

**Fermi National Accelerator Laboratory**

**FERMILAB-FN-664**

## **Pulse Formation in a Hybrid Photodetector**

Dan Green

*Fermi National Accelerator Laboratory  
P.O. Box 500, Batavia, Illinois 60510*

January 1998

# **Pulse Formation in a Hybrid Photodetector**

Dan Green

Fermilab

## Introduction:

The US CMS experiment plans to use hybrid photodetectors (HPD) in the hadron calorimeter (HCAL) as the photon transducer. These devices consist of a vacuum package with a photocathode and a PIN diode contained within. The photoelectrons emitted by the cathode are accelerated by about 10 kV and impinge on a PIN diode. These devices are supplied in “pixel” mode by DEP.[1] They are low noise, reasonable gain, and operate in a high magnetic field. The PIN diode can also be separated into individual pixel readouts, making the device a low cost/channel baseline device for the CMS HCAL.

The LHC accelerator has bunch crossings every 25 nsec. Given the high rates, it is important to investigate the pulse formation in the HPD and to insure that the device is sufficiently fast to be used in CMS.

## Fields in the PIN Diode:

The depleted Si diode is characterized by an ion charge density  $\rho$ , which is constant. The electric field  $E$  is then proportional to the coordinate  $x$  along the depth of the diode, where  $d = 300 \mu\text{m}$  is the total depth of the diode. Just at depletion, the field is zero at the physical boundary. The field can be raised beyond the depletion point, as shown in Fig.1. The field  $E_0$  is the maximum field which exists just at depletion. The field  $\Delta E$  exists if the diode is driven over depletion. It is equivalent to characterize the overdrive as a distance,  $x_d$ , beyond the physical distance  $d$  over which the field extends (virtually).

$$\begin{aligned}\nabla \cdot E &= \rho = dE/dx & (1) \\ \Delta E &= E_0(x_d/d) \\ E &= E_0/d(d-x) + \Delta E \\ E &= E_0/d[(d-x)+x_d]\end{aligned}$$

The potential which is applied to the diode, the bias  $V_0$ , is derivable from the field  $E(x)$ , Under the boundary conditions,  $V(0) = 0$ ,  $V(d) = V_0$ .

$$\begin{aligned}E &= -\nabla V & (2) \\ V_0 &= -E_0[d/2+x_d]\end{aligned}$$

## The Motion of the Charge, $x(t)$ :

Assume that the HDP is constructed such that the accelerated photoelectrons impinge on the PIN diode. The range of a 10 kV electron in Si is about  $1.5 \mu\text{m}$ . We label the initial location of the photoelectron by  $x_0$ . In the DEP device, the diode is backside illuminated. In terms of Fig.1, that means  $x_0 \sim d$ . The liberated electrons move to  $x = d$ . Since they move a very short distance, at low velocity due to the low fields at that location, they induce only a small current on the electrodes. Therefore, we ignore the electron motion, and concentrate completely in what follows of the motion of the holes.

The holes move in the electric field  $E$  set up by the bias voltage  $V_0$ . Note that the device acts like a “solid state ionization chamber” except for the fact that the electric field is not uniform, as it is for a liquid argon calorimeter, for example. The holes are assumed to have a constant mobility,  $\mu$ , in the Si. The equation of motion of the holes is:

$$dx/dt = -\mu E \quad (3)$$

This equation is solved with the boundary condition,  $x(0) = x_0$ .

$$\begin{aligned} \tau &= d/\mu E_0 \\ x(t) &= (d+x_d) - (d+x_d-x_0)e^{t/\tau} \end{aligned} \quad (4)$$

Note that there is a characteristic time,  $\tau$ , over which the holes are swept by the field to the electrode located at  $x = 0$ . The exponential behavior of  $x(t)$  follows from the  $E \sim x$  behavior of the field. The total time to go to  $x = 0$  can be easily found from the general hole trajectory, Eq.3.

$$\begin{aligned} 0 < x_0 < d \\ x(t_0) &= 0 \\ (d+x_d)/(d+x_d-x_0) &= \exp(t_0/\tau) \end{aligned} \quad (5)$$

### Induced Current;

The signal taken off the Si diode is that induced on the electrodes by capacitive coupling. It can be found easily by appealing to energy conservation. The fields in the Si do work on the moving holes and that work must be supplied by the energy stored in the capacitor formed by the PIN diode electrode structure. The electrode current is  $I$  and the electrode charge is  $Q$ . The stored energy is  $U = Q^2/2C = QV^2/2$ .

$$\begin{aligned} I &= dQ/dt \\ dU &= V_0 dQ = F dx = (qE)(\mu E dt) \\ I(t) &= q\mu E^2/V_0 \end{aligned} \quad (6)$$

Substituting in the expression for  $x(t)$  we can find a compact form for the current  $I(t)$ .

$$\begin{aligned} I(t) &= I_{\max} \exp[2(t-t_0)/\tau] \\ I_{\max} &= (2q/\tau)[(1 + \alpha)^2/(1 + 2\alpha)] \\ \alpha &= x_d/d \end{aligned} \quad (7)$$

Note that the current is largest in the largest field which occurs at  $x \sim 0$  or  $t \sim t_0$ . Aside from factors of order 1, the maximum current is  $2e/\tau$ .

The current would be the pulse shape observed from a transimpedance amplifier. It is also common practice to integrate the current in a charge sensitive amplifier so that the output is the induced charge  $Q(t)$ . The charge can be found by integrating Eq.7 under the

boundary condition that  $Q(0) = 0$ . The maximum charge is collected when the holes sink on the electrode at  $x = 0$  at time  $t = t_o$ .

$$Q_{\max} = Q(t_o) = e[(1 + \alpha)^2 / (1 + 2\alpha)][1 - \exp(-2t_o/\tau)] \quad (8)$$

Clearly, if the time to sink on the electrodes is small with respect to  $\tau$  the total charge is small. On the other hand, if the time is comparable to  $\tau$  the total charge  $e$  is collected on the electrode.

### Numerology and Examples:

Suppose the PIN diode is  $300 \mu\text{m} = d$  deep. Assume that the resistivity of the Si is such that the depletion voltage is 50 V. Using Eq.2, we find that the field just at depletion is,  $E_o = 3.3 \text{ kV/cm}$ . Using the value for the mobility,  $\mu = 400 \text{ cm}^2/\text{V}\cdot\text{sec}$ , we find that  $\tau \sim 22 \text{ nsec}$ . In what follows we assume that  $x_o$  is the  $\sim$  the range of a 10 kV electron in Si, or  $2 \mu\text{m}$ . These numerical values result in the values of the parameters shown in Table 1.

Table 1: PIN Diode Parameters

$V_o$ (Volts)	$x_d$ ( $\mu\text{m}$ )	$\Delta E$ (kV/cm)	$t_o$ (nsec)	$I_{\max}$ (pA)
55	16	0.18	65	13.95
75	77	0.85	36	14.52
100	153	1.68	24.7	15.70
150	304	3.35	15.7	18.64
200	456	5.02	11.6	21.87

Clearly, just near depletion,  $V_o \sim 50\text{V}$ , the fields near the  $x = d$  electrode are small. At an applied voltage of 150V, the field is everywhere  $>$  the peak voltage just at depletion, 3.3 kV/cm, and  $x_d \sim d$ . The time for the holes to cross the gap from  $x_o \sim d - 2 \mu\text{m}$  to  $x = 0$  is large near depletion voltage and shrinks to 25 nsec (LHC bunch crossing) at 100 V. The current  $I_{\max}$  in Table 1 refers to the current induced by the motion of a single hole. Since a single photoelectron releases about 2000 e-h pairs, the peak current for a single photoelectron is  $\sim 30 \text{ nA}$ . This peak current is rather insensitive to the applied bias voltage  $V_o$ .

The pulse shapes for  $V_o = 75, 100$  and  $200 \text{ V}$  are shown in Fig.2. Note the shortening of the pulse width,  $t_o$ , as  $V_o$  is raised. Note also that  $I(o)$  increases with  $V_o$  as the fields are increased at  $x(o) \sim d$  in the overdepleted mode of operation. Comparing to Table 1, the values of  $t_o$  and  $I_{\max}$  can be read off of Fig.1. The basic exponential behavior of  $I(t)$  is seen, with  $I(t)$  peaking at a value of  $I_{\max}$  at  $t = t_o$ . The charge  $Q(t)$  is just the integral of

the current pulse shapes shown in Fig.2. The analysis given above has been experimentally confirmed in recent tests with a DEP device with many pixels. [2] Pulse shapes very similar to those shown in Fig.2, suitably rounded by rise time effects of the electronics, have been seen. That data supports the simplifying assumptions ( e.g. electron motion ignored) made in this note.

**References:**

1. Hybrid Photomultiplier Tubes, DEP – Delft Instruments, the Netherlands
2. Anatoly Rhonzin – private communication

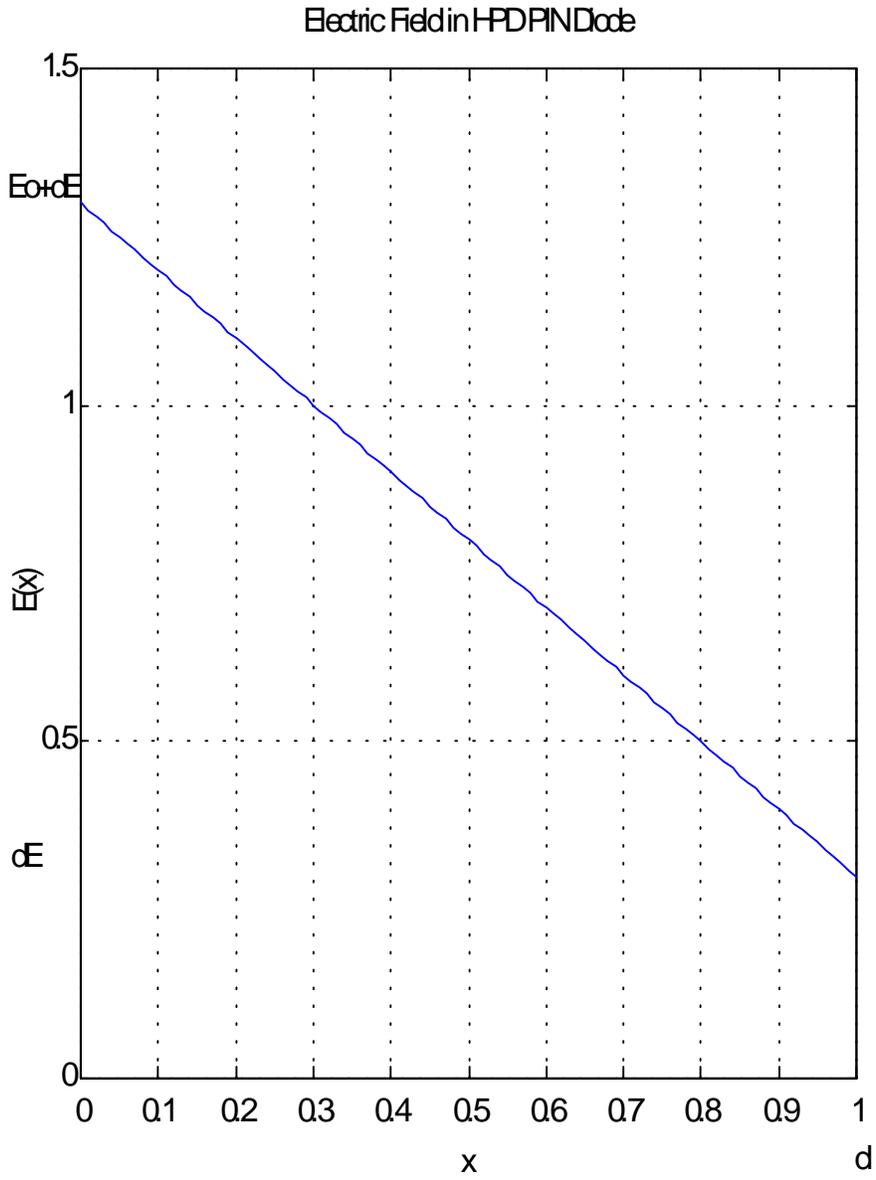


Figure 1: Behavior of  $E(x)$  in the general case. The potential is greater than the depletion potential, so that the field at  $x = d$  is  $\Delta E$ . The intercept at  $E = 0$  occurs at  $d + x_d$ , outside the PIN diode.

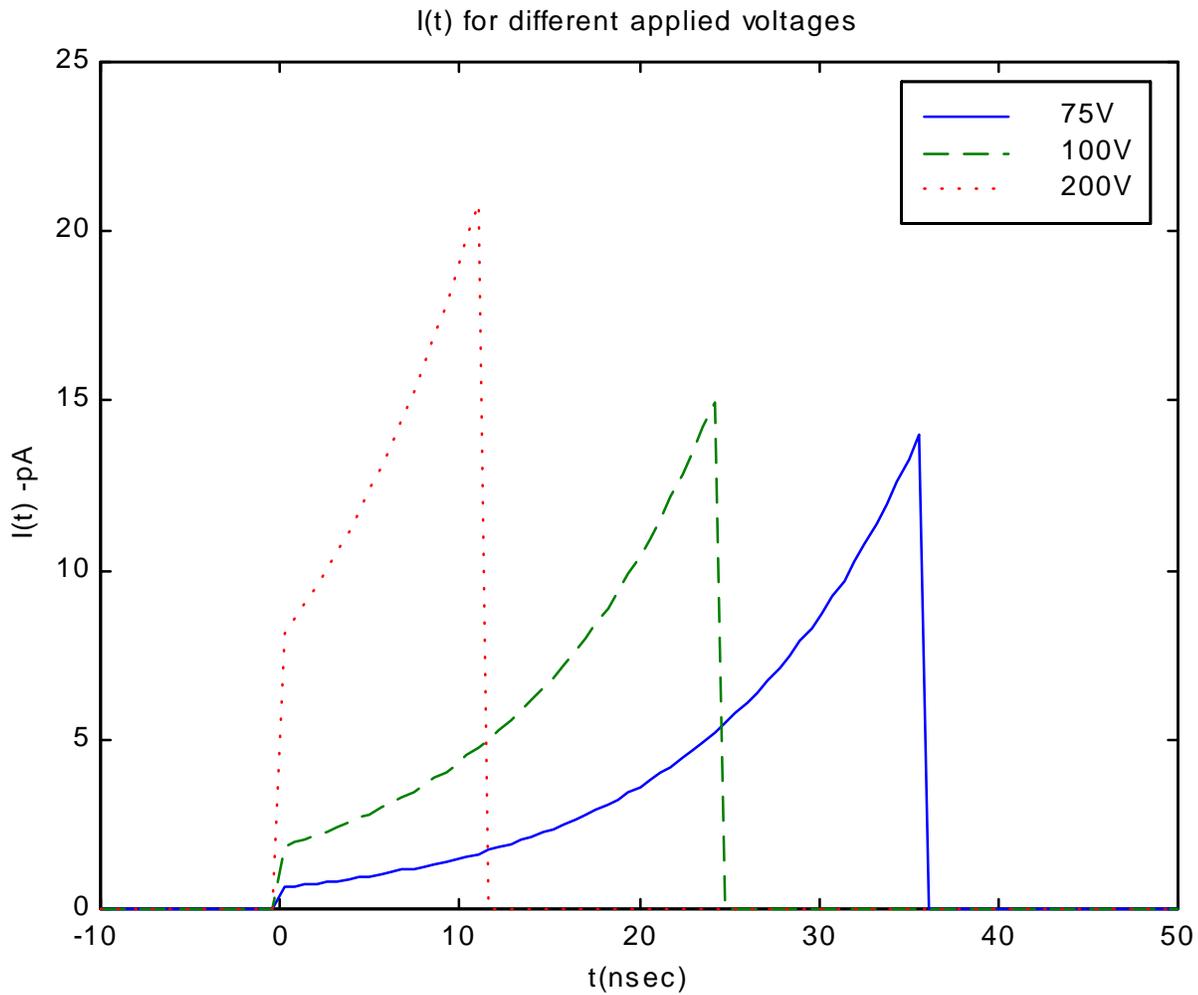


Figure 2: Pulse shape,  $I(t)$ , for three different bias voltages. The current pulse takes a time  $t_0$  to fully develop, starting with a finite value at  $t = 0$ , rising exponentially, and achieving a maximum at  $t = t_0$ .