Fermi National Accelerator Laboratory

FERMILAB-Conf-98/053

Model for Pion Production in Proton-Nucleus Interactions

N.V. Mokhov and S.I. Striganov

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

February 1998

Published Proceedings of the *Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider*, Fermilab, Batavia, Illinois, November 6-9, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Model for Pion Production in Proton-Nucleus Interactions*

N. V. Mokhov and S. I. Striganov+

Fermi National Accelerator Laboratory, P.O. Box 500, Batavia, Illinois 60510

+*Now at Institute for High-Energy Physics, Protvino, Moscow region, Russia*

February 5, 1998

Abstract

A new phenomenological model has been developed to describe pion production in high-energy proton-nucleus interactions. Special attention is paid to low-momentum pions (0.1< p <2 GeV/c) for intermediate proton momenta 5< p_0 <30 GeV/c. It is shown that the model predictions are in an excellent agreement with data in the entire kinematic region. Comparisons to other models are also presented. The model is embedded into the MARS13 code.

Introduction

Reliable prediction of pion yield in hadron-nucleus (hA) collisions is vital in numerous applications, particularly in the planning of future experiments and accelerators. The newest examples include a $\mu^+\mu^-$collider project [1] and neutrino experiments at Fermilab Main Injector [2] and Booster [3]. There are a few models capable of generating pions in $pA \rightarrow \pi^{\pm}X$ reactions, e. g., [4, 5, 6, 7, 8, 9]. Theoretical calculations based on the intranuclear cascade model are reliable at proton momenta p_0 <5 GeV/c, but drastically overestimate hadron yield at higher energies. Microscopic models, such as DPMJET [5] (based on the dual topological unitarization approach) and FRITIOF [6] (based on the LUND model) were developed mainly for high energies $\gtrsim$50 GeV/c. As it is shown in [1], there is an uncertainty up to a factor of 5 in the pion yield at p <1 GeV/c on heavy nuclei for proton momenta 5< p_0 <30 GeV/c – the region that is especially interesting for the $\mu^+\mu^-$collider project. On the other hand, there are many data on inclusive charged pion production in hA collisions obtained over the last three decades. Based on those data and

*Submitted to *Proceedings of the Workshop on Physics at the First Muon Collider and at the Front End of a Muon Collider*, Fermilab, Batavia, IL, November 6-9, 1997

our original model [4, 9], we develop a phenomenological model for a reliable description of inclusive pion production in the entire kinematic range for pA collisions at 5 GeV/c$< p_0 <$10 TeV/c.

Phenomenological Model

Many reliable data and parameterizations exist on pion yield in pp-collisions. We can compensate for the lack of data for pA reactions by using the following form (see, e. g., [4, 9]) for the double differential cross section of the $pA \to \pi^\pm X$ reaction:

$$\frac{d^2\sigma^{pA\to\pi^\pm X}}{dpd\Omega} = R^{pA\to\pi^\pm X}(A, p_0, p, p_\perp)\frac{d^2\sigma^{pp\to\pi^\pm X}}{dpd\Omega}, \quad (1)$$

where p and $p_\perp$ are the total and transverse momenta of $\pi^\pm$, and A is an atomic mass of the target nucleus. The function $R^{pA\to\pi^\pm X}$, measured with much higher precision than the absolute yields, is almost independent of $p_\perp$ and its dependence on p_0 and p is much weaker than for the differential cross-section itself. Because of rather different properties of pion production on nuclei in the forward ($x_F \gtrsim$0) and backward ($x_F \lesssim$0) hemispheres, where x_F is the Feynman's longitudinal variable, we treat these two regions differently.

R at $x_F \gtrsim$0.05. In this region we assume $R^{pA\to\pi^\pm X} \sim A^\alpha$. The power α is almost independent of the pion sign. The following parameterization was proposed in [10] for $p_0 \geq$70 GeV/c:

$$\alpha_g = 0.8 - 0.75 \cdot x_F + 0.45 \cdot x_F^3/|x_F| + 0.1 \cdot p_\perp^2. \quad (2)$$

Fig. 1(a) shows our compilation of data [11, 12, 13, 14, 15] on α for π^--production. It turns out that (2) describes data very well at $p_0 \geq$24 GeV/c and can be successfully used at lower momenta (5$\leq p_0 \leq$24 GeV/c) if it is replaced with (see Fig. 1(a)):

$$\alpha = \alpha_g - 0.0087 \cdot (24 - p_0). \quad (3)$$

The $R^{pA\to\pi^\pm X} \sim A^\alpha$ form doesn't extrapolate well to A=1 because of the difference in the π-yield in proton-proton and proton-neutron collisions. This difference can be taken into account if one uses the following form for $R^{pA\to\pi^\pm X}$ [10]:

$$R^{pA\to\pi^\pm X} = (\frac{A}{2})^\alpha \cdot f(p_0, Y), \quad (4)$$

where $f(p_0, Y) = \frac{d\sigma}{dp}(pd \to \pi^\pm)/\frac{d\sigma}{dp}(pp \to \pi^\pm)$. It turns out that pion yields in pd and pp collisions are not very different, i. e. $f(p_0, Y) \approx$1. Using FRITIOF results, we found that $f(p_0, Y)_{\pi^-} = 1 + 0.225/N_{\pi^-} - a_{\pi^-} \cdot Y_{cms}$, where N_{π^-} is mean π^- multiplicity in pp collisions and Y_{cms} is pion rapidity in the center-of-mass system (CMS). Data [16] show linear dependence of N_{π^-} on *free energy* $W = \frac{(\sqrt{s}-2\cdot m_p)^{0.75}}{\sqrt{s}^{0.25}}$, where $\sqrt{s}$ is the CMS collision energy. Our fit to the data gives $N_{\pi^-} = 0.81 \cdot (W - 0.6)$. The other parameter $a_{\pi^-} = 0.16$ for $p_0 \leq 20$ GeV/c, and depends on energy for higher momenta as $a_{\pi^-} = -0.055 + 0.747/log(s)$. $f(p_0, Y)_{\pi^-}$ is forced to be 1 if it becomes less than 1. For π^+ production the approximation is much simpler $f(p_0, Y)_{\pi^+} = 0.85 + 0.005 \cdot p_0$ for $p_0 \leq 30$ GeV/c and $f(p_0, Y)_{\pi^+} = 1$ for higher momenta.

R at x_F $\lesssim$**0.05**. In this region, due to the lack of experimental data on α, we use the following expression for the function R in (1):

$$R^{pA \to \pi^\pm X} = \frac{dN/dY(pA)}{dN/dY(pp)} \quad (5)$$

The following scaling law was proposed in [17] for charged shower particle ($\beta > 0.7$) production in pA collisions at 20 – 400 GeV/c:

$$\frac{Y_0}{< N_s >} \cdot \frac{dN}{d\eta} = f(A, \frac{\eta}{Y_0}), \quad (6)$$

where $< N_s >$ is a mean multiplicity of shower particles, Y_0 is rapidity of primary proton and $\eta = -log(tan(\theta/2))$ is pseudorapidity of a secondary particle. We found that this approximation is in a reasonable agreement with data at p_0 >7.5 GeV/c [18]. Unfortunately, η is not a convenient variable to describe forward pion production (θ ≈0). Our analysis of the $pA \to \pi^- X$ data [19, 20] at 10< p_0 <100 GeV/c shows that replacing η in (6) with rapidity Y

$$\frac{dN}{dY} = \frac{< N_\pi >}{Y_0} \cdot F(A, \frac{Y}{Y_0}) \quad (7)$$

provides better description of the pion yield in the entire kinematic range. Here $< N_\pi >$ is mean pion multiplicity, $Y_0 = log(\frac{E_0 + p_0}{m_p})$ is rapidity of incident proton and $Y = log(\frac{E_\pi + p_z}{m_\perp})$ is π rapidity, $m_\perp = \sqrt{p_\perp^2 + m_\pi^2}$. We choose the Gaussian form for the scaling function:

$$F(A, \frac{Y}{Y_0}) = c_1 \cdot exp(-(\frac{Y}{Y_0} - c_2)^2/c_3), \quad (8)$$

where for π^-: $c_1 = 1.149 \cdot A^{0.0479}$, $c_2 = 0.492 \cdot A^{-0.0565}$, and $c_3 = 0.214 \cdot A^{-0.121}$. Reliable rapidity distributions for π^+ at x_F <0 are measured only for $p_0 \geq 100$ GeV/c. Assuming that the scaling (7)-(8) is valid for π^+ also, we found the following parameters from data [20]: c_1=1.6, $c_2 = 0.521 \cdot A^{-0.0416}$, and c_3=0.12. The data on dN/dY for $pp \to \pi^\pm X$ reaction is well described by a Gaussian [21]: $dN/dY = C_{pp} \cdot exp(-Y_{cm}^2/2\sigma^2)$, where $\sigma_{\pi^+} = 0.402 + 0.198 \cdot log(p_0)$ and $\sigma_{\pi^-} = 0.465 + 0.157 \cdot log(p_0)$. The normalization parameter in (5), combined of $< N_\pi >$, C_{pp} etc, is chosen to match the functions (4) and (5) at x_F=0.05.

Table 1: Parameters in formula (9).

	A	B	C	D	E	F
π^+	60.1	1.9	0.18	0.3	12	2.7
π^-	51.2	2.6	0.17	0.3	12	2.7

pp$\rightarrow$ $\pi^{\pm}$**X**. To describe the invariant cross section of charged pion production in pp collisions we use the form proposed in [22] that we modified at low [13, 23] and high [4] $p_\perp$:

$$E\frac{d^3\sigma^{pp\rightarrow\pi^{\pm}X}}{dp^3} = A(1-\frac{p^*}{p^*_{max}})^B exp(-\frac{p^*}{C\sqrt{s}})V_1(p_\perp)V_2(p_\perp), \qquad (9)$$

where p^* and p^*_{max} are pion momentum and maximum momentum transfer in CMS and parameters are given in Table 1. The best description of the $p_\perp$ dependence is obtained with:

$$V_1(p_\perp) = \begin{cases} (1-D)exp(-Ep_\perp^2) + Dexp(-Fp_\perp^2), & p_\perp \leq 0.933 \text{ GeV/c}, \\ 0.2625/(p_\perp^2+0.87)^4, & p_\perp > 0.933 \text{ GeV/c}, \end{cases}$$

$$V_2(p_\perp) = \begin{cases} 0.7363\, exp(0.875p_\perp), & p_\perp \leq 0.35 \text{ GeV/c}, \\ 1, & p_\perp > 0.35 \text{ GeV/c}. \end{cases}$$

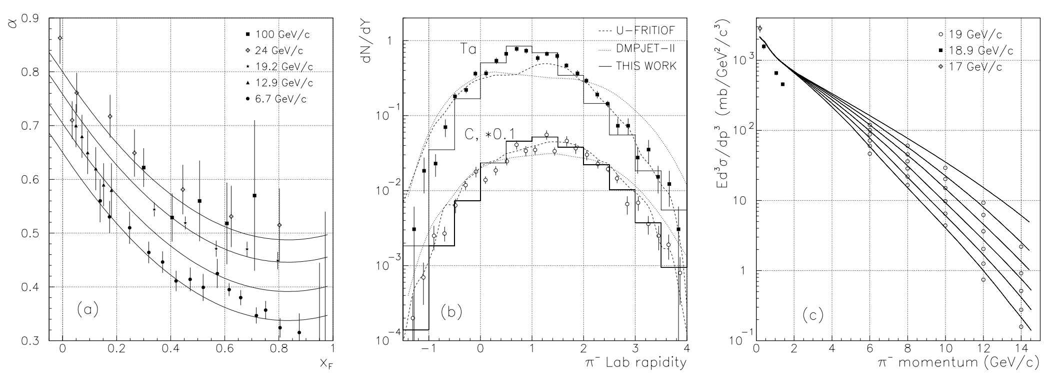

Figure 1: (a) Parameter α (2)-(3) calculated for p_0=6.7, 12.9, 19.2 and $>$24 GeV/c (from bottom up) in comparison with data [11, 12, 13, 14, 15]; (b) rapidity distributions of π^- in pC and pTa interactions at 10 GeV/c as calculated with FRITIOF, DPMJET and the model developed in this paper in comparison with data [19]; (c) $pCu \rightarrow \pi^- X$ at p_0=17 – 19 GeV/c for θ=12.5, 30, 40, 50, 60 and 70 mrad (from top down), data from [13, 25, 26].

Comparison to Data

The model developed agrees very well with available data and reliable DPMJET-II [5] predictions at proton momenta 50 GeV/c$< p_0 <$10 TeV/c (as used in the original model [4, 9]). In this section we compare pion yield predicted by the new model and other models with data available at lower proton momenta down to $p_0 \approx$5 GeV/c on thin and thick nuclear targets. Fig. 1(b) shows comparison of calculated π^- rapidity distributions in pC and pTa collisions at p_0=10 GeV/c with data [19]. One sees that our model gives much better results than the DPMJET and FRITIOF codes for soft pions at 0$< Y <$2. Production of energetic pions is also nicely described by our model (Fig. 1(b),(c)). Fig. 2 shows that the developed model gives a reliable description of pions generated at $\theta \approx$0 in the very 'difficult' for other models region of intermediate momenta 5$< p_0 <$19 GeV/c.

The 'mystery' with the soft pion production began with the analysis of the pHg reaction at 24 GeV/c [1]. As Figs. 3(a),(b) show, the model developed gives a good agreement with data (calculations for nuclei other than hydrogen and copper nuclei at this energy are not shown but agree very well with data), and give guidance on the other codes in this considered case (see Fig. 3(c),(d)). Calculations with the MARS13(97) code (with the new model) of the pion double differential spectra with cascading in the thick copper and lead targets at p_0=6 and 8 GeV/c agree nicely with data [25, 30] in the momentum region 0.1$< p <$5 GeV/c (so crucial for $\mu^+\mu^-$ collider applications) (see Figs. 4 and 5). At the same time, GEANT, even in the most appropriate FLUKA mode, certainly has some problem here.

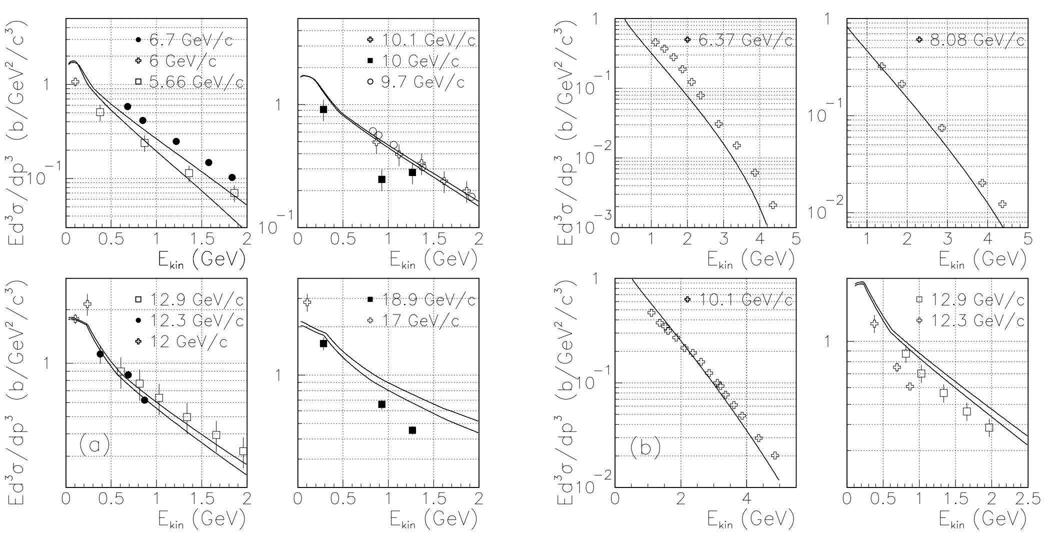

Figure 2: (a) $pCu \rightarrow \pi^- X$ ($\theta \approx 0$), data from [11, 12, 24, 25, 26, 27, 28]; (b) $pCu \rightarrow \pi^+ X$ ($\theta \approx 0$), data from [12, 27, 28, 29]. Two curves presented in some plots correspond to higher (top) and lower (bottom) p_0 shown in those plots.

Conclusion

This model development began while one of us (NM) pointed out that there is a great degree of uncertainty in central (x_F $\approx$0) pion production on medium and heavy nuclei in the medium proton momentum range 5< p_0 <30 GeV/c [1]. Successful benchmarking, performed with the MARS13(97) code with the new model embeded, against data in the wide kinematic range for nuclei ranging from hydrogen to lead, assure that we have now a tool for reliable prediction of pion yield for the proton momentum range estimated as 5 GeV/c< p_0 <10 TeV/c.

References

[1] *$\mu^+\mu^-$Collider: A Feasibility Study*, The $\mu^+\mu^-$Collider Collaboration, BNL–52503; Fermilab–Conf–96/092; LBNL–38946, July 1996.

[2] http://www-numi.fnal.gov:8875/.

[3] Church, E., et al., 'A proposal for BooNE Neutrino Experiment at the Fermilab Booster', Fermilab-P-898 (1997).

[4] Kalinovskii, A. N., Mokhov, N. V., and Nikitin, Yu. P., 'Passage of High-Energy Particles through Matter', AIP, New York (1989).

[5] Ranft, J., Phys. Rev., **D51**, p. 64 (1995); Gran Sasso report INFN/AE-97/45 (1997).

[6] Uzhinskii, V.V., JINR-E2-96-192, Dubna (1996)

[7] Mokhov, N. V., 'The MARS code system Users Guide, version 13(95)', Fermilab–FN–628 (1995).

[8] Kahana, D., and Torun, Y., BNL-61983, Brookhaven (1995).

[9] Mokhov, N. V., Striganov, S. I., and Uzunian, A. V., IHEP-87-59, Serpukhov (1987).

[10] Geist, W. M., Nucl. Phys., **A525**, p. 149 (1991).

[11] Bayukov, Y. D., et al., Yad. Fiz., **29**, p. 947 (1979).

[12] Yamamoto, A., KEK 81-13, Tsukuba (1981)

[13] Allaby, J. V., et al., CERN 70-12 (1970).

[14] Eichten, T., et al., Nucl. Phys., **B44**, p. 333 (1972).

[15] Barton, D. S., et al., Phys.Rev., **D27**, p. 2580 (1983).

[16] Gazdzicki, M., and Rohrich, D., Z. Phys., **C65**, p. 215 (1995).

[17] Stenlund, E., and Otterlund, I., CERN-EP/82-42 (1982).

[18] Bayukov, Y. D., et al., Yad. Fiz., **42**, p. 1414 (1985).

[19] Armutliysky, D., et al., Yad. Fiz., **48**, p. 161 (1988).

[20] Whitmore, J. J., et al., Z. Phys., **C62**, p. 199 (1994).

[21] Uvarov, V. A., and Shlyapnikov, P. V., Sov. J. Nucl. Phys., **50**, p. 1048 (1982).

[22] Folomeshkin, V. N., IHEP-71-22, Serpukhov (1971).

[23] Smith, D. B., et al., Phys. Rev. Lett., **23**, p. 1064 (1969).

[24] Papp, J., LBL-3633, Berkeley (1975).

[25] Berley, D., et al., IEEE Trans. Nucl. Sc., **23**, p. 997 (1973).

[26] Amman, J.F., et al., LA-9486-MS, Los Alamos (1982).

[27] Vorontsov, I. A., et al., ITEP-11, Moscow (1988).

[28] Marmer, G. J., et al., Phys. Rev., **179**, p. 1294 (1969).

[29] Arefiev, A. S., et al., ITEP-25, Moscow (1985).

[30] Audus, M. F. et al., Nuovo Cimento, **A46**, p. 502 (1966)

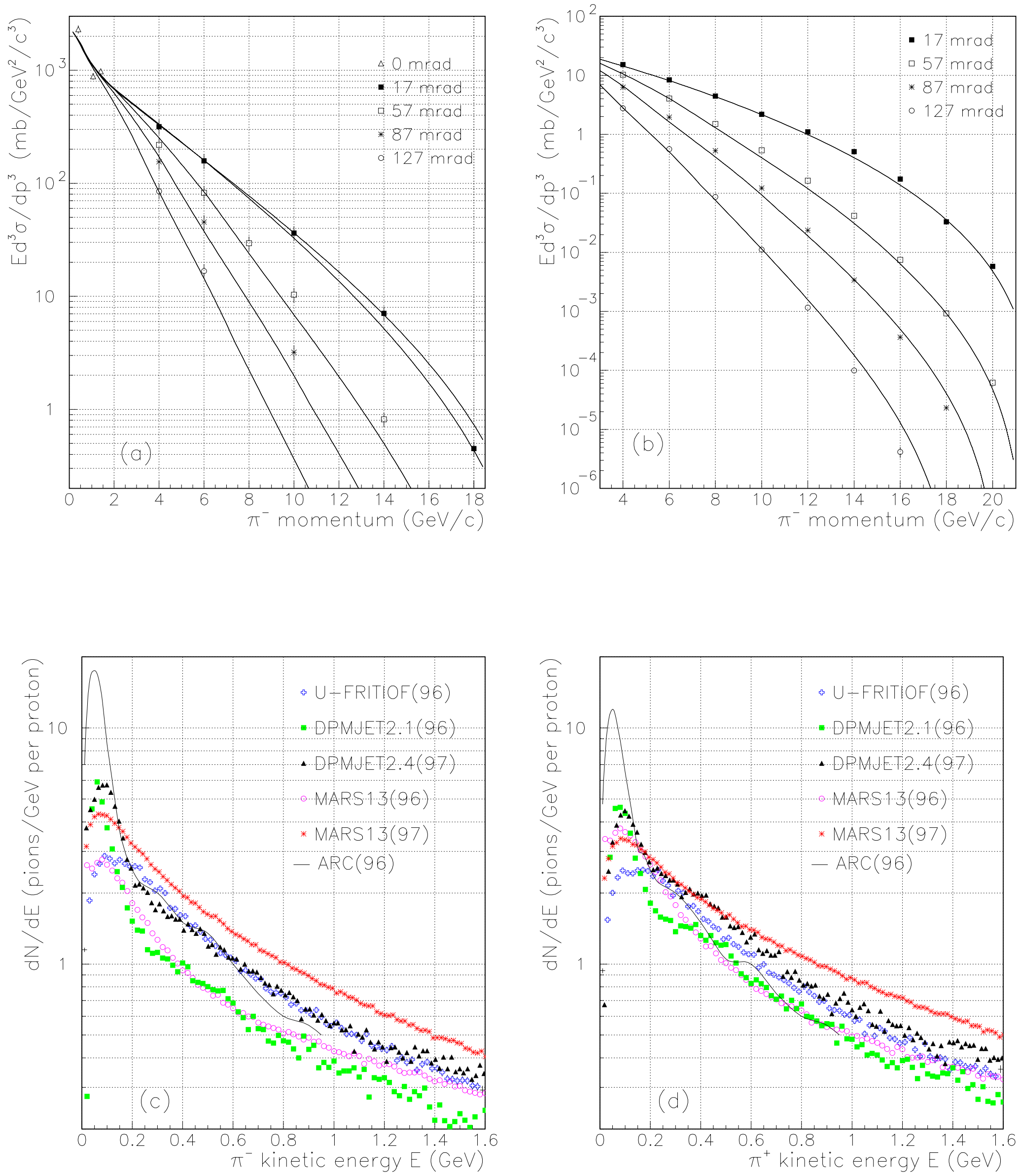

Figure 3: Pion spectra at p_0=24 GeV/c: (a) $pCu \to \pi^- X$ at 5 angles, data from [14, 26]; (b) $pp \to \pi^- X$ at 4 angles, data from [14]; (c) $pHg \to \pi^- X$ and (d) $pHg \to \pi^+ X$ according to several codes (MARS13(97) with the new model).

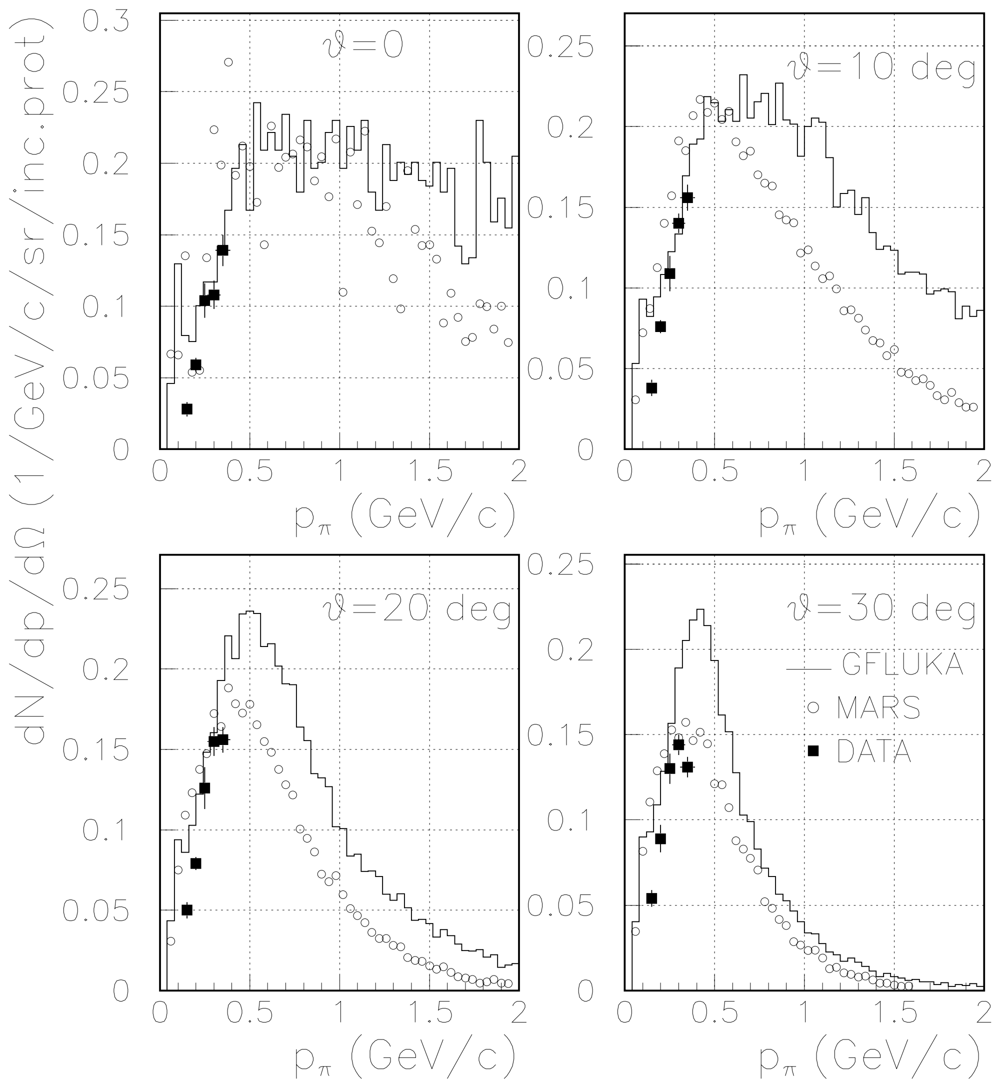

Figure 4: Double differential π^- spectra from a 6-inch thick copper target at p_0=6 GeV/c as calculated with GEANT-FLUKA and MARS13(97) and measured in [25].

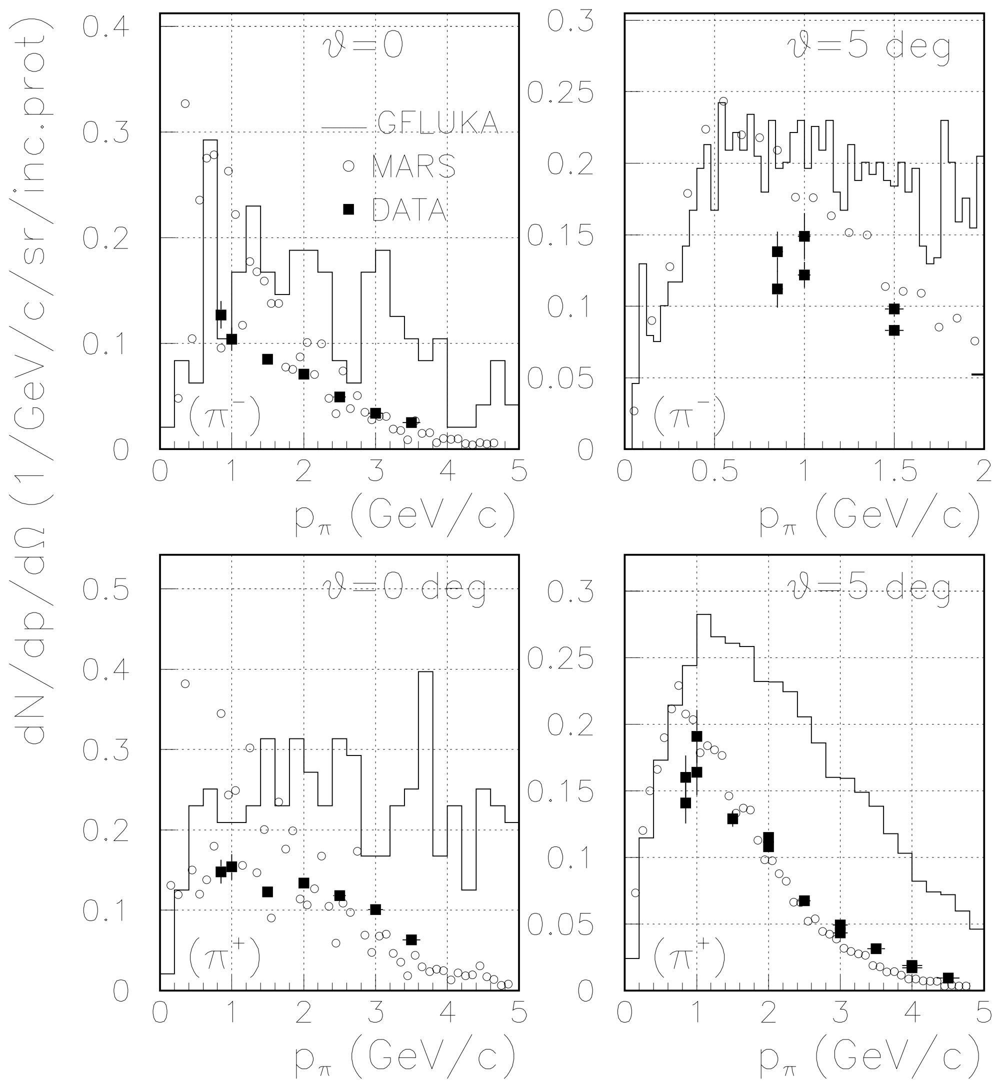

Figure 5: Double differential π^- (top) and π^+ (bottom) spectra from a 10-cm thick lead target at p_0=8 GeV/c as calculated with GEANT-FLUKA and MARS13(97) and measured in [30].