

Cosmological Parameters

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Abstract. The discussion of cosmological parameters used to be a source of embarrassment to cosmologists. Today, measurements of the cosmological parameters are leading the way into the era of precision cosmology. The CMB temperature is measured to four significant figures, $T_0 = 2.7277 \pm 0.002$ K; the Hubble constant is now determined with a reliable error estimate, $H_0 = (65 \pm 5)$ km sec⁻¹ Mpc⁻¹; the mass density of baryons is precisely determined by big-bang nucleosynthesis $\Omega_B = (0.019 \pm 0.001)h^{-2}$; and the age of the Universe inferred from the ages of the oldest stars is 14 ± 1.5 Gyr, which is consistent the expansion age. Further, we have the first full accounting of matter and energy in the Universe, complete with a self consistency check. Expressed a fraction of the critical density it goes like this: neutrinos, between 0.3% and 15%; stars, between 0.3% and 0.6%; baryons (total), $5\% \pm 0.5\%$; matter (total), $40\% \pm 10\%$; smooth, dark energy, $80\% \pm 20\%$; totaling to the critical density (within the errors).

INTRODUCTION

Five years ago “precision cosmology” would have been an oxymoron. That is not true today. Our knowledge of the cosmological parameters reflects that change. Once an embarrassment, they are quickly becoming a source of pride.

The COBE FIRAS team led by John Mather determined the temperature of the cosmic microwave background radiation to four significant figures, $T_0 = (2.7277 \pm 0.0001 \pm 0.002)$ K. Here, the first error is the error relative to the internal reference black body and the second error, which dominates the error budget, is the uncertainty in determining the temperature of the internal reference black body (Fixsen et al, 1996). Had the on-board thermometers performed better, they might have had another couple of significant figures (which boggles the mind!). In any case, this is a truly impressive measurement, cosmological or otherwise.

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For the first time in thirty years, there is agreement (within the uncertainties) on the value of the Hubble constant and a reliable estimate of the uncertainty. *My summary* of the recent measurements is:² $H_0 = (65 \pm 5) \text{ km s}^{-1} \text{ Mpc}^{-1}$ (see Freedman, 1999 for a more expert assessment of the situation). The improvement is due in large measure to the Hubble Key Project, which has established Cepheid calibrations for the secondary distance indicators and just as importantly stimulated renewed interest in the examining the distance scale. Systematic uncertainties remain; foremost are the distance to the LMC and reddening and metallicity effects on the Cepheid period-luminosity relation.

Similar progress has been made in dating the age of the oldest objects in the Universe: globular clusters, the heavy elements and white dwarfs (see e.g., Chaboyer, 1998). Allowing for a time of a 2 Gyr until the formation of the first generation of stars, the age of the Universe is $t_0 = (14 \pm 1.5) \text{ Gyr}$. While the hard lower limit to the age of 10 Gyr derives from all three methods, the central value and error is driven by age dating of globular clusters. Here progress has occurred due to better stellar modelling, improvement in the distance scale and better microphysics. Regarding the distance scale; the parallax distance measurements made by Hipparcos played a role, though, probably more important was the fact that it spurred scrutiny of the entire enterprise.

I devote the rest of this article to a summary of our knowledge of the matter and energy content of the Universe.

AN INVENTORY OF MATTER AND ENERGY

Preliminaries

The quantity and composition of matter and energy in the Universe is of fundamental importance in cosmology. The fraction of the critical energy density contributed by all forms of matter and energy,

$$\Omega_0 \equiv \frac{\rho_{\text{tot}}}{\rho_{\text{crit}}} = \sum_i \Omega_i, \quad (1)$$

determines the geometry of the Universe:

$$R_{\text{curv}}^2 = \frac{H_0^{-2}}{\Omega_0 - 1}. \quad (2)$$

Here, subscript ‘0’ denotes the value at the present epoch, $\rho_{\text{crit}} = 3H_0^2/8\pi G \simeq 1.88h^2 \times 10^{-29} \text{ g cm}^{-3}$, Ω_i is the fraction of critical density contributed by component i (e.g., baryons, photons, stars, etc) and $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$. The sign of R_{curv}^2

²⁾ I use standard conventions for the meaning of error bars: 1σ bet your graduate student’s reputation; 2σ bet the family pet; 3σ bet a child (other than first born male).

MATTER / ENERGY in the UNIVERSE

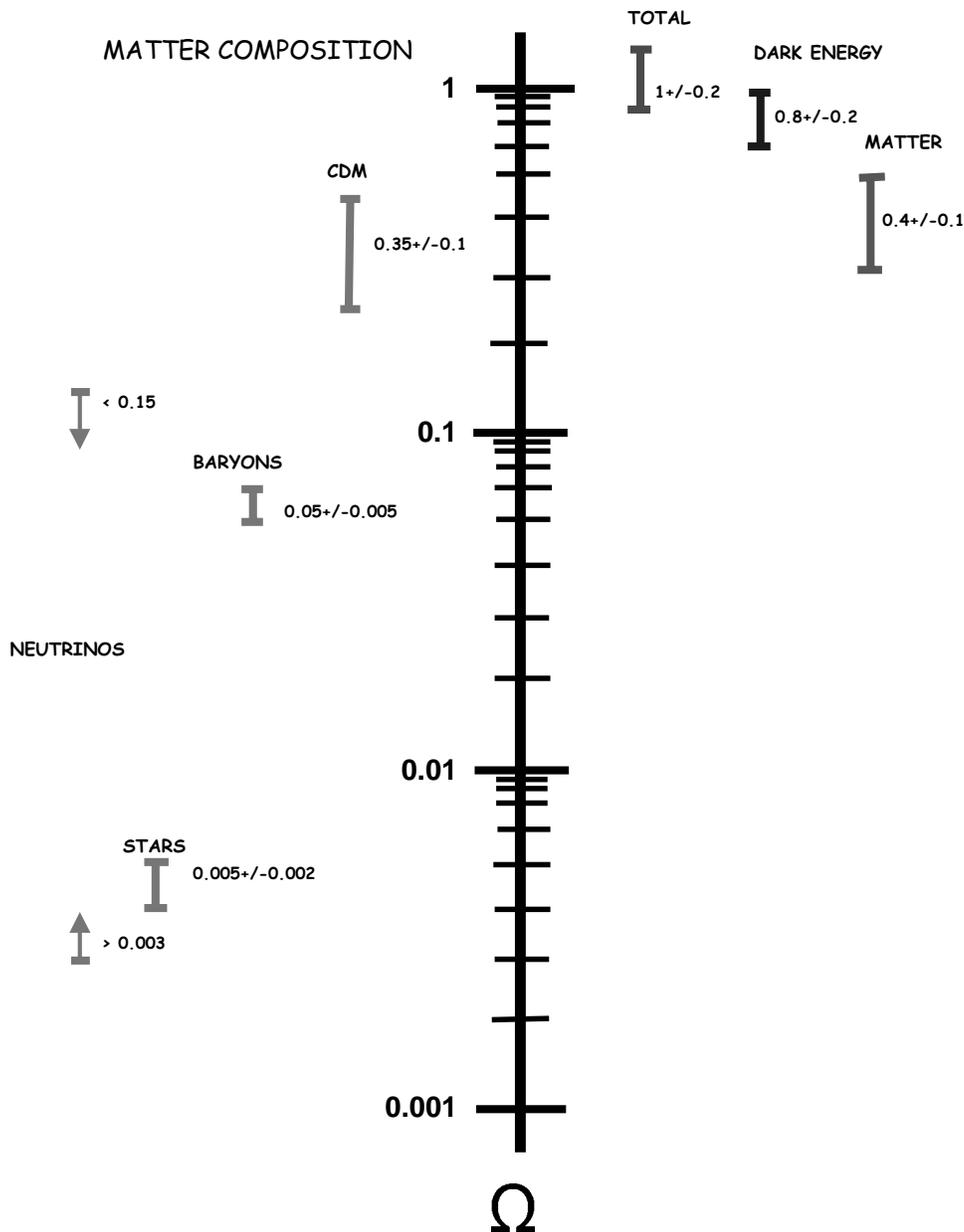


FIGURE 1. Summary of matter/energy in the Universe. The right side refers to an overall accounting of matter and energy; the left refers to the composition of the matter component. The contribution of relativistic particles, CBR photons and neutrinos, $\Omega_{\text{rel}} h^2 = 4.170 \times 10^{-5}$, is not shown. The upper limit to mass density contributed by neutrinos is based upon the failure of the hot dark matter model of structure formation (White, Frenk & Davis, 1983; and Dodelson et al, 1996) and the lower limit follows from the evidence for neutrino oscillations (Fukuda et al, 1998). Here H_0 is taken to be $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$.

specifies the spatial geometry: positive for a 3-sphere, negative for a 3-saddle and 0 for the flat space.

The present rate of deceleration of the expansion depends upon Ω_0 as well as the composition of matter and energy,

$$q_0 \equiv \frac{(\ddot{R}/R)_0}{H_0^2} = \frac{1}{2}\Omega_0 + \frac{3}{2}\sum_i \Omega_i w_i. \quad (3)$$

The pressure of component i , $p_i \equiv w_i \rho_i$; e.g., for baryons $w_i = 0$, for radiation $w_i = 1/3$, and for vacuum energy $w_i = -1$.

The fate of the Universe – expansion forever or recollapse – is not directly determined by Ω_0 and/or q_0 . It depends upon precise knowledge of the composition of *all* components of matter and energy, for all times in the future. Recollapse occurs only if there is a future turning point, that is an epoch when the expansion rate,

$$H^2 = \frac{8\pi G}{3} \sum_i \rho_i - \frac{1}{R_{\text{curv}}^2}, \quad (4)$$

vanishes and

$$\frac{\ddot{R}}{R} = -\frac{4\pi G}{3} \sum_i \rho_i [1 + w_i] \quad (5)$$

is less than zero. In a universe comprised of matter alone, a positively curved universe ($\Omega_0 > 1$) eventually recollapses and a negatively curved universe ($\Omega_0 < 1$) expands forever. However, exotic components complicate matters: a positively curved universe with positive vacuum energy can expand forever, and a negatively curved universe with negative vacuum energy can recollapse.

The quantity and composition of matter and energy in the Universe is also crucial for understanding the past. It determines the relationship between age of the Universe and redshift, when the Universe ended its early radiation dominated era, and the growth of small inhomogeneities in the matter. Ultimately, the formation and evolution of large-scale structure and even individual galaxies depends upon the composition of the dark matter and energy.

Measuring the quantity and composition of matter and energy in the Universe is a challenging task. Not just because the scale of inhomogeneity is so large, around 10 Mpc; but also, because there may be components that remain exactly or relatively smooth (e.g., vacuum energy or relativistic particles) and only reveal their presence by their influence on the evolution of the Universe itself.

Radiation

Because the cosmic background radiation (CBR) is known to be black-body radiation to very high precision (better than 0.005%) and its temperature is known

to four significant figures, $T_0 = 2.7277 \pm 0.002 \text{ K}$, its contribution is very precisely known, $\Omega_\gamma h^2 = 2.480 \times 10^{-5}$. If neutrinos are massless or very light, $m_\nu \ll 10^{-4} \text{ eV}$, their energy density is equally well known because it is directly related to that of the photons, $\Omega_\nu = \frac{7}{8}(4/11)^{4/3}\Omega_\gamma$ (per species) (there is a small 1% positive correction to this number; see Dodelson & Turner, 1992).

It is possible that additional relativistic species contribute significantly to the energy density, though big-bang nucleosynthesis (BBN) severely constrains the amount (the equivalent of less than 0.4 of a neutrino species; see e.g., Burles et al, 1999) unless they were produced by the decay of a massive particle after the epoch of BBN.

In any case, we can be confident that the radiation component of the energy density today is small. The matter contribution (denoted by Ω_M), consisting of particles that have negligible pressure, is the easiest to determine because matter clumps and its gravitational effects are thereby enhanced (e.g., in rich clusters the matter density typically exceeds the mean density by a factor of 1000 or more). With this in mind, I will decompose the present matter/energy density into two components, matter and vacuum energy,

$$\Omega_0 = \Omega_M + \Omega_\Lambda, \quad (6)$$

ignoring the contribution of the CBR and ultrarelativistic neutrinos. I will use vacuum energy as a stand in for a smooth component (more later). Vacuum energy and a cosmological constant are indistinguishable: a cosmological constant corresponds to a uniform energy density of magnitude $\rho_{\text{vac}} = \Lambda/8\pi G$.

Total Matter/Energy Density: $\Omega_0 = 1 \pm 0.2$

There is a growing consensus that the anisotropy of the CBR offers the best means of determining the curvature of the Universe and thereby Ω_0 , cf., Eq. (2). This is because the method is intrinsically geometric – a standard ruler on the last-scattering surface – and involves straightforward physics at a simpler time (see e.g., Kamionkowski et al, 1994).

At last scattering baryonic matter (ions and electrons) was still tightly coupled to photons; as the baryons fell into the dark-matter potential wells the pressure of photons acted as a restoring force, and gravity-driven acoustic oscillations resulted. These oscillations can be decomposed into their Fourier modes; Fourier modes with $k \sim lH_0/2$ determine the multipole amplitudes a_{lm} of CBR anisotropy. Last scattering occurs over a short time, making the CBR is a snapshot of the Universe at $t_{\text{ls}} \sim 300,000 \text{ yrs}$. Each mode is “seen” in a well defined phase of its oscillation. (For the density perturbations predicted by inflation, all modes the have same initial phase because all are growing-mode perturbations.) Modes caught at maximum compression or rarefaction lead to the largest temperature anisotropy; this results in a series of acoustic peaks beginning at $l \sim 200$ (see Fig. 2). The wavelength of the lowest frequency acoustic mode that has reached maximum compression,

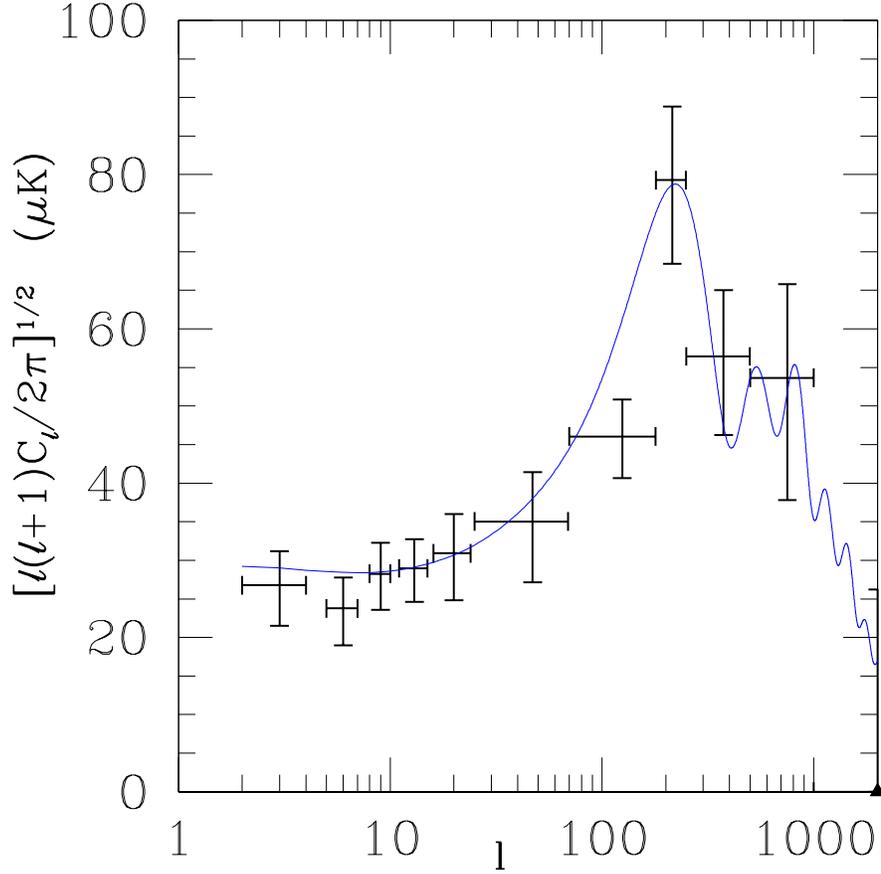


FIGURE 2. Summary of CBR anisotropy measurements, binned to reduce error bars and visual confusion. The temperature variations across the sky have been expanded in spherical harmonics, $\delta T(\theta, \phi) = \sum_i a_{lm} Y_{lm}(\theta, \phi)$ and $C_l \equiv \langle |a_{lm}|^2 \rangle$. In plain language, this plot shows the size of the temperature variation between two points on the sky separated by angle θ (ordinate) vs. multipole number $l = 200^\circ/\theta$ ($l = 2$ corresponds to 100° , $l = 200$ corresponds to $\theta = 1^\circ$, and so on). The theoretical curve is for the Λ CDM model with $H_0 = 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_M = 0.4$, which is a good fit to the data. Note also the evidence for the first of a series of “acoustic peaks.” The presence of these acoustic peaks is a key signature of the density perturbations of quantum origin predicted by inflation (Figure courtesy of L. Knox).

$\lambda_{\max} \sim v_s t_{\text{ls}}$, is the standard ruler on the last-scattering surface. Both λ_{\max} and the distance to the last-scattering surface depend upon Ω_0 , and the position of the first peak $l \simeq 200/\sqrt{\Omega_0}$. This relationship is insensitive to the composition of matter and energy in the Universe.

CBR anisotropy measurements, shown in Fig. 2, now cover three orders of magnitude in multipole number and involve more than twenty experiments. COBE is the most precise and covers multipoles $l = 2 - 20$; the other measurements come from balloon-borne, Antarctica-based and ground-based experiments using both low-frequency ($f < 100$ GHz) HEMT receivers and high-frequency ($f > 100$ GHz) bolometers. Taken together, all the measurements are beginning to define the position of the first acoustic peak, at a value that is consistent with a flat Universe. Various analyses of the extant data have been carried out, indicating $\Omega_0 \sim 1 \pm 0.2$ (see e.g., Lineweaver, 1998). It is certainly too early to draw definite conclusions or put too much weight in the error estimate. However, a strong case is developing for a flat Universe and more data is on the way (Python V, Viper, MAT, Maxima, Boomerang, DASI, and others). Ultimately, the issue will be settled by NASA's MAP (launch late 2000) and ESA's Planck (launch 2007) satellites which will map the entire CBR sky with 30 times the resolution of COBE (around 0.1°) (see Page & Wilkinson, 1999).

Matter

Baryons

For more than twenty years big-bang nucleosynthesis has provided a key test of the hot big-bang cosmology as well as the most precise determination of the baryon density. Careful comparison of the primeval abundances of D, ^3He , ^4He and ^7Li with their big-bang predictions defined a concordance interval, $\Omega_B h^2 = 0.007 - 0.024$ (see e.g., Copi et al, 1995; for another view, see Hata et al, 1995).

Of the four light elements produced in the big bang, deuterium is the most powerful "baryometer" – its primeval abundance depends strongly on the baryon density ($\propto 1/\rho_B^{1/7}$) – and the evolution of its abundance since the big bang is simple – astrophysical processes only destroy deuterium. Until recently deuterium could not be exploited as a baryometer because its abundance was only known locally, where roughly half of the material has been through stars with a similar amount of the primordial deuterium destroyed. In 1998, the situation changed dramatically, launching BBN into the precision era of cosmology.

Over the past four years there have been claims of upper limits, lower limits, and determinations of the primeval deuterium abundance, ranging from $(\text{D}/\text{H}) = 10^{-5}$ to $(\text{D}/\text{H}) = 3 \times 10^{-4}$. In short, the situation was confusing. In 1998 Burles and Tytler clarified matters and established a strong case for $(\text{D}/\text{H})_P = (3.4 \pm 0.3) \times 10^{-5}$ (Burles & Tytler, 1998a,b). That case is based upon the deuterium abundance measured in four high-redshift hydrogen clouds seen in absorption against distant

QSOs, and the remeasurement and reanalysis of other putative deuterium systems. In this enterprise, the Keck I 10-meter telescope and its HiRes Echelle Spectrograph have played the crucial role.

The Burles – Tytler measurement turns the previous factor of three concordance range for the baryon density into a 10% determination of the baryon density, $\rho_B = (3.8 \pm 0.4) \times 10^{-31} \text{ g cm}^{-3}$ or $\Omega_B h^2 = 0.02 \pm 0.002$ (see Fig. 3), with about half the error in ρ_B coming from the theoretical error in predicting the BBN yield of deuterium. [A very recent analysis has reduced the theoretical error significantly, and improved the accuracy of the determination of the baryon density, $\Omega_B h^2 = 0.019 \pm 0.0012$ (Burles et al, 1999).]

This precise determination of the baryon density, based upon the early Universe physics of BBN, is consistent with two other measures of the baryon density, based upon entirely different physics. By comparing measurements of the opacity of the Lyman- α forest toward high-redshift quasars with high-resolution, hydrodynamical simulations of structure formation, several groups (Meiksin & Madau, 1993; Rauch et al, 1997; Weinberg et al, 1997) have inferred a lower limit to the baryon density, $\Omega_B h^2 > 0.015$ (it is a lower limit because it depends upon the baryon density squared divided by the intensity of the ionizing radiation field). The second test involves the height of the first acoustic peak: it rises with the baryon density (the higher the baryon density, the stronger the gravitational force driving the acoustic oscillations). Current CBR measurements are consistent with the Burles – Tytler baryon density; the MAP and Planck satellites should ultimately provide a 5% or better determination of the baryon density, based upon the physics of gravity-driven acoustic oscillations when the Universe was 300,000 yrs old. This will be an important cross check of the BBN determination.

Weighing the dark matter: $\Omega_M = 0.4 \pm 0.1$

Since the pioneering work of Fritz Zwicky and Vera Rubin, it has been known that there is far too little material in the form of stars (and related material) to hold galaxies and clusters together, and thus, that most of the matter in the Universe is dark (see e.g. Trimble, 1987). Weighing the dark matter has been the challenge. At present, I believe that clusters provide the most reliable means of estimating the total matter density.

Rich clusters are relatively rare objects – only about 1 in 10 galaxies is found in a rich cluster – which formed from density perturbations of (comoving) size around 10 Mpc. However, because they gather together material from such a large region of space, they can provide a “fair sample” of matter in the Universe. Using clusters as such, the precise BBN baryon density can be used to infer the total matter density (White et al, 1993). (Baryons and dark matter need not be well mixed for this method to work provided that the baryonic and total mass are determined over a large enough portion of the cluster.)

Most of the baryons in clusters reside in the hot, x-ray emitting intracluster gas

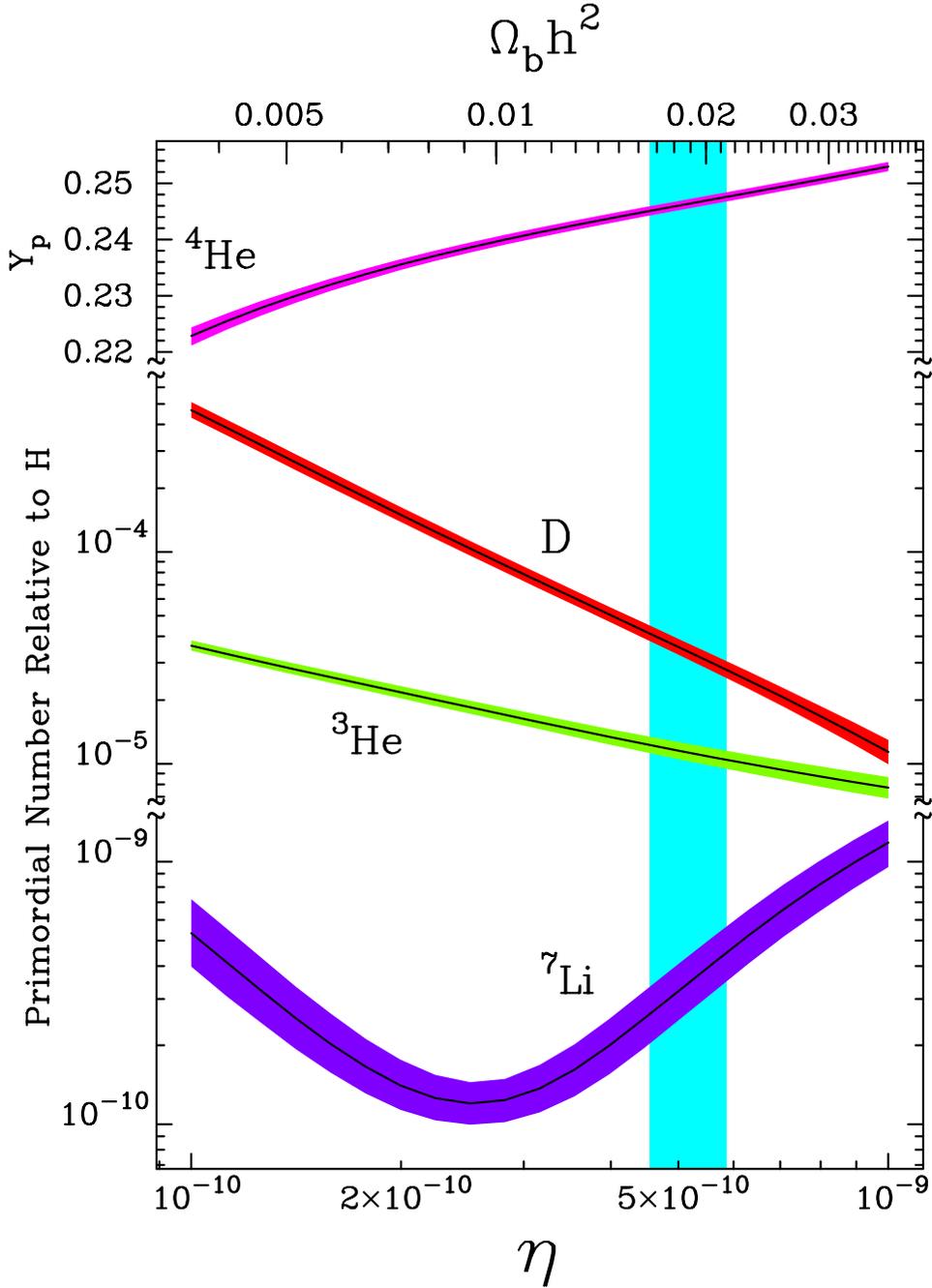


FIGURE 3. Predicted abundances of ${}^4\text{He}$ (mass fraction), D, ${}^3\text{He}$, and ${}^7\text{Li}$ (number relative to hydrogen) as a function of the baryon density; widths of the curves indicate “ 2σ ” theoretical uncertainty. The dark band highlights the determination of the baryon density based upon the recent measurement of the primordial abundance of deuterium (Burles & Tytler, 1998a,b), $\Omega_B h^2 = 0.019 \pm 0.0024$ (95% cl); the baryon density is related to the baryon-to-photon ratio by $\rho_B = 6.88\eta \times 10^{-22} \text{ g cm}^{-3}$ (from Burles et al, 1999).

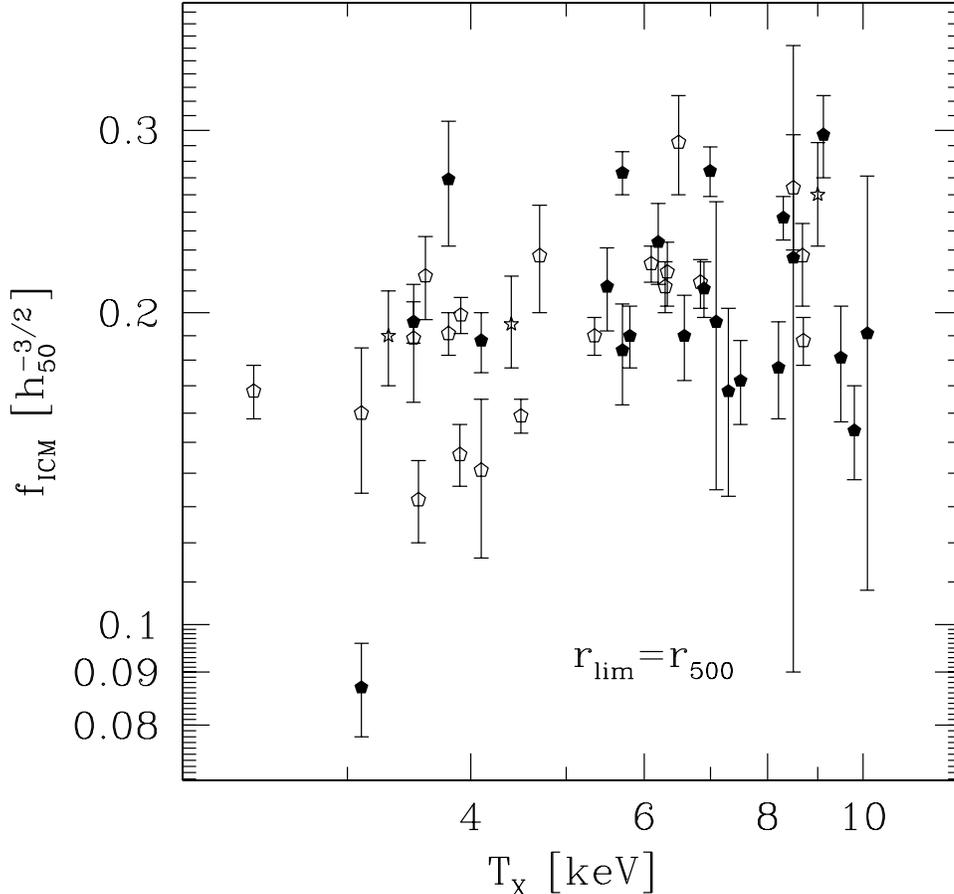


FIGURE 4. Cluster gas fraction as a function of cluster gas temperature for a sample of 45 galaxy clusters (Mohr et al, 1998). While there is some indication that the gas fraction decreases with temperature for $T < 5$ keV, perhaps because these lower-mass clusters lose some of their hot gas, the data indicate that the gas fraction reaches a plateau at high temperatures, $f_{\text{gas}} = 0.212 \pm 0.006$ for $h = 0.5$ (Figure courtesy of Joe Mohr).

and not in the galaxies themselves, and so the problem essentially reduces to determining the gas-to-total mass ratio. The gas mass can be determined by two methods: 1) measuring the x-ray flux from the intracluster gas and 2) mapping the Sunyaev - Zel'dovich CBR distortion caused by CBR photons scattering off hot electrons in the intracluster gas. The total cluster mass can be determined three independent ways: 1) using the motions of clusters galaxies and the virial theorem; 2) assuming that the gas is in hydrostatic equilibrium and using it to infer the underlying mass distribution; and 3) mapping the cluster mass directly by gravitational lensing (Tyson, 1999). Within their uncertainties, and where comparisons can be made, the three methods for determining the total mass agree (see e.g., Tyson, 1999); likewise, the two methods for determining the gas mass are consistent.

Mohr et al (1998) have compiled the gas to total mass ratios determined from x-ray measurements for a sample of 45 clusters; they find $f_{\text{gas}} = (0.075 \pm 0.002)h^{-3/2}$.

Carlstrom (1999), using his S-Z gas measurements and x-ray measurements for the total mass for 27 clusters, finds $f_{\text{gas}} = (0.06 \pm 0.006)h^{-1}$. (The agreement of these two numbers means that clumping of the gas, which could lead to an overestimate of the gas fraction based upon the x-ray flux, is not a problem.) Invoking the “fair-sample assumption,” the mean matter density in the Universe can be inferred:

$$\begin{aligned}\Omega_M &= \Omega_B / f_{\text{gas}} = (0.3 \pm 0.05)h^{-1/2} \text{ (Xray)} \\ &= (0.25 \pm 0.04)h^{-1} \text{ (S - Z)} \\ &= 0.4 \pm 0.1 \text{ (my summary)}. \end{aligned} \tag{7}$$

I believe this to be the most reliable and precise determination of the matter density. It involves few assumptions, and most of them have now been tested (clumping, hydrostatic equilibrium, variation of gas fraction with cluster mass).

Supporting evidence for $\Omega_M = 0.4 \pm 0.1$

This result is consistent with a variety of other methods that involve different physics. For example, based upon the evolution of the abundance of rich clusters with redshift, Henry (1999) finds $\Omega_M = 0.45 \pm 0.1$ (also see, Bahcall & Fan, 1998 and N. Bahcall, 1999). Dekel and Rees (1994) place a low limit $\Omega_M > 0.3$ (95% cl) derived from the outflow of material from voids (a void effectively acts as a negative mass proportional to the mean matter density).

The analysis of the peculiar velocities of galaxies provides an important probe of the mass density averaged over very large scales (of order several hundred Mpc). By comparing measured peculiar velocities with those predicted from the distribution of matter revealed by redshift surveys such as the IRAS survey of infrared galaxies, one can infer the quantity $\beta = \Omega_M^{0.6}/b_I$ where b_I is the linear bias factor that relates the inhomogeneity in the distribution of IRAS galaxies to that in the distribution of matter (in general, the bias factor is expected to be in the range 0.7 to 1.5; IRAS galaxies are expected to be less biased, $b_I \approx 1$). Recent work by Willick & Strauss (1998) finds $\beta = 0.5 \pm 0.05$, while Sigad et al (1998) find $\beta = 0.9 \pm 0.1$. The apparent inconsistency of these two results and the ambiguity introduced by bias preclude a definitive determination of Ω_M by this method. However, Dekel (1994) quotes a 95% confidence lower bound, $\Omega_M > 0.3$, and the work of Willick & Strauss seems to strongly indicate that Ω_M is much less than 1; both are consistent with $\Omega_M \sim 1$.

Finally, there is strong, but circumstantial, evidence from structure formation that Ω_M is around 0.4 and significantly greater than Ω_B . It is based upon three different lines of reasoning. First, there is no viable model for structure formation without a significant amount of nonbaryonic dark matter. The reason is simple: in a baryons-only model, density perturbations grow only from the time of decoupling, $z \sim 1000$, until the Universe becomes curvature dominated, $z \sim \Omega_B^{-1} \sim 20$; this is not enough growth to produce all the structure seen today with the size of density

perturbations inferred from CBR anisotropy. With nonbaryonic dark matter and $\Omega_M \gg \Omega_B$, dark-matter perturbations begin growing at matter – radiation equality and continue to grow until the present epoch, or nearly so, leading to significantly more growth and making the observed large-scale structure consistent with the size of the density perturbations inferred from CBR anisotropy.

Second, the transition from radiation domination at early times to matter domination at around 10,000 yrs leaves its mark on the shape of the present power spectrum of density perturbations, and the redshift of matter – radiation equality depends upon Ω_M . Measurements of the shape of the present power spectrum based upon redshift surveys indicate that the shape parameter $\Gamma = \Omega_M h \simeq 0.25 \pm 0.05$ (see e.g., Peacock & Dodds, 1994). For $h \sim 2/3$, this implies $\Omega_M \sim 0.4$. (If there are relativistic particles beyond the CBR photons and relic neutrinos, the formula for the shape parameter changes and $\Omega_M \gg 0.4$ can be accommodated; see Dodelson et al, 1996). Third, the Ly- α forest, seen in the absorption spectra of high-redshift quasars, gives us a unique view of the Universe at a time when structure was just becoming nonlinear. Based upon a comparison of high-resolution N -body simulations with 28 Keck HIRES spectra, Weinberg et al (1998) infer $\Omega_M = 0.34 \pm 0.1$.

Mass-to-light ratios and Ω_M

The most mature approach to estimating the matter density involves the use of mass-to-light ratios, the measured luminosity density, and the deceptively simple equation,

$$\langle \rho_M \rangle = \langle M/L \rangle \mathcal{L}, \quad (8)$$

where $\mathcal{L} = 2.4h \times 10^8 L_{B\odot} \text{Mpc}^{-3}$ is the measured (B-band) luminosity density of the Universe. Once the average mass-to-light ratio for the Universe is determined, Ω_M follows by dividing it by the critical mass-to-light ratio, $(M/L)_{\text{crit}} = 1200h$ (in solar units). Though it is tantalizingly simple – and it is far too easy to take any measured mass-to-light ratio and divide it by $1200h$ – this method does not provide an easy and reliable method of determining Ω_M .

Based upon the mass-to-light ratios of the bright, inner regions of galaxies, $(M/L)_* \sim \text{few}$, the fraction of critical density in stars (and closely related material) has been determined, $\Omega_* \simeq (0.003 \pm 0.001)h^{-1}$ (see e.g., Faber & Gallagher, 1979). Persic & Salucci (1992) derive a similar value based upon the observed stellar-mass function. Luminous matter accounts for only a tiny fraction of the total mass density and only about a tenth of the baryons.

CNOC (Carlberg et al, 1996, 1997) have done a very careful job of determining a mean cluster mass-to-light ratio, $(M/L)_{\text{cluster}} = 240 \pm 50$, which translates to an estimate of the mean matter density, $\Omega_{\text{cluster}} = 0.20 \pm 0.04$. Because clusters contain thousands of galaxies and cluster galaxies do not seem *radically* different from field galaxies, one is tempted to take this estimate of the mean matter density

seriously. However, it is significantly smaller than the value I advocated earlier, $\Omega_M = 0.4 \pm 0.1$. Which estimate is right?

I believe the higher number, based upon the cluster baryon fraction, is more reliable and that we should be surprised that the CNOC number is so close, closer than we had any right to expect! After all, only a small fraction of galaxies are found in clusters and the luminosity density \mathcal{L} itself evolves strongly with redshift and corrections for this effect are large and uncertain. (We are on the tail end of star formation in the Universe: 80% of star formation took place at a redshift greater than unity.) While the value for Ω_M derived from the cluster baryon fraction also relies upon clusters, the underlying assumption is far weaker and much more justified, namely that clusters provide a fair sample of matter in the Universe.

Even if mass-to-light ratios were measured in the red (they typically are not), where the light is dominated by low-mass stars and reflects the integrated history of star formation rather than the current star-formation rate as it does in the blue, one would still require the fraction of baryons converted into stars in clusters to be identical to that in the field to have agreement between the CNOC estimate and that based upon the cluster baryon fraction. Apparently, the fraction of baryons converted into stars in the field and in clusters is similar, but not identical.

To put this in perspective and to emphasize the shortcomings of the mass-to-light technique, had one used the cluster mass-to-x-ray ratio and the x-ray luminosity density, one would have inferred $\Omega_M \sim 0.05$. A factor of two discrepancy based upon optical mass-to-light ratios does not seem so bad. Enough said.

Missing energy?

The results $\Omega_0 = 1 \pm 0.2$ and $\Omega_M = 0.4 \pm 0.1$ are in apparent contradiction, suggesting that one or both are wrong. However, prompted by a strong belief in a flat Universe, theorists have explored the remaining logical possibility: a dark, exotic form of energy that is smoothly distributed and contributes 60% of the critical density (Turner et al, 1984; Peebles, 1984). Being smoothly distributed its presence would not have been detected in measurements of the matter density. The properties of this missing energy are severely constrained by other cosmological facts, including structure formation, the age of the Universe, and CBR anisotropy. So much so, that a smoking-gun signature for the missing energy was predicted (see e.g., Turner, 1991).

To begin, let me parameterize the bulk equation of state of this unknown component: $w = p_X/\rho_X$. This implies that its energy density evolves as $\rho_X \propto R^{-n}$ where $n = 3(1 + w)$. The development of the structure observed today from density perturbations of the size inferred from measurements of the anisotropy of the CBR requires that the Universe be matter dominated from the epoch of matter – radiation equality until very recently. Thus, to avoid interfering with structure formation, the dark-energy component must be less important in the past than it is today. This implies that n must be less than 3 or $w < 0$; the more negative w is, the

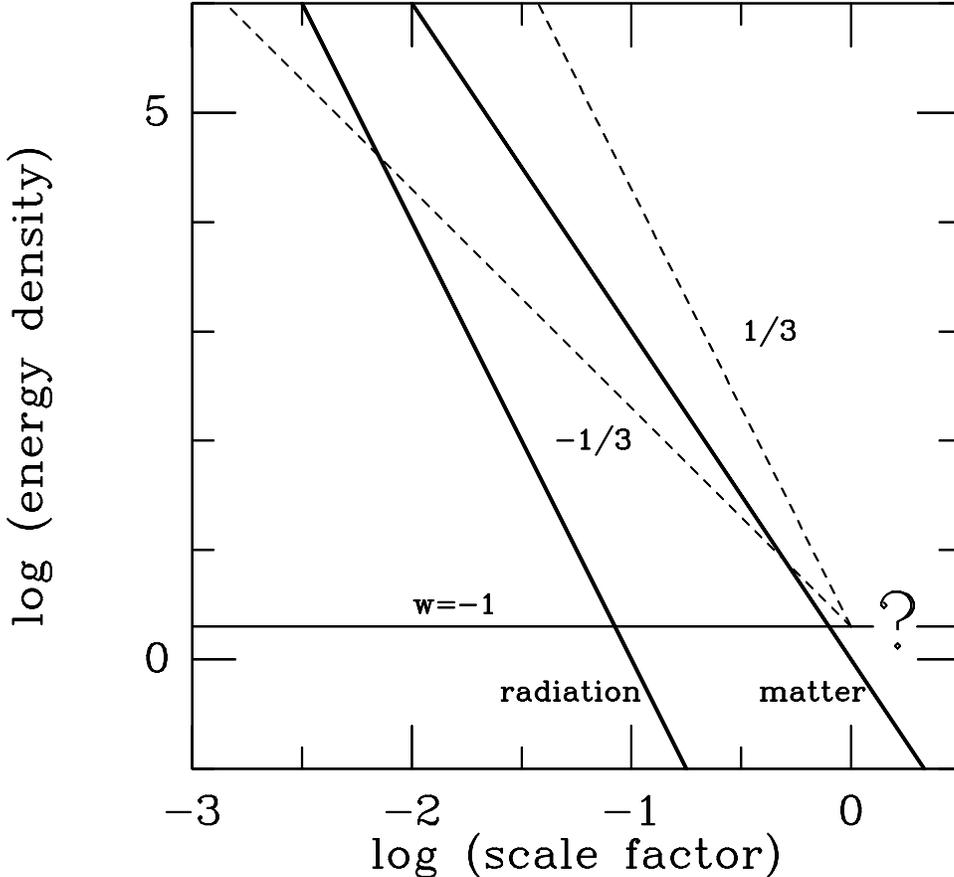


FIGURE 5. Evolution of the energy density in matter, radiation (heavy lines), and different possibilities for the dark-energy component ($w = -1, -\frac{1}{3}, \frac{1}{3}$) vs. scale factor. The matter-dominated era begins when the scale factor was $\sim 10^{-4}$ of its present size (off the figure) and ends when the dark-energy component begins to dominate, which depends upon the value of w : the more negative w is, the longer the matter-dominated era in which density perturbations can go into the large-scale structure seen today. These considerations require $w < -\frac{1}{3}$ (Turner & White, 1997).

faster this component gets out of the way (see Fig. 5). More careful consideration of the growth of structure implies that w must be less than about $-\frac{1}{3}$ (Turner & White, 1997).

Next, consider the constraint provided by the age of the Universe and the Hubble constant. Their product, $H_0 t_0$, depends the equation of state of the Universe; in particular, $H_0 t_0$ increases with decreasing w (see Fig. 6). To be definite, I will take $t_0 = 14 \pm 1.5$ Gyr and $H_0 = 65 \pm 5$ km s $^{-1}$ Mpc $^{-1}$ (see e.g., Chaboyer, 1998 and Freedman, 1999); this implies that $H_0 t_0 = 0.93 \pm 0.13$. Fig. 6 shows that $w < -\frac{1}{2}$ is preferred by age/Hubble constant considerations.

To summarize, consistency between $\Omega_M \sim 0.4$ and $\Omega_0 \sim 1$ along with other cosmological considerations implies the existence of a dark-energy component with bulk pressure more negative than about $-\rho_X/2$. The simplest example of such is

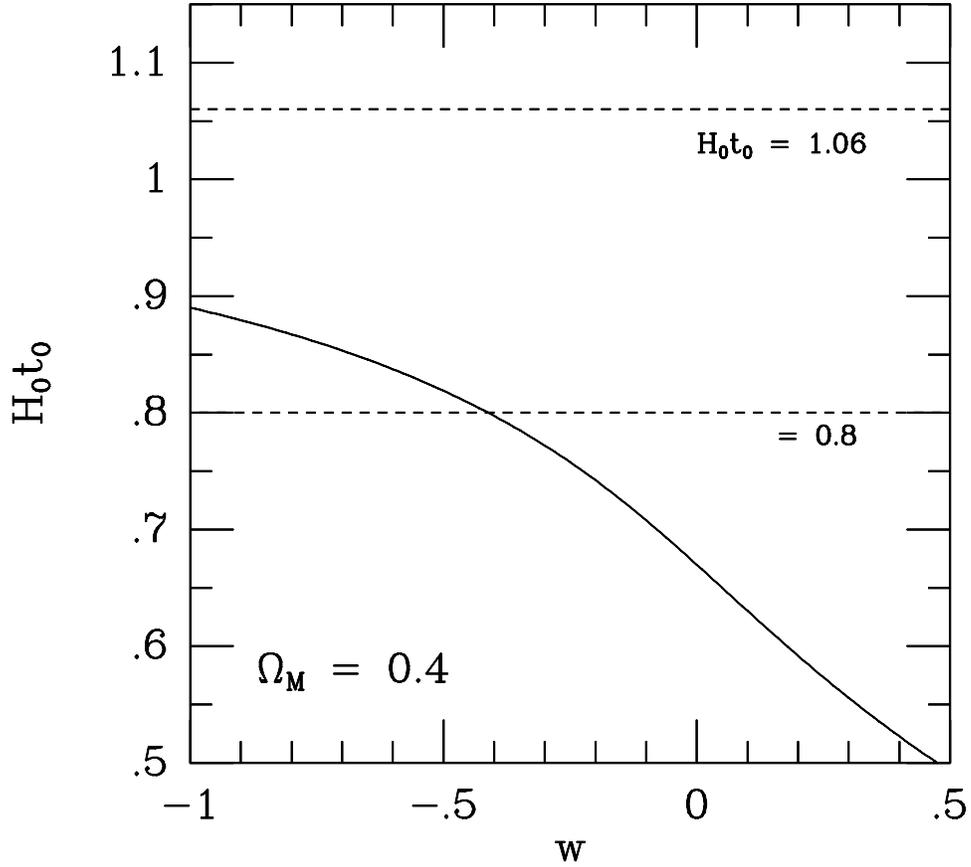


FIGURE 6. $H_0 t_0$ vs. the equation of state for the dark-energy component. As can be seen, an added benefit of a component with negative pressure is an older Universe for a given Hubble constant. The broken horizontal lines denote the 1σ range for $H_0 = 65 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $t_0 = 14 \pm 1.5 \text{ Gyr}$, and indicate that $w < -\frac{1}{2}$ is preferred.

vacuum energy (Einstein's cosmological constant), for which $w = -1$. The smoking-gun signature of a smooth, dark-energy component is accelerated expansion since $q_0 = 0.5 + 1.5w\Omega_X \simeq 0.5 + 0.9w < 0$ for $w < -\frac{5}{9}$.

Missing energy found!

In 1998 evidence for accelerated expansion was presented in the form of the magnitude – redshift (Hubble) diagram for fifty-some type Ia supernovae (SNe Ia) out to redshifts of nearly 1. Two groups, the Supernova Cosmology Project (Perlmutter et al, 1998) and the High-z Supernova Search Team (Riess et al, 1998; Schmidt et al, 1998), working independently and using different methods of analysis, each found evidence for accelerated expansion. Perlmutter et al (1998) summarize their results as a constraint to a cosmological constant (see Fig. 7),

$$\Omega_\Lambda = \frac{4}{3}\Omega_M + \frac{1}{3} \pm \frac{1}{6}. \quad (9)$$

For $\Omega_M \sim 0.4 \pm 0.1$, this implies $\Omega_\Lambda = 0.85 \pm 0.2$, or just what is needed to account for the missing energy!

(A simple explanation of the SN Ia results may be useful. If galactic distances and velocities were measured today they would obey a perfect Hubble law, $v_0 = H_0 d$, because the expansion of the Universe is simply a rescaling. Because we see distant galaxies at an earlier time, their velocities should be larger than predicted by the Hubble law, provided the expansion is slowing due to the attractive force of gravity. Using SNe Ia as standard candles to determine the distances to faraway galaxies, the two groups in effect found the opposite: distant galaxies are moving slower than predicted by the Hubble law, implying the expansion is speeding up!)

Recently, two other studies, one based upon the x-ray properties of rich clusters of galaxies (Mohr et al, 1999) and the other based upon the properties of double-lobe radio galaxies (Guerra et al, 1998), have reported evidence for a cosmological constant (or similar dark-energy component) that is consistent with the SN Ia results (i.e., $\Omega_\Lambda \sim 0.7$).

There is another powerful test of an accelerating Universe whose results are more ambiguous. It is based upon the fact that the frequency of multiply lensed QSOs is expected to be significantly higher in an accelerating universe (Turner, 1990). Kochanek (1996) has used gravitational lensing of QSOs to place a 95% cl upper limit, $\Omega_\Lambda < 0.66$; and Waga and Miceli (1998) have generalized it to a dark-energy component with negative pressure: $\Omega_X < 1.3 + 0.55w$ (95% cl), both results for a flat Universe. On the other hand, Chiba and Yoshii (1998) claim evidence for a cosmological constant, $\Omega_\Lambda = 0.7_{-0.2}^{+0.1}$, based upon the same data. From this I conclude: 1) lensing excludes Ω_Λ larger than 0.8, and 2) when larger objective surveys of gravitational-lensed QSOs are carried out (e.g., the Sloan Digital Sky Survey), there is the possibility of uncovering another smoking-gun for accelerated expansion.

By far, the strongest evidence for dark energy is the SN Ia data. The statistical errors reported by the two groups are smaller than possible systematic errors. Thus, the believability of the results turn on the reliability of SNe Ia as one-parameter standard candles. SNe Ia are thought to be associated with the nuclear detonation of Chandrasekhar-mass white dwarfs. The one parameter is the rate of decline of the light curve: The brighter ones decline more slowly (the so-called Phillips relation; see Phillips, 1993). The lack of a good theoretical understanding of this (e.g., what is the physical parameter?) is offset by strong empirical evidence for the relationship between peak brightness and rate of decline, based upon a sample of nearby SNe Ia. It is reassuring that in all respects studied, the distant sample of SNe Ia appear to be similar to the nearby sample. For example, distribution of decline rates and dispersion about the Phillips relationship. The local sample spans a range of metallicity, likely spanning that of the distant sample, and further, suggesting that metallicity is not an important second parameter.

At this point, it is fair to say that if there is a problem with SNe Ia as standard candles, it must be subtle. Cosmologists are even more inclined to believe the SN Ia results because of the preexisting evidence for a “missing-energy component” that led to the prediction of accelerated expansion.

Cosmic concordance

With the SN Ia results we have for the first time a complete and self-consistent accounting of mass and energy in the Universe (see Fig. 1). The consistency of the matter/energy accounting is illustrated in Fig. 7. Let me explain this exciting figure. The SN Ia results are sensitive to the acceleration (or deceleration) of the expansion and constrain the combination $\frac{4}{3}\Omega_M - \Omega_\Lambda$. (Note, $q_0 = \frac{1}{2}\Omega_M - \Omega_\Lambda$; $\frac{4}{3}\Omega_M - \Omega_\Lambda$ corresponds to the deceleration parameter at redshift $z \sim 0.4$, the median redshift of these samples). The (approximately) orthogonal combination, $\Omega_0 = \Omega_M + \Omega_\Lambda$ is constrained by CBR anisotropy. Together, they define a concordance region around $\Omega_0 \sim 1$, $\Omega_M \sim 1/3$, and $\Omega_\Lambda \sim 2/3$. The constraint to the matter density alone, $\Omega_M = 0.4 \pm 0.1$, provides a cross check, and it is consistent with these numbers. Cosmic concordance!

But there is more. We also have a consistent and well motivated picture for the formation of structure in the Universe, Λ CDM. The Λ CDM model, which is the cold dark matter model with $\Omega_B \sim 0.05$, $\Omega_{\text{CDM}} \sim 0.35$ and $\Omega_\Lambda \sim 0.6$, is a very good fit to all cosmological constraints: large-scale structure, CBR anisotropy, age of the Universe, Hubble constant and the constraints to the matter density and cosmological constant; see Fig. 8 (Krauss & Turner, 1995; Ostriker & Steinhardt, 1995; Turner, 1997). Further, as can be seen in Fig. 2, CBR anisotropy measurements are beginning to show evidence for the acoustic peaks characteristic of the Gaussian, curvature perturbations predicted by inflation. Until 1998, Λ CDM’s only major flaw was the absence of evidence for accelerated expansion. Not now.

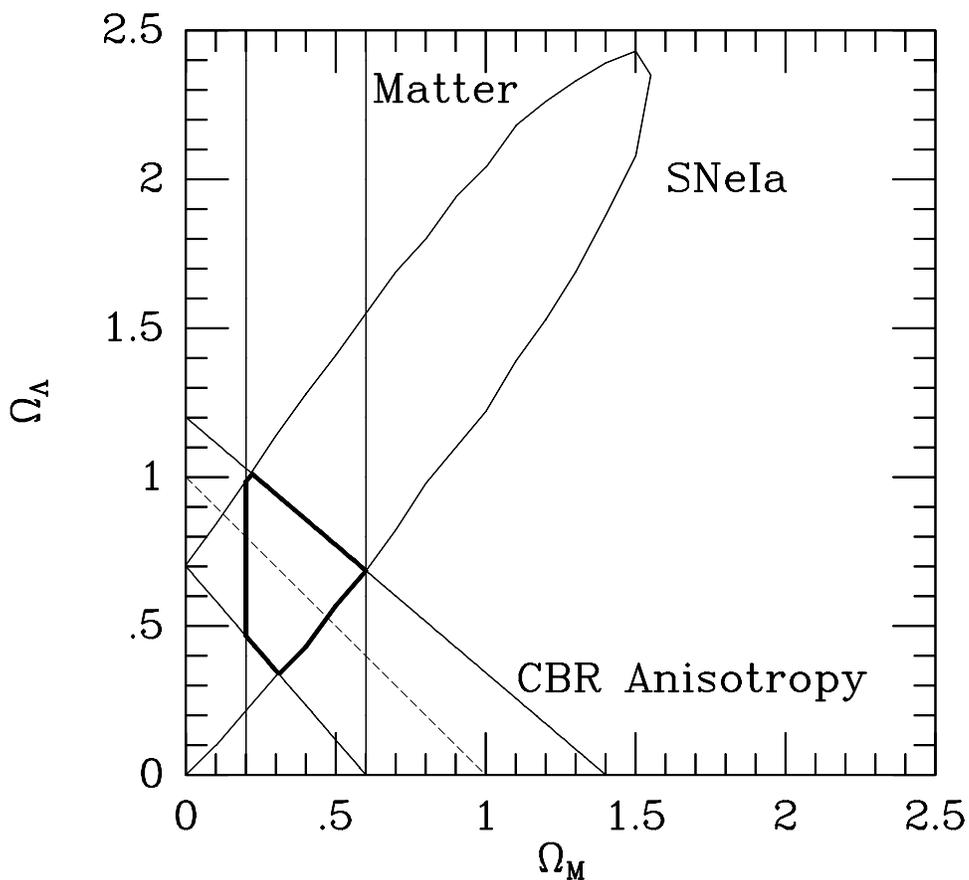


FIGURE 7. Two- σ constraints to Ω_M and Ω_Λ from CBR anisotropy, SNe Ia, and measurements of clustered matter. Lines of constant Ω_0 are diagonal, with a flat Universe shown by the broken line. The concordance region is shown in bold: $\Omega_M \sim 1/3$, $\Omega_\Lambda \sim 2/3$, and $\Omega_0 \sim 1$. (Particle physicists who rotate the figure by 90° will recognize the similarity to the convergence of the gauge coupling constants.)

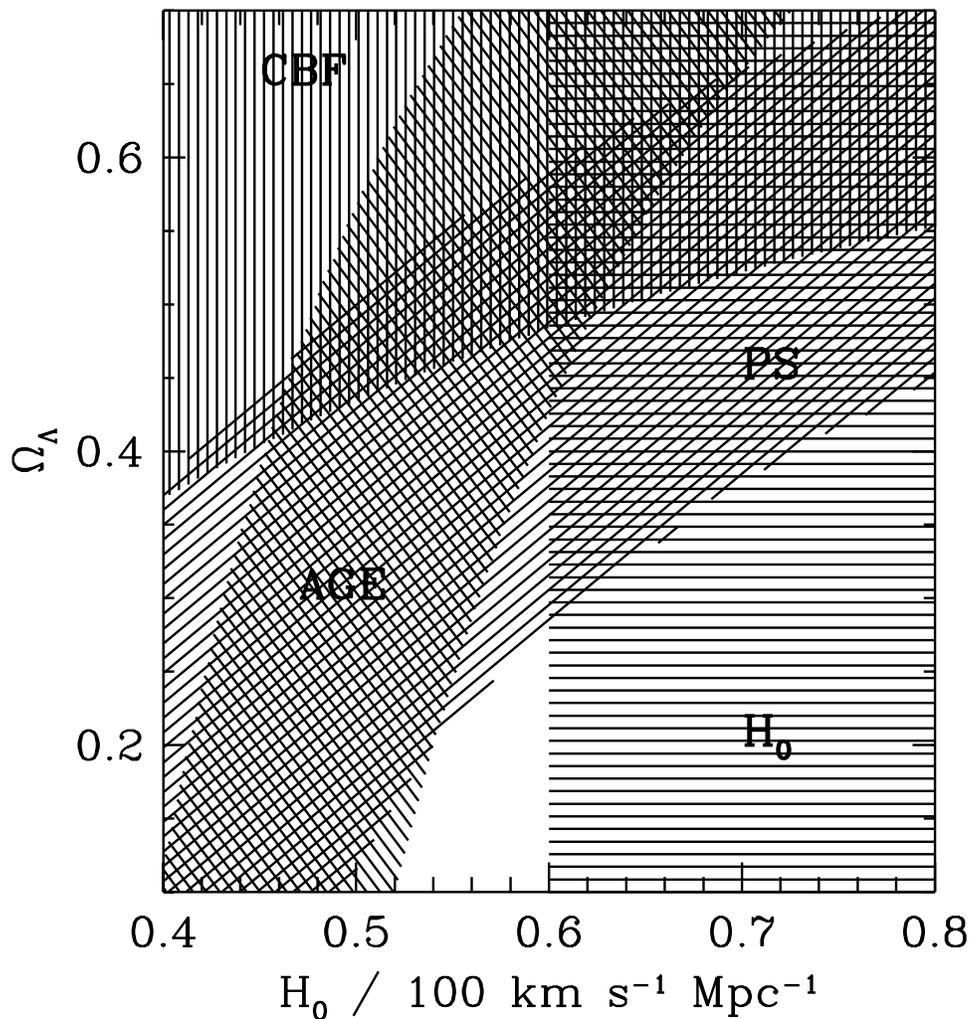


FIGURE 8. Constraints used to determine the best-fit CDM model: PS = large-scale structure + CBR anisotropy; AGE = age of the Universe; CBF = cluster-baryon fraction; and H_0 = Hubble constant measurements. The best-fit model, indicated by the darkest region, has $H_0 \simeq 60 - 65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_\Lambda \simeq 0.55 - 0.65$. Evidence for its smoking-gun signature – accelerated expansion – was presented in 1998 (adapted from Krauss & Turner, 1995 and Turner, 1997).

THREE DARK MATTER PROBLEMS

While stars are very interesting and pretty to look at – and without them, astronomy wouldn't be astronomy and we wouldn't exist – they represent a tiny fraction of the cosmic mass budget, only about 0.5% of the critical density. As we have known for several decades, the bulk of the matter and energy in the Universe is dark. The present accounting defines three dark matter/energy problems; none is yet fully addressed.

Dark Baryons

By a ten to one margin, the bulk of the baryons are dark and not in the form of stars. With the exception of clusters, where the “dark” baryons exist as hot, x-ray emitting intracluster gas, the nature of the dark baryons is not known. Clusters only account for around 10% or so of the baryons in the Universe (Persic & Salucci, 1992) and the (optically) dark baryons elsewhere, which account for 90% or more of all the baryons, could take on a different form.

The two most promising possibilities for the dark baryons are diffuse hot gas and “dark stars” (white dwarfs, neutron stars, black holes or objects of mass around or below the hydrogen-burning limit). I favor the former possibility for a number of reasons. First, that's where the dark baryons in clusters are. Second, the cluster baryon fraction argument can be turned around to infer Ω_{gas} at the time clusters formed, redshifts $z \sim 0 - 1$,

$$\Omega_{\text{gas}} h^2 = f_{\text{gas}} \Omega_M h^2 = 0.023 (\Omega_M/0.4)(h/0.65)^{1/2}. \quad (10)$$

That is, at the time clusters formed, the mean gas density was essentially equal to the baryon density (unless $\Omega_M h^{1/2}$ is very small), thereby accounting for the bulk of baryons in gaseous form. Third, numerical simulations suggest that most of the baryons should still be in gaseous form today (Rauch et al, 1997).

There are two arguments for dark stars as the baryonic dark matter. First, the gaseous baryons not associated with clusters have not been detected. Second, the results of the microlensing surveys toward the LMC and SMC (Spiro, 1999) are consistent with about one-third of our halo being in the form of half-solar mass white dwarfs.

I find neither argument compelling; gas outside clusters will be cooler ($T \sim 10^5 - 10^6$ K) and difficult to detect, either in absorption or emission. There are equally attractive explanations for the Magellanic Cloud microlensing events (e.g., self lensing by the Magellanic Clouds, lensing by stars in the spheroid, or lensing due to disk material that, due to flaring and warping of the disk, falls along the line of sight to the LMC; see Sahu, 1994; Evans et al, 1998; Gates et al, 1998; Zaritsky & Lin, 1997; Zhao, 1998). The white-dwarf interpretation for the halo has a host of troubles: Why haven't the white dwarfs been seen (Graff et al, 1998)? The star formation rate required to produce these white dwarfs – close to $100 \text{ yr}^{-1} \text{ Mpc}^{-3}$ –

far exceeds that measured at any time in the past or present (see Madau, 1999). Where are the lower-main-sequence stars associated with this stellar population and the gas, expected to be 6 to 10 times that of the white dwarfs, that didn't form into stars (Fields et al, 1997)? Finally, there is evidence that the lenses for both SMC events are stars within the SMC (Alcock et al, 1998; EROS Collaboration, 1998a,b) and at least one of the LMC events is explained by an LMC lens.

The SMC/LMC microlensing puzzle can be stated another way. The lenses have all the characteristics of ordinary, low-mass stars (e.g., mass and binary frequency). If this is so, they cannot be in the halo (they would have been seen); the puzzle is to figure out where they are located.

Cold Dark Matter

The second dark-matter problem follows from the inequality $\Omega_M \simeq 0.4 \gg \Omega_B \simeq 0.05$: There is much more matter than there are baryons, and thus, nonbaryonic dark matter is the dominant form of matter. The evidence for this very profound conclusion has been mounting for almost two decades, and this past year, the Burles – Tytler deuterium measurement anchored the baryon density and allowed the cleanest determination of the matter density, through the cluster baryon fraction.

Particle physics provides an attractive solution to the nonbaryonic dark matter problem: relic elementary particles left over from the big bang (see Ellis, 1999). Long-lived or stable particles with very weak interactions can remain from the earliest moments of particle democracy in sufficient numbers to account for a significant fraction of critical density (very weak interactions are needed so that their annihilations cease before their numbers are too small). The three most promising candidates are a neutrino of mass 30 eV or so (or $\sum_i m_{\nu_i} \sim 30$ eV), an axion of mass $10^{-5 \pm 1}$ eV, and a neutralino of mass between 50 GeV and 500 GeV. All three are motivated by particle-physics theories that attempt to unify the forces and particles of Nature. The fact that such particles can also account for the nonbaryonic dark matter is either a big coincidence or a big hint. Further, the fact that these particles interact with each other and ordinary matter very weakly, provides a simple and natural explanation for dark matter being more diffusely distributed.

At the moment, there is significant circumstantial evidence against neutrinos as the bulk of the dark matter. Because they behave as hot dark matter, structure forms from the top down, with superclusters fragmenting into clusters and galaxies (White, Frenk & Davis, 1983), in stark contrast to the observational evidence that indicates structure formed from the bottom up. (Hot + cold dark matter is still an outside possibility, with $\Omega_\nu \sim 0.15$ or less; see Dodelson et al, 1996 and Gawiser & Silk, 1998.) Second, the evidence for neutrino mass based upon the atmospheric (Totsuka, 1999) and solar-neutrino (Kirsten, 1999 and Bahcall, 1999) data suggests a neutrino mass pattern with the tau neutrino at 0.1 eV, the muon neutrino at 0.001 eV to 0.01 eV and the electron neutrino with an even smaller mass. In particular, the factor-of-two deficit of atmospheric muons neutrinos with its

dependence upon zenith angle is very strong evidence for a neutrino mass difference squared between two of the neutrinos of around 10^{-2} eV^2 (Fukuda et al, 1998). This sets a *lower* bound to neutrino mass of about 0.1 eV, implying neutrinos contribute at least as much mass as bright stars. WOW!

Both the axion and neutralino behave as cold dark matter; the success of the cold dark matter model of structure formation makes them the leading particle dark-matter candidates. Because they behave as cold dark matter, they are expected to be the dark matter in our own halo; in fact, there is nothing that can keep them out (Gates & Turner, 1994). As discussed above, 2/3 of the dark matter in our halo – and probably all the halo dark matter – cannot be explained by baryons in any form. The local density of halo material is estimated to be $10^{-24} \text{ g cm}^{-3}$, with an uncertainty of slightly less than a factor of 2 (Gates et al, 1995). This makes the halo of our galaxy an ideal place to look for cold dark matter particles! An experiment at Livermore National Laboratory with sufficient sensitivity to detect halo axions is currently taking data (van Bibber et al, 1998; Rosenberg and van Bibber, 1999) and experiments at several laboratories around the world are beginning to search for halo neutralinos with sufficient sensitivity to detect them (Sadoulet, 1999). The particle dark-matter hypothesis is compelling, but very bold, and most importantly, it is now being tested.

Finally, while the axion and the neutralino are the most promising particle dark-matter candidates, neither one is a “sure thing.” Moreover, any sufficiently heavy particle relic (mass greater than a GeV or so) will behave as cold dark matter. A host of more exotic possibilities have been suggested, from solar-mass primordial black holes produced at the quark/hadron transition (see e.g., Jedamzik, 1998 and Jedamzik & Niemeyer, 1998) that masquerade as MACHOs in our halo to super-massive (mass greater than 10^{10} GeV) particles produced by nonthermal processes at the end of inflation (see e.g., Kolb, 1999). Lest we become overconfident, we should remember that Nature has many options for the particle dark matter.

Dark Energy

I have often used the term exotic to refer to particle dark matter. That term will now have to be reserved for the dark energy that is causing the accelerated expansion of the Universe – by any standard, it is more exotic and more poorly understood. Here is what we do know: it contributes about 60% of the critical density; it has pressure more negative than about $-\rho/2$; and it does not clump (otherwise it would have contributed to estimates of the mass density). The simplest possibility is the energy associated with the virtual particles that populate the quantum vacuum; in this case $p = -\rho$ and the dark energy is absolutely spatially and temporally uniform.

This “simple” interpretation has its difficulties. Einstein “invented” the cosmological constant to make a static model of the Universe and then he discarded it; we now know that the concept is not optional. The cosmological constant corresponds

to the energy associated with the vacuum. However, there is no sensible calculation of that energy (see e.g., Zel'dovich, 1967; Bludman and Ruderman, 1977; and Weinberg, 1989), with estimates ranging from 10^{122} to 10^{55} times the critical density. Some particle physicists believe that when the problem is understood, the answer will be zero. Spurred in part by the possibility that cosmologists may have actually weighed the vacuum (!), particle theorists are taking a fresh look at the problem (see e.g., Harvey, 1998; Sundrum, 1997). Sundrum's proposal, that the energy of the vacuum is close to the present critical density because the graviton is a composite particle with size of order 1 cm, is indicative of the profound consequences that a cosmological constant has for fundamental physics.

Because of the theoretical problems mentioned above, as well as the checkered history of the cosmological constant, theorists have explored other possibilities for a smooth, component to the dark energy (see e.g., Turner & White, 1997). Wilczek and I pointed out that even if the energy of the true vacuum is zero, as the Universe as cooled and went through a series of phase transitions, it could have become hung up in a metastable vacuum with nonzero vacuum energy (Turner & Wilczek, 1982). In the context of string theory, where there are a very large number of energy-equivalent vacua, this becomes a more interesting possibility: perhaps the degeneracy of vacuum states is broken by very small effects, so small that we were not steered into the lowest energy vacuum during the earliest moments.

Vilenkin (1984) has suggested a tangled network of very light cosmic strings (also see, Spergel & Pen, 1997) produced at the electroweak phase transition; networks of other frustrated defects (e.g., walls) are also possible. In general, the bulk equation-of-state of frustrated defects is characterized by $w = -N/3$ where N is the dimension of the defect ($N = 1$ for strings, $= 2$ for walls, etc.). The SN Ia data almost exclude strings, but still allow walls.

An alternative that has received a lot of attention is the idea of a “decaying cosmological constant”, a term coined by the Soviet cosmologist Matvei Petrovich Bronstein in 1933 (Bronstein, 1933). (Bronstein was executed on Stalin's orders in 1938, presumably for reasons not directly related to the cosmological constant; see Kragh, 1996.) The term is, of course, an oxymoron; what people have in mind is making vacuum energy dynamical. The simplest realization is a dynamical, evolving scalar field. If it is spatially homogeneous, then its energy density and pressure are given by

$$\begin{aligned}\rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi)\end{aligned}\tag{11}$$

and its equation of motion by (see e.g., Turner, 1983)

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0\tag{12}$$

The basic idea is that energy of the true vacuum is zero, but not all fields have evolved to their state of minimum energy. This is qualitatively different from that

of a metastable vacuum, which is a local minimum of the potential and is classically stable. Here, the field is classically unstable and is rolling toward its lowest energy state.

Two features of the “rolling-scalar-field scenario” are worth noting. First, the effective equation of state, $w = (\frac{1}{2}\dot{\phi}^2 - V)/(\frac{1}{2}\dot{\phi}^2 + V)$, can take on any value from 1 to -1 . Second, w can vary with time. These are key features that may allow it to be distinguished from the other possibilities. In fact, there is some hope that more SNe Ia will be able to do this and perhaps even permit the reconstruction of the scalar-field potential (Huterer & Turner, 1998). At moment, taking the SNe Ia data together with large-scale structure data we can already constrain the average value of w to be less than -0.60 (95% cl) (Perlmutter, White & Turner, 1999).

The rolling scalar field scenario (aka mini-inflation or quintessence) has received a lot of attention over the past decade (Freese et al, 1987; Ozer & Taha, 1987; Ratra & Peebles, 1988; Frieman et al, 1995; Coble et al, 1996; Turner & White, 1997; Caldwell et al, 1998). It is an interesting idea, but not without its own difficulties. First, one must *assume* that the energy of the true vacuum state (ϕ at the minimum of its potential) is zero; i.e., it does not address the cosmological constant problem. Second, as Carroll (1998) has emphasized, the scalar field is very light and can mediate long-range forces. This places severe constraints on it. Finally, with the possible exception of one model (Frieman et al, 1995), none of the scalar-field models address how ϕ fits into the grander scheme of things and why it is so light ($m \sim 10^{-33}$ eV).

CONCLUDING REMARKS

The cosmological parameters T_0 , H_0 , t_0 , and Ω_i are now all measured with reliable and respectable error estimates. This is a remarkable achievement for a science about which it used to be said, “the errors are always in the exponents.” Already, the consistency between H_0 , t_0 and Ω_i provides an important crosscheck on the consistency of the hot big-bang framework, which it passes with flying colors.

Over the next two decades we can expect to see even more progress. After the MAP and Planck missions and the completion of the Sloan Digital Sky Survey, it is not unreasonable to expect a 1% determination of H_0 and Ω_0 , and a 5% determination of Ω_M and Ω_B , with significant crosschecks. In addition, we should have a much better understanding of the mysterious dark energy. The COBE FIRAS measurement of T_0 has set a standard that is unlikely to be exceeded for a long time. These precision measurements of the cosmological parameters will test the big-bang framework and general relativity as well as testing the paradigm that aspires to extend it, Inflation + Cold Dark Matter.

These are exciting times in cosmology!

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