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Topcolor and the First Muon Collider¹

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Abstract. We describe a class of models of electroweak symmetry breaking that involve strong dynamics and top quark condensation. A new scheme based upon a seesaw mechanism appears particularly promising. Various implications for the first-stage muon collider are discussed.

TOPCOLOR I

The top quark mass may be large because it is a combination of a *dynamical condensate component*, $(1 - \epsilon)m_t$, generated by a new strong dynamics [1], together with a small *fundamental component*, ϵm_t , i.e., $\epsilon \ll 1$, generated by something else. The most obvious “handle” on the top quark for new dynamics is the color index. Invoking new dynamics involving the top quark color index leads directly to a class of Technicolor-like models incorporating “Topcolor”. We expect in such schemes that the new strong dynamics occurs primarily in interactions that involve $\bar{t}t\bar{t}t$, $\bar{t}t\bar{b}b$, and $\bar{b}b\bar{b}b$.

In Topcolor I the dynamics at the ~ 1 TeV scale involves the following structure at the TeV scale (or a generalization thereof) [2]:

$$SU(3)_1 \times SU(3)_2 \times U(1)_{Y_1} \times U(1)_{Y_2} \times SU(2)_L \rightarrow SU(3)_{QCD} \times U(1)_{EM} \quad (1)$$

where $SU(3)_1 \times U(1)_{Y_1}$ ($SU(3)_2 \times U(1)_{Y_2}$) generally couples preferentially to the third (first and second) generations. The $U(1)_{Y_i}$ are just strongly rescaled versions of electroweak $U(1)_Y$.

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The fermions are then assigned $(SU(3)_1, SU(3)_2, Y_1, Y_2)$ quantum numbers in the following way:

$$\begin{aligned}
(t, b)_L &\sim (3, 1, 1/3, 0) & (t, b)_R &\sim (3, 1, (4/3, -2/3), 0) & (2) \\
(\nu_\tau, \tau)_L &\sim (1, 1, -1, 0) & \tau_R &\sim (1, 1, -2, 0) \\
(u, d)_L, (c, s)_L &\sim (1, 3, 0, 1/3) & (u, d)_R, (c, s)_R &\sim (1, 3, 0, (4/3, -2/3)) \\
(\nu, \ell)_L \ell = e, \mu &\sim (1, 1, 0, -1) & \ell_R &\sim (1, 1, 0, -2)
\end{aligned}$$

Topcolor must be broken, which we describe by an (effective) scalar field:

$$\Phi \sim (3, \bar{3}, y, -y) \quad (3)$$

When Φ develops a VEV, it produces the simultaneous symmetry breaking

$$SU(3)_1 \times SU(3)_2 \rightarrow SU(3)_{QCD} \quad \text{and} \quad U(1)_{Y_1} \times U(1)_{Y_2} \rightarrow U(1)_Y \quad (4)$$

$SU(3)_1 \times U(1)_{Y_1}$ is assumed to be strong enough to form chiral condensates which will be “tilted” in the top quark direction by the $U(1)_{Y_1}$ couplings. The theory is assumed to spontaneously break down to ordinary QCD $\times U(1)_Y$ at a scale of ~ 1 TeV, before it becomes confining. The isospin splitting that permits the formation of a $\langle \bar{t}t \rangle$ condensate but disables the $\langle \bar{b}b \rangle$ condensate is due to the $U(1)_{Y_i}$ couplings. The b -quark mass in this scheme can arise from a combination of ETC effects and instantons in $SU(3)_1$. The θ -term in $SU(3)_1$ may manifest itself as the CP-violating phase in the CKM matrix. Above all, the new spectroscopy of such a system should begin to materialize indirectly in the third generation, perhaps at the Tevatron in top and bottom quark production, or possibly in a muon collider.

The symmetry breaking pattern outlined above will generically give rise to three (pseudo)-Nambu-Goldstone bosons $\tilde{\pi}^a$, or “top-pions”, near the top mass scale. *This is the smoking gun of Topcolor.* [We were led to Topcolor by considering how strong dynamics might produce the analog of the decay $t \rightarrow H^+ + b$, considered to be a SUSY signature for a charged Higgs-boson H . This is an example of “SUSY-Technicolor/Topcolor duality”.] If the Topcolor scale is of the order of 1 TeV, the top-pions will have a decay constant of $f_\pi \approx 50$ GeV, and a strong coupling given by a Goldberger-Treiman relation, $g_{tb\pi} \approx m_t/\sqrt{2}f_\pi \approx 2.5$, potentially observable in $\tilde{\pi}^+ \rightarrow t + \bar{b}$ if $m_{\tilde{\pi}} > m_t + m_b$.

We assume presently that ESB can be primarily driven by a Higgs sector or Technicolor, with gauge group $G_{TC}[3]$ [4]. This gives the $\mathcal{O}(\epsilon)$ component of m_t . Technicolor can also provide condensates which generate the breaking of Topcolor to QCD and $U(1)_Y$.

The coupling constants (gauge fields) of $SU(3)_1 \times SU(3)_2$ are respectively h_1 and h_2 ($A_{1\mu}^A$ and $A_{2\mu}^A$) while for $U(1)_{Y_1} \times U(1)_{Y_2}$ they are respectively q_1

and q_2 , $(B_{1\mu}, B_{2\mu})$. The $U(1)_{Y_i}$ fermion couplings are then $q_i \frac{Y_i}{2}$, where Y_1, Y_2 are the charges of the fermions under $U(1)_{Y_1}, U(1)_{Y_2}$ respectively.

Topcolor I produces new gauge heavy bosons Z' , and “colorons” B^A with couplings to fermions given by:

$$\mathcal{L}_{Z'} = g_1(Z' \cdot J_{Z'}) \quad \mathcal{L}_B = g_3 \cot \theta (B^A \cdot J_B^A) \quad (5)$$

where the currents $J_{Z'}$ and J_B in general involve all three generations of fermions

$$\begin{aligned} J_{Z'} &= -(J_{Z',1} + J_{Z',2}) \tan \theta' + J_{Z',3} \cot \theta' \\ J_B &= -(J_{B,1} + J_{B,2}) \tan \theta + J_{B,3} \cot \theta \end{aligned} \quad (6)$$

For example, for the third generation the currents read explicitly (in a weak eigenbasis):

$$\begin{aligned} J_{Z',3}^\mu &= \frac{1}{6} \bar{t}_L \gamma^\mu t_L + \frac{1}{6} \bar{b}_L \gamma^\mu b_L + \frac{2}{3} \bar{t}_R \gamma^\mu t_R - \frac{1}{3} \bar{b}_R \gamma^\mu b_R \\ &\quad - \frac{1}{2} \bar{\nu}_{\tau L} \gamma^\mu \nu_{\tau L} - \frac{1}{2} \bar{\tau}_L \gamma^\mu \tau_L - \bar{\tau}_R \gamma^\mu \tau_R \\ J_{B,3}^{A,\mu} &= \bar{t} \gamma^\mu \frac{\lambda^A}{2} t + \bar{b} \gamma^\mu \frac{\lambda^A}{2} b \end{aligned} \quad (7)$$

where λ^A is a Gell-Mann matrix acting on color indices. We ultimately demand $\cot \theta \gg 1$ and $\cot \theta' \gg 1$ to select the top quark direction for condensation.

The attractive Topcolor interaction, for sufficiently large $\kappa = g_3^2 \cot^2 \theta / 4\pi$, would by itself trigger the formation of a low energy condensate, $\langle \bar{t}t + \bar{b}b \rangle$, which would break $SU(2)_L \times SU(2)_R \times U(1)_Y \rightarrow U(1) \times SU(2)_c$, where $SU(2)_c$ is a global custodial symmetry. On the other hand, the $U(1)_{Y_1}$ force is attractive in the $\bar{t}t$ channel and repulsive in the $\bar{b}b$ channel. Thus, to make $\langle \bar{b}b \rangle = 0$ and $\langle \bar{t}t \rangle \neq 0$ we can have in concert critical and subcritical values of the combinations:

$$\kappa + \frac{2\kappa_1}{9N_c} > \kappa_{crit}; \quad \kappa_{crit} > \kappa - \frac{\kappa_1}{9N_c}; \quad (8)$$

Here N_c is the number of colors and $\kappa_1 = g_1^2 \cot^2 \theta' / 4\pi$. (It should be mentioned that our analyses are performed in the context of a large- N_c approximation). This leads to “tilted” gap equations in which the top quark acquires a constituent mass, while the b quark remains massless. Given that both κ and κ_1 are large there is no particular fine-tuning occurring here, only “rough-tuning” of the desired tilted configuration. Of course, the NJL approximation is crude, but as long as the associated phase transitions of the real strongly coupled theory are approximately second order, analogous rough-tuning in the full theory is possible. The full phase diagram of the model is shown in Fig. 1. of [5].

TOPCOLOR II

If the above described “Topcolor I” is the analog of Weinberg’s original version of the SM, incorporating standard fermions and the Z -boson, then Topcolor II is the analog of the original Georgi-Glashow model, which incorporated no new Z boson, but rather included additional fermions. [This is an example of “Weinberg—Georgi-Glashow” duality.] The strong $U(1)$ is present in the previous scheme to avoid a degenerate $\langle \bar{t}t \rangle$ with $\langle \bar{b}b \rangle$. However, we can give a model in which there is: (i) a Topcolor $SU(3)$ group but (ii) no strong $U(1)$ with (iii) an anomaly-free representation content. In fact the original model of [2] was of this form, introducing a new quark of charge $-1/3$. Let us consider a generalization of this scheme which consists of the gauge structure $SU(3)_Q \times SU(3)_1 \times SU(3)_2 \times U(1)_Y \times SU(2)_L$. We require an additional triplet of fermions fields (Q_R^a) transforming as $(3, 3, 1)$ and Q_L^a transforming as $(3, 1, 3)$ under the $SU(3)_Q \times SU(3)_1 \times SU(3)_2$.

The fermions are then assigned the following quantum numbers in $SU(2) \times SU(3)_Q \times SU(3)_1 \times SU(3)_2 \times U(1)_Y$:

$$\begin{aligned}
 (t, b)_L \quad (c, s)_L &\sim (2, 1, 3, 1) & Y = 1/3 & \quad (9) \\
 (t)_R &\sim (1, 1, 3, 1) & Y = 4/3; & \\
 (Q)_R &\sim (1, 3, 3, 1) & Y = 0 & \\
 \\
 (u, d)_L &\sim (2, 1, 1, 3) & Y = 1/3 & \\
 (u, d)_R \quad (c, s)_R &\sim (1, 1, 1, 3) & Y = (4/3, -2/3) & \\
 (\nu, \ell)_L \quad \ell = e, \mu, \tau &\sim (2, 1, 1, 1) & Y = -1; & \\
 (\ell)_R &\sim (1, 1, 1, 1) & Y = -2 & \\
 b_R &\sim (1, 1, 1, 3) & Y = 2/3; & \\
 (Q)_L &\sim (1, 3, 1, 3) & Y = 0; &
 \end{aligned}$$

Thus, the Q fields are electrically neutral. One can verify that this assignment is anomaly free.

The $SU(3)_Q$ confines and forms a $\langle \bar{Q}Q \rangle$ condensate which acts like the Φ field and breaks the Topcolor group down to QCD dynamically. We assume that Q is then decoupled from the low energy spectrum by its large constituent mass. There is a lone $U(1)$ Nambu–Goldstone boson $\sim \bar{Q}\gamma^5 Q$ which acquires a large mass by $SU(3)_Q$ instantons.

TRIANGULAR TEXTURES

The texture of the fermion mass matrices will generally be controlled by the symmetry breaking pattern of a horizontal symmetry. In the present case

we are specifying a residual Topcolor symmetry, presumably subsequent to some initial breaking at some scale Λ , large compared to Topcolor, e.g., the third generation fermions in Model I have different Topcolor assignments than do the second and first generation fermions. Thus the texture will depend in some way upon the breaking of Topcolor [5] [3].

Let us study a fundamental Higgs boson, which ultimately breaks $SU(2)_L \times U(1)_Y$, together with an effective field Φ breaking Topcolor as in eq.(4). We must now specify the full Topcolor charges of these fields. As an example, under $SU(3)_1 \times SU(3)_2 \times U(1)_{Y1} \times U(1)_{Y2} \times SU(2)_L$ let us choose:

$$\Phi \sim (3, \bar{3}, \frac{1}{3}, -\frac{1}{3}, 0) \quad H \sim (1, 1, 0, -1, \frac{1}{2}) \quad (10)$$

The effective couplings to fermions that generate mass terms in the up sector are of the form

$$\begin{aligned} \mathcal{L}_{\mathcal{M}U} = & m_0 \bar{t}_L t_R + c_{33} \bar{T}_L t_R H \frac{\det \Phi^\dagger}{\Lambda^3} + c_{32} \bar{T}_L c_R H \frac{\Phi}{\Lambda} + c_{31} \bar{T}_L u_R H \frac{\Phi}{\Lambda} \\ & + c_{23} \bar{C}_L t_R H \Phi^\dagger \frac{\det \Phi^\dagger}{\Lambda^4} + c_{22} \bar{C}_L c_R H + c_{21} \bar{C}_L u_R H \\ & + c_{13} \bar{F}_L t_R H \Phi^\dagger \frac{\det \Phi^\dagger}{\Lambda^4} + c_{12} \bar{F}_L c_R H + c_{11} \bar{F}_L u_R H + \text{h.c.} \end{aligned} \quad (11)$$

Here $T = (t, b)$, $C = (c, s)$ and $F = (u, d)$. The mass m_0 is the dynamical condensate top mass. Furthermore $\det \Phi$ is defined by

$$\det \Phi \equiv \frac{1}{6} \epsilon_{ijk} \epsilon_{lmn} \Phi_{il} \Phi_{jm} \Phi_{kn} \quad (12)$$

where in Φ_{rs} the first(second) index refers to $SU(3)_1$ ($SU(3)_2$). The matrix elements now require factors of Φ to connect the third with the first or second generation color indices. The down quark and lepton mass matrices are generated by couplings analogous to (11).

To see what kinds of textures can arise naturally, let us assume that the ratio Φ/Λ is small, $O(\epsilon)$. The field H acquires a VEV of v . Then the resulting mass matrix is approximately triangular:

$$\begin{pmatrix} c_{11}v & c_{12}v & \sim 0 \\ c_{21}v & c_{22}v & \sim 0 \\ c_{31}O(\epsilon)v & c_{32}O(\epsilon)v & \sim m_0 + O(\epsilon^3)v \end{pmatrix} \quad (13)$$

where we have kept only terms of $O(\epsilon)$ or larger.

This is a triangular matrix (up to the c_{12} term). When it is written in the form $U_L \mathcal{D} U_R^\dagger$ with U_L and U_R unitary and \mathcal{D} positive diagonal, there automatically result restrictions on U_L and U_R . In the present case, the elements $U_L^{3,i}$ and $U_L^{i,3}$ are vanishing for $i \neq 3$, while the elements of U_R are not constrained

by triangularity. Analogously, in the down quark sector $D_L^{i,3} = D_L^{3,i} = 0$ for $i \neq 3$ with D_R unrestricted. The situation is reversed when the opposite corner elements are small, which can be achieved by choosing $H \sim (1, 1, -1, 0, \frac{1}{2})$.

These restrictions on the quark mass rotation matrices have important phenomenological consequences. For instance, in the process $B^0 \rightarrow \bar{B}^0$ there are potentially large contributions from top-pion and coloron exchange. However, these contributions are proportional to the product $D_L^{3,2} D_R^{3,2}$. The same occurs in $D^0 - \bar{D}^0$ mixing, where the effect goes as products involving U_L and U_R off-diagonal elements. Therefore, triangularity can naturally select these products to be small.

The precise selection rules depend upon the particular symmetry breaking that occurs. This example is merely illustrative of the systematic effects that can occur in such schemes.

TOP-PIONS; INSTANTONS; THE B-QUARK MASS.

Since the top condensation is a spectator to the TC (or Higgs) driven ESB, there must occur a multiplet of top-pions. A chiral Lagrangian can be written:

$$L = i\bar{\psi}\not{\partial}\psi - m_t(\bar{\psi}_L\Sigma P\psi_R + h.c.) - \epsilon m_t\bar{\psi}P\psi, \quad P = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \quad (14)$$

and $\psi = (t, b)$, and $\Sigma = \exp(i\tilde{\pi}^a\tau^a/\sqrt{2}f_\pi)$. With $\epsilon = 0$ this is invariant under $\psi_L \rightarrow e^{i\theta^a\tau^a/2}\psi_L$, $\tilde{\pi}^a \rightarrow \tilde{\pi}^a + \theta^a f_\pi/\sqrt{2}$. Hence, the relevant currents are left-handed, $j_\mu^a = \psi_L\gamma_\mu\frac{\tau^a}{2}\psi_L$, and $\langle \tilde{\pi}^a | j_\mu^b | 0 \rangle = \frac{f_\pi}{\sqrt{2}}p_\mu\delta^{ab}$. The Pagels-Stokar relation, eq.(1), then follows by demanding that the $\tilde{\pi}^a$ kinetic term is generated by integrating out the fermions. The top-pion decay constant estimated from eq.(1) using $\Lambda = M_B$ and $m_t = 175$ GeV is $f_\pi \approx 50$ GeV. The couplings of the top-pions take the form:

$$- \frac{m_t}{\sqrt{2}f_\pi} \left[i\bar{t}\gamma^5 t\tilde{\pi}^0 + \frac{i}{\sqrt{2}}\bar{t}(1 - \gamma^5)b\tilde{\pi}^+ + \frac{i}{\sqrt{2}}\bar{b}(1 + \gamma^5)t\tilde{\pi}^- \right] \quad (15)$$

and the coupling strength is governed by the relation $g_{bt\tilde{\pi}} \approx m_t/\sqrt{2}f_\pi$.

The small ETC mass component of the top quark implies that the masses of the top-pions will depend upon ϵ and Λ . Estimating the induced top-pion mass from the fermion loop yields:

$$m_{\tilde{\pi}}^2 = \frac{N\epsilon m_t^2 M_B^2}{8\pi^2 f_\pi^2} = \frac{\epsilon M_B^2}{\log(M_B/m_t)} \quad (16)$$

where the Pagels-Stokar formula is used for f_π^2 (with $k = 0$) in the last expression. For $\epsilon = (0.03, 0.1)$, $M_B \approx (1.5, 1.0)$ TeV, and $m_t = 180$ GeV this predicts $m_{\tilde{\pi}} = (180, 240)$ GeV. The bare value of ϵ generated at the ETC

scale Λ_{ETC} , however, is subject to very large radiative enhancements by Topcolor and $U(1)_{Y_1}$ by factors of order $(\Lambda_{ETC}/M_B)^p \sim 10^1$, where the $p \sim O(1)$. Thus, we expect that even a bare value of $\epsilon_0 \sim 0.005$ can produce sizeable $m_{\tilde{\pi}} > m_t$. Note that $\tilde{\pi}$ will generally receive gauge contributions to its mass; these are at most electroweak in strength, and therefore of order ~ 10 GeV.

Top-pions can be as light as ~ 150 GeV, in which case they would emerge as a detectable branching fraction of top decay [6]. However, there are dangerous effects in $Z \rightarrow b\bar{b}$ with low mass top pions and decay constants as small as ~ 60 GeV [8]. A more comfortable phenomenological range is slightly larger than our estimates, $m_{\tilde{\pi}} \gtrsim 300$ GeV and $f_{\tilde{\pi}} \gtrsim 100$ GeV.

The b quark receives mass contributions from ETC of $O(1)$ GeV, but also an induced mass from instantons in $SU(3)_1$. The instanton effective Lagrangian may be approximated by the 't Hooft flavor determinant (we place the cut-off at M_B):

$$L_{eff} = \frac{k}{M_B^2} e^{i\theta_1} \det(\bar{q}_L q_R) + h.c. = \frac{k}{M_B^2} e^{i\theta_1} [(\bar{b}_L b_R)(\bar{t}_L t_R) - (\bar{t}_L b_R)(\bar{b}_L t_R)] + h.c. \quad (17)$$

where θ_1 is the $SU(3)_1$ strong CP -violation phase. θ_1 cannot be eliminated because of the ETC contribution to the t and b masses. It can lead to induced scalar couplings of the neutral top-pion [5], and an induced CKM CP -phase, however, we will presently neglect the effects of θ_1 .

We generally expect $k \sim 1$ to 10^{-1} as in QCD. Bosonizing in fermion bubble approximation $\bar{q}_L t_R \sim \frac{N}{8\pi^2} m_t M_B^2 \Sigma_1^i$, where $\Sigma_j^i = \exp(i\tilde{\pi}^a \tau^a / \sqrt{2} f_{\tilde{\pi}})_j^i$ yields:

$$L_{eff} \rightarrow \frac{N k m_t}{8\pi^2} e^{i\theta_1} [(\bar{b}_L b_R) \Sigma_1^1 + (\bar{t}_L b_R) \Sigma_1^2 + h.c.] \quad (18)$$

This implies an instanton induced b -quark mass:

$$m_b^* \approx \frac{3 k m_t}{8\pi^2} \sim 6.6 k \text{ GeV} \quad (19)$$

This is not an unreasonable estimate of the observed b quark mass, as we might have feared it would be too large.

TOP SEE-SAW

EWSB may occur via the condensation of the top quark in the presence of an extra vectorlike, weak-isoscalar quark [7]. The mass scale of the condensate is large, of order 0.6 TeV corresponding to the electroweak scale $f_{\tilde{\pi}} \approx 175$ GeV. The vectorlike iso-scalar then naturally admits a seesaw mechanism, yielding the physical top quark mass, which is then adjusted to the experimental value. The choice of a natural \sim TeV scale for the topcolor dynamics

then determines the mass of the weak-isoscalar see-saw partner. The scheme is economical, requiring no additional weak-isodoublets, and therefore easily satisfies the constraints upon the S parameter using estimates made in the large- N approximation. The constraints on custodial symmetry violation, i.e., the value of the $\delta\rho$ or equivalently, T parameter, are easily satisfied, being principally the usual m_t contribution, plus corrections that are suppressed by the see-saw mechanism.

The dynamical fermion masses that are induced can be written as:

$$\mathcal{L} = -(\bar{t}_L, \bar{\chi}_L) \begin{pmatrix} 0 & m_{t\chi} \\ m_{\chi t} & m_{\chi\chi} \end{pmatrix} \begin{pmatrix} t_R \\ \chi_R \end{pmatrix} + \text{h.c.} \quad (20)$$

Typically $\bar{\chi}_L\chi_R$ is the most attractive channel, and it is possible to arrange the $\langle\bar{\chi}_L\chi_R\rangle$ condensate to be significantly larger than the other ones, such that $m_{\chi\chi}^2 \gg m_{\chi t}^2 > m_{t\chi}^2$. As a result the physical top mass is suppressed by a seesaw mechanism:

$$m_t \approx \frac{m_{\chi t}m_{t\chi}}{m_{\chi\chi}} \left[1 + O\left(m_{\chi t,t\chi}^2/m_{\chi\chi}^2\right) \right]. \quad (21)$$

The electroweak symmetry is broken by the $m_{t\chi}$ dynamical mass. Therefore, the electroweak scale is estimated to be given by

$$v^2 \approx \frac{3}{16\pi^2} m_{t\chi}^2 \ln\left(\frac{M}{m_{t\chi}}\right). \quad (22)$$

Thus, $v \approx 174$ GeV requires a dynamical mass $m_{t\chi} \sim 620$ GeV for $M \sim 5$ TeV (and $m_{t\chi} = 520$ GeV for $M \sim 10$ TeV). From eq. (21) follows then that a top mass of 175 GeV requires $m_{\chi t}/m_{\chi\chi} \approx 0.29$. The electroweak T parameter can be estimated in fermion-bubble large- N approximation as:

$$T \approx \frac{3m_t^2}{16\pi^2\alpha(M_Z^2)v^2} \frac{m_{t\chi}^2}{m_{\chi t}^2} \left[1 + O\left(m_{\chi t,t\chi}^2/m_{\chi\chi}^2\right) \right], \quad (23)$$

where α is the fine structure constant. Moreover, we obtain the usual Standard Model result for the S parameter. Requiring that our model does not exceed the 1σ upper bound on S and T , we obtain $m_{t\chi}/m_{\chi t} \leq 0.55$.

It should be emphasized that these results do not require excessive fine-tuning. The top-seesaw is therefore a plausible natural theory of dynamical EWSB with a minimal number of new degrees of freedom. This model also implies the existence of pseudo-Nambu-Goldstone bosons (pNGB's). A cursory discussion of that is given in ref.[7].

OBSERVABLES

There are several classes of possible experimental implications of the kinds of models we described above that may be relevant to the muon collider. We

will describe them here briefly as lines to be developed further. These may be enumerated as follows:

1. $\mu\bar{\mu} \rightarrow Z'$; this is the province of high energy machine, since we expect $M_{Z'} \gtrsim 0.5$ TeV.
2. $\mu\bar{\mu} \rightarrow \pi_{top}$; the notion that the muon collider can see technipions, or other PNGB's, such as top-pions has emerged from discussions in this workshop, prompted by MacKenzie and myself. Lane has presented the multi-scale technicolor signal [4].
3. Effects in Z physics involving the third generation, such as $Z \rightarrow b\bar{b}$ [8].
4. Effects in top-quark pair production at threshold, e.g., see [11] for analogous case in e^+e^- and $p\bar{p}$ collider physics.
5. Induced GIM violation in low energy processes such as $K^+ \rightarrow \pi^+\nu\bar{\nu}$; we discuss this below as an example of a potential signature that can be enhanced by Topcolor wrt the Standard Model (this result was anticipated in ref[5] before the observation of the single event at Brookhaven E787).
6. Induced lepton family number violation, e.g. $\mu\bar{\mu} \rightarrow \tau\bar{\mu}$.
7. Flavor dependent production effects, e.g. anomalous $\mu\bar{\mu} \rightarrow b\bar{s}$, etc.
8. New physics in e.g. μp collisions, such as $d(u) + \bar{\mu} \rightarrow b(t) + \bar{\tau}$.

GIM and lepton family number violation arise because of the generational structure of topcolor. (It is actually more general than topcolor; the mere statement that the top mass is largely dynamical implies effects like this) In going to the mass eigenbasis, quark (and lepton) fields are rotated, e.g., by the matrices U_L, U_R (for the up-type left and right handed quarks) and D_L, D_R (for the down-type left and right handed quarks). For example, for the b-quark we make the replacement

$$b_L \rightarrow D_L^{bb} b_L + D_L^{bs} s_L + D_L^{bd} d_L \quad (24)$$

and analogously for b_R . Thus there will be induced FCNC interactions. This provides constraints and opportunities. Thus, induced effects like $\mu\bar{\mu} \rightarrow b\bar{s}$ may be enhanced, and effects like $\mu\bar{\mu} \rightarrow \tau\bar{\mu}$ may occur. Since the muon is presumably closer in affiliation to the third generation than is the electron, such effects may show up in muon collider physics, but be inaccessible in electron linear colliders! Similarly, induced effects like $\mu\bar{\mu} \rightarrow b\bar{s}$ may be enhanced.

For the FMC, sensitive probes arise in e.g., K-physics. there is a Z' induced contact term at low energies of the form $b\bar{b}\nu_\tau\bar{\nu}_\tau$ (this assumes that the τ is associated with the third generation; nothing fundamentally compels this, but we shall assume it to be true in the following). The above mass rotation

induces a $s\bar{d}\nu_\tau\bar{\nu}_\tau$ which contributes to $K^+ \rightarrow \pi^+\nu\bar{\nu}$. The ratio of the Topcolor amplitude to the SM is then

$$\frac{\mathcal{A}^{TC}}{\mathcal{A}^{SM}} = - \left(\frac{g_1 \cot \theta'}{M_{Z'}} \right)^2 \frac{\sqrt{2}\pi \sin^2 \theta_W}{24\alpha G_F} \frac{\delta_{ds}}{\sum_j V_{js}^* V_{jd} D_j(x_j)} \sim -3 \times 10^9 \delta_{ds} \frac{\kappa_1}{M_{Z'}^2} \quad (25)$$

where $\delta_{ds}^* = D_L^{bs} D_L^{bd*} - 2D_R^{bs} D_R^{bd*}$. The form-factor $f_+(q^2)$ is experimentally well known. We expect, $|\delta_{ds}| \sim \lambda^{10}$ where λ is the Wolfenstein CKM parameter. For $M_{Z'} = 500$ GeV and $\kappa_1 = 1$ the ratio of amplitudes is about ~ 4.0 , and the branching ratio is between 0.3 to $O(10)$, times the SM result, depending on the sign of the interference. The recent observation of one event by the Brookhaven E787 Collaboration [10] makes this an exciting channel in which to search for new physics. High sensitivity experiments are possible at the front-end muon collider with its copious K-meson yields.

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