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S. Dodelson

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510*

C. Wiegert

*Department of Physics, The University of Chicago
Chicago, Illinois 60637-1433*

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Bending of Light by Vector Perturbations

S. Dodelson¹ and C. Wiegert²

¹ *NASA/Fermilab Astrophysics Center, P. O. Box 500, Batavia, Illinois 60510, USA*

² *Department of Physics, The University of Chicago, Chicago, IL 60637-1433, USA*

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A number of different mechanisms exist which can produce vector perturbations to the metric. One might think that such perturbations could deflect light rays from distant sources, producing observable effects. Indeed, this is expected to be the case for scalar perturbations. Here we show that the deflection from vector perturbations is very small, remaining constant over large distances, similar to the deflection due to tensor perturbations (gravity waves).

I. INTRODUCTION

Light travels along geodesics. As such its path is determined by the gravitational metric. When the metric deviates from a simple Minkowski form, light ceases to travel in straight lines. Were we to detect deviations from straight-line travel, we would be probing the very structure of the gravitational metric. In many cases, particularly in cosmology, knowledge of the metric is invaluable. For example, if we detected scalar perturbations to the metric, we would learn something about the source of these perturbations, the mass distribution. Since we are otherwise limited to information about the distribution of luminous objects such as stars and galaxies, direct information about the mass is tremendously important. There have been numerous successful efforts along these lines recently, among them maps of the distribution of mass in clusters of galaxies [1] and detections of massive compact objects in the halo of our galaxy [2]. Future efforts will likely reveal much about the mass distribution in the universe [3], offering independent estimates of quantities such as the power spectrum.

Over the years a number of groups [4,5] have explored the possibility of detecting tensor perturbations to the metric in this fashion. Among other reasons, tensor perturbations are interesting because they are produced during inflation [6]. Thus a direct probe of tensor perturbations in principle gives us information about inflation. However, the consensus is now that such perturbations are much harder to detect than are scalar perturbations. A scalar perturbation to the metric can coherently add to the displacement of light over very large distances. This is not true for tensor perturbations.

These studies naturally lead to the question of whether vector perturbations to the metric can be detected. This is more than an academic question: Recent studies have shown that defect theories of structure formation produce large vector perturbations to the metric [7,8]. Measuring light deflection via vector perturbations is one way then to search for elusive topological defects. Other, less speculative, ways of producing vector perturbations include magnetic fields which should excite vector modes.

Here we study the effect of vector perturbations on light propagation. In a random field, the deflection due to scalar perturbations grows as $D^{3/2}$ where D is the distance travelled. The deflection due to tensor modes was shown [5] to grow only logarithmically as D gets large. For vector modes, we find even less of an effect; the rms deflection is constant over large distances. Thus we expect light deflection to be an inefficient way to search for vector perturbations. Section II presents a handwaving summary of the Kaiser-Jaffe argument for why tensor modes do not produce deviations and extends this argument to vector perturbations. Section III makes this argument more rigorous.

II. LIGHT DEFLECTION IN THE PRESENCE OF PLANE WAVE METRIC PERTURBATION

We write the metric as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{2.1}$$

where the Minkowski metric $\eta_{\mu\nu} = (1, -1, -1, -1)$. We will use conventions in which Greek indices run over all four space-time coordinates and Latin indices are spatial; also we set $c = 1$.

Consider light travelling in the \hat{z} direction in the presence of a single plane wave $h_{\mu\nu}$ with wavevector

$$\vec{k} = k \left(\sqrt{1 - \mu^2}, 0, \mu \right). \quad (2.2)$$

Scalar perturbations are standing waves so $h_{\mu\nu} \propto e^{i\vec{k}\cdot\vec{x}}$, while tensor perturbations travel at the speed of light so $h_{\mu\nu} \propto e^{i(kz - \vec{k}\cdot\vec{x})}$. Kaiser and Jaffe showed that the displacements in the directions perpendicular to the direction of propagation (\hat{z}) obey the geodesic equations

$$\ddot{d}_{\text{Scalar}} \propto k e^{ikz\mu} \quad (2.3)$$

$$\ddot{d}_{\text{Tensor}} \propto k(1 - \mu)^{3/2} e^{ikz(1-\mu)}. \quad (2.4)$$

Here dots denote derivative with respect to position z and the perpendicular displacement is d . The change in the direction, or the displacement angle, after the photons have travelled a distance D is therefore

$$\dot{d}_{\text{Scalar}} \propto \frac{1}{\mu} [e^{ikD\mu} - 1] \quad (2.5)$$

$$\dot{d}_{\text{Tensor}} \propto (1 - \mu)^{1/2} [e^{ikD(1-\mu)} - 1]. \quad (2.6)$$

In the scalar case, \vec{k} -modes perpendicular to the \hat{z} direction in which light is travelling (i.e. those with $\mu = 0$) produce an abnormally large displacement angle. Expanding out the exponential, we find $\dot{d}_{\text{Scalar}} \sim kD$ as long as $\mu < 1/(kD)$. As the light travels further and further, its displacement angle gets larger and larger. This is physically reasonable in this simple case where the metric consists of only one plane wave. For, the photon can indeed get a large kick by simply travelling perpendicular to the direction in which the field is changing. It then experiences a constant force, getting a constant kick and corresponding boost in the displacement angle. For tensors the situation is completely different. In that case, for the light to see a constant field, it needs to travel along with the gravity wave. That is, to experience a coherent push, the photon needs to travel in the direction along which the field is changing, $\mu = 1$. Due to the $(1 - \mu)^{1/2}$ factor in front, though, the displacement when travelling in this direction is zero. So the typical displacement angle of light travelling in a tensor perturbation will be of order the field strength. It will *not* be enhanced by a factor of order kD as it travels a long distance. Tensor perturbations do not produce observable light deflections because distortions are suppressed when light travels in the resonant direction.

How does light behave in the presence of vector modes? Consider the following vector field

$$h_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -\sqrt{1 - \mu^2} & 0 \\ 0 & -\sqrt{1 - \mu^2} & 0 & -\mu \\ 0 & 0 & -\mu & 0 \end{pmatrix} e^{i\vec{k}\cdot\vec{x}}, \quad (2.7)$$

where \vec{k} is again defined as in 2.2. Borrowing from the next section, we write down the geodesic equation for light travelling in the presence of this metric:

$$\ddot{d}_{\text{Vector}} = -\frac{\partial h_{iz}}{\partial z} = \delta_{iy} ik\mu^2 e^{ikz\mu}. \quad (2.8)$$

Comparing 2.8 with 2.3, we see that the change in the deflection is suppressed by a relative factor μ^2 in the resonant direction. Thus, even if $kz\mu$ is small, the deflection is still small; there is no resonance. All modes contribute an equal (small) amount; there is nothing special about the $\mu = 0$ mode. There will be no accumulated displacement as light travels long distances.

III. DEFLECTION IN RANDOM VECTOR FIELD

We now make the argument of the previous section more rigorous, introducing a general vector perturbation as a random sum over plane waves, and solving the geodesic equation for a light ray.

In synchronous gauge, a general vector perturbation has only space-space components ($h^{0i} = h^{00} = 0$). If we Fourier transform h , the spatial components are

$$h_{ij}(\vec{x}) = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}} \left[h_1(\vec{k}) \left(\hat{m}_{1i}\hat{k}_j + \hat{k}_i\hat{m}_{1j} \right) + ih_2(\vec{k}) \left(\hat{m}_{2i}\hat{k}_j + \hat{k}_i\hat{m}_{2j} \right) \right] \quad (3.1)$$

where \hat{m}_1 and \hat{m}_2 are two unit vectors orthogonal to wave direction \hat{k} and to each other. The factor of i is inserted here to ensure that reality implies $h_a(\vec{k}) = h_a^*(-\vec{k})$ for $a = 1, 2$. With no loss of generality, we can choose the propagation direction of light to be in the \hat{z} direction. Then, write the three orthogonal vectors as

$$\hat{k} = \left(\sqrt{1-\mu^2} \cos \phi, \sqrt{1-\mu^2} \sin \phi, \mu \right) \quad (3.2)$$

$$\hat{m}_1 = (\sin \phi, -\cos \phi, 0) \quad (3.3)$$

$$\hat{m}_2 = \left(-\mu \cos \phi, -\mu \sin \phi, \sqrt{1-\mu^2} \right). \quad (3.4)$$

Note that we have assumed here that the vector field is time independent. We argue that this is a conservative assumption. In a cosmological setting, vectors fields decay over time; the only way they can be important is if they are continuously seeded and so remain relatively constant with time.

Consider a photon travelling through this field with direction $\hat{z} + \dot{\vec{d}}$, where the dot denotes $d/dt = \hat{n}^\mu \partial / \partial x^\mu$ and the zero order direction is $\hat{n}^\mu = (1, \hat{z}) = (1, 0, 0, 1)$. Since the vector field is assumed to be time independent, $d/dt = \partial / \partial z$. The geodesic equation for the light is then

$$\begin{aligned} \ddot{d}^i &= -, \quad \hat{n}^\mu \hat{n}^\nu = -\eta^{ij} \hat{n}^\mu \hat{n}^\nu \left[\frac{\partial h_{j\mu}}{\partial x^\nu} - \frac{1}{2} \frac{\partial h_{\mu\nu}}{\partial x^j} \right] \\ &= - \left[\frac{\partial h_{iz}}{\partial z} - \frac{1}{2} \frac{\partial h_{zz}}{\partial x^i} \right] \end{aligned} \quad (3.5)$$

The last equality follows since $h_{\mu\nu}$ has no time components. Upon inserting our expression for $h_{\mu\nu}$ into 3.5 we find

$$\ddot{d}^i = \dot{H}^i \quad (3.6)$$

with

$$H^i \equiv \int \frac{d^3k}{(2\pi)^3} \mu e^{ikz\mu} (-h_1 \sin \phi + ih_2 \mu \cos \phi, h_1 \cos \phi + ih_2 \mu \sin \phi). \quad (3.7)$$

Since the displacement angle $\dot{d}^i \propto H^i \propto \mu$, it does *not* get a large contribution from modes with $\mu = 0$; in fact these contribute relatively little. Rather, \dot{d} is of order the field strength h_1, h_2 .

If $h_1(\vec{k})$ and $h_2(\vec{k})$ are random fields, then the change in the direction of a photon travelling from $z = 0$ to $z = D$ will be zero on average. The mean square direction change can be calculated:

$$\begin{aligned} \langle |\delta \dot{\vec{d}}|^2 \rangle &= \langle (\dot{d}^i(D) - \dot{d}^i(0)) (\dot{d}^i(D) - \dot{d}^i(0)) \rangle \\ &= \frac{1}{2\pi^2} \int_0^\infty dk k^2 \int_{-1}^1 d\mu (\mu^2 P_1(k) + \mu^4 P_2(k)) (1 - \cos(k\mu D)) \end{aligned} \quad (3.8)$$

where the power spectra are defined so that

$$\langle h_a(\vec{k}) h_b^*(\vec{k}') \rangle = \langle h_a(\vec{k}) h_b(-\vec{k}') \rangle = (2\pi)^3 \delta(\vec{k} - \vec{k}') \delta_{ab} P_a(k). \quad (3.9)$$

The oscillatory term in 3.8 is irrelevant for large distances, and we are left with

$$\langle |\delta \vec{d}|^2 \rangle = \frac{2}{3} \langle h_1^2 \rangle + \frac{2}{5} \langle h_2^2 \rangle \quad (3.10)$$

So, even after travelling large distances in a vector field, light has experienced little directional change. This is identical to the case of a tensor field but dramatically different than the scalar field, for which $\langle |\delta \vec{d}|^2 \rangle \propto D$.

Kaiser and Jaffe showed that many observables are governed by the power spectrum

$$P_H(k) \equiv \int_{-\infty}^{\infty} dz e^{-ikz} \langle \vec{H}(0) \cdot \vec{H}(z) \rangle. \quad (3.11)$$

Typically the mean square displacement after a distance k^{-1} in a random field is given by P_H/k which is proportional to k^{-3} for scalar perturbations but k^0 for tensor perturbations.

We now calculate this power spectrum for vector modes:

$$\begin{aligned} P_H(k) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} dz \int_0^{\infty} dk' \int_{-1}^1 d\mu k'^2 (\mu^2 P_1(k') + \mu^4 P_2(k')) e^{iz(k'\mu - k)} \\ &= \frac{1}{2\pi} \int_0^{\infty} k'^2 dk' \int_{-1}^1 d\mu (\mu^2 P_1(k') + \mu^4 P_2(k')) \delta(k'\mu - k) \\ &= \frac{1}{2\pi} \int_k^{\infty} k' dk' \left[\left(\frac{k}{k'} \right)^2 P_1(k') + \left(\frac{k}{k'} \right)^4 P_2(k') \right] \end{aligned} \quad (3.12)$$

On large scales (small k) then, $P_H(k)/k \propto k$ for vector modes. Thus in the physical cases where tensor modes produce a logarithmic divergence, vector modes produce no such divergence.

One example of this is the question of the angular deflection of an image from its unperturbed location. On average, the image is unchanged, but the rms angular deviation is

$$\delta\theta_{rms} = \frac{1}{D} \left\langle \left(\int_{z_0}^{z_1} dz H(z) \right)^2 \right\rangle^{1/2}, \quad (3.13)$$

where the light starts at z_0 and travels a distance D to z_1 . Kaiser and Jaffe showed that this angular deviation is proportional to $D^{1/2}$ for scalar perturbations and $\sqrt{\ln(D)}/D$ for tensor perturbations. To calculate it for vector perturbations, we again follow Kaiser and Jaffe to write

$$\begin{aligned} \delta\theta_{rms} &= \frac{1}{D} \left(\frac{2}{\pi} \int \frac{dk}{k^2} P_H(k) \sin^2(kD/2) \right)^{1/2} \\ &= \frac{1}{D} \left(\frac{2}{D\pi^2} \int_0^{\infty} \frac{dk'}{k'} \int_0^{k'D/2} dx \sin^2 x [P_1(k') + (2x/k'D)^2 P_2(k')] \right)^{1/2}, \end{aligned} \quad (3.14)$$

where in the last line here, we have used 3.12, changed the order of integration, and introduced the dummy variable $x = kD/2$. Performing the x integral, but keeping only terms to highest order in $k'D$ leads to

$$\delta\theta_{rms} = \frac{1}{D} \left(\frac{1}{2\pi^2} \int_0^{\infty} dk' \left[P_1(k') + \frac{1}{3} P_2(k') \right] \right)^{1/2}. \quad (3.15)$$

So the rms displacement angle is of order $\langle h^2 \rangle^{1/2} / k_V D$ where k_V is the wavenumber where the power spectrum peaks. This is even smaller than the corresponding displacement angle for tensor modes, which was enhanced (slightly) by a logarithm.

IV. CONCLUSIONS

Even if there is a background of vector modes perturbing the gravitational metric, light should travel virtually undeflected over large distances. This conclusion is markedly different than what we expect if there are scalar

perturbations to the metric, since scalar perturbations can act coherently over large distances. Even in the best case, where vector modes remain constant, any coherent action is defused by a suppression of the perturbation in the resonant direction (i.e. the μ^2 factor in 2.8). Vector modes, just like tensor modes, do not bend light.

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