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I. GENERAL THEORY

For a beam pipe of perfectly rectangular cross section (no rounded corner), the longitudinal wall resistive impedance for a length L at frequency $\omega/(2\pi)$ is [1]

$$Z_{\parallel} = [1 - i \operatorname{sgn}(\omega)] \frac{\rho L}{\pi \delta h} F\left(\frac{w}{h}\right), \quad (1)$$

where h is the height and w the width of the cross section, ρ is the wall resistivity, and the form factor $F(\frac{w}{h})$ depends only on the ratio of width to height. For a squared cross section where $w = h$, $F = 1$ up to a few significant figures. In the above, the skin depth at angular frequency ω is given by

$$\delta = \sqrt{\frac{2\rho}{|\omega|\mu}} = \sqrt{\frac{2\rho}{|\omega|Z_0}}, \quad (2)$$

where the magnetic permeability of the wall will be taken as $\mu = 1$ and $Z_0 \approx 377$ Ohms is the impedance of free space.

For a beam pipe of circular cross section of radius b , the expression for longitudinal wall resistive impedance is exactly the same as Eq. (1), with the form factor $F = 1$ and $h = 2b$. Therefore, we can carry out the analysis with Eq. (1) and make the correct substitution of h or b at the end, depending on whether the cross section is square or circular.

The power loss to the wall by a bunch of linear charge distribution $\lambda(z)$ is given by

$$P = \left(\frac{\omega_0}{2\pi}\right)^2 \sum_{n=-\infty}^{\infty} |\tilde{\lambda}(n\omega_0)|^2 \operatorname{Re} Z_{\parallel}(n\omega_0), \quad (3)$$

where the summation is over all the revolution harmonic n , ω_0 is the revolution angular frequency, and $\tilde{\lambda}$ the Fourier spectrum of the bunch:

$$\tilde{\lambda}(\omega) = \int_{-\infty}^{\infty} \frac{dz}{c} \lambda(z) e^{-i\omega z/c}, \quad (4)$$

c being the velocity of light. Assuming a Gaussian distribution for the bunch containing N protons with rms length σ_z , we have

$$\tilde{\lambda}(\omega) = eN e^{-\frac{1}{2}(\omega\sigma_z/c)^2}. \quad (5)$$

Then the power loss for M bunches is given by

$$P = FM \left(\frac{eNc}{2\pi} \right)^2 \left(\frac{2}{\sigma_z^{3/2} h} \right) \left(\frac{\rho Z_0}{2} \right)^{1/2} \sum_{n=-\infty}^{\infty} n \left(\frac{\sigma_z}{R} \right)^{3/2} e^{-(n\sigma_z/R)^2}, \quad (6)$$

where R is the mean radius of the accelerator ring. The summation can be approximated by an integral

$$\sum \longrightarrow \int_0^{\infty} dx x^{1/4} e^{-x} = \Gamma\left(\frac{3}{4}\right) = 1.2254167. \quad (7)$$

For a Tevatron bunch of length $\sigma_z = 14$ cm, the step size of the integration, $\sigma_z/R = 1.4 \times 10^{-4}$, is small, and the approximation should be extremely accurate. In fact, numerical summation agrees with the integration approximation up to the 10th figure. Note that the summand has maximum contribution when $n \approx R/(2\sigma_z) = 3571$, and gives negligible contribution when n is small. Thus, we do not need to worry at all whether the thickness of the beam pipe is larger than the skin depth at low frequencies. The final formula for the power loss is therefore

$$P = FM \left(\frac{eNc}{2\pi} \right)^2 \left(\frac{2}{\sigma_z^{3/2} h} \right) \left(\frac{\rho Z_0}{2} \right)^{1/2} \Gamma\left(\frac{3}{4}\right). \quad (8)$$

It is interesting to point out that the power loss is independent of the circumference or revolution frequency of the accelerator ring, because this is the loss into the length of the beam-pipe wall which the beam sees in unit time.

TeV33 will be working with $M = 100$ bunches each of intensity $N = 2.7 \times 10^{11}$, and rms length $\sigma_z = 14$ cm. The beam pipe is of stainless steel with a resistivity of $\rho = 7.4 \times 10^{-7}$ Ohm-m. For a square beam pipe of side $h = 6$ cm throughout the ring, we obtain the power loss $P = 3.922$ kwatt.

II. POWER LOSS IN COLD SECTIONS

The Tevatron beam pipe in the Tevatron is mostly of a square cross section with rounded corners. Each side is of width $h = 6$ cm. The beam pipe in the quadrupoles is of circular cross section of radius $b = 3.485$ cm. According to the design report [2], the various types, number, and lengths of quadrupoles are listed in Table I. Although some

Table I: Quadrupole information in the Tevatron.

| | Length (m) | Number | Total Length (m) |
|---------------------------------------------|---------------|--------|---------------------|
| Standard quad | 1.679 | 180 | 302.209 |
| Long straight inner quad | 2.525 | 12 | 30.297 |
| Normal long straight short quad (48, 12) | 0.815 | 8 | 6.517 |
| Outer quad | 2.101 | 8 | 16.809 |
| High beta long straight short quad (48, 12) | 0.648 | 4 | 2.591 |
| Outer quad | 2.291 | 4 | 9.163 |
| Total | | | 367.586 |

quadrupoles in the Tevatron may have been different from the original design, especially those in the low-beta sections, the result of this analysis will not be affected by very much because, first, the width of the square beam pipe is not much different from the diameter of the circular beam pipe in the quadrupoles, and, second, the total length of quadrupoles only amounts to about 5.6% of the circumference of the whole ring.

We worry only about the heat load in the cold sections of the beam pipe. Table II lists all the warm sections in the Tevatron. Information concerning the low-beta sections is supplied by Gelfan [3] and the rest is taken from the design report [2]. We want to emphasize that usually there is a length of about 1 m for each transition from cold to warm sections, and these transition lengths have been included in the lengths of the warm sections tabulated.

The computation of the power loss is tabulated in Table III. The power loss in the cold sections is found to be 3.604 kw. Note that the $\rho = 7.4 \times 10^{-7}$ Ohm-m we used is the resistivity of stainless steel at room temperature. According to the SSC design report [4], this resistivity will drop to 5.0×10^{-7} Ohm-m at superconducting temperatures. If this is true, the power loss in the cold sections will be lowered to 2.96 kw.

Table II: The warm sections in the Tevatron.

| | Length (m) | Number | Total Length (m) |
|-------------------------------------------|---------------|--------|---------------------|
| Median location 17 | 14.539 | 6 | 87.234 |
| Normal beta median location 48 | 8.192 | 5 | 40.958 |
| High beta median location 48 | 8.136 | 1 | 8.136 |
| Normal beta doublet space 49 or 11 | 3.819 | 6 | 22.915 |
| High beta doublet space 49 or 11 | 3.852 | 2 | 7.704 |
| Normal or high beta long straight section | 53.194 | 4 | 212.776 |
| Low beta double space 49 or 11 | 11.959 | 4 | 47.836 |
| Low beta long straight section | 15.248 | 2 | 30.496 |
| Total | | | 458.053 |

Table III: Power loss in different sections using $\rho = 7.4 \times 10^{-7}$ Ohm-m.

| | Warm | Cold Square | Quadrupoles |
|------------------------------------------------|--------|-------------|-------------|
| Fraction of whole ring | 0.0729 | 0.8686 | 0.0585 |
| Power loss if pipe is all square or round (kw) | | 3.9221 | 3.3763 |
| Actual power loss (kw) | | 3.4067 | 0.1975 |
| Total power loss in cold sections | | | 3.6042 kw |

References

- [1] K.Y. Ng, Part. Accel. **16**, 63 (1984).
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- [3] N. Gelfan, private communication.
- [4] *Conceptual Design of the Superconducting Super Collider*, SSC-SR-2020, SSC Central Design Group, ed. J.D. Jackson, March, 1986, p. 169.