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FERMILAB-Conf-98/098-T

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April 1998

Published Proceedings of the *XXXIst Rencontres de Moriond, QCD and High Energy Hadronic Interactions*, Les Arcs, France, March 21-28, 1998

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

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FERMILAB–CONF–98/098–T
hep-ph/yymmnnn

B DECAYS INTO LIGHT MESONS

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Abstract:

I calculate the form factors describing semileptonic and penguin induced decays of B mesons into light pseudoscalar and vector mesons. The form factors are calculated from QCD sum rules on the light-cone including contributions up to twist 4, radiative corrections to the leading twist contribution and SU(3) breaking effects. The theoretical uncertainty is estimated to be $\sim$(15–20)%.

To be published in the proceedings of XXXI. Rencontres de Moriond: QCD and high energy hadronic interactions, Les Arcs, France, 21–28 Mar 1998.

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Decays of B mesons into light mesons offer the possibility to access the less well known entries in the CKM quark mixing matrix like V_{ub} and V_{ts}. The measurement of rare penguin induced B decays may also give hints at new physics in the form of loop-induced effects. With new data of hitherto unknown precision from the new experimental facilities BaBar at SLAC and Belle at KEK expected to be available in the near future, the demands at the accuracy of theoretical predictions are ever increasing. The central problem of all such predictions, our failure to solve nonperturbative QCD, is well known and so far prevents a rigorous calculation of form factors from first principles. Theorists thus concentrate on providing various approximations. The maybe most prominent of these, simulations of QCD on the lattice, have experienced considerable progress over recent years; the current status for B decays is summarized in [1]. It seems, however, unlikely that lattice calculations will soon overcome their main restriction in describing $b \to u$ and $b \to s$ transitions, namely the effective upper cut-off that the finite lattice size imposes on the momentum of the final state meson. The cut-off restricts lattice predictions of B decay form factors to rather large momentum transfer q^2 of about $15\,\mathrm{GeV}^2$ or larger. The physical range in B decays, however, extends from 0 to about $20\,\mathrm{GeV}^2$, depending on the process; for radiative decays like $B \to K^*\gamma$ it is exactly $0\,\mathrm{GeV}^2$. Still, one may hope to extract from the lattice data some information on form factors in the full physical range, as their behaviour at large q^2 restricts the shape at small q^2 via the analytical properties of a properly chosen vacuum correlation function. The latter function, however, also contains poles and multi-particle cuts whose exact behaviour is not known, which limits the accuracy of bounds obtained from such unitarity constraints and until now has restricted their application to $B \to \pi$ transitions [2, 3]. The most optimistic overall theoretical uncertainty one may hope to obtain from this method is the one induced by the input lattice results at large q^2, which to date is around 30% [4, 2]. A more model-dependent extension of the lattice form factors into the low q^2 region is discussed in [5].

An alternative approach to heavy-to-light transitions is offered by QCD sum rules on the light-cone. In contrast and complementary to lattice simulations, it is just the fact that the final state meson *does* have large energy and momentum of order $\sim m_B/2$ in a large portion of phase-space that is used as starting point (which restricts the method to not too large momentum transfer, to be quantified below). The key idea is to consider $b \to u$ and $b \to s$ transitions as hard exclusive QCD processes and to combine the well-developed description of such processes in terms of perturbative amplitudes and nonperturbative hadronic distribution amplitudes [6] (see also [7] for a nice introduction) with the method of QCD sum rules [8] to describe the decaying hadron. The idea of such "light-cone sum rules" was first formulated and carried out in [9] in a different context for the process $\Sigma \to p\gamma$, its first application to B decays was given in [10]. Subsequently, light-cone sum rules were considered for many B decay processes, see [11, 12] for reviews.[2] As light-cone sum rules

[2]There also exists an extended literature on a more "direct" extension of QCD sum rules to heavy-to-light transitions, which is based on three-point correlation functions, see e.g. [13]. The conceptional restrictions of these sum rules are discussed in Ref. [14]. They fail to give a viable description of form factors at small and moderate momentum transfer.

are based on the light-cone expansion of a correlation function, they can be systematically improved by including higher twist contributions and radiative corrections to perturbative amplitudes. The first calculations in [10, 14] were done at tree-level and to leading twist 2 accuracy. In [15, 16], twist 3 and 4 contributions to $B \to \pi$ were included, in [17], one-loop radiative corrections to the twist 2 contribution to the form factor f_+^π were calculated, and in [18], twist 3 and 4 contributions and next-to-leading corrections to all $B \to$ pseudoscalar form factors were calculated. In these proceedings, I present the results of Refs. [18, 19] for $B \to$ pseudoscalar and $B \to$ vector transitions, which rely on recent results for twist 3 and 4 vector meson distribution amplitudes [20, 21].

Let me begin by defining the form factors. Let P be a light pseudoscalar meson, i.e. π or K, and V be a vector meson, i.e. ρ, ω, K^* or ϕ; V_μ and A_μ are the appropriate vector and axialvector currents, respectively. Semileptonic form factors are defined by ($q = p_B - p$)

$$\langle P(p)|V_\mu|B(p_B)\rangle = f_+^P(q^2)\left\{(p_B+p)_\mu - \frac{m_B^2-m_P^2}{q^2}\, q_\mu\right\} + \frac{m_B^2-m_P^2}{q^2}\, f_0^P(q^2)\, q_\mu, \quad (1)$$

$$\text{with } f_+^P(0) = f_0^P(0),$$

$$\langle V(p)|(V-A)_\mu|B(p_B)\rangle = -i\epsilon^*_\mu(m_B+m_V)A_1^V(q^2) + i(p_B+p)_\mu(\epsilon^* p_B)\,\frac{A_2^V(q^2)}{m_B+m_V}$$

$$+\, iq_\mu(\epsilon^* p_B)\,\frac{2m_V}{q^2}\,\left(A_3^V(q^2) - A_0^V(q^2)\right) + \epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\,\frac{2V^V(q^2)}{m_B+m_V} \quad (2)$$

$$\text{with } A_3^V(q^2) = \frac{m_B+m_V}{2m_V}\,A_1^V(q^2) - \frac{m_B-m_V}{2m_V}\,A_2^V(q^2), \qquad A_0^V(0) = A_3^V(0) \quad (3)$$

The penguin form factors are defined as

$$\langle K(p)|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle \equiv \langle K(p)|\bar{s}\sigma_{\mu\nu}q^\nu b|B(p_B)\rangle$$

$$= i\left\{(p_B+p)_\mu q^2 - q_\mu(m_B^2-m_K^2)\right\}\,\frac{f_T^K(q^2)}{m_B+m_K} \quad (4)$$

$$\langle K^*|\bar{s}\sigma_{\mu\nu}q^\nu(1+\gamma_5)b|B(p_B)\rangle = i\epsilon_{\mu\nu\rho\sigma}\epsilon^{*\nu}p_B^\rho p^\sigma\, 2T_1(q^2)$$

$$+\, T_2(q^2)\left\{\epsilon^*_\mu(m_B^2-m_{K^*}^2) - (\epsilon^* p_B)\,(p_B+p)_\mu\right\}$$

$$+\, T_3(q^2)(\epsilon^* p_B)\left\{q_\mu - \frac{q^2}{m_B^2-m_{K^*}^2}\,(p_B+p)_\mu\right\} \quad (5)$$

$$\text{with } T_1(0) = T_2(0). \quad (6)$$

The physical range in q^2 is $0 \le q^2 \le (m_B - m_{P,V})^2$. Although there are of course no semileptonic decays $B \to Ke\nu$, the above form factors contribute to e.g. $B \to K\ell\bar{\ell}$. Recalling the results of perturbative QCD for the π electromagnetic form factor as summarized in [7], one may suppose that the dominant contribution to the above form factors be the exchange of a hard perturbative gluon between e.g. the u quark and the antiquark, which possibility was advocated for instance in [22]. This is, however, not the case, and it was pointed out already in Ref. [10] that the dominant contribution comes from the so-called Feynman mechanism, where the quark created in the weak decay carries nearly all of the final state meson's momentum, while all other quarks are soft, and which bears no perturbative suppression by factors α_s/π. In an expansion in the inverse b quark mass, the contribution from the Feynman mechanism is of the same order as the gluon exchange contribution with momentum fraction of the quark of order $1-1/m_b$, but it dies off in the strict limit $m_b \to \infty$ due to Sudakov effects. This means that — unlike in the case of the electromagnetic π form factor — knowledge of the hadron distribution amplitudes

$$\phi(u,\mu^2) \sim \int_0^{\mu^2} dk_\perp^2\, \Psi(u,k_\perp),$$

where Ψ is the full Fock-state wave function of the B and $\pi(K)$, respectively, u is the longitudinal momentum fraction carried by the (b or $u(s)$) quark, $k_\perp$ is the transverse quark momentum, is not sufficient to calculate the form factors in the form of overlap integrals

$$F \sim \int_0^1 du\, dv\, \phi^*_{\pi(K)}(u)\, T_{\rm hard}(u,v;q^2)\, \phi_B(v)$$

(with $T_{\rm hard} \propto \alpha_s$).[3] Instead, in the method of light-cone sum rules, only the light meson is described by distribution amplitudes. Logarithms in $k_\perp$ are taken into account by the evolution of the distribution amplitudes under changes in scale, powers in $k_\perp$ are taken into account by higher twist distribution amplitudes. The B meson, on the other hand, is described like in QCD sum rules by the pseudoscalar current $\bar{d}i\gamma_5 b$ in the unphysical region with virtuality $p_B^2 - m_b^2 \sim O(m_b)$, where it can be treated perturbatively. The real B meson, residing on the physical cut at $p_B^2 = m_B^2$, is then traced by analytical continuation, supplemented by the standard QCD sum rule tools to enhance its contribution with respect to that of higher single- or multi-particle states coupling to the same current.

The starting point for the calculation of the form factors are thus the correlation functions ($j_B = \bar{d}i\gamma_5 b$):

$$\mathrm{CF}_V = i\int d^4y e^{iqy} \langle P(p)|T[\bar{q}\gamma_\mu b](y) j_B^\dagger(0)|0\rangle \; = \; \Pi_+^P (q+2p)_\mu + \Pi_-^P q_\mu, \tag{7}$$

$$\mathrm{CF}_T = i\int d^4y e^{iqy} \langle P(p)|T[\bar{q}\sigma_{\mu\nu}q^\nu b](y) j_B^\dagger(x)|0\rangle \; = \; 2iF_T^P(p_\mu q^2 - (pq)q_\mu), \tag{8}$$

[3] Note also that not much is known about ϕ_B, whereas the analysis of light meson distribution amplitudes is facilitated by the fact that it can be organized in an expansion in conformal spin, much like the partial wave expansion of scattering amplitudes in quantum mechanics in rotational spin.

Table 1: Form factors in a three parameter fit. Renormalization scale for T_i is $\mu = m_b = 4.7\,\mathrm{GeV}$.

	$F(0)$	a_F	b_F	$F(0)$	a_F	b_F	
f_+^π	0.30 ± 0.04	1.35	0.27	0.35 ± 0.05	1.37	0.35	f_+^K
f_0^π	0.30 ± 0.04	0.39	0.62	0.35 ± 0.05	0.40	0.41	f_0^K
f_T^π	0.30 ± 0.04	1.34	0.26	0.39 ± 0.05	1.37	0.37	f_T^K
A_1^ρ	0.27	0.11	-0.75	0.35	0.54	-0.02	$A_1^{K^*}$
A_2^ρ	0.23	0.77	-0.40	0.30	1.02	0.08	$A_2^{K^*}$
A_0^ρ	0.37	1.42	0.50	0.47	1.64	0.94	$A_0^{K^*}$
V^ρ	0.34	1.32	0.19	0.47	1.50	0.51	V^{K^*}
T_1^ρ	0.29	1.36	0.24	0.39	1.53	1.77	$T_1^{K^*}$
T_2^ρ	0.29	0.08	-0.94	0.39	0.36	-0.49	$T_2^{K^*}$
T_3^ρ	0.20	0.96	-0.31	0.26	1.07	-0.16	$T_3^{K^*}$

and similar ones for vector mesons, which are calculated in an expansion around the light-cone $x^2 = 0$. The expansion goes in inverse powers of the b quark virtuality, which, in order for the light-cone expansion to be applicable, must be of order m_b. This restricts the accessible range in q^2 to $m_b^2 - q^2 \lesssim O(m_b)$ parametrically. For physical B mesons, I choose $m_b^2 - q^2 \leq 18\,\mathrm{GeV}^2$. The technical details of the calculation are described in [19]. Important is that in [18, 19] for the first time twist 3 and 4 contributions and radiative corrections to the twist 2 contributions were included. The impact of these corrections is small: both radiative corrections and twist 4 contributions are at the 5% level, which shows that both the light-cone and the perturbative expansion are under control.

In Fig. 1 I show the form factors for several $B \to$ vectormeson transitions as functions of q^2 with input parameters as stated in the caption. SU(3) breaking effects are included by different hadron distribution amplitudes and amount up to $\sim 10\%$. The remaining theoretical uncertainty of these form factors is mainly systematic and dominated by the error introduced by isolating the ground state B meson contribution. This error is estimated to be $\sim 10\%$, and together with the other uncertainties introduced by the choice of m_b, the QCD sum rule parameters and the hadronic distribution amplitudes [23, 24, 25, 20, 21], I arrive at a $\sim 20\%$ uncertainty.

The form factors as depicted in the figure lend themselves to a convenient parametrization in terms of three parameters:

$$F(q^2) = \frac{F(0)}{1 - a_F\,\frac{q^2}{m_B^2} + b_F\left(\frac{q^2}{m_B^2}\right)^2}. \tag{9}$$

The corresponding parameters for specific form factors are tabulated in Tab. 1. The parametrization is acurate to within 1% for $0 \leq q^2 \leq 18\,\mathrm{GeV}^2$.

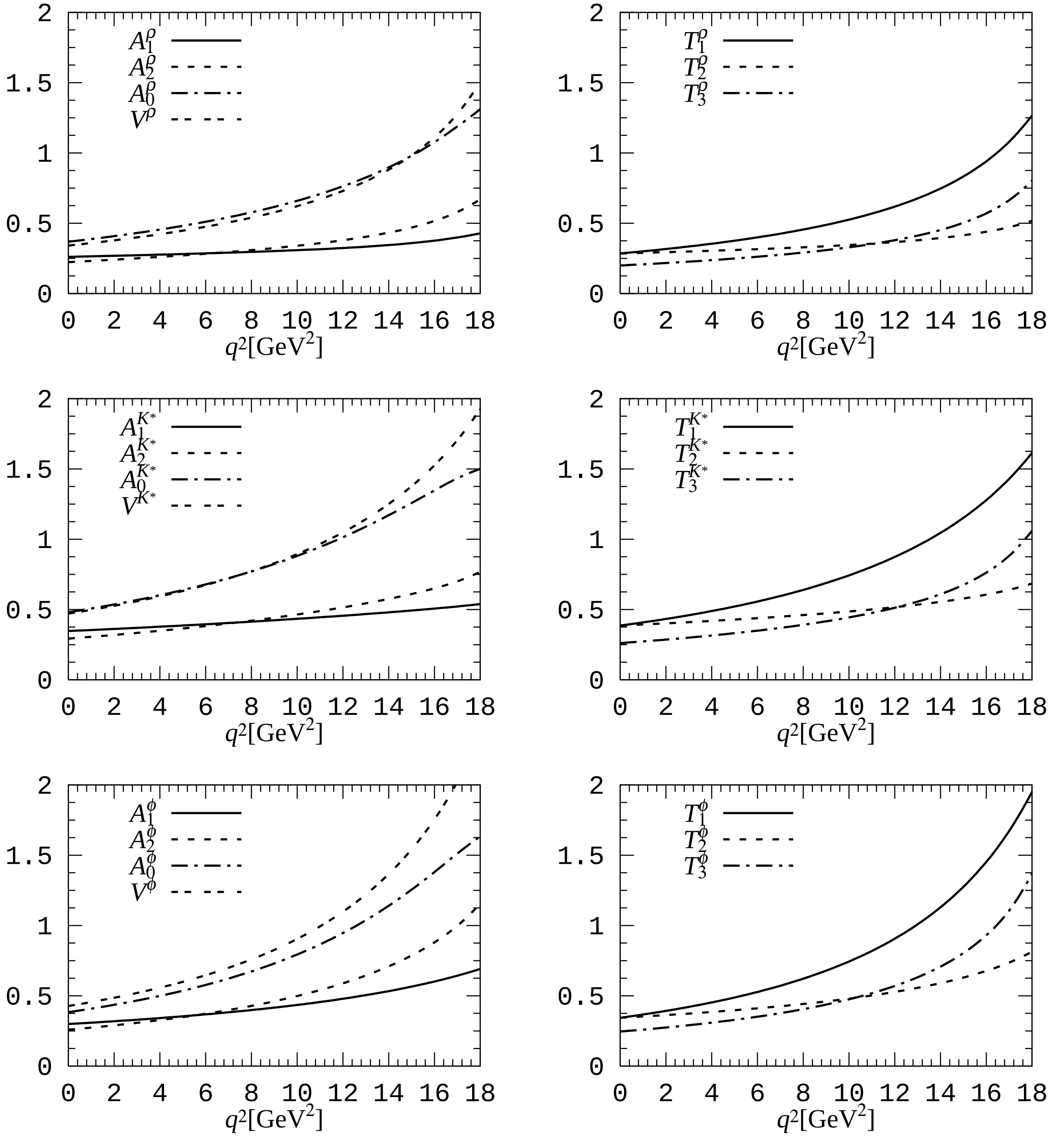

Figure 1: Light-cone sum rule results for $B \to$ vector meson form factors. Renormalization scale for T_i is $\mu = m_b = 4.7\,\mathrm{GeV}$. Further parameters: $m_b = 4.7\,\mathrm{GeV}$, $s_0 = 35\,\mathrm{GeV}^2$, $M^2 = 7\,\mathrm{GeV}^2$.

Further improvement and refinement of the above results within the method of light-cone sum rules is difficult and requires in particular better control over distribution amplitudes. I thus conclude with the request at the lattice community to feel challenged by the uncertainty of the old results [26] and to improve them by making full use if the refined computational implementation of lattice QCD that has been developed in recent years.

Acknowledgments

Fermilab is operated by Universities Research Association, Inc., under contract no. DE–AC02–76CH03000 with the U.S. Department of Energy.

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