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# BAYESIAN ANALYSIS OF THE SOLAR NEUTRINO ANOMALY[a]

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We present an analysis of the recent solar neutrino data from the five experiments using Bayesian approach. We extract quantitative and easily understandable information pertaining to the solar neutrino problem. The probability distributions for the individual neutrino fluxes and, discrepancy distribution for B and Be fluxes, which include theoretical and experimental uncertainties have been extracted. The analysis carried out assuming that the neutrinos are unaltered during their passage from the sun to earth, clearly indicate that the observed PP flux is consistent with the 1995 standard solar model predictions of Bahcall and Pinsonneault within $2\sigma$ (standard deviation), whereas the $^8B$ flux is down by more than $12\sigma$ and the $^7Be$ flux is maximally suppressed. We also deduce the experimental survival probability for the solar neutrinos as a function of their energy in a model-independent way. We find that the shape of that distribution is in qualitative agreement with the MSW oscillation predictions.

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## 1 Introduction

Discrepancy between the measured and the predicted solar neutrino fluxes [1] has prompted a great deal of activity in particle physics as well as in astrophysics over the last two decades. Solar models [1] that predict the solar neutrino fluxes use the *standard model* of elementary particles and the theory of stellar evolution. Table I gives a comparison between the neutrino fluxes measured by the four pioneering experiments [2] and the standard solar model (SSM) predictions [1]. In the past, these data have been analyzed using $\chi^2$ methods [3] and, attempts are made to explain the discrepancy between the measured and the predicted fluxes using MSW/vacuum oscillation models for neutrinos.

Recently, we have analyzed the solar neutrino data using Bayesian approach [4]. In this paper, we present the latest results from our analyses including the survival probability extracted in a model-independent way.

## 2 Bayesian Analysis and Results

Bayes' theorem gives us a prescription for calculating the posterior probability $P(\phi|s,I)$ for certain hypotheses about fluxes $\phi$, given the measured quantities $s$ and the prior information $I$. According to the Bayes' theorem,

$$P(\phi|s,I) = \frac{\mathcal{L}(s|\phi,I)\mathcal{P}(\phi|I)}{\int_{\phi}\mathcal{L}(s|\phi,I)\mathcal{P}(\phi|I)}, \tag{1}$$

where $\mathcal{L}$ is the likelihood function assigned to $s$ and $\mathcal{P}(\phi|I)$ is the prior probability function for $\phi$. The integration in the denominator guarantees the sum of the probabilities to be unity.

### *2.1 Solar Neutrino Flux*

The measured solar neutrino rate $s_i$ in an experiment conducted on earth is a function of experimental detection cross section $\sigma_i(E_\nu)$ and the neutrino flux $\Phi_j \cdot (\phi_N(E_\nu))_j$. The quantity $\Phi_j$ is the energy-averaged neutrino flux due to a particular source from the core of the sun and $(\phi_N(E_\nu))_j$ is its energy dependent normalized flux. Thus

$$s_i = \sum_j \Phi_j \int_{E_{th_i}} \sigma_i(E_\nu)\cdot(\phi_N(E_\nu))_j dE_\nu \tag{2}$$

where $E_{th_i}$ is the threshold energy for the experiment $i$. There are at least eight known sources of neutrinos in the sun. However, the neutrinos detected in the three types of experiments listed in Table I, are predominantly from $^7Be$, $PP$ and $^8B$ sources. Hence, to extract [4] the individual fluxes from experimental data, we

Table 1: Measured and predicted solar neutrino fluxes in units of SNU ($=10^{-36}$ $\nu$/atom/sec). Kamiokande data are relative to the Bahcall and Pinsonneault[1] predictions.

| Experiment [2] | Detection Technique and Threshold Energy $E_{th}$ | Flux Rates | |
|---|---|---|---|
| | | Measured | Predicted [1] |
| Homestake | $\nu_e + {}^{37}Cl \rightarrow e^- + {}^{37}Ar$, $E_{th}$=0.814 MeV | 2.59±0.20 | 9.30±1.30 |
| GALLEX | $\nu_e + {}^{71}Ga \rightarrow e^- + {}^{71}Ge$, $E_{th}$=0.233 MeV | 69.7±7 | 137±8 |
| SAGE | " | 69±12 | " |
| Super-Kamiokande | $\nu_e + e^- \rightarrow e^- + \nu_e$, $E_{th}$=7.5 MeV | 0.38±0.03 | 1.0±0.17 |
| Kamiokande | " | 0.42±0.06 | " |

simplify the problem by treating these three explicitly and the contributions from all other sources added together. (To extract the survival probability, explained in Sec. 2.2, the contributions from all eight individual neutrino sources in the sun have been considered.) Thus, Eq. 2 can be written as,

$$s \;=\; R \cdot \phi, \text{ with } R = \begin{pmatrix} 7.35 & 1.24 & 0.0 & 0.75 \\ 16.09 & 37.1 & 69.7 & 15.03 \\ 1.0 & 0.0 & 0.0 & 0.0 \end{pmatrix}. \qquad (3)$$

The four columns in the matrix $R$ correspond to $B$, $Be$, $PP$ and *other* neutrino fluxes, respectively, and the three rows correspond to Cl, Ga, and $H_2O$ experiments, respectively. We assume that the data are uncorrelated so that the error matrix $\Sigma$ associated with the neutrino rates $s^T = (s_{Cl}, s_{Ga}, s_{H_2O}) = (2.59, 69.50, 0.39)$ is diagonal, with $\Sigma^T = (0.20, 6.70, 0.03)$. We assume the likelihood function $\mathcal{L}$ to be of Gaussian form,

$$\mathcal{L}(s|\phi, I) = \exp(-\frac{1}{2} z^T z), \quad \text{with} \quad z = \Sigma^{-1} \cdot (s - R \cdot \phi). \qquad (4)$$

We take the prior probability function $\mathcal{P}(\phi|I) = \textit{constant}$. Our studies show that any reasonable function for $\mathcal{P}(\phi|I)$ is acceptable. The posterior probability function $P(\phi_B, \phi_{Be}, \phi_{PP}, \phi_O)$, written as in Eq. (1), can be used to get the probability for one or more of the fluxes by marginlizing it with respect to all other fluxes.

Figures 1(a), (b) and (c) show, respectively, the normalized probability distributions for B, PP and Be neutrino fluxes obtained by using data from three experiments shown in Table 1. They indicate that all of them deviate quite significantly from the standard solar model predictions. The PP flux is consistent with the standard model prediction within $2\sigma$, whereas the $^8B$ flux deviates by more than $10\sigma$. In the case of the $^7Be$, the probability distribution peaks at zero

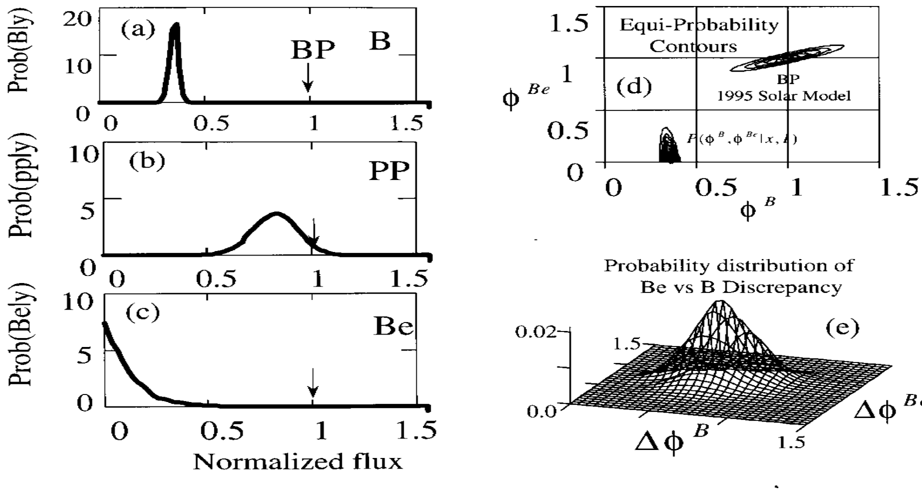


Figure 1: (a),(b), and (c) show the normalized probability distributions for the solar neutrino fluxes from Bayesian analyses. The arrows indicate the standard solar model [1] predictions. The 2D probability distribution for $^8B$ and $^7Be$ and their discrepancy plots are shown in (d) and (e), respectively.

or at a negative value (which is unphysical). We find that $\phi_{Be}$ is less than 0.4., at the 95% confidence level.

By marginalizing the posterior probability function (Eq. (1)) over PP and all other solar neutrino fluxes, we obtain a two dimensional probability distribution function for B and Be. The equi-probability contours for $^7Be$ *vs.* $^8B$ fluxes are shown in Fig. 1(d). The results from the 1995 standard solar model [1] are also shown for comparison. By treating the previously determined 2D $^8B$ and $^7Be$ posterior probability distribution function as a prior probability function, the discrepancy distribution between theory and experiment is extracted and the result is displayed in Fig. 1(e). In this analysis we take into account both the theoretical and the experimental errors. This result clearly indicates the discrepancy between SSM predictions and the experimental results is larger than $3\sigma$.

### *2.2 Survival Probability*

In the past, attempts have been made to explain the observed neutrino deficits in the three experiments by using the predicted neutrino fluxes from SSM and assuming MSW effects and/or vacuum oscillation model for the neutrinos. Thereby the survival probabilities of the neutrinos have been extracted. In this section,

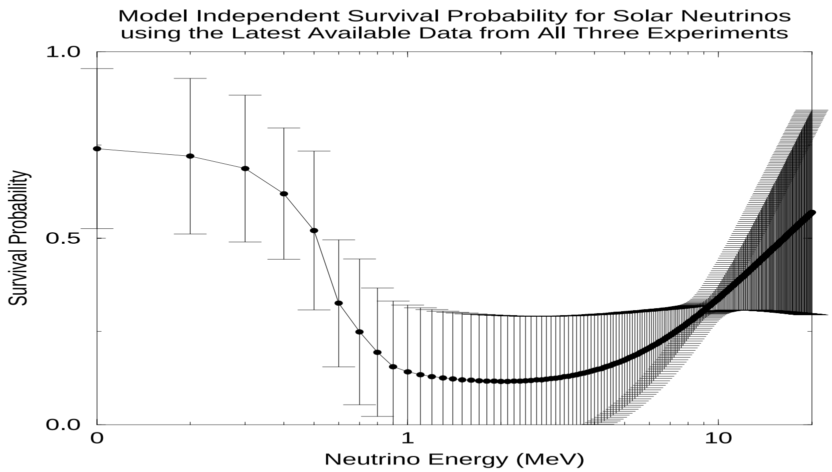


Figure 2: Solar neutrino survival probability as a function of neutrino energy. The one sigma errors shown in the figure arise from the experimental data alone. The analysis do not include theoretical uncertainties [1].

we extract the survival probability in a model-independent way using a Bayesian approach. The sensitivity of the survival probability at different regions of the solar neutrino spectrum is also studied. In this approach, the measured data is represented by,

$$s_i = \sum_j \Phi_j \int_{E_{th_i}} \sigma_i(E_\nu) \cdot (\phi_N(E_\nu))_j P(E_\nu) dE_\nu. \quad (5)$$

The quantity $P(E_\nu)$ is the survival probability. Other variables are similar to the ones shown in Eq. (2). A close examination of the results presented in the section 2.1 suggests a non-linear form for the function $P(E_\nu)$, which we parameterize as

$$P(E_\nu|a_1, a_2..a_5) = \frac{a_1}{1 + e^{\frac{E_\nu - a_2}{a_3}}} + a_4 e^{\frac{a_3}{E_\nu}}, \quad (6).$$

with five parameters, $a_1, a_2, ..., a_5$. One may also use a very high order polynomial for parameterization. The current parameterization makes the problem computationally far less intense and, convergence on optimal values of parameters is quite rapid.

With the parametric form of $P$ given in Eq. 6, the likelihood function $\mathcal{L}$ and the posterior probability function $P(a_1, a_2, .., a_5|D)$ have been generated taking all eight different neutrino sources in the sun and three measured integrated neutrino fluxes. The parameters $a_1, a_2, .., a_5$ are eliminated by marginalization. Thus, the survival probability as a function of neutrino energy becomes ,

$$P(E_\nu|D) = \int_{a_1,a_2,..,a_5} P(E_\nu|a_1, a_2, .., a_5) Post(a_1, a_2, .., a_5|D). \quad (7)$$

Figure 2 displays the extracted survival probability which clearly demonstrates suppression of the neutrino flux around 1 MeV, i.e., near $^7B$ and *pep* region of the neutrino spectrum. However, the survival probability has quite large errors[d] throughout the energy range considered here. The overall shape of the survival probability distribution as a function of the neutrino energy is consistent with the predictions made assuming MSW effects [5]. However, our study clearly accommodates a very large range of MSW parameters. From our study we find that even if the precision in the standard solar model as well as in the experiments are improved by a large factor, one cannot draw useful conclusions about its survival probability over the energy range between 1.5 MeV to 6.5 MeV with certainty. Perhaps this gap needs to be filled in by accelerator based neutrino experiments where the energy of the neutrino spectrum can be tuned.

It is unquestionable that by the method of marginalization, the effect of parameterization can be eliminated to first order. We find that in the test carried out assuming SSM predictions in the place of measured data gives unit survival probability for the entire energy range considered here. However, the survival probability extracted here has some bias because of the particular functional form (see Eq. 6) assumed in the analysis. In the region where there is no data one expects the probability of the survival probability to be flat with its mean at $0.5\pm0.3$. The present analysis shows that in this energy range the results are consistent within errors though mean value is lower than expected.

In conclusion, we have performed a Bayesian analysis of the existing solar neutrino experimental data. The analysis carried out assuming the neutrinos to be unaltered during their passage from the sun to earth, clearly indicate that the PP flux is consistent with the 1995 standard solar model predictions within two standard deviations, whereas the $^8B$ flux is suppressed by more than $10\sigma$ and the probability distribution for the $^7Be$ flux is maximally suppressed. We also deduce the survival probability for the solar neutrinos as a function of their energy in a model independent way. We find that the shape of the probability distribution is in qualitative agreement with the MSW oscillation predictions.

## 3 Acknowledgments

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[d] We have not included the theoretical errors, here.

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