, S_β^μ , and a fermionic field $\bar{\psi}_{\dot{\alpha}}$. Since the current is conserved, we have:

$$\int d^4x \partial_\mu G_{\beta\dot{\alpha}}^\mu = \langle 0| \{Q_\beta, \bar{\psi}_{\dot{\alpha}}(0)\} |0\rangle, \quad (15)$$

(the supercharge is, as usual, the integral of the zeroth component of the supercurrent, $Q_\beta = \int d^3x S_\beta^0(x)$), or, equivalently in momentum space:

$$i P_\mu G_{\beta\dot{\alpha}}^\mu(P) \Big|_{P_\mu \rightarrow 0} = \langle 0| \{Q_\beta, \bar{\psi}_{\dot{\alpha}}(0)\} |0\rangle. \quad (16)$$

The anticommutator above can be non-zero only if supersymmetry is broken. Further, if it is nonvanishing, we find from eq. (16) that there must be a massless particle in the spectrum, giving rise to a pole in the Green's function $G_{\beta\dot{\alpha}}^\mu$ at zero momentum. By inserting a complete set of states on the right hand side it becomes clear that this particle must be a fermion, $\bar{\eta}$, with coupling to the supercurrent $\langle 0|S_\alpha^\mu|\bar{\eta}_{\dot{\alpha}}\rangle = f \sigma_{\alpha\dot{\alpha}}^\mu$.⁴

There are two kinds of multiplets in an $N = 1$ theory in 4 dimensions—chiral multiplets and vector multiplets. Let us denote the corresponding fermions by ψ and λ . Under supersymmetry these transform as [38]:

$$\delta_\zeta \psi = i\sqrt{2}\sigma^\mu \bar{\zeta} \partial_\mu \phi + \sqrt{2}\zeta F, \quad (17)$$

⁴Taking the supercurrent $\bar{S}_{\dot{\alpha}}^\mu$ instead of $\bar{\psi}_{\dot{\alpha}}$ in (16) and using the supersymmetry algebra $\{Q_\alpha, \bar{S}_{\dot{\alpha}}^\nu(x)\} = -2\sigma_{\alpha\dot{\alpha}}^\mu T_\mu^\nu(x)$, see eq. (14), one can also relate the vacuum energy density, E_0 , to the goldstino coupling, $E_0 = f^2$ [1].

$$\delta_\zeta \lambda = \sigma^{\mu\nu} \zeta F_{\mu\nu} + i\zeta D, \quad (18)$$

where F and D are the auxiliary fields of the chiral and vector multiplet, respectively, ϕ is the scalar component of the chiral multiplet, and $F_{\mu\nu}$ is the field strength of the gauge field of the vector multiplet. Either of these two kinds of fermions can be present in the anticommutator in (16). The first two terms in eqs. (17) and (18) cannot acquire vacuum expectation values, since Lorentz invariance is unbroken. Thus the condition for supersymmetry breaking is that some auxiliary component, either F or D , must acquire a vacuum expectation value. In general both F and D terms could get such vevs, correspondingly the goldstino will generally be a combination of the fermions ψ and λ .

One distinction between bosonic symmetries and supersymmetry is worth pointing out. For a broken bosonic symmetry, the Goldstone boson is associated with long wavelength fluctuations (“spin waves”) along the flat direction of the potential associated with the global symmetry. In contrast, for broken supersymmetry, a goldstino arises even when the vacuum is unique.

4.3 Simple examples of F - and D -type supersymmetry breaking.

It is useful to begin the study of supersymmetry breaking in field theory by studying a simple example, called an O’Raifeartaigh model, which does not involve any gauge fields—in this case supersymmetry breaking will occur because an auxiliary F component acquires a vev. As we will see below, the low-energy dynamics in more complicated situations will often reduce to a model of this type. The example we consider here [43], has three fields, ϕ_1, ϕ_2, ϕ_3 , with a conventional Kähler potential, $K = \sum_{i=1}^3 \phi_i^\dagger \phi_i$, and a superpotential given by:

$$W = m \phi_1 \phi_2 + \lambda (\phi_1^2 - a^2) \phi_3. \quad (19)$$

The corresponding scalar potential can then be shown to be, using (7):

$$V = |m \phi_1|^2 + |\lambda (\phi_1^2 - a^2)|^2 + |m \phi_2 + 2 \lambda \phi_3 \phi_1|^2, \quad (20)$$

where the three terms on the right hand side are the squares of the F components of ϕ_2, ϕ_3 , and ϕ_1 , respectively. One can see immediately that the first two terms on the right hand side cannot both be zero, thus the vacuum energy must be non-zero and supersymmetry is broken. If $|m|^2 > 2|\lambda^2 a^2|$, one finds that the global minimum lies at $\phi_1 = \phi_2 = 0$, correspondingly F_3 —the auxiliary component of ϕ_3 —acquires a vev. The potential, eq. (20) has a flat direction which corresponds to varying ϕ_3 ; it can therefore take any value. Vacua corresponding to different values of ϕ_3 are physically different; for example, the spectrum of the theory depends on $\langle \phi_3 \rangle$.

Since supersymmetry is broken, we do not expect, in general, bosons and their fermionic partners to be equal in mass. It is easy to see that this is in fact true. For example, working in the vacuum where $\langle\phi_3\rangle = 0$ one finds the following excitations. In the bosonic spectrum, ϕ_3 has zero mass, ϕ_2 has mass m , and two real scalar fields which arise as combinations of ϕ_1 and ϕ_1^\dagger have masses, $|m|^2 \pm 2|\lambda^2 a^2|$. In the fermionic spectrum, ψ_3 (the fermionic partner of ϕ_3) is massless, while ψ_1 and ψ_2 pair together with a Dirac mass m . We see thus that the degeneracy between ϕ_1 and ψ_1 is lifted. Notice further that there is one massless fermion, ψ_3 , it is the goldstino. This is accord with the fact that F_3 acquired a vev in this vacuum.

We saw above that the potential, eq. (20), does not uniquely determine $\langle\phi_3\rangle$ and the classical theory has a flat direction. Since supersymmetry is broken we expect quantum effects to lift this flat direction and to pick out a unique value of $\langle\phi_3\rangle$. The quantum effects enter through the Kähler potential which is perturbatively renormalized. The classical vacuum energy (20) of the O’Raifeartaigh model, in the vacuum with $\phi_1 = \phi_2 = 0$ and ϕ_3 —arbitrary, is:

$$V_{class} = \lambda^2 a^4 . \tag{21}$$

The leading dependence of the quantum effects is incorporated in eq. (21) by noting that λ is a running coupling that depends on the scale of the expectation value. Since the Yukawa coupling is not asymptotically free, it increases logarithmically upon increasing ϕ_3 . Thus it turns out, after a one-loop effective potential calculation is performed [44], that the minimum of the potential is attained when $\phi_3 = 0$. We note (and we will point out examples later) that the stabilization of classical flat directions by perturbative corrections to the Kähler potential has important model building applications [45, 46, 47, 48, 49].

Finally, we give an example of D -type (“Fayet-Iliopoulos type”) supersymmetry breaking. D -type breaking can only occur in Abelian gauge theories—it is possible to show that (at tree level) supersymmetry breaking in non-Abelian theories is controlled by F-terms only [38]. We will not give other examples of D -type breaking in this review. We would only like to stress that Fayet-Iliopoulos-type supersymmetry breaking may be of phenomenological relevance. The occurrence of $U(1)$ factors of the gauge group with Fayet-Iliopoulos terms is common in string theory compactifications [50]. The relevant $U(1)$ factors are usually anomalous (the anomalies are canceled by the Green-Schwarz mechanism) and generate Fayet-Iliopoulos terms at one loop. The model discussed below illustrates this rather generic mechanism of supersymmetry breaking.

As an example of D -type breaking we consider a $U(1)$ supersymmetric gauge theory with two “electrons”—two chiral superfields, Q and \bar{Q} , with $U(1)$ charge $+1$ and -1 , respectively. The Kähler potential and the superpotential are:

$$K = Q^\dagger e^V Q + \bar{Q}^\dagger e^{-V} \bar{Q} + 2 \xi_{FI} V ,$$

$$W = m Q \bar{Q} , \quad (22)$$

where V denotes the $U(1)$ vector superfield and ξ_{FI} is the Fayet-Iliopoulos term (which has dimension of mass squared and can be easily seen to be gauge invariant, see [38]). The scalar potential of the model is:

$$V = |m Q|^2 + |m \bar{Q}|^2 + \frac{1}{8} |Q^\dagger Q - \bar{Q}^\dagger \bar{Q} + 2\xi_{FI}|^2 . \quad (23)$$

The first two terms in V are the F terms of \bar{Q} and Q , respectively, while the last term is the square of the D term of the vector multiplet. It is easy to see from eq. (23), that if the Fayet-Iliopoulos term vanishes, $\xi_{FI} = 0$ (and $m \neq 0$) the vacuum occurs for $Q = \bar{Q} = 0$ and supersymmetry is unbroken. On the other hand, if both the mass and the Fayet-Iliopoulos term are nonvanishing, supersymmetry is clearly broken. The breaking of supersymmetry is D type if $m^2 > \xi_{FI}/2$ and the $U(1)$ gauge symmetry is unbroken (the goldstino field then is the gaugino, as is clear from its supersymmetry transformation law, eq. (18)). On the other hand, when $m^2 < \xi_{FI}/2$ supersymmetry breaking is of mixed $F - D$ type and the gauge symmetry is broken (the goldstino field is then a mixture of the gaugino and the fermionic components of Q, \bar{Q} [38]). We note that the model discussed above is an example of a model with vanishing Witten index—it breaks supersymmetry for nonvanishing $\xi \neq 0$, while for $\xi = 0$ supersymmetry is unbroken.

4.4 Broken global symmetries and supersymmetry breaking.

It is also useful to comment at this stage on the relation between R symmetries and supersymmetry breaking. Symmetries which do not commute with the supersymmetry generators are called R symmetries.⁵ Consider a situation where the dynamics responsible for supersymmetry breaking can be described by an effective theory involving only chiral superfields. We denote these fields by $\phi_i, i = 1, \dots, n$, in the discussion below. Supersymmetry is unbroken if

$$F_i = \frac{\partial W}{\partial \phi_i} = 0 , \quad (24)$$

for all fields ϕ_i . Eq. (24) imposes n holomorphic conditions on n complex variables (the ϕ_i). If the superpotential $W(\phi_i)$ is generic, it should be possible to satisfy all these conditions and supersymmetry is not broken.

We now investigate how things change if the superpotential preserves a global symmetry. For a non- R symmetry one can show that that the above argument goes through essentially

⁵We use the convention of [38], where the R charge of the superspace coordinates, θ_α , equals 1. Thus, the fermion component of a chiral superfield of R charge q has R charge $q - 1$, the superpotential in an R symmetric theory has R charge 2, the fermions in vector multiplets (gauginos) have R charge 1, etc.

unchanged. The global symmetry means that the superpotential only depends on appropriate combinations of the ϕ_i which are singlets of the global symmetry. In terms of these reduced number of degrees of freedom, eq. (24) imposes an equally reduced number of conditions, again leading to unbroken supersymmetry.

However, for an R symmetry things can be different. In this case the superpotential is not invariant and has an R charge 2. If the field ϕ_1 is charged under the R symmetry, W can be written as (we are assuming here that ϕ_1 has a expectation value, breaking thus the R symmetry):

$$W = \phi_1^{2/q_1} f(X_i), \quad X_i = \phi_i \phi_1^{-\frac{q_i}{q_1}}. \quad (25)$$

Now, for supersymmetry to be unbroken we have the conditions $W_i = 0$, or, equivalently:

$$\frac{\partial f}{\partial X_i} = 0, \quad (26)$$

and

$$f(X_i) = 0. \quad (27)$$

Notice, that these are n equations but in $n - 1$ variables. Generically they will not be met and supersymmetry is broken [51].

The above discussion leads us to believe that a broken R symmetry is necessary for supersymmetry breaking. One way in which this conclusion can be avoided is if, unlike what was assumed above, the superpotential is not generic. The superpotential is, after all, protected from corrections in perturbation theory by a non-renormalization theorem. Corrections can be generated non-perturbatively but these are often of a very special form. Thus in several instances the superpotential is non-generic and supersymmetry is broken even in the absence of an R symmetry. Another way in which this conclusion is avoided is if the underlying theory does not possess an R symmetry, but the R symmetry arises as an accidental symmetry in the superpotential of the effective theory—involving the fields ϕ_i —discussed above [13], [26]. Once again, this can happen because non-perturbative effects lead to corrections to the superpotential of restricted form. In this case the R symmetry will be broken by higher dimensional operators in the Kähler potential. In this context we should also mention that upon coupling to supergravity, the continuous R symmetry is always broken by the constant term in the superpotential needed to cancel the cosmological constant [54]. Finally, our discussion assumed that the relevant effective theory only contained chiral superfields. This is not true in general, as we will see below in our discussion of non-calculable models. The argument above does not apply to such situations, although in several cases of this type an R symmetry is present and in fact the corresponding 't Hooft anomalies play an important role in establishing the breaking of supersymmetry [11].

Another relation between broken global symmetries and broken supersymmetry is the following [8]: if the theory has no classical flat directions and has a broken global symmetry, then supersymmetry is broken. To see this, note that if a global symmetry is broken while supersymmetry is unbroken, the Goldstone boson of the broken symmetry has a massless scalar supersymmetric partner. Since the Goldstone boson is the phase of the order parameter, its supersymmetry partner corresponds to a dilatation of the order parameter, and thus represents a flat direction of the theory.⁶ But the theory has no classical flat directions, and it is unlikely that strong coupling dynamics will lead to their appearance. One thus concludes that supersymmetry is broken in a theory with no classical flat directions and a broken global symmetry.

5 Models of dynamical supersymmetry breaking.

In the previous section, we studied some general features of supersymmetry breaking in field theory and gave some examples, where supersymmetry breaking occurred at tree level. As was discussed in the Introduction, both from the theoretical and phenomenological points of view it is much more interesting to explore the problem of non-perturbative supersymmetry breaking. This is the question to which we now turn.

We saw in our discussion of supersymmetric QCD that supersymmetric gauge theories generically have flat directions at the classical level. Non-renormalization theorems tell us that these directions are not lifted in perturbation theory but, as we saw in Section 2, they can be lifted by non-perturbative effects. The basic idea in dynamical supersymmetry breaking is to involve these non-perturbative effects in an essential way in the lifting of flat directions, leading to a non-zero vacuum energy and thus supersymmetry breaking.

There has been a great deal of research in dynamical supersymmetry breaking in the recent past and many new examples exhibiting this phenomenon have been constructed [13]-[28]. It would be inappropriate to discuss all of them here. Instead, we will talk about a few illustrative examples and content ourselves by providing references to the rest of the literature. In organizing the discussion, it is useful to begin by thinking about the various energy scales involved in the problem. The non-perturbative effects are characterized by the strong coupling scale, Λ , of a gauge theory. In studying supersymmetry breaking, it is helpful if the scale of supersymmetry breaking can be made much lower than the scale Λ . There are a few good reasons for this. First, in many cases, once the two scales are

⁶Unless the low energy theory is described by a compact supersymmetric nonlinear sigma model. Such sigma models, however, can only be coupled to gravity if Newton's constant is quantized [52]. Renormalizable gauge theories can be coupled to gravity for all, even arbitrarily small, values of the Newton constant. This should also hold for their low-energy effective theories. Thus, we conclude that the low energy theory of a renormalizable theory can not be a compact supersymmetric sigma model.

separated, at energies lower than Λ the gauge degrees of freedom can be integrated out giving rise to a much simpler non-linear sigma model. Second, as was mentioned in Section 2, most of the recent advances in the study of non-perturbative supersymmetric theories have been restricted to the infra-red, i.e. energies much lower than Λ . Once the supersymmetry breaking scale lies in this region, these powerful tools can be brought into play. In this review we will mainly discuss examples where such a separation of scales can be arranged.

The separation between the strong coupling scale Λ and the supersymmetry breaking scale can be arranged as follows. It turns out that in many cases for supersymmetry breaking to occur, a tree level superpotential is required. By making the coupling constant of these tree level terms small enough the supersymmetry breaking scale can be lowered. In some cases we discuss below, these terms will be non-renormalizable and the corresponding couplings will be naturally small (i.e. suppressed by a small ratio of scales). In others, we will have to adjust some dimensionless Yukawa coupling to be small instead.

As was mentioned above, once the supersymmetry breaking scale can be lowered, one can integrate the gauge degrees of freedom out, at the scale Λ , giving rise to a non-linear sigma model. In Section 2, we saw that such a sigma model is characterized by both a Kähler potential and a superpotential. In all the cases we study, the full superpotential will be determined. However, it will not always be possible to determine the Kähler potential.

In Section 5.1, we will first study “calculable” models, in which the Kähler potential can be determined as well. This will allow us to explicitly determine where the supersymmetry breaking vacuum lies and ask more detailed questions about it. We study calculable models where supersymmetry breaking occurs due to instanton-induced superpotentials (Sections 5.1.1 and 5.1.2) or gaugino condensation (Sections 5.1.3 and 5.1.4). Finally, we briefly mention the calculable models with flat directions (“plateau” models) in Section 5.1.5.

In Section 5.2, we will then turn to theories where the Kähler potential cannot be determined, but where we will still be able to establish that supersymmetry is broken. In Sections 5.2.1-5.2.3, we consider examples that demonstrate how the techniques of holomorphy and duality come into play in studying supersymmetry breaking. Finally, in Section 5.3, we consider some models where the scale of supersymmetry breaking is of the same order as the strong coupling scale. By using the global symmetries and ’t Hooft anomaly cancellation arguments, we will see how supersymmetry breaking can be established in these cases as well.

5.1 Calculable models.

We begin our discussion by considering models where the low-energy effective theory responsible for supersymmetry breaking can be completely determined. In turn this will allow us

to explicitly find where the supersymmetry breaking vacuum lies, ask how the other global symmetries of the theory are realized, and calculate the masses of the low-energy excitations. Because of the detailed information that can be extracted from such calculable models, they have played a very useful role in phenomenological studies, see Section 6. They have also served as an important starting point for constructing entire new classes of supersymmetry breaking theories.

We will consider two examples in detail here. They have an $SU(3) \times SU(2)$ and $SU(4) \times U(1)$ gauge symmetry, respectively, and are referred to as the (3,2) and (4,1) models. In both cases by adjusting a Yukawa coupling we will make the supersymmetry breaking scale low compared to the relevant strong coupling scale of gauge dynamics. The Kähler potential will be calculable in both cases, although the reasons behind this will be somewhat different. We will also comment on various generalizations of these examples.

5.1.1 Instanton-driven supersymmetry breaking: the (3,2) model.

The (3,2) model was first studied by Affleck, Dine and Seiberg [8, 9]. It consists of a theory with an $SU(3) \times SU(2)$ gauge group. In addition, the theory has two anomaly free global symmetries, $U(1)_Y$ and $U(1)_R$. Under these various symmetries the matter content transforms as follows ($\bar{Q}^i_\alpha \equiv (\bar{D}^i, \bar{U}^i)$):⁷

| | | | | | |
|--------------|-----------------|-----------|----------|----------|------|
| | $SU(3)$ | $SU(2)$ | $U(1)_Y$ | $U(1)_R$ | |
| Q_i^α | \square | \square | $1/6$ | 1 | |
| \bar{U}^i | $\bar{\square}$ | 1 | $-2/3$ | 0 | (28) |
| \bar{D}^i | $\bar{\square}$ | 1 | $1/3$ | 0 | |
| L^α | 1 | \square | $-1/2$ | -3 | |

Let us first study the classical behavior of this theory. The $SU(3)$ and $SU(2)$ D-flatness conditions are given by:

$$Q_\alpha^\dagger{}^m Q_l^\alpha - \bar{Q}_\alpha^m \bar{Q}_l^\dagger{}^\alpha = 0, \quad (29)$$

and,

$$Q_\alpha^\dagger{}^i Q_i{}^\beta + L_\alpha^\dagger L^\beta = \frac{1}{2} \delta_\alpha^\beta (Q^\dagger Q + L^\dagger L), \quad (30)$$

respectively. These conditions do not select a unique vacuum, rather there are 3 (complex) flat directions in the theory which can be parametrized by the gauge invariant chiral superfields ("moduli"):

$$X_1 = Q \bar{D} L, \quad X_2 = Q \bar{U} L, \quad X_3 = \det \bar{Q}_\alpha Q^\beta. \quad (31)$$

⁷The reader might have noticed that this theory is quite similar to the one generation standard model, with two differences: $U(1)_Y$ is a global symmetry and the positron field is missing. In fact, the theory has $U(1)_Y$ and $U(1)_Y^3$ anomalies.

At a generic point along these flat directions the $SU(3)$ and $SU(2)$ gauge symmetries are completely broken, the corresponding gauge bosons and their superpartners are heavy, and the low-energy dynamics can be described by an effective theory containing the X_i chiral superfields.

Now let us turn to the quantum behavior of this theory. One finds that instanton effects lift the flat directions and give rise to a superpotential in the low-energy effective theory of the form:

$$W_{dyn} = \frac{\Lambda_3^7}{X_3}. \quad (32)$$

Eq. (32) is determined in the following way. It is the only term that is allowed by holomorphy and the symmetries of the theory. Further, an explicit (constrained) instanton calculation shows that it does arise. The non-perturbative superpotential, eq. (32), gives rise to a potential energy that is minimized when some fields acquire large expectation values and "run away" to infinity. Thus we find that the quantum theory does not have a stable ground state.

To avoid this problem, we can add a tree level superpotential, preserving the $U(1)_Y \times U(1)_R$ symmetry, of the form:

$$W_{tree} = \lambda Q \cdot \bar{D} \cdot L = \lambda X_1. \quad (33)$$

Classically, one now finds that the F term conditions following from this superpotential, along with the D term conditions, (29), (30), lift all the flat directions giving rise to a unique vacuum with all the fields set to zero.

The behavior of the quantum theory is much more interesting. One can show that in this case the exact superpotential in the low-energy effective theory is given by a sum of the two terms, eq. (32) and (33), to be:

$$W = W_{dyn} + W_{tree} = \frac{\Lambda_3^7}{X_3} + \lambda X_1. \quad (34)$$

To show this, one uses, following [3], holomorphy, symmetries, and various limits. Note first that when $\lambda \rightarrow 0$ and $X_3 \rightarrow \infty$ the superpotential is reliably given by eq. (34). By holomorphy and symmetries, the most general form of the superpotential is $W_{dyn} \times f(t)$, with f an arbitrary function of $t \equiv \lambda X_1 X_3 / \Lambda_3^7$. Now, we see that any value of t can be obtained by taking $\lambda \rightarrow 0$ and $X_3 \rightarrow \infty$ appropriately. Thus, eq. (34) should be exact.

From eq. (34) it follows that the F -flatness condition for the field X_1 , $dW/dX_1 = \lambda = 0$ can not be satisfied. Thus one concludes that supersymmetry is broken in the quantum theory.

We have assumed in this analysis that an effective theory in terms of the three fields X_i correctly describes the low-energy dynamics. If extra massless degrees of freedom were to

enter the low-energy theory at some points in moduli space, the description in terms of the fields X_i would break down. This would manifest itself, for example, in singularities in the Kähler metric K_{ij^*} . At points where these singularities are present, the energy which goes like $W_{i^*}^* K^{-1i^*j} W_j$ could go to zero and supersymmetry would not be broken.

In fact, as we will see in a moment, for small enough λ the vacuum lies in a region of field space where both the $SU(3)$ and $SU(2)$ groups are higgsed and weakly coupled. Thus, we will be able to explicitly compute the spectrum and show that it is consistent with the absence of extra massless particles. But before doing so, it is worth noting that this result also follows quite generally from duality. The $(3, 2)$ model (for small enough λ) can be shown, by adding extra vectorlike flavors, to be dual to a weakly coupled magnetic theory. This dual theory is completely higgsed and one can show that the low-energy spectrum corresponds to the X_i fields with no additional massless particles.

Let us now study the supersymmetry breaking vacuum in more detail. As was mentioned at the outset this model is calculable—one can determine the expectation values of the fields and the low-energy spectrum explicitly. We will not be able to provide the full details here, see refs. [9, 54], and will content ourselves with sketching out the general picture and providing some of the steps in the calculations.

The basic idea is to do a self consistent analysis. One begins by assuming that the field expectation values break both the $SU(3)$ and $SU(2)$ groups at energies much above their strong coupling scale. This allows us to compute the Kähler potential and determine the full non-linear sigma model. The energy can then be explicitly minimized to determine these expectation values and verify that the starting assumption was in fact correct. Before going further, let us note that the assumption one begins with is very plausible. The non-perturbative superpotential pushes the vacuum out to large field strengths. In contrast, the tree level superpotential results in a contribution to the energy that grows at large field strengths. The minimum should lie where these two terms balance each other (see Fig. 2). From eq. (34) it follows that the corresponding vacuum expectation value, v , should roughly go like

$$v \sim \frac{\Lambda_3}{\lambda^{1/7}} . \quad (35)$$

For small enough λ , v can be made large and if enough fields get expectation values both $SU(3)$ and $SU(2)$ should be broken.

The Kähler potential in terms of the fields X_i can be calculated. At tree level it can be determined to be:

$$K = 24 \frac{A + B x}{x^2} , \quad (36)$$

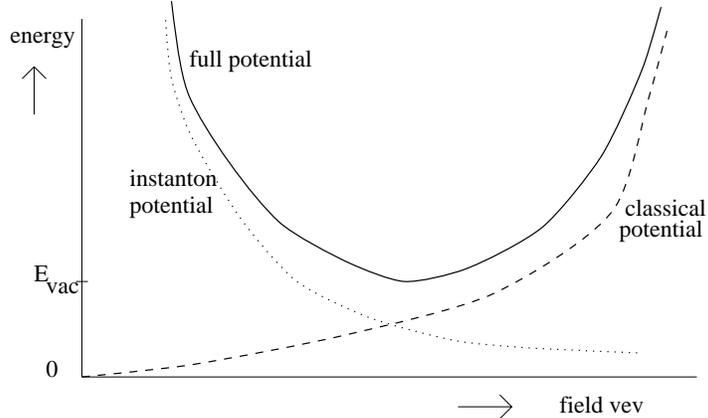


Figure 2: The scalar potential in the 3-2 model: instanton vs. tree-level contribution.

where

$$A = \frac{1}{2} (X_1^\dagger X_1 + X_2^\dagger X_2), \quad B = \frac{1}{3} \sqrt{X_3^\dagger X_3}, \quad x = 4\sqrt{B} \cos\left(\frac{1}{3} \text{Arccos} \frac{A}{B^{3/2}}\right). \quad (37)$$

Although its form is complicated there is a straightforward way to determine the Kähler potential. One starts with the canonical Kähler potential for the Q, \bar{Q}, L fields and projects onto the D-flat directions, (30), (29), see [9]. Equivalently, one can integrate out the vector fields that become heavy along the flat directions [53], [54].⁸

Let us stress that the tree level Kähler potential will in general receive both perturbative and non-perturbative corrections. However, if v is large enough, eq. (35), and both the $SU(2)$ and $SU(3)$ gauge groups are broken, these can be neglected.

With the Kähler potential and superpotential, eqs. (36), (34), at hand, the non-linear sigma model is completely determined. The energy can now be found from eq. (7) and minimized. We omit some of the details here. On doing so one finds that the full $SU(3) \times SU(2)$ gauge symmetry is indeed broken at a scale or order v , eq. (35). Thus the starting assumption is validated. Furthermore, as expected from the above discussion, supersymmetry is broken. The vacuum energy is of order $E \sim \lambda^{10/7} \Lambda^4$, as one expects from eqs. (35) and (34).

It is also worth discussing briefly how the other global symmetries are realized in this vacuum. It turns out that the $U(1)_Y$ global symmetry is unbroken in the ground state, while the R symmetry is broken. The reader might recall that once the tree level superpotential (33) is added the theory has no flat directions at tree level. Thus from the general considerations of Section 4.4 we expect that once the R symmetry is broken, supersymmetry is broken as well. This is indeed what we have found.

The massless spectrum consists of a massless goldstino, an R -axion (goldstone boson of

⁸In the mathematical literature the above procedure of constructing the tree-level effective theory of the D-flat moduli is known as the “Kähler quotient,” see, e.g. [55].

the spontaneously broken R symmetry), and an additional massless fermion, with charge -1 under $U(1)_Y$ (its existence can be inferred from 't Hooft anomaly matching for $\text{tr } U(1)_Y$ and $\text{tr } U(1)_Y^3$). Note that all these fields arise from the X_i fields; as promised above, there are no additional massless particles. Further, all other components of the X_i chiral superfields have masses of order λv . Finally, by considering the full $SU(3) \times SU(2)$ theory, one can also determine the spectrum and supersymmetry-breaking mass splittings of the heavy vector multiplets.

Finally, we note here that our consideration of the $(3, 2)$ model applies to a limited region of parameter space—our considerations are valid whenever the expectation value (35) $v \gg \Lambda_2, \Lambda_3$, i.e. $\lambda \ll 1$ and,

$$\Lambda_3 \gg \lambda^{1/7} \Lambda_2 . \quad (38)$$

The analysis above showed that the Witten Index vanishes. This is of course true more generally, and so the theory could break supersymmetry for other values of the parameters as well. In fact, for $\Lambda_2 \gg \Lambda_3$ (with λ still $\ll 1$), when (38) is not obeyed, the description of supersymmetry breaking changes [19], but supersymmetry remains broken. Later we will discuss an example of how sometimes different (in the case described in Section 5.2.4, “electric” and “magnetic”) descriptions of supersymmetry breaking are relevant in different in regions of parameter space.

5.1.2 Generalizations of the $(3, 2)$ model.

The $(3, 2)$ model has a number of interesting generalizations. One can think of constructing this model by starting with two flavor QCD, gauging an $SU(2)$ flavor symmetry and adding an extra L lepton field to cancel anomalies. Some generalizations of this construction are the $SU(N) \times SU(2)$ models in [14], the $SU(2M + 1) \times SP(2M)$ models discussed in [14] and in [19], the $SU(N) \times SU(N - 1)$ models of [22], the $SU(N) \times SU(N - 2)$ theories of [23], and the models of [37]. While these are analogous, in their field content, to the $(3, 2)$ model, the dynamics leading to supersymmetry breaking in many of them is quite different. We will have more to say about some of them in the following sections.

We should also comment on some other calculable models in the literature, which are analogous to the $(3, 2)$ model, and which break supersymmetry. One example is the $SU(5)$ model with two \square and two $\bar{\square}$ representations. This $SU(5)$ “two generations” model has dynamics that is very similar to that of the 3-2 model [10]. Its ground state has been recently analyzed in detail [56]. Using the recent work [57] on classifying $N = 1$ supersymmetric gauge theories with a simple gauge group and with $\mu_{matter} < \mu_{adj}$ (where $\mu_{matter(adj)}$ is the index of the matter (adjoint) representation), one can show that among the theories with a simple gauge group (including both classical and exceptional groups) and “purely chiral”

matter content (i.e. such that no mass terms can be added for any fields), the $SU(5)$ “two generations” model of [10] is the only one with completely calculable dynamics.⁹

Another example in this class, which can be constructed using product groups is the $SP(4) \times U(1)$ model of [18]. Finally, additional calculable models can be found upon “deforming” noncalculable chiral models by adding the “right” amount of massive vectorlike matter (see the discussion in the Section 5.3).

5.1.3 Supersymmetry breaking by gaugino condensation: the $(4, 1)$ model.

We now turn to considering another example of a calculable model with supersymmetry breaking. It is based on a gauge theory with $SU(4) \times U(1)$ gauge symmetry [14], [17]. The $(3, 2)$ model we discussed in the previous section was calculable because one could arrange for the vacuum to lie in a region of moduli space where the full $SU(3) \times SU(2)$ gauge group was completely higgsed and weakly coupled. In contrast, as we will see, in the $(4, 1)$ model the $SU(4) \times U(1)$ gauge symmetry is only partially broken to an $SU(2)$ subgroup which gets strongly coupled and confines. Nevertheless, we will argue that at low enough energies the resulting sigma model is weakly coupled and calculable.

The model has the following matter fields and charge assignments:

| | | | |
|-------------------|-----------------|--------|------|
| | $SU(4)$ | $U(1)$ | |
| $A_{\alpha\beta}$ | \square | 2 | |
| Q_α | \square | -3 | |
| \bar{Q}^α | $\bar{\square}$ | -1 | |
| S | 1 | 4 | (39) |

In studying the quantum behavior of this theory it is convenient to first ignore the effects of the $U(1)$ gauge symmetry. The $SU(4)$ flat directions can then be described by the following moduli:¹⁰

$$M = \bar{Q} \cdot Q \sim -4, \text{ Pf}A \sim 4, \text{ and } S \sim 4, \quad (40)$$

where for later convenience we have also shown the $U(1)$ charges of the $SU(4)$ moduli. Along a generic flat direction the $SU(4)$ gauge symmetry is broken to an $SU(2)$ subgroup. There is no matter charged under the unbroken $SU(2)$. In the quantum theory, non-perturbative dynamics in this $SU(2)$ theory leads to confinement. At scales below the $SU(2)$ confining scale the confined degrees of freedom, e.g., the glueballs and their superpartners, can be integrated out. The effective theory below the $SU(2)$ strong coupling scale involves the moduli $M, \text{Pf}A$, and S . Gaugino condensation in the $SU(2)$ theory gives rise to a non-perturbative superpotential in this effective theory, proportional to the scale of the unbroken

⁹We thank W. Skiba for a guided tour of ref. [57] and discussions.

¹⁰We are using a notation where Pf stands for the Pfaffian. For example, $\text{Pf} A = \epsilon^{ijkl} A_{ij} A_{kl} / 8$.

$SU(2)$:

$$W \sim \Lambda_{SU(2)}^3 = \frac{\Lambda_{SU(4)}^5}{\sqrt{M \text{ Pf} A}} . \quad (41)$$

The $U(1)$ gauge symmetry can be incorporated in the effective theory by "turning on" its gauge coupling. In terms of the moduli, S , M , and $\text{Pf} A$, one finds that the flat directions of the $U(1)$ D-term potential are described by two moduli, $M \text{ Pf} A$ and SM . From this point onwards the analysis has many similarities with that of the $(3,2)$ model. We will consequently only sketch out the details.

The non-perturbative superpotential, eq. (41), only involves the first modulus, $M \text{ Pf} A$. It results in an energy which is minimized when some fields are pushed out to infinity; thus the quantum theory has a runaway vacuum. To cure this problem we introduce a tree level superpotential:

$$W_{tree} = \lambda S \bar{Q} \cdot Q = S M . \quad (42)$$

One can show that this tree level superpotential lifts all the $SU(4) \times U(1)$ flat directions. The full superpotential in the low energy effective theory is now given by the sum of eqs. (41) and (42):

$$W_{exact} = \frac{\Lambda_{SU(4)}^5}{\sqrt{M \text{ Pf} A}} + \lambda S M . \quad (43)$$

From eq. (43) it follows that the F term condition for SM (which is one of the two moduli) cannot be met and thus supersymmetry is broken.

In fact, the resulting vacuum can be explicitly determined since the effective theory is weakly coupled in the relevant region of moduli space. This might come as a bit of a surprise to the reader. What about the surviving $SU(2)$ gauge theory which, as we have mentioned above, has strong dynamics associated with it and confines? The corrections to the superpotential were incorporated in eq. (43). In the Kähler potential one expects non-perturbative effects associated with the strongly coupled $SU(2)$ to give rise to corrections that go like $\Lambda_{SU(2)}/v$. Here v is the scale of a typical expectation value, which can be estimated by balancing the two terms in eq. (43) and goes like,

$$v \sim \frac{\Lambda_{SU(2)}}{\lambda^{1/3}} . \quad (44)$$

Thus, for small enough λ , $\Lambda_{SU(2)}/v \ll 1$ and the corrections to the classical Kähler potential are suppressed.

The resulting ground state and spectrum of low-energy excitations can now be explicitly calculated, in a manner very similar to the $(3,2)$ model. We will not go into the details here. Let us instead briefly review the picture of the underlying physics that was responsible for supersymmetry breaking. One starts with an $SU(4) \times U(1)$ theory in the ultraviolet. At a

scale of order v , eq. (44), this is broken to an $SU(2)$ gauge group. The $SU(2)$ theory confines at the scale $\Lambda_{SU(2)} \sim \Lambda_{SU(4)} \lambda^{2/15}$ giving rise to the low-energy sigma model. In particular, gaugino condensation in the $SU(2)$ theory gives rise to a non-perturbative superpotential in the sigma model. Finally, supersymmetry breaking takes place in this sigma model, at a scale, $E \sim \lambda^{3/10} \Lambda_{SU(4)}$.

We end with one remark. The tree level superpotential (42), which lifts all flat directions, preserves an R symmetry. Thus, provided the R symmetry is broken, we could have concluded at the outset from the general considerations of section 4.4, that supersymmetry breaking must occur.

5.1.4 Generalizations of the $(4, 1)$ model.

The $(4, 1)$ model can be generalized in a straightforward way to an entire class of theories with $SU(2l) \times U(1)$ gauge symmetry and with matter consisting of a single antisymmetric tensor representation, $A \sim \square$, $2l - 3$ antifundamentals, \bar{Q} , one fundamental field Q and $2l - 3$ fields, S_i , which are uncharged under the $SU(2l)$ symmetry but carry an $U(1)$ charge [14], [17]. In all these theories an $SU(2)$ gauge symmetry is left unbroken. Gaugino condensation in this group, together with an appropriate tree level superpotential then lift all flat directions giving rise to supersymmetry breaking. For reasons analogous to the $(4, 1)$ case by adjusting appropriate couplings one can arrange for the resulting supersymmetry breaking dynamics to be governed by a calculable theory.

In turn, the $SU(2l) \times U(1)$ models can be further generalized. The matter content in these theories can be thought of [14], [17] as arising by starting with an $SU(2l + 1)$ theory with an antisymmetric tensor, $A \sim \square$ and $2l - 3$ antifundamentals, \bar{Q} , and breaking the gauge symmetry down to $SU(2l) \times U(1)$. There are, of course, other possible breakings of $SU(2l + 1)$. It is natural to ask if they give rise to supersymmetry breaking theories as well. This question was addressed by [26]. They added a heavy adjoint field superfield Σ to the theory, with a superpotential, $W \sim Tr \Sigma^{k+1}$. This allows the $SU(2l + 1)$ to be generically broken to $SU(2l + 1) \rightarrow U(1)^{k-1} \prod_{s=1}^k SU(n_s)$, with $\sum_s n_s = 2l + 1$. The matter content of the $U(1)^{k-1} \prod_{s=1}^k SU(n_s)$ theory can be obtained by decomposing the \square and $2l - 3$ $\bar{\square}$ s of $SU(2l + 1)$ into representations of the unbroken gauge group. The authors of [26] performed a comprehensive analysis of supersymmetry breaking in this class of models. A description of their analysis would take us far from the objective of this article; we only note that the deconfinement method of [58] and the duality in SQCD with adjoint matter and superpotential $Tr \Sigma^{k+1}$ [59] were essential in understanding supersymmetry breaking. Ref. [26] concluded that for $k = 2$, supersymmetry is broken, once appropriate Yukawa couplings are added to the superpotential (to lift the flat directions), while for $k > 2$, supersymmetry is generically

(for exceptions, see [26]) not broken. We see that this construction relates many seemingly different models, for which supersymmetry breaking can be studied in a unified manner. For example, in the simplest case for $l = 2, k = 2$, starting with the $SU(5)$ model, one obtains, choosing $n_1 = 4, n_2 = 1$, the $SU(4) \times U(1)$ model, discussed above, while choosing $n_1 = 3, n_2 = 2$, the $SU(3) \times SU(2)$ model of Section 5.1.1 is obtained.

Many other models of dynamical supersymmetry breaking exhibit behavior, similar to the $SU(4) \times U(1)$ model. Examples are the $SU(N) \times SU(N - k)$ theories [22], [23] (see also Section 5.2.3), with $k = 1, 2$ (whose light spectrum has been analyzed in detail along the above lines in [60], [61]), the models of [16], and many of the models in [14], [26].

5.1.5 Calculable models with classical flat directions: “plateau” models.

These calculable models are based on some of the models considered in Section 5.2.2, in particular the models based on quantum-modified moduli spaces of [19],[20]. The models have classical flat directions and have been shown to break supersymmetry. However, the vacuum expectation value can not be calculated in a controlled approximation. The models can be made calculable by weakly gauging a global symmetry and using the perturbative corrections to the Kähler potential to stabilize the flat direction [46], [47] at a large expectation value, in a variant of the “inverse hierarchy” mechanism [62].

5.2 Noncalculable models.

The calculable models are the simplest class of theories breaking supersymmetry. They form an important starting point in the study of this phenomenon, and as we have seen, teach us a lot. In pursuing this study further, we would like to complicate things in stages. Accordingly, in this Section, we continue to study theories in which by adjusting a parameter, the supersymmetry breaking scale is made lower than the underlying scale of non-perturbative dynamics. Thus below the strong coupling scale, but above the supersymmetry breaking scale we can use a supersymmetric effective theory to describe the dynamics. But here, unlike the previous examples, we will consider situations in which the non-perturbative effects are important in both correcting the superpotential and the Kähler potential. We see below how in many cases one can still argue that supersymmetry is broken. But in the absence of more information about the Kähler potential one cannot find in detail where the vacuum lies.

In Section 5.2.1 we consider a simple model, where supersymmetry breaking is due to confinement. Section 5.2.2 is devoted to models of supersymmetry breaking with classical flat directions that are lifted by nonperturbative effects. In Section 5.2.3, we give an example of how duality can be used to study supersymmetry breaking.

Finally, in Section 5.3, we turn to the case where the scale of non-perturbative dynamics and supersymmetry breaking are comparable. This is of course the generic situation. The full complexity of these theories makes a controlled analysis difficult. Even so, we show how in some cases the various global symmetries and associated 't Hooft anomalies, and criteria like the Witten index make it quite plausible that supersymmetry is broken.

5.2.1 Supersymmetry breaking by confinement: the ISS model.

The first model we look at was studied by Intriligator, Seiberg and Shenker [13]. Its simplicity makes it a good point to begin.

The model is an $SU(2)$ gauge theory with a single chiral superfield in the three-index symmetric representation (i.e. the four-dimensional “spin-3/2” representation). This theory is chiral—no holomorphic mass term can be written for the single spin-3/2 representation. To see this, denote the matter field by $q_{\alpha\beta\gamma}$ (with $q_{\alpha\beta\gamma} = q_{\beta\alpha\gamma} = \dots$). It is easy to see then, that the quadratic invariant $q \cdot q$ (indices contracted with ϵ symbols) vanishes identically, because of the symmetry of q . There is no cubic invariant; the only independent invariant is then¹¹ $u = q^4$.

The theory thus has a one dimensional moduli space and along the flat direction parametrized by u , the $SU(2)$ symmetry is totally broken. Classically, the moduli space is singular at the origin: the classical Kähler potential is $K \sim q^\dagger q|_{D-flat} \sim (u^\dagger u)^{1/4}$ and the Kähler metric is singular at $u = 0$. When $u \rightarrow 0$ the $SU(2)$ symmetry is restored and extra vector multiplets become massless.

The quantum theory is asymptotically free,¹² and one expects non-trivial dynamics in the infra-red. It is also useful to note that the model has an anomaly free R symmetry, under which the field q has charge $3/5$.

The authors of ref. [13] argued that quantum-mechanically, the theory confines in the vicinity of the origin of moduli space. As a result, the classical singularity at the origin is smoothed out without the appearance of any new massless particles. While one cannot prove this assertion, it meets one non-trivial check. At the origin, the global $U(1)_R$ symmetry mentioned above is unbroken. The u field saturates the 't Hooft anomaly matching conditions for this $U(1)_R$ symmetry. This is easy to see: the relevant anomalies are $\text{Tr}R = 7/5$ and $\text{Tr}R^3 = (7/5)^3$; these are obviously saturated by the fermionic component of u which has R -charge $7/5$.

In the following discussion we will accept the above assertion that the effective theory

¹¹More precisely, $u = q_{\alpha_1\beta_1\gamma_1} \epsilon^{\alpha_1\alpha_2} \epsilon^{\beta_1\beta_2} q_{\alpha_2\beta_2\gamma_2} \epsilon^{\gamma_1\gamma_3} q_{\alpha_3\beta_3\gamma_3} \epsilon^{\alpha_3\alpha_4} \epsilon^{\beta_3\beta_4} q_{\alpha_4\beta_4\gamma_4} \epsilon^{\gamma_2\gamma_4}$.

¹²For the spin-3/2 representation, $T(R) = 5$, the one-loop beta function of the gauge coupling is then $b_0 \sim 3T(G) - T(R) = 1$.

in terms of the field u is valid everywhere in moduli space. It follows then that the Kähler potential is smoothed out near the origin and can be approximated as $K \simeq u^\dagger u |\Lambda|^{-6}$, for $u \ll \Lambda^4$. We now add a tree-level superpotential to the theory:

$$W_{tree} = \frac{u}{M_{UV}} . \quad (45)$$

This term is nonrenormalizable; the theory therefore should be considered as a low-energy effective theory valid at scales below M_{UV} . We note that this term lifts the one classical flat direction in this theory; it also breaks the R symmetry.¹³ In the presence of eq. (45) it follows that the F term condition for u cannot be met and supersymmetry is broken.

A few comments are in order at this stage. First, we cannot say with certainty where the vacuum lies. In the vicinity of the origin the leading term in the Kähler potential $K \simeq u^\dagger u |\Lambda|^{-6}$ gives rise to an energy that goes like

$$E_0 = K_{u^*u}^{-1} |W_u|^2 \simeq \frac{|\Lambda|^6}{|M_{UV}|^4} , \quad (46)$$

and behaves like a constant. Thus, in determining the minimum, higher terms in the Kähler potential, which are difficult to estimate, need to be kept. Second, regardless of exactly where the minimum lies, eq. (46) gives a good estimate of the energy. So, the supersymmetry breaking scale is smaller than Λ , justifying the use of the effective theory in terms of u . Finally, as was mentioned above, the microscopic theory one starts with breaks down at the scale M_{UV} . However, by taking $\Lambda \ll M_{UV}$, one can be quite sure that there is a supersymmetry breaking minimum in the region where the effective theory is valid. This is because classically the superpotential, eq. (45), lifts all the flat directions. It follows then that in the region $\Lambda \ll (u)^{1/4} \ll M_{UV}$, where the classical approximation is trustworthy, the energy must rise, leading to the conclusion that a local minimum must lie in the region $(u)^{1/4} \leq O(\Lambda)$. It is, of course, possible that the full theory has a global supersymmetry preserving minimum with $(u)^{1/4} \geq M_{UV}$, but that can not be decided without knowledge of the underlying theory at scales above M_{UV} .

The mechanism by which this model breaks supersymmetry is, in fact, more general and occurs in more complicated theories. For example, the $SU(7)$ "s-confining" theory with matter consisting of two sets of $\square + 3 \times \bar{\square}$ breaks supersymmetry after addition of an appropriate tree level superpotential [27].

¹³ As an aside, we note that the superpotential in the effective low energy theory, eq. (45), has an accidental R symmetry, which is a combination of the R symmetry of the microscopic theory and the accidental $U(1)$ of the low-energy theory; this accidental symmetry is broken by higher dimensional terms in the Kähler potential.

5.2.2 Models with classical flat directions: the ITIY model.

We now turn to discussing the theory first studied by Intriligator and Thomas [19], and Izawa and Yanagida [20]. This model has two remarkable features. First, it breaks supersymmetry even though it is non-chiral. Second, the classical theory has flat directions, which are lifted by non-perturbative quantum effects leading to supersymmetry breaking.

The ITIY model is an $SU(2)$ gauge theory with 4 fundamentals Q^i and six singlets S_{ij} , with a tree-level superpotential,

$$W = \lambda S_{ij} Q^i \cdot Q^j . \quad (47)$$

For further reference we note that the global symmetries of the theory include an $SU(4)$ flavor symmetry under which the Q^i transform as a \square and the S_{ij} as an $\bar{\square}$.

Let us begin by studying the classical behavior of this theory. The $SU(2)$ D-flat directions can be described by gauge invariant chiral superfields, “mesons,” which we denote by $M^{ij} \sim Q^i \cdot Q^j$. There are six of these meson fields but they are not all independent; classically they satisfy a constraint:

$$\epsilon_{ijkl} M^{ij} M^{jk} = 0 . \quad (48)$$

It follows from the superpotential, eq. (47), that all the meson flat directions are lifted, since the F -term equations for the S_{ij} fields set all the mesons to zero. However the singlet flat directions remain unlifted. Along these flat directions, the mesons are zero but the singlet fields S_{ij} are free to vary.

Let us now turn to the quantum behavior of this theory. As in the classical case, the quantum dynamics is described by the mesons M^{ij} and the singlets S_{ij} . The difference is that in the quantum case non-perturbative effects modify the constraint eq. (48) by the addition of a term dependent on the strong coupling scale of the $SU(2)$ gauge theory,^[3]¹⁴ Λ . This gives rise to the following full superpotential in the effective theory:

$$W_{eff} = \lambda S_{ij} M^{ij} + A \left(\epsilon_{ijkl} M^{ij} M^{jk} - \Lambda^4 \right) , \quad (49)$$

where A is a Lagrange multiplier that implements the constraint.

We now find that the theory breaks supersymmetry! The F term conditions for the S fields still set all the mesons, M^{ij} , to zero, but now this is in conflict with the quantum modified constraint (which follows from the F -term condition of the Lagrange multiplier A).

The argument above tells us that supersymmetry is broken, but it does not tell us where the resulting ground state lies. We saw above that classically the theory had flat directions.

¹⁴This was discussed in Section 2. In the language of supersymmetric QCD we have a situation with $N_c = N_f = 2$ here.

One might wonder if the energy goes to a minimum (which is non-zero) at infinity, resulting in an unstable runaway ground state. In fact, one can show that this does not happen. Only the flat directions in the classical theory are relevant for this discussion—along the other directions the energy grows very large as we go out to infinity and one does not expect quantum effects to turn this around. Consider such direction along which the $SU(4)$ global symmetry is broken to $SP(4) \equiv SO(5)$ by giving a vev

$$S_{ij} = s J_{ij}, \quad (50)$$

where $J = \text{diag}\{i\sigma_2, i\sigma_2\}$ is the $SP(4)$ invariant tensor. If s is large all the quarks get a large mass, of order $\lambda s \gg \Lambda$, and can be integrated out at a scale much above Λ . At low-energies this gives rise to a theory which contains an $SU(2)$ gauge field (and superpartners) with no matter, and the fields S_{ij} . The strong coupling scale of the low-energy theory is determined by threshold corrections to be:

$$\Lambda_{low}^6 = \Lambda^4 \lambda^2 s^2. \quad (51)$$

Gaugino condensation in the $SU(2)$ group now gives rise to a superpotential, which goes like Λ_{low}^3 . Substituting from eq. (51) then gives:

$$W = \Lambda^2 \lambda s. \quad (52)$$

The energy along this direction can now be calculated; it is given by:

$$E = \left| \frac{\partial W}{\partial s} \right|^2 \sim |\lambda \Lambda|^2. \quad (53)$$

We see that the energy is a constant, independent of s . In this discussion, we have so far assumed that the Kähler potential for s is classical. In fact this is true, to leading order, but there are small corrections [45], [48], [47]. For large enough s , the leading corrections arise because the coupling λ is dependent on s , and being non-asymptotically free increases with s logarithmically. Thus from eq. (52) we see that the energy increases (although only logarithmically) with s and the runaway behavior is avoided.¹⁵ Along other directions where S_{ij} gets a vev breaking the flavor symmetry to $SU(2) \times SU(2)$ one finds similarly that the energy increases and there is no runaway behavior.

The running of the Yukawa coupling thus pushes the expectation value of s to small values, where the theory is not weakly coupled. This makes it difficult to reliably calculate the ground state of the model. We should mention that it is possible [46], [47], to stabilize the vacuum at large expectation values upon gauging (part of) the flavor symmetry of the model,

¹⁵As s increases, at some point one hits a Landau pole singularity and other degrees of freedom must come into play. However this scale can be made much higher than Λ . Also, we note that for the present purpose the running of the coupling λ is governed by the $\beta_\lambda = 8\lambda^3/(16\pi^2)$.

by a variant of the inverse hierarchy mechanism [62]. This results in a class of calculable models, the “plateau” models mentioned in Section 5.1.5.

We began the discussion of the ITIY model by noting that it was not chiral. The reader might wonder how this is consistent with the breaking of supersymmetry. Specifically, one can add mass terms for both the quarks and the S_{ij} fields. This lifts all the flat directions. For large enough masses, the low-energy theory is a pure $SU(2)$ Yang-Mills which has two vacua and a Witten index of 2 [41]. What happens now when the masses are taken to zero? On adding mass terms to the superpotential and incorporating the non-perturbative constraint we find:

$$W = \lambda S_{ij} M^{ij} + m_{ij} M^{ij} + \frac{1}{2} \tilde{m} \text{Pf}S + A \left(\text{Pf}M - \Lambda^4 \right). \quad (54)$$

From here it follows that the expectation values are given by:

$$M^{ij} \sim \epsilon^{ijkl} m_{kl} \left(\frac{\Lambda^4}{\text{Pf}m} \right)^{\frac{1}{2}}, \quad (55)$$

and,

$$S_{ij} \sim \frac{m_{ij}}{\tilde{m}} \left(\frac{\Lambda^4}{\text{Pf}m} \right)^{\frac{1}{2}}. \quad (56)$$

The square root above can take two values—this corresponds to the two vacua of $SU(2)$ SYM we expect. Now we can take \tilde{m} and m_{ij} to go to zero (keeping the relative ratio of masses fixed). We see that M^{ij} has a finite limit, but $S_{ij} \rightarrow \infty$. Thus the supersymmetry preserving vacua run off to infinity in the limit of vanishing mass. The Witten index changes discontinuously from 2 to 0, because the mass terms change the behavior of the Hamiltonian at large field strengths.

We close this section by noting that the ITIY model has several generalizations, see [19]; for simplicity we have focussed on the simplest example of this class here.

5.2.3 Supersymmetry breaking and duality: the (5, 3) model.

In this section, we will consider an example of the $SU(N) \times SU(N - k)$ models. They were studied in [22, 23], where it was shown that a large class of these theories broke supersymmetry. As was mentioned in Section 5.1.2, in terms of their matter content, these models can be thought of as generalizations of the the (3, 2) model.

Here, for simplicity, we will discuss a particular model in this class. It has an $SU(5) \times SU(3)$ gauge symmetry. In the following discussion we will often refer to this theory as the electric theory. We will also construct another theory, based on an $SU(5) \times SU(2)$ gauge group, which we call the “magnetic” dual theory. The electric and magnetic theories will be

equivalent to each other in the infra-red and both are useful in learning about the low-energy behavior, especially supersymmetry breaking. In particular, it will be interesting to study the behavior of the theory by varying a Yukawa coupling. The electric theory will yield a calculable theory of supersymmetry breaking in some restricted region of parameter space. In contrast, the magnetic description will not be calculable, but it will allow us to establish that supersymmetry is broken in a much larger region of parameter space.

The matter content of the $SU(5) \times SU(3)$ theory is given as follows:

| | | | |
|-------------------------|-----------------|-----------------|------|
| | $SU(5)$ | $SU(3)$ | |
| $Q_{\alpha\dot{\beta}}$ | \square | \square | (57) |
| $L^{\alpha i}$ | $\bar{\square}$ | $\mathbf{1}$ | |
| $R_a^{\dot{\beta}}$ | $\mathbf{1}$ | $\bar{\square}$ | |

where $i = 1, 2, 3$ and $a = 1, \dots, 5$ are flavor indices under the global $SU(3)_L \times SU(5)_R$ symmetry, while dotted (undotted) greek letters denote the indices under the $SU(5)$ ($SU(3)$) gauge groups. The theory has a renormalizable tree level superpotential:

$$W_{tree} = \tilde{\lambda}_i^a R_a \cdot Q \cdot L^i + \tilde{\alpha}_{ab} R_c \cdot R_d \cdot R_e \epsilon^{abcde} . \quad (58)$$

This superpotential lifts all classical flat directions and preserves a diagonal, anomaly-free $SU(2)$ subgroup of the global flavor symmetry, provided the superpotential couplings are of the form:

$$\tilde{\lambda} = \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \tilde{\alpha} = \begin{pmatrix} 0 & \alpha & 0 & 0 & 0 \\ -\alpha & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \alpha & 0 \\ 0 & 0 & -\alpha & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} . \quad (59)$$

Eq. (58) with the couplings (59) can be shown to preserve a flavor-dependent, anomaly free R symmetry.¹⁶ Thus, since there are no flat directions, according to the general criterion of Section 4.4, one expects that if the R symmetry is broken supersymmetry is also broken.

This is indeed what we will find below when we try to understand the supersymmetry breaking dynamics in more detail. Since the analysis is quite involved, it is useful to first sketch out in words the general idea. Throughout this discussion we consider the theory for $\Lambda_3 \gg \Lambda_5$. We then vary the parameter α and ask about supersymmetry breaking.¹⁷ It will

¹⁶ To see this one can start by assigning different R charges to Q , L , $R_{a<5}$, and R_5 . These charges have to satisfy four conditions: two ensuring that the superpotential terms are invariant, and two—that the symmetry is anomaly free; it is easy to see then that there is a solution with nonvanishing charges of all fields.

¹⁷ In general, the behavior of this theory also depends on λ . For simplicity, we will keep λ fixed of order ≤ 1 here.

turn out that the magnetic theory mentioned above will allow us to establish, for $\alpha < 1$, that supersymmetry is broken. However, the magnetic theory is not calculable and we will not be able to learn much more about the resulting ground state. When $\alpha \ll 1$ though, we will see that the vacuum lies in a region of moduli space where the electric theory provides a weakly coupled sigma model. By using this effective theory we will independently be able to see that supersymmetry is broken and also learn a great deal about the resulting ground state.

Calculable limit: “electric” description of supersymmetry breaking.

Since the calculable limit is simpler, we start with the situation $\alpha \ll 1$ and first consider the electric theory. To understand this case it is useful to first consider what happens when $\alpha = 0$. The “baryonic term” in eq. (58) is absent in this limit and the classical theory has flat directions. Let us consider one such direction along which the baryon, $b^{45} = \epsilon^{45abc} R_c \cdot R_d \cdot R_e$ acquires an expectation value. Along this direction $R_a^\alpha = v \delta_a^\alpha$, $a = 1, 2, 3$, and the $SU(3)$ gauge symmetry is completely broken. We will be interested in what happens for large values of v . In particular we will assume that $v \gg \Lambda_3, \Lambda_5$ and discuss the low-energy effective theory in this region of moduli space. The supersymmetry breaking vacuum will then lie in this region thereby making our analysis self-consistent. If $v \gg \Lambda_3$, the $SU(3)$ gauge symmetry is broken while it is still weak and non-perturbative effects coming from it can be neglected. Furthermore, the Yukawa coupling—the first term in eq. (58)—gives a large mass, $\sim \lambda v$ to the Q and L fields, which transform under the $SU(5)$ gauge symmetry. Thus, these matter fields can be integrated out, leaving a pure $SU(5)$ group at low-energies. The strong coupling scale of this theory is given by $\Lambda_{5L}^{15} = \lambda^3 b^{45} \Lambda_5^{12}$. Gaugino condensation in the low-energy pure $SU(5)$ theory now gives rise to a superpotential:

$$W_{eff} \sim \Lambda_{5L}^3 \sim \lambda^{\frac{3}{5}} \Lambda_5^{\frac{12}{5}} (b^{45})^{\frac{1}{5}}. \quad (60)$$

It is easy to see that this superpotential results in runaway behavior, with the R fields being pushed out to infinity. The behavior described above along the $b^{45} \neq 0$ direction is in fact true along a general baryonic flat direction, $b^{ab} \neq 0$ as well.

We now return to the original theory, with α , eq. (58), (59), non-zero but small. As was mentioned above, the flat directions are now all lifted with the choice (59). However, if α is small enough we still expect the vacuum to lie far out along the baryonic directions. The low-energy effective theory is then described by an independent set made out of the baryon fields, b^{ab} , and is calculable for reasons analogous to the (4, 1) model case studied in Section 5.1.3. In particular, gaugino condensation in the unbroken $SU(5)$ group gives rise to a non-perturbative superpotential of the form (60) in this theory. The resulting sigma

model can be explicitly analyzed, [60], and shows that supersymmetry is indeed broken. We will not go into the details here.

Instead let us only note that the expectation values for the R fields can be estimated by balancing the non-perturbative term, eq. (58) with the second term in the tree level superpotential, $\tilde{\alpha}_{ab} R_c \cdot R_d \cdot R_e \epsilon^{abcde}$. This gives an estimate for v :

$$v \sim \left(\frac{\lambda^3}{\alpha^5} \right)^{\frac{1}{12}} \Lambda_5 . \quad (61)$$

The supersymmetry breaking scale then goes like:

$$M_{SUSY} \sim (\lambda^3 \alpha)^{1/12} \Lambda_5 \quad (62)$$

Finally, the scale at which the low-energy pure $SU(5)$ theory confines is

$$\Lambda_{5L} \sim \left(\frac{\lambda^3}{\alpha} \right)^{1/12} \Lambda_5 . \quad (63)$$

We can now check that the assumptions in the above analysis are consistent. For small enough α , $v \gg \Lambda_3, \Lambda_5$; moreover the vacuum energy, $M_{SUSY} \ll \Lambda_{5L}$. Thus, the breaking of supersymmetry could be studied in a low-energy effective theory which neglects the non-perturbative effects of the $SU(3)$ group and incorporates them for the $SU(5)$ group as discussed above.

Now let us ask what happens when α is increased. We see from eq. (61) that as α increases, v decreases. Thus at some point, while α is still much less than one, we come to a situation where $\Lambda_5 \ll v \sim \Lambda_3$. At this stage we can no longer reliably use the description above. In particular, we cannot neglect the dynamical effects in the $SU(3)$ gauge theory.

The “magnetic” description of supersymmetry breaking.

We now turn to constructing the dual theory with $SU(5) \times SU(2)$ gauge symmetry. This description will not be calculable, but it allows one to show quite generally, as long as $\alpha < 1$, that supersymmetry is broken.

Before discussing the dual theory it is important to state one assumption. Strictly speaking, so far, Seiberg’s duality has only been used to relate the electric and magnetic theories at zero momentum. Here we will assume that the two are equivalent for some range of non-zero momentum as well. This is not unreasonable—the two theories should approximate each other for small enough values of momentum. As long as this is true our analysis will be valid—the supersymmetry breaking can be brought within this range by tuning the parameters λ, α .

Note that, as was described at the beginning, we take $\Lambda_3 \gg \Lambda_5$. Thus, it is useful in constructing the dual theory to first “turn off” the $SU(5)$ coupling. In addition, to begin, we disregard the tree level Yukawa couplings, eq. (58). Since the $SU(3)$ theory has now 5 flavors, i.e. $N_f = N_c + 2$, the appropriate description of the infrared physics is in terms of a dual $SU(2)$ theory, with the following matter content:

| | | | |
|----------------------------|--------------------|--------------------|------|
| | $SU(5)$ | $SU(2)$ | |
| $q_{\dot{\beta}}^{\alpha}$ | $\bar{\mathbf{5}}$ | $\mathbf{5}$ | |
| $r^{\beta a}$ | $\mathbf{1}$ | $\bar{\mathbf{5}}$ | (64) |
| $M_{\alpha a}$ | $\mathbf{5}$ | $\mathbf{1}$ | |
| $L^{\alpha i}$ | $\bar{\mathbf{5}}$ | $\mathbf{1}$ | |

and a superpotential

$$W = M_{\alpha a} r^a \cdot q^{\alpha} . \quad (65)$$

Now we turn back on the “spectator” $SU(5)$ coupling and observe that in this dual description, the $SU(5)$ theory has five flavors of quarks (M_a) and antiquarks ($L^i, q^{\dot{\alpha}}$). It is therefore confining, with a quantum modified moduli space (supersymmetric QCD with $N_f = N_c$). Below the confining scale of the $SU(5)$ theory, the appropriate degrees of freedom are the baryons and mesons:

$$N_{a\dot{\alpha}} \sim M_a \cdot q_{\dot{\alpha}}, K_a^i \sim M_a \cdot L^i, B \sim \det M, \bar{B} \sim q^2 \cdot L^3 . \quad (66)$$

Hereafter we omit various scale factors that appear in the duality map; for details, see [23]. The superpotential (65) of the theory then becomes:

$$W = N \cdot r + A \left(N^2 \cdot K^3 - B\bar{B} - \bar{\Lambda}_5^{10} \right) , \quad (67)$$

where A is a Lagrange multiplier enforcing the quantum modified constraint. The scale $\bar{\Lambda}_5$ is the scale of the $SU(5)$ theory in the dual; it can be found using the duality scale matching and the symmetries of the problem. Below that scale, the appropriate degrees of freedom are the mesons and baryons (66) and the $SU(5)$ singlets r_a . We see that now, upon crossing the Λ_{5L} threshold, the matter content of the $SU(2)$ theory has changed: the $SU(2)$ theory has now 5 flavors, N_a and r^a . These flavors, however, are massive: below the scale $\bar{\Lambda}_5$, the Yukawa coupling in (65) turns into a mass term. Thus, at low enough energies, the $SU(2)$ theory confines as well.

The following analysis to find the confined degrees of freedom in this low-energy theory is straightforward, but rather tedious, and we give only the main points, omitting various details. For simplicity, let us collectively denote the mesons of the $SU(2)$ theory by V_{ij} , with $i, j = 1, \dots, 10$ (thus, the matrix V is antisymmetric and has elements $N_a \cdot N_b, N_a \cdot r^b, r^c \cdot r^d$).

Along the flat directions of the $SU(2)$ theory, the nonperturbative superpotential is

$$W_{dyn} = \left(\frac{\text{Pf}V}{\bar{\Lambda}_{2L}} \right)^{1/3}. \quad (68)$$

This superpotential also exists in supersymmetric QCD with $N_f > N_c$; upon adding mass terms and integrating out the flavors, it gives rise to the usual superpotential induced by gaugino condensation.¹⁸

Thus, following the intricate renormalization group flow, we arrive at a low-energy description in terms of chiral superfields only. The moduli of this low-energy theory are $K_a^i, (r^2)^{ab}, (N \cdot r)_b^a, (N^2)_{ab}, B$, and \bar{B} . The superpotential of this effective theory is the sum of (67) and (68). We now turn on the tree level superpotential (58), written in terms of the appropriate variables in the confined low-energy description,

$$W_{tree} = \tilde{\lambda}_i^a R_a \cdot Q \cdot L^i + \tilde{\alpha}_{ab} R_c \cdot R_d \cdot R_e \epsilon^{abcde} = \tilde{\lambda}_i^a K_a^i + \tilde{\alpha}_{ab} (r^2)^{ab}. \quad (69)$$

Note that the trilinear couplings in the tree level superpotential are mapped, by the strong coupling dynamics, into linear terms in the low-energy superpotential. We can now analyze the F term conditions in the effective theory that follow from its superpotential:

$$W = (N \cdot r)_a^a + A \left(N^2 \cdot K^3 - B \bar{B} - \bar{\Lambda}_5^{10} \right) + \left(\frac{\text{Pf}V}{\bar{\Lambda}_{2L}} \right)^{1/3} + \tilde{\lambda}_i^a K_a^i + \tilde{\alpha}_{ab} (r^2)^{ab}. \quad (70)$$

Extremizing with respect to $A, K, r^2, N \cdot r, N^2, B$, and \bar{B} , we find that there is no extremum of the superpotential (70), establishing thus that supersymmetry is broken (for more details, see [23]). Thus, as promised, the theory breaks supersymmetry. Unfortunately, we cannot say more about the resulting vacuum. As mentioned above, the $SU(5) \times SU(2)$ dual description is non-calculable.

We conclude this section with one comment. The model discussed here, like many others in this review are based on a non-simple, product group gauge theory. In fact many of the recently found theories with dynamical supersymmetry breaking have product gauge groups [14, 19, 21, 22, 23, 24, 25, 26, 28]. This is in large part because the non-perturbative behavior of such theories is reasonably well understood—for example, as we saw above, often dual theories can be constructed by applying Seiberg duality to each factor of the product in turn [22]. At the same time, as the examples here have shown, the interplay between the various groups can lead to interesting non-perturbative dynamics including supersymmetry breaking.

¹⁸ We note an additional subtlety here: the scale of the $SU(2)$ dual theory below the confining scale of $SU(5)$ is field dependent, $\bar{\Lambda}_{2L} \sim B$; for details, see [22, 23].

5.3 The $SU(5)$ model and related examples.

We end this Section by discussing some examples of models where the scale of supersymmetry breaking and strong dynamics are comparable. The particular models we study were discovered long ago by Affleck, Dine, and Seiberg [11], [12]. Of course in many examples discussed in the previous sections there is generically (i.e. in the absence of a small coupling) no separation between the various scales. Some of our comments will be applicable to them as well.

The theory we look at is a “one generation” $SU(5)$ model. It has an $SU(5)$ gauge symmetry with a single antisymmetric tensor, $A \sim \mathbf{\bar{10}}$, and antifundamental, $\bar{Q} \sim \mathbf{\bar{5}}$, matter representation. Affleck, Dine, and Seiberg argued that this theory probably breaks supersymmetry.

One can show that the theory has two anomaly free global symmetries, $U(1)_{\mathcal{A}} \times U(1)_{\mathcal{R}}$, under which the superfields transform as: $A \sim (-1, 1)$, $\bar{Q} \sim (3, -9)$. These global symmetries will play an important role in the subsequent discussion. No holomorphic gauge invariant can be constructed from the matter fields in this theory. From this it follows that the theory has no classical flat directions. It also follows that no superpotential is allowed by the gauge symmetries.

The reasoning leading to the conclusion that supersymmetry is broken in this model goes as follows. If the global symmetry of the model is unbroken there should be massless fermions in the spectrum to saturate 't Hooft's anomaly matching conditions. The authors of [11] performed a search for solutions of the anomaly conditions and found that the simplest solutions were extremely complicated. To illustrate this point, we give one of the simplest solutions of the anomaly matching conditions, $\text{tr}R = -26$, $\text{tr}\mathcal{A} = 5$, $\text{tr}\mathcal{A}^3 = 125$, $\text{tr}R^3 = -4976$, $\text{tr}R\mathcal{A}^2 = -450$, $\text{tr}\mathcal{A}R^2 = 1500$. The minimal solution [11] requires the existence of five massless Weyl fermions with charges $(-5, 26)$, $(5, -20)$, $(5, -24)$, $(0, 1)$, and $(0, -9)$ under the global $U(1)_{\mathcal{A}} \times U(1)_{\mathcal{R}}$ symmetry (this is to be contrasted with the solution of the nonsupersymmetric version of the model, where a single massless fermion saturates 't Hooft anomaly matching [63]). The difficulty in satisfying anomaly matching leads to the conclusion that some (or all) of the global symmetry is broken. Now we can apply the general reasoning (Section 4.4) that if a global symmetry is broken in a theory without classical flat directions, then supersymmetry is broken. One thus concludes that supersymmetry is broken in this theory. Additional arguments that supersymmetry is broken, based on considering correlators in instanton backgrounds, appear in [64]. The scale of supersymmetry breaking is, presumably, of the order of the strong coupling scale of $SU(5)$, Λ_5 , and the massless spectrum should include a goldstino and Goldstone boson(s) for the broken global symmetry.

There exists a whole class of models, whose low-energy dynamics reduces to that of the

$SU(5)$ model. These are the $SU(2k + 1)$ theories with matter representations $A \sim \square$ and $\bar{Q}^i \sim \bar{\square}$, with $i = 1, \dots, 2k - 3$. These theories have classically flat directions, parametrized by the gauge invariant holomorphic polynomials $X^{ij} = A \cdot \bar{Q}^i \cdot \bar{Q}^j$. By studying the D-term equations, it is easy to see that along a generic flat direction, $\text{rank}\langle X^{ij} \rangle = 2k - 4$, the gauge symmetry is broken to an $SU(5)$ with a $\square + \bar{\square}$ matter representation [9]. This theory breaks supersymmetry at a scale $\Lambda_{SU(5)}^{13} \sim \Lambda_{SU(2k+1)}^{4k+5} / X^{(4k-8)/3}$. The potential of the theory is, presumably, proportional to $\Lambda_{SU(5)}^4$, and the theory has a runaway vacuum. The runaway behavior can be avoided if tree level terms are added to the superpotential, $W_{tree} = \lambda_{ij} X^{ij}$, to lift the classically flat directions. The theory then has a stable supersymmetry breaking vacuum.

Another theory with very similar behavior to that of the $SU(5)$ “one-generation” model is the $SO(10)$ theory with a single spinor representation [12].

It is worth mentioning that the Witten index for the $SU(5)$ and $SO(10)$ theories can be calculated and vanishes, consistent with supersymmetry breaking as we discussed in Section 3.2. The basic idea is to add extra vectorlike flavors (e.g. for the $SU(5)$ theory pairs of \square and $\bar{\square}$) [16]. The resulting theory now has D-flat directions. One can add small mass terms for the extra vector like flavors and analyze the low-energy dynamics in an effective supersymmetric field theory [16], [17], [65], [66]. One finds that supersymmetry is broken and thus that the Witten index is zero. On increasing the masses, the extra heavy flavors decouple but the Witten index stays unchanged. These theories with extra light flavors are interesting in their own right as examples of supersymmetry breaking. For example, the $SU(5)$ theory with two extra pairs of \square and $\bar{\square}$ is a completely calculable model [17]. The resulting theory is very similar to the 3-2 model: the gauge symmetry is totally broken, there is a nonperturbative superpotential due to instanton effects, and for appropriately chosen parameters, the vacuum occurs for large expectation values, where the whole spectrum is under perturbative control.

6 Phenomenological applications: gauge mediated supersymmetry breaking.

Finally we end this review by briefly discussing the application of dynamical supersymmetry breaking to the construction of phenomenological models of supersymmetry breaking. For a detailed review, a complete list of references, and discussion of phenomenological signatures, we recommend [47].

As we discussed in the Introduction, supersymmetric extensions of the standard model offer an attractive solution to the hierarchy problem. In order to explain the hierarchy

between the electroweak and Planck scales, supersymmetry has to break dynamically and generate the electroweak scale $m_W \sim 10^{-17} M_{Planck}$. Achieving dynamical supersymmetry breaking requires the addition of a new sector of the theory—the supersymmetry breaking sector. Some of the models we discussed in the previous sections could play the roles of such a sector.

In order to generate masses for the scalar partners of the quarks and leptons and the fermion partners of the gauge bosons (and the soft parameters), the breaking of supersymmetry has to be communicated to the standard model. At present, theoretically speaking, there exist two main candidates for this messenger interaction.

An obvious candidate for such a messenger interaction is supergravity. Until recently, theories where supergravity is the messenger of supersymmetry breaking were the most studied ones. There are good reasons for this: since gravity is an universal interaction, once supersymmetry is broken in any sector of the theory, it is automatically transmitted to all other sectors, generating soft masses to the scalar superpartners of the quarks and leptons. The soft masses are of order

$$m_{soft} \sim \frac{M_{SUSY}^2}{M_{Planck}} \sim 10^{2-3} \text{GeV} , \quad (71)$$

where M_{SUSY} is the supersymmetry breaking scale. The above equation can be derived based on dimensional grounds: the soft masses have to vanish in the limit $M_{Planck} \rightarrow \infty$, while the power of M_{Planck} follows from the fact that the communication is a tree-level effect in the supergravity lagrangian. The requirement that the the soft mass parameters are of order the electroweak scale follows from phenomenological and naturalness considerations. From eq. (71), we can deduce that the scale of the supersymmetry breaking in supergravity mediated models is of order $M_{SUSY} \sim 10^{10-12}$ GeV. Thus, the supersymmetry breaking scale is rather high, beyond direct experimental reach. We will not discuss here the pros and cons of supergravity mediated models of supersymmetry breaking, but only mention an important drawback: the communication of supersymmetry breaking involves dynamics at scales of order M_{Planck} , which is not well understood at present (many other shortcomings, such as generically large flavor changing neutral currents, can be related to this fact).

An economical alternative to gravity, as the messenger interaction, are the gauge interactions of the standard model. This scenario, called gauge mediated supersymmetry breaking, has received considerable attention recently.

In their simplest incarnation, gauge mediated models postulate the existence of new particles with standard model charges—the messenger quarks and leptons. These messenger particles are heavy, with mass of order M_{mess} . They interact with the supersymmetry breaking sector and thereby acquire supersymmetry breaking mass splittings of order ΔM_{mess} .

Since they carry standard model gauge quantum numbers, the supersymmetry breaking mass splittings are transmitted to the standard model squarks, sleptons, and gauginos at the loop level. Typically, a soft mass parameter is of order

$$m_{soft} \sim \frac{g^2}{16\pi^2} \Delta M_{mess} \sim 10^{2-3} \text{GeV}, \quad (72)$$

where g is a standard model gauge coupling and $1/16\pi^2$ is the standard one-loop suppression factor. We see that the scale of the messengers is a lot smaller than the relevant scale in supergravity models: $\Delta M_{mess} \sim 10^{4-5} \text{GeV}$. A plausible possibility is that the scale of supersymmetry breaking is of the same order as the scale of mass splittings of the messenger supermultiplets, i.e. $M_{SUSY} \sim \Delta M_{mess}$. We see then, that gauge mediated models of supersymmetry breaking could involve physics at scales much smaller than the scales in supergravity; one also makes no use of the ill-understood dynamics at the Planck scale. The lower scale offers hope that the supersymmetry breaking dynamics may be amenable to direct experimental studies in a foreseeable future. We saw in our discussion in Section 4.2 that the breaking of global supersymmetry gives rise to a goldstino. In the presence of gravity the goldstino is “eaten” by the gravitino. Because of the low-scale of supersymmetry breaking in gauge mediated models the gravitino is often the lightest R charged particle. This can give rise to distinct experimental signatures.

Historically, gauge mediated models provided the first phenomenological framework of supersymmetry breaking. After the advent of supergravity (in the early 1980’s) they were abandoned, mostly because of the alluring simplicity of supergravity models (with the almost automatic generation of all soft parameters at tree level), and also because supersymmetric gauge dynamics was not well understood at the time.

Gauge mediated supersymmetry breaking was resurrected in 1994, when the first phenomenologically viable model was built [67]. This model has served to refocus attention on the possibility of gauge mediation and provided an important “existence proof”. But it has some drawbacks—one of them being its rather complicated structure. The supersymmetry breaking sector in this model uses the (3,2) model discussed in Section 5.1.1. The anomaly free unbroken $U(1)_Y$ global symmetry is gauged; it is called the “messenger $U(1)$ ” interaction. The messenger $U(1)$ transmits supersymmetry breaking to some other fields, which in turn give a supersymmetry breaking expectation value to a gauge singlet field (the “messenger singlet”). The messenger singlet finally interacts with the messenger quarks and leptons and gives them the desired supersymmetry breaking mass splitting. One reason for the complicated nature of this model is that at the time it was constructed only a few theories of dynamical supersymmetry breaking were known.

As we have seen in this review, recent studies of dynamical supersymmetry breaking

have brought many new theories exhibiting this phenomenon to light. These have proved instrumental in constructing new examples of gauge mediated models. In particular, one idea that has been explored is to construct models where the unification of the supersymmetry breaking and messenger sectors (thereby getting rid of the messenger $U(1)$ gauge interaction) is achieved. The idea is to identify the standard model gauge group with the unbroken global symmetry group of the supersymmetry breaking sector, which had to be large enough to accommodate the whole $SU(3) \times SU(2) \times U(1)$. The states in the supersymmetry breaking sector now also carry standard model gauge quantum numbers. Thus the messengers are identified with fields in the supersymmetry breaking sector; their supersymmetry breaking mass splittings are transmitted to the standard model squarks, sleptons, and gauginos at the loop level.

The first “direct gauge mediation” models that were constructed used the $SU(N) \times SU(N-k)$, N -odd, $k = 1, 2$ theories of [22], [23]. For appropriate choices of N these theories have a ground state with broken supersymmetry and a sufficiently large global symmetry group to allow for embedding the standard model gauge group in it [60], [61]. We also mention here that the “plateau” models of ref. [46], [47] have proven useful in constructing phenomenological models of supersymmetry breaking. For a discussion of the status of the various models mentioned here, and other recent developments in the field, we refer the reader to [49].

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