Fermi National Accelerator Laboratory

FERMILAB-Conf-98/029

Beam-Beam Interactions

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January 1998

Published Proceedings of the *International Symposium on Near Beam Physics*,
Fermilab, Batavia, Illinois, September 22-24, 1997

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

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Abstract

This paper gives a brief review of the beam-beam interactions of the hadron beams. Some recent results on the Pacman effect and dynamic aperture studies are also included.

1 Introduction

No matter how "perfectly" a collider could be built (*e.g.,* good vacuum, small magnet errors, little non-linearity and low coupling impedance, *etc.*), beam-beam interactions will be the ultimate limit of its performance. These interactions will cause particle losses, emittance growth, tune shifts, orbit displacements, beam instabilities, non-linear resonances and will limit the dynamic aperture and the beam current and beam lifetime. Because interactions of hadron beams are quite different from that of lepton beams, we will content ourselves with the study of hadron beams in this paper.

There have been extensive machine studies on beam-beam interactions at the Tevatron at Fermilab and the S$p\bar{p}$S at CERN. There were also intensive theoretical and computational beam-beam studies at the former SSC and for the future LHC. [1, 2, 3] We will briefly review these results. We will also discuss some new results recently obtained from the LHC work, mainly on the Pacman effect and dynamic aperture.

2 Strong beam-beam interactions

2.1 Inelastic scattering

This is what a collider is built for. This process generates the events that detectors will record and the experimentalists will analyze. It also results in particle losses. The loss rate is:

$$\frac{dN}{dt} = \mathcal{L}\sigma_{\rm inel} \tag{1}$$

which gives the beam lifetime due to luminosity. Take the LHC as an example. The total number of particles per beam is 2.8×10^{14}, the luminosity $\mathcal{L}$ is 10^{34} cm^{-2}s^{-1}, the inelastic cross section $\sigma_{\rm inel}$ is about 60 mb, and there are two high luminosity interaction points (IPs). These numbers give a beam lifetime of about 65 hours.

2.2 Elastic scattering

The proton-proton elastic scattering contributes to the emittance growth. The growth rate is given by (per IP):

$$\frac{d\epsilon}{dt} = \frac{N_{\rm B} f_0}{4\pi\epsilon}\sigma_{\rm el}\sigma_\alpha^2 \tag{2}$$

The meaning of the symbols can be found in the Glossary. The RMS value of pp elastic scattering angle in the center of mass system, σ_α, is

$$\sigma_\alpha = \frac{hc}{E_{\rm c.m.}\sqrt{2\pi\sigma_{\rm T}}} \tag{3}$$

In the LHC, for colliding beams with $E_{\rm c.m.} = 14$ TeV and $\sigma_{\rm T} \approx 100$ mb, one finds $\sigma_\alpha = 11$ μrad. Using $N_{\rm B} = 1 \times 10^{11}$, $f_0 = 11.2$ kHz, $\epsilon = 5 \times 10^{-10}$ m-rad, $\sigma_{\rm el} = 40$ mb, one gets a growth rate of about 1×10^{-16} m-rad/s per IP.

3 Electromagnetic beam-beam interactions

There are two types of interactions: head-on and long range (which is also called parasitic crossings). The characteristic quantity of these interactions is the beam-beam parameter ξ. It is sometimes also called the Amman-Ritson parameter to honor the two physicists who first investigated it in 1960. Consider two counter-circulating round bunches. At small amplitude, the opposing bunch looks like a lens with the strength:

$$f = \frac{N_B r_p}{\gamma \sigma_x^2} = \frac{N_B r_p}{\gamma \epsilon \beta^*} \tag{4}$$

The tune shift per IP is:

$$\xi = \frac{\beta^* f}{4\pi} = \frac{\beta^* N_B r_p}{4\pi\gamma\epsilon\beta^*} = \frac{r_p}{4\pi} \cdot \frac{N_B}{\epsilon_N} \tag{5}$$

Note that this parameter is independent of the beam energy and the beta-function and, apart from a constant, is equivalent to the beam brightness N_B/ϵ_N. This perhaps surprisingly simple result makes this parameter very useful. It is one of the basic parameters in the design of any collider. (Note that the brightness is also limited by the space charge effect in the first circular accelerator in the injector chain.) The design value of ξ is 0.0034 for the LHC and 0.0009 for the SSC.

3.1 Tune shift and tune spread

The most significant beam-beam effect observed at the Tevatron and S$p\bar{p}$S is the slow diffusion, which is believed to be caused by high order betatron resonances. It leads to particle losses that in turn decrease the beam lifetime and create background in detectors. The head-on tune shift per IP (which is also the tune spread) is:

$$\Delta\nu_{\rm HO} = \xi \left(\frac{2R_{\rm re}^2}{1 + R_{\rm re}} \right) \tag{6}$$

where $R_{\rm re}$ is the luminosity reduction factor due to the crossing angle and equals:

$$R_{\rm re} = \left(1 + \left(\frac{\theta\sigma_s}{2\sigma_x} \right)^2 \right)^{-1/2} \tag{7}$$

For long range interactions, the tune shift per IP is:

$$\Delta\nu_{\rm LR} = \xi \cdot \frac{N_p}{n^2} \tag{8}$$

where n is the full crossing angle in units of $\sigma_{x'}$, N_p is the number of parasitic crossings and equals:

$$N_p = \frac{4L^*}{S_B} \tag{9}$$

The long range tune spread per IP is:

$$\delta\nu_{\mathrm{LR}} = \xi \cdot \frac{3N_p a^2}{2n^4} \tag{10}$$

where a is the betatron oscillation amplitude in units of σ_x. It is seen that long range interactions are more complicated and are dependent upon many parameters, in particular, on the crossing angle n. As a matter of fact, the introduction of a crossing angle is mainly for the purpose of reducing long range beam-beam effects.

In order to control the slow diffusion, it is required to keep the total tune spread (head-on + long range + non-linear magnetic field effects) within a "tune budget," which is usually about 0.02. The working point is so chosen such that all the resonances below the 10th order can be avoided when the total tune spread is kept within this budget. There are several such regions on the tune diagram near the diagonal that one can choose from. It is interesting that different machines seem to have different preferences. For example, the Tevatron chooses a tune near 0.415, the former SSC near 0.285, and the S$p\bar{p}$S near 0.31 (which is also likely to be the choice for the LHC).

The linear tune shift can be compensated by retuning the quadrupoles. Alternate crossing planes at 90° relative to each other (*e.g.,* alternate horizontal and vertical crossings, or 45° tilted crossing planes) can also effectively cancel the tune spread. But the Pacman effect makes it difficult, see Section 3.4 below.

3.2 Orbit distortion

Long range interactions will also cause orbit distortion, which is given by:

$$\Delta x = \frac{8\pi\xi N_p}{n} \tag{11}$$

Therefore, fine steering is desired near the IP's for orbit corrections. But again, the Pacman effect further complicates the corrections (see 3.4).

3.3 Coherent effects

Both head-on and long range interactions can produce coherent beam-beam effects. The rigid dipole modes (π-mode and σ-mode) and higher order multipole modes can be studied by theoretical modelling and by computer simulations. The results are usually expressed in terms of the stability boundary in the (ξ, ν_β) space for checking if there would be enough room for the working area during normal machine operations.

3.4 Pacman effects

In a collider, the bunch train contains several injection gaps and an abort gap. Bunches that in the interaction regions are circulating past gaps of missing bunches in the counter-circulating beam are called the Pacman bunches. Such bunches will suffer anomalous tune shifts and orbit displacements different from the "average" bunches circulating relative to a locally fully filled beam. Therefore if the machine is optimized for average bunches the Pacman bunches will not be in an optimized environment and may suffer enhanced losses. However, loss of a Pacman bunch will create new Pacman bunches in the counter-circulating beam, and over the course of time holes will develop in both beams and eventually the beams may be destroyed. When the IPs are symmetrically placed with separations of half the ring circumference, a circulating bunch encounters the identical pattern of counter-circulating bunches at each IP. For this special case the Pacman effects at the paired IPs are related and the IPs can be configured to cancel or minimize the Pacman anomalies. Irrespective of the phase advance between the IPs, the anomalous tune shift is cancellable by crossing planes at 90° relative to each other at the two IPs. However, the anomalous orbit shifts can at best be minimized by a "best" choice of phase separations between the IPs, namely, separated in phase by half the phase advance around the ring. Ref. [4] shows that the orbit distortion at the two IPs, A and B, is:

$$|\Delta x_A| \text{ and/or } |\Delta x_B| \geq \frac{1}{2}\Delta x \tag{12}$$

For the symmetric case one has:

$$|\Delta x_A| = |\Delta x_B| = \frac{1}{2}\Delta x \tag{13}$$

Thus the symmetric case represents the optimum configuration.

At the LHC using a β^* of 50 cm, an emittance of 5×10^{-8} cm-rad, a θ of 200 μrad, a $\Delta\nu_{\mathrm{HO}}$ of 0.0034 per IP, and N_p equal to 9 (for the so-called run away Pacman effect), the orbit displacement in the symmetric case is 0.06 σ_x or 1 μm for a beam with a σ_x of 16 μm. Such an orbit displacement is very small and will contribute minimally to instability.

3.5 Dynamic aperture

The dynamic aperture during collisions is mainly determined by the beam-beam interactions as well as by the multipole errors of the low-β quads in the interaction regions. Among other factors, it has a strong dependence on the crossing angle. On the one hand, larger crossing means less long range beam-beam interactions. Thus, the dynamic aperture limited by beam-beam would become bigger. On the other hand, however, the dynamic aperture limited by the low-β quads would be smaller because of poor field qualities when beams move further away from the magnet axis. Therefore, when the crossing angle increases, the dynamic aperture would at first increase (which is the beam-beam dominated region); after reaching a maximum value, it would decrease (which is the field error dominated region). Numerical studies by long term tracking for the LHC have confirmed this prediction. [5]

This study is important because it plays a big role in the requirement of the low-β quad aperture. If the aperture is too small, one will not be able to open up the crossing angle to the preferred size. As a consequence, the dynamic aperture could be severely limited by the beam-beam effects. Use the LHC as an example. Its low-β quad aperture is 70 mm. The design value of the crossing angle is 200 μrad. [3] But tracking studies show that, in order to have a dynamic aperture of 7-8 σ_x, the crossing angle needs to be increased to about 300 μrad. [5] This lead to a new space budget of the quad aperture and a re-design of the shielding inside the quads for making a larger crossing possible.

Table 1. Comparison of Machine Parameters

Machine	DORIS I	HERA	SSC	LHC
ξ	0.01	0.0006	0.0009	0.0034
ν_s	0.03	0.01	0.0012	0.0021
$\theta\sigma_s/\sigma_x$	0.7	4	0.45	0.48

3.6 Synchro-betatron resonance

The crossing angle may excite synchro-betatron resonances. There are three key parameters that will determine the strength of these resonances, namely, the beam-beam parameter ξ, the synchrotron tune ν_s, and the normalized crossing angle $\theta\sigma_s/\sigma_x$. Table 1 is a comparison of these parameters in four machines: the DORIS I, the HERA, the SSC and the LHC (of which $\theta = 200$ μrad is used). The synchro-betatron resonance was a major concern of the two DESY machines. However, it is seen from the table that this effect should not be as critical in the SSC or the LHC. For example, based on Piwinski's theory [6], simulations were done for the SSC and showed that, with $\theta = 150$ μrad, only the satellites of the resonances up to the order of six could be harmful to the beams. Between these resonances there was enough space for the working area. [7]

4 Discussions

The strong beam-beam interactions give rise to particle losses and emittance growth. These interactions and other effects (*e.g.,* intrabeam scattering, synchrotron radiation, residual gas scattering, beam collimation and external excitations) lead to the evolution of machine luminosity, which can readily be calculated. [8]

The electromagnetic beam-beam interactions have been studied in the past four decades. One has achieved relatively good understanding of the effects on the tune shift, orbit distortion, dynamic aperture and synchro-betatron resonances by means of the weak-strong or weak-weak model. However, less successful is the strong-strong model, which is more complicated and is a real challenge in the investigations. Because it is one of the main causes of the formation of the beam halo, it certainly deserves more attention in the future study of the near beam physics.

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Glossary

N	Total number of particles in a beam
t	Time
$\mathcal{L}$	Luminosity
$\sigma_{\rm inel}$	Inelastic cross section
$\sigma_{\rm el}$	Elastic cross section
$\sigma_{\rm T}$	Total cross section
ϵ	RMS transverse emittance
$N_{\rm B}$	Number of protons per bunch
f_0	Revolution frequency
σ_α	RMS value of pp elastic scattering angle in the center of mass system
h	Planck's constant
c	Speed of light
$E_{\rm c.m.}$	Energy in the center of mass system
$\epsilon_{\rm N}$	Normalized RMS transverse emittance
$r_{\rm p}$	Classical proton radius
γ	Relativistic factor
β^*	β-function at the interaction point
ξ	Beam-beam parameter (Amman-Ritson parameter)
θ	Full crossing angle in unit radian
n	Full crossing angle in units of $\sigma_{x'}$
a	Betatron oscillation amplitude in units of σ_x
$R_{\rm re}$	Luminosity reduction factor due to the crossing angle
L^*	Effective interaction distance (on one side of the IP)
$S_{\rm B}$	Bunch spacing
ν_β	Betatron tune
ν_s	Synchrotron tune
σ_x	RMS beam transverse spatial size
$\sigma_{x'}$	RMS beam transverse angular size
σ_s	RMS bunch length
$\Delta\nu_{\rm HO}$	Head-on beam-beam tune shift
$\Delta\nu_{\rm LR}$	Long range beam-beam tune shift
$\delta\nu_{\rm LR}$	Long range beam-beam tune spread
N_p	Number of parasitic crossings
Δx	Orbit distortion in units of σ_x