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DYNAMICAL CONSTRAINTS ON DARK COMPACT OBJECTS

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Abstract

Many of the baryons in the Universe are dark and at least some of the dark baryons could be in the form of compact objects. Such objects could be in various locations - galactic discs, galactic halos, clusters of galaxies or intergalactic space - and each of these is associated with a dark matter problem. For each site we consider the various dynamical constraints which can be placed on the fraction of the dark matter in compact objects of different mass. Small compact objects in the Galaxy are constrained by upper limits on their encounter rate with the Earth and Solar System since they would resemble meteors or comets. Larger objects are constrained by the disruptive or disturbing effects they would have on various astronomical systems. For disc objects, the most interesting constraints come from the disruption of binary stars or open star clusters. For halo objects, they come from the disruption of globular clusters, the heating of the Galactic disc and their accumulation in the Galactic nucleus as a result of dynamical friction. For cluster objects, they come from the tidal distortion and disruption of galaxies. For intergalactic objects, they come from the upper limit on the peculiar motions induced in galaxies. We also apply these limits to the situation in which the compact objects are clusters of smaller objects.

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1. Introduction

The aim of this paper is to bring together all the dynamical constraints on the mass of compact objects - or clusters of compact objects - residing in various astronomical locations associated with dark matter. Such compact objects would probably have to be baryonic, in the sense that they derive from ordinary atomic matter rather than exotic relics of the Big Bang. We therefore start by reviewing the evidence that some of the baryons in the Universe are dark and the likelihood that they are in the form of compact objects. We also discuss where the objects could be located, how they might have originated and what form they might take. The other possibility is that the compact objects are black holes which were formed in the early Universe. Although such "primordial" black holes should not be regarded as baryonic, they have very similar dynamical consequences, so much of our discussion will also apply to them.

1.1 Evidence for Baryonic Dark Matter

Evidence for dark matter has been claimed in four different contexts (Carr 1994): There may be local dark matter in the Galactic *disc* with a mass comparable to that in visible form ($M_{dark} \sim M_{vis}$). There may be dark matter in the *halo* of our own and other galaxies with a mass which depends upon the (uncertain) halo radius R_h and is of order $M_{dark} \sim 10 M_{vis}(R_h/100\mathrm{kpc})$. There may be dark matter associated with *clusters* of galaxies ($M_{dark} \sim 10 M_{vis}$). In the inflationary scenario, there may also be smoothly distributed *background* dark matter, required in order that the total cosmological density have the critical value which separates ever-expanding models from recollapsing ones ($M_{dark} \sim 100 M_{vis}$). The form of the dark matter need not be the same in all these contexts: some of it may be non-baryonic (eg. in elementary particle relics from the early Universe) but some of it may also be baryonic.

The main argument for both baryonic and non-baryonic dark matter comes from Big Bang nucleosynthesis. This is because the success of the standard picture in explaining the primordial light element abundances only applies if the baryon density parameter Ω_b lies in the range $0.007h^{-2}$ to $0.022h^{-2}$, where h is the Hubble parameter in units of 100 km $s^{-1} Mpc^{-1}$ (Copi et al. 1995). The upper limit is well below 1, which suggests that no baryonic candidate could provide the critical density required in the inflationary scenario. This conclusion also applies if one invokes inhomogeneous nucleosynthesis since one requires $\Omega_b<0.09h^{-2}$ even in this case (Mathews et al. 1993). The standard scenario therefore assumes that the total density parameter is 1, with only the fraction Ω_b being baryonic. On the other hand, the lower limit to Ω_b almost certainly exceeds the density of visible baryons Ω_v. A careful inventory by Persic & Salucci (1992) shows that the contributions to Ω_v are 0.0007 from spirals, 0.0015 from ellipticals and spheroidals, $0.00035h^{-1.5}$ from hot gas within an Abell radius for rich clusters, and $0.00026h^{-1.5}$ from hot gas out to a virialization radius in groups and poor clusters. This gives a total of $(2.2+0.6h^{-1.5})\times10^{-3}$, although neutral hydrogen in galaxies gives another contribution of $0.2\times10^{-3}h^{-1}$ (Rao & Briggs 1993). The fraction of baryons in dark form must therefore be in the range 60% to 95% for $0.5<h<1$. Dar (1995) gets a larger value, $\Omega_v=(4.5+0.9h^{-1.5})\times10^{-3}$, but this is still below Ω_b. Thus it seems that one needs both *non-baryonic* and baryonic dark matter.

Which of the dark matter problems could be baryonic? Baryons would certainly suffice to explain the dark matter *in galactic discs*: even if all discs have the dark fraction envisaged for our Galaxy, this only corresponds to $\Omega_d\approx0.001$, well below the value required by cosmological nucleosynthesis. On the other hand, the *cluster* dark matter has a density $\Omega_c\approx0.1\text{-}0.2$ and this cannot be baryonic unless one invokes inhomogeneous nucleosynthesis. We have seen that even inhomogeneous nucleosynthesis would not permit the background dark matter to be baryonic. The more intriguing question is whether dark baryons could suffice to explain galactic halos. If the Milky Way is typical, the density associated with halos would be $\Omega_h\approx0.02h^{-1}$ $(R_h/70\text{kpc})$, so the nucleosynthesis upper limit on Ω_b implies that *all* the dark matter in halos could be baryonic only for $R_h < 70h^{-1}$kpc. For our own halo the minimum radius consistent with rotation curve measurements, the local escape speed, the kinematics of globular clusters and the dynamics of the Local Group is 70kpc (Fich & Tremaine 1991). More generally, gravitational lensing of background galaxies by foreground ones suggest that R_h is at least $30h^{-1}$kpc (Brainerd *et al.* 1996). Both these limits would just be compatible with baryonic halos. However, observations of the satellites of other galaxies (Zaritsky et al. 1993, Zaritsky & White 1994) suggest that R_h is at least 200 kpc, which would not be. Generally the baryonic fraction could be at most $(R_h/70h^{-1}\text{kpc})^{-1}$.

Although the standard scenario assumes $\Omega_v << \Omega_b <<1$, two problems have recently arisen with this point of view. Firstly, X-ray data suggest that the ratio of visible baryon mass (in stars and hot gas) to total mass in clusters is anomalously high compared to the mean cosmic ratio implied by Big Bang nucleosythesis. For example, ROSAT observations of Coma suggest that the baryon fraction within the central 3 Mpc is about 25%, which is a least 5 times the cosmological ratio (White et al. 1993), and there is now evidence that this is a very widspread phenomenon (White & Fabian 1995). It is hard to understand how the extra baryon concentration would come about, since dissipation should be unimportant on these scales. Unless one invokes a cosmological constant, this suggests that either the cosmological density is well below the critical value or the baryon density is higher than allowed by the homogeneous nucleosynthesis scenario. The latter possibility would stengthen the case for baryonic dark matter since the gap between Ω_v and Ω_b would be increased.

Secondly, recent measurements of the deuterium abundance in quasar absorption systems suggest a primordial value of around 2×10^{-4} and this is an order of magnitude larger than is usually assumed (Songaila et al. 1994, Carswell et al. 1994, Rugers & Hogan 1996). In this case, the upper bound on Ω_b from nucleosynthesis is reduced to $0.005h^{-2}$, which is only marginally larger than the Persic-Salucci estimate of Ω_v, so the need for dark baryons may be removed altogether. However, the evidence for such a high deuterium abundance is disputed (Tytler et al. 1996). Indeed the most recent observations suggest that the deuterium line in the first high-abundance cloud may have been misidentified (Tytler at al. 1997). In any case, the resolution of this issue is crucial to the status of baryonic dark matter.

1.2 Location and Origin of Dark Baryons

Although this paper focusses on the possibility that the dark baryons are in compact objects, it must be stressed that this is not a necessary consequence of the condition $\Omega_v << \Omega_b$. There are at least five possible locations for the dark baryons, as summarized in Table (1), only some of which involve compact objects. Indeed some dark baryons must reside in each of these, so the crucial question is where most of them are. On the other hand, baryonic objects could still have interesting dynamical consequences even if their contribution to the dark matter density is small.

* *Hot Intergalactic Medium.* The discrepancy between Ω_b and Ω_v could be resolved if the missing baryons were in a hot intergalactic medium but, in this case, the temperature T would need to be finely tuned. Since the Gunn-Peterson test requires the neutral hydrogen density to be

$\Omega(HI)<10^{-8}h^{-1}$ out to a redshift of 3 (Steidel & Sargent 1988) and since the COBE limit on the Compton distortion of the microwave background ($y<3\times10^{-5}$) requires the ionized hydrogen density to be $\Omega(HII)<0.1(T/10^7K)^{-1}h^{-1}$ at that redshift (Mather et al. 1994), the temperature must lie in the range 10^4-10^6K if $\Omega_{IGM}\sim\Omega_b$. Positive evidence for intergalactic gas may come from the recent detection of helium absorption (Jacobsen et al. 1993, Davidsen et al. 1996).

* *Lyman-α Clouds*. Although the density parameter associated with "damped" clouds is probably around $0.003h^{-2}$ (Lanzetta et al. 1991), comparable to the density in galaxies and therefore consistent with the idea that these are protogalactic discs, the density associated with undamped systems is unknown and - depending on the ionized fraction - could be much larger (Rees 1986). Indeed simulations suggest that, in the CDM scenario, the undamped systems are distributed on filaments and sheets at a redshift of 2 with a baryon density comparable to the nucleosynthesis bound (Weinberg et al. 1996). In this case, *most* the intergalactic medium must be in the form of Lyman-a clouds. However, by the present epoch the undamped systems could still have fragmented into stars, so this does not exclude the other options discussed below.

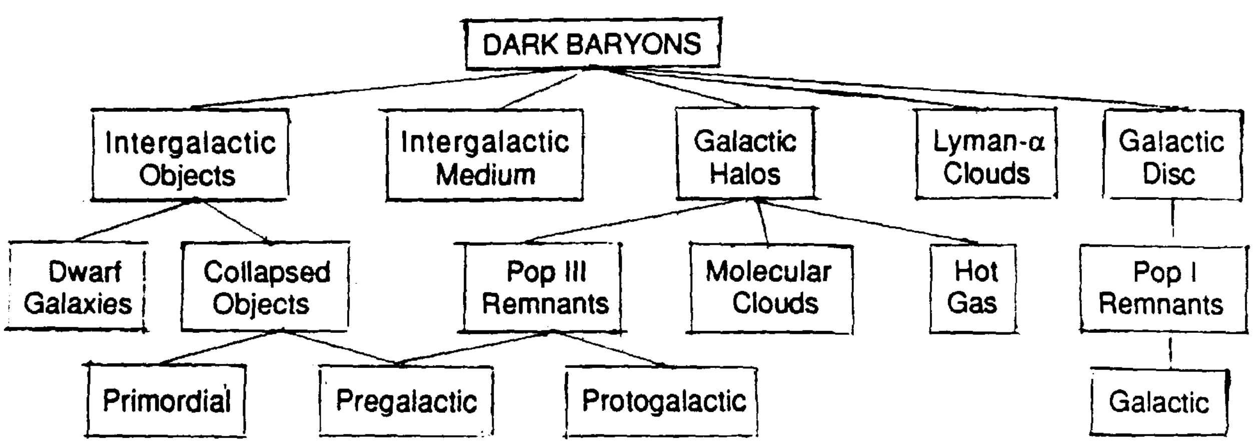
* *Galactic Discs*. At least some dark baryons must be in the remnants of Population I stars (i.e. in galactic discs). For our own galaxy Bahcall et al. (1992) estimate the dark fraction as 60%, although this has been disputed by Kuijken & Gilmore (1989, 1991). Flynn & Fuchs (1995) get an upper limit of 10%. Even if one accepts the Bahcall et al. estimate, the associated cosmological density is much less than Ω_v, so most of the dark baryons must be elsewhere.

* *Intergalactic Objects*. The usual estimate of Ω_v does not include the contribution from an intergalactic population of dark objects, such as dwarf galaxies (Bristow & Phillipps 1994, Loveday et al. 1996) or low surface brightness galaxies (McCaugh 1994, 1997). Indeed it has recently been claimed that such galaxies may provide all of the missing baryons (Impey & Bothun 1997). This paper emphasizes the possibility that there could also be baryons in the form of "Dark Intergalactic Compact Objects" (DICOs): either the remnants of a first generation of pregalactic stars or primordial black holes which formed before cosmological nucleosynthesis. Only the latter could have the critical density required by inflation. Otherwise most of the intergalactic dark matter would have to be in the form of "Weakly Interacting Massive Particles" or "WIMPs".

* *Galactic Halos.* We have seen that galactic halos could contain all the dark baryons if the typical halo radius R_h is less than $70h^{-1}$kpc. In this case, one might consider three possible forms for the dark baryons: hot gas, the remnants of pregalactic or protogalactic stars, or cold molecular clouds. The first possibility would appear to be inconsistent with X-ray observations, since the gas would need to have the virial temperature of 10^6K, although there may be evidence for some gas with this temperature (Suto et al. 1996). The second possibility corresponds to the "Massive Compact Halo Object" or "MACHO" scenario and has attracted considerable interest recently as a result of the microlensing detections (Alcock et al. 1993, 1996; Auborg et al. 1993). The third possibility requires that the molecular clouds be distributed in a disk (Pfenniger et al. 1994); although such clouds should not really be classified as compact objects, they would have many of the dynamical consequences considered here.

Both the MACHO and DICO scenarios require the existence of what are termed "Population III" stars; this is to distinguish them from the "Population I" and "Population II" stars which reside in the discs and spheroids of galaxies. Although there are no observations which unambiguously require that most of the baryons in the Universe were processed through Population III stars, there are theoretical reasons for anticipating their formation. This is because the existence of galaxies and clusters of galaxies implies that there must have been density fluctuations in the early Universe and, in some scenarios, depending on the nature of the fluctuations and the nature of the dominant dark matter, these fluctuations would also give rise to a population of bound clouds in the period betwen decoupling and galaxy formation (White & Rees 1978, Carr et al. 1984, Ashman & Carr 1991).

Table (1): *Possible locations of dark baryons and formation epochsof compact objects*



The first bound clouds could face various possible fates (Carr 1994). They might just turn into ordinary stars and form objects like globular clusters. On the other hand, the conditions of star formation could have been very different at early times and several alternatives have been suggested. Some people propose that the first stars could have been much *smaller* than at present (Palla et al. 1983, Yoshii & Saio 1986, Ashman & Carr 1991), others that they could have been much larger (Silk 1977, Kashlinsky & Rees 1983, Tohline 1980), others that they may have collapsed directly to supermassive black holes (Gnedin & Ostriker 1992, Loeb 1993). In the baryon-dominated "isocurvature" scenario (Peebles 1987), with highly non-linear fluctuations on small scales, the first clouds may even have collapsed to black holes before decoupling (Hogan 1993). In this case, the baryon density could be higher than the usual nucleosynthesis value because the nucleosynthetic products in the high density regions would be locked up inside remnants (Gnedin et al., 1995). One certainly needs the first clouds to fragment into stars which are very different from the ones forming today if they are to produce a lot of dark matter.

Although intergalactic dark objects would have to be pregalactic (or primordial) in origin, there is no necessity for halo objects to be pregalactic since even clouds which bind before galaxies need not fragment until much later. They may just remain as Lyman-α clouds which undergo no further evolution until the epoch of galaxy formation, as indicated by the studies of Weinberg et al. (1996). The epoch of Population III formation will be very important for the relative distribution of baryonic and non-baryonic dark matter, especially if the non-baryonic dark matter is "cold" so that it can cluster in galactic halos. In this case, if the Population III stars form *before* galaxies, one might expect their remnants to be distributed throughout the Universe, with the ratio of the non-baryonic and baryonic densities being the same everywhere and of order 10. If they form at the same time as galaxies, perhaps in the first phase of protogalactic collapse, one would expect the remnants to be confined to halos and clusters. In this case, their contribution to the halo density could be larger since the baryons would probably dissipate and become more concentrated. However, angular momentum considerations require that halos must be essentially in place by the time galactic discs form, so there must not be too much dissipation (Fall & Efstathiou 1981). Compact objects in galactic discs presumably form after the discs themselves and must therefore be of galactic origin. The various possible epochs for compact object formation are indicated in Table (1).

1.3 Nature of Compact Objects

We have seen that dark objects of baryonic or primordial origin could reside in galactic discs, galactic halos or the background Universe. In this paper the term "compact object" will used to cover all these cases. There could also be compact objects in clusters of galaxies but these are unlikely to be a separate population; probably they would have to comprise either accreted intergalactic objects or objects which derived from disrupted halos. We now discuss the possible form of these compact objects. The first possibility is the most speculative and assumes that the objects form in the first few moments of the Big Bang. The other possibilities all correspond to variants of the "Population III" scenario.

* *Primordial Black Holes (PBHs).* Black holes may have formed in the early Universe, either from initial inhomogeneities or at some sort of cosmological phase transition (Carr 1996). Those forming at time t after the Big Bang would have a mass of order the horizon mass at that epoch $\sim 10^5 (t/s)\ M_\odot$. Since there could not have been large-amplitude horizon-scale inhomogeneities at the epoch of cosmological nucleosynthesis ($t \sim 1s$), PBHs forming via the first mechanism are unlikely to be larger than $10^5 M_\odot$. On the other hand, since there is no phase transition after the quark-hadron era at 10^{-5}s, those forming via the second are unlikely to be larger than $1 M_\odot$. In fact, the possibility that PBHs may have formed at the quark-hadron transition (Crawford & Schramm 1982) has attracted considerable attention recently (Schmid 1996, Jedamzik 1996) because they would naturally have a mass comparable to that required by the microlensing results. However, the PBH scenario has one disadvantage: because the ratio of PBH to radiation density increases as the cosmological scale factor, the fraction of the Universe going into PBHs at time t could only have been $10^{-6}(t/s)^{1/2}$ even if their density today is close to critical (Carr 1975). It therefore requires fine-tuning of the collapse fraction to explain any of the dark matter problems. In this respect the Population III explanation is more attractive since one expects a fraction of order unity of the Universe's baryonic mass to end up in the dark objects.

* *Low Mass Objects (LMOs).* This term will be used to cover all objects below 0.8 $M_\odot$ which have either not completed their nuclear burning phase or not passed through one at all. Stars in the range 0.08-0.8 $M_\odot$ are still on the main-sequence. Those below below 0.1 $M_\odot$ are termed M-dwarfs; although they are very dim, source count constraints already

exclude them from providing all of the disc or halo dark matter (Bahcall et al. 1996). Objects in the range 0.001-0.08 $M_{\odot}$ would never burn hydrogen, although those above 0.01 $M_{\odot}$ could still burn deuterium, and are termed brown dwarfs (BDs). They represent a balance between gravity and degeneracy pressure. Objects below 0.001$M_{\odot}$, being held together by intermolecular rather than gravitational forces, have atomic density and are here termed "snowballs".

* *Intermediate Mass Objects (IMOs).* This term will be used to describe stars between 0.8 $M_{\odot}$ and about 100 $M_{\odot}$ which have already completed their evolution. Those in the range 0.8-4 $M_{\odot}$ would leave white dwarf remnants, while those between 8 $M_{\odot}$ and some mass M_{BH} would leave neutron star remnants. In either case, the remnants would eventually cool and become dark. Stars in the mass range 4-8 M_{O} may be disrupted entirely during their carbon-burning stage but this is not certain. Stars larger than M_{BH} would evolve to black holes; estimates of M_{BH} range from 25 $M_{\odot}$ to 50 $M_{\odot}$ (Maeder 1992). Both neutron stars and stellar black holes would appear to be implausible dark matter candidates because their precursors would produce too much enrichment (Carr et al. 1984). White dwarfs would therefore appear to be the most conservative IMO candidate (Ryu et al.1993), especially in view of the results from the microlensing experiments. However, this scenario also has problems since it requires galaxies to go through a very bright initial phase and this may be inconsistent with galactic number counts (Charlot & Silk 1996). It also requires the initial mass function to be restricted to 2-8 $M_{\odot}$ to avoid excessive background light and heavy element production.

* *Very Massive Objects (VMOs).* Stars with initial mass in the range above 100$M_{\odot}$ would experience the pair-instability during their oxygen-burning phase (Fowler & Hoyle 1964). This would lead to disruption below some initial mass $M_{C} \approx 200 M_{\odot}$ but complete collapse above it (Woosley & Weaver 1982, Ober et al. 1983, Bond et al. 1984). VMO black holes may therefore be more plausible dark matter candidates than ordinary stellar black holes. However, stars with an initial mass above 100$M_{\odot}$ are radiation-dominated and therefore unstable to pulsations. This leads to considerable mass loss during hydrogen-burning, although the pulsations are unlikely to be completely disruptive. The main problem with the VMO scenario is that the background light generated by the very luminous precursors exceeds the observational upper limits for most parameters (Bond et al. 1991, Wright et al. 1994). It must be stressed that there is little evidence for VMOs forming at the present epoch, so they are invoked specifically to provide dark matter.

**Supermassive Objects (SMOs).* Metal-free stars larger than $10^5 M_\odot$ would collapse directly to black holes due to general relativistic instabilities before any nuclear burning (Fowler 1966). They would therefore have no nucleosynthetic consequences, although they could explode in some mass range above $10^5 M_\odot$ if they had non-zero metallicity (Fuller et al. 1986). SMOs would also generate very little radiation, emitting only 10^{-11} of their rest-mass energy in photons. It should be stressed that it is very difficult for supermassive objects to collapse directly to black holes, at least at the present epoch. They are much more likely to fragment into smaller objects, with black holes only forming subsequently in the core as a result of relaxation. This means that only a small fraction of the mass of the Universe is likely to be in SMOs. For example, while there is certainly good evidence that SMO holes reside in galactic nuclei, perhaps with masses as large as $10^9 M_\odot$, these would only have a negligible cosmological density. Huge intergalactic objects, if they exist, are therefore unlikely to be single compact objects like black holes.

* *Dark Clusters.* In many scenarios one expects the Population III stars to form in clumps of about $10^6 M_\odot$ (Carr & Lacey 1986). This mass arises naturally as the mass of the first bound clouds in either a baryon-dominated Universe with isothermal fluctuations or a cold dark matter Universe with adiabatic fluctuations (Carr & Rees 1984). It also arises if the first clouds form protogalactically through some type of two-phase instability (Fall & Rees 1985). However, one could also envisage scenarios in which the cluster mass is well below $10^6 M_\odot$ (Ashman 1990, Kerins & Carr 1994, Wasserman & Salpeter 1994, Moore & Silk 1995, De Paolis et al. 1995). Provided the clusters maintain their integrity, it makes very little difference to their dynamical effects whether they consist of brown dwarfs, white dwarfs, black holes, or even single supermassive holes. However, in many circumstances, one would *not* expect the clusters to maintain their integrity, since they would be disrupted by collisions or Galactic tidal effects and this places constraints on the mass and radius of any surviving clusters. For the reasons discussed above, supermassive dark objects are also more likely to be dark clusters (eg. dark galaxies) than single black holes.

All sorts of constraints can be placed on the density of compact objects in different locations (disc, halo, cluster, background) and these are reviewed by Carr (1994). In this paper we will focus exclusively on constraints associated with dynamical effects. Some of these effects have been discussed before - either in the published literature or in unpublished form (Sakellariadou 1984) - and all of them could be treated in more detail than we do here. However, we believe this is the first time the constraints have all been brought together and a more careful treatment would preclude their being covered in a single paper. For the purposes of this paper, the mass of the compact object in each location will be taken to be arbitrary, even though non-dynamical constraints may already exclude some ranges. Our conclusions will also be largely independent of the nature of the compact objects. Although some of our calculations assume that they are black holes, since black holes represent the only stable candidate in the high mass range, we have seen that the compact objects could also be clusters of smaller objects. The results are summarized in Figure (7). A preliminary version of this figure also appeared in Carr (1994) but without derivation.

We discuss the constraints in order of increasing mass. Very low mass objects would be numerous enough to encounter the Earth occasionally and the associated constraints are discussed in §2. Many of the dynamical limits are associated with the destruction of star clusters or galaxies by compact objects. §3 presents a general discussion of this problem and the results are then applied in various astronomical contexts in §4. Dynamical friction limits are covered in §5 and disc heating limits in §6. The most plausible dark matter candidates are probably too small to have interesting dynamical effects if smoothly distributed. We therefore focus on the dark cluster scenario in §7, concluding that the mass and radius of the clusters must be very tightly constrained. The dynamical consequences of huge intergalactic dark objects is discussed in §8. We bring all the limits together and draw some general conclusions in §9.

2. Direct Encounter Constraints

Compact objects smaller than Jupiter - here termed "snowballs" - would have none of the disruptive effects on astronomical systems discussed later. However, if there were a large population of such objects in the disc or halo of the Galaxy, one would expect some of them to enter the solar system or impact the Earth occasionally and this would have observable consequences. Objects in the mass range $10^{-6}g<M<10^{7}g$ would resemble meteorites, those with $10^{15}g<M<10^{22}g$ would resemble comets, and those in the intermediate mass range would leave impact craters on Earth. Upper limits on the frequency of meteors, comets and the number of impact craters therefore provide constraints on compact objects too small to be eliminated by any other type of observation.

Of course, primordial snowballs would be very different from conventional comets and meteorites in terms of their composition. They would probably be composed of pure hydrogen, since no primordial helium would be expected to condense (Phinney 1995) and they would contain no metals. However, this would be irrelevant for the encounter effects discussed here. The prime difference in this context would be the larger velocities expected for dark matter snowballs since this would make their effects more pronounced. Hills (1986) has studied the encounter limits in detail and we summarize and update his results here. We express the limits as constraints on the fraction of the disc and halo dark matter in objects of mass M and summarize them in Figure (1). It should be stressed that non-dynamical effects may exclude some of the mass ranges considered here. For example, within the age of the Universe, compact objects smaller than $10^{22}g$ might be evaporated by the microwave background (Hegyi & Olive 1983, Phinney 1985) and those smaller than $10^{26}g$ might be evaporated by their own heat (De Rujula et al. 1992). Objects above $10^{26}g$ may also be excluded by microlensing limits (Auborg et al. 1995). However, these arguments are not definitive - the first two have been criticized by White (1996) and the third is based on fairly limited data - so it is useful to obtain independent dynamical constraints.

In discussing the encounter constraints, one first needs to determine the average speed $<V>$ of the objects relative to the Earth. For objects in the Galactic disc one expects $<V>=(V_d^2+V_\odot^2+V_\oplus^2+V_e^2)^{1/2}$, where $V_\odot=42kms^{-1}$ is the solar escape velocity at 1AU, $V_\oplus=30kms^{-1}$ is the orbital velocity of the Earth, $V_e=11kms^{-1}$ is the escape velocity from the Earth, and V_d is the 3-dimensional velocity dispersion within the disc. $<V>$ is an average speed over periods exceeding a year, so that seasonal effects associated with Earth's motion around the Sun are eliminated. The value of V_d is problematic since the velocity dispersion of disc stars depends upon their age (see §6). Hills took V_d=30kms^{-1} but here we take V_d=60kms^{-1} since this is probably more appropriate for older objects. This gives $<V>$=80kms^{-1}, compared to an average speed of 40kms^{-1}for Solar System objects. For halo objects one expects $<V>\approx(V_h^2+V_g^2)^{1/2}$, where V_h is the 3-dimensional halo velocity dispersion, V_g=220kms^{-1} is the velocity of the Sun around the Galactic centre, and the other velocity components are neglected in this case. For a spheroidal halo, $V_h=\sqrt{3/2}V_g$=270kms^{-1} and this gives $<V>\approx$350kms^{-1}. For comparison, Hills took V_g=250kms^{-1} and V_h=200kms^{-1}, corresponding to $<V>$=320kms^{-1}. Of course, there will be considerable variation in the encounter speed about these average values - partly due to the direction of the object's motion relative to the Earth's orbit and partly due to the (presumably Maxwellian) distribution in the pre-encounter velocity.

To calculate the rate at which objects hit the Earth, one must allow for the gravitational focussing of the Sun. This increases the flux at the orbital radius of the Earth by a factor $[1+(V_\odot/V_d)^2]$=1.5 for disc objects or $[1+(V_\odot/V_h)^2]$=1.02 for halo objects. The gravitational focussing effect of the Earth can be neglected, so the effective cross-section for encounters is just $\pi R_\oplus^2$. The mass flux on Earth is then

$$dM/dt = \{1.5,1\}\pi R_\oplus^2 f(M)\rho V_* = \{5\times10^9, 9\times10^8\}\ f(M)\ \mathrm{g\ y^{-1}}\{\text{disc, halo}\} \quad (2.1)$$

where $V_*=[<V>^2-V_e^2]^{1/2}\approx<V>$ is the average pre-encounter velocity, f(M) is the fraction of the dark matter in objects of mass M and ρ is the dark matter density. We have taken the local halo and disc densities to be 0.01$M_\odot$pc^{-3} and 0.15$M_\odot$pc^{-3}, respectively, the latter being based on the estimate of Bahcall et al. (1992). This compares to the smaller values of 0.008$M_\odot$pc^{-3} and 0.1$M_\odot$pc^{-3} assumed by Hills. Although the local density of the halo objects is only 7% that of the disc objects, their mass flux is 18% because of their larger velocities.

Observations imply that meteors with absolute magnitude down to M_V=15 (defined as the apparent magnitude at 100 km) have a flux of 8×10^6 s^{-1} (Allen 1973). The mass is related to M_V and V by

$$\log(M/g) = 2.5 - 1.7\log(V/\mathrm{kms}^{-1}) - 0.4M_V \qquad (M<1g) \qquad (2.2)$$

(Opik 1958), so this corresponds to masses exceeding 10^{-7}g or 10^{-8}g for objects with velocities appropriate for the disc or halo respectively. Halo objects are brighter for a given mass because of their larger velocities. However, at most 1% of these meteors could be of interstellar origin (i.e. on hyperbolic orbits) and at most 0.01% could be of halo origin (Jones & Sarma 1985). Allowing for these factors, eqn (2.1) gives a limit

$$f(M) < \begin{cases} 5\times10^{-5}(M/10^{-7}g) & (M>10^{-7}g) \quad (\mathrm{disc}) \\ 3\times10^{-7}(M/10^{-8}g) & (M>10^{-8}g) \quad (\mathrm{halo}) \end{cases} \qquad (2.3)$$

The limit on the meteor flux down to $M_V=6$, corresponding to masses exceeding 10^{-3}g for disc objects and $M=10^{-4}$g for halo objects, is 460 s^{-1} (Allen 1973) and the same argument as above then implies

$$f(M) < \begin{cases} 3\times10^{-5}(M/10^{-3}g) & (M>10^{-3}g) \quad (\mathrm{disc}) \\ 2\times10^{-7}(M/10^{-4}g) & (M>10^{-4}g) \quad (\mathrm{halo}) \end{cases} \qquad (2.4)$$

For given M this is stronger than limit (2.3) but it does not extend to such low values. Meteors with magnitude down to $M_V=-5$, corresponding to masses exceeding 10 g for disc objects and 1 g for halo objects, would produce fireballs and the fact that no fireballs of interstellar origin were observed in a large-area survey over an "effective" (i.e. scaled to the full area of the Earth) period of 30 hours led Hills to infer a limit

$$f(M) < \begin{cases} 5\times10^{-7}(M/10g) & (M>10g) \quad (\mathrm{disc}) \\ 3\times10^{-7}(M/1g) & (M>1g) \quad (\mathrm{halo}) \end{cases} \qquad (2.5)$$

He anticipated that satellite observations of the Earth or Jupiter could strengthen these limits by a factor of 10^3 or 10^5, respectively.

Hills also considered the limits which come from considering impact craters on Earth. The time between impacts is known to be 1.6y for $M\sim10^8$g, 2.5×10^3y for $M\sim10^{12}$g and 2×10^6y for $M\sim10^{16}$g (Allen 1973), these masses being appropriate for objects in the disc. For halo objects, the corresponding masses would be smaller by 20 (scaling as

V^2) because their higher velocities would result in larger impact craters. One can infer the following limits:

$$f(M) < \begin{cases} 0.01(M/10^8 g) & (M\sim 10^8 g) \\ 0.08(M/10^{12} g) & (M\sim 10^{12} g) \\ (M/10^{16} g) & (M\sim 10^{16} g) \end{cases} \qquad \text{(disc)} \qquad (2.6)$$

$$f(M) < \begin{cases} 0.003(M/5x10^6 g) & (M\sim 5x10^6 g) \\ 0.02(M/5x10^{10} g) & (M\sim 5x10^{10} g) \\ 0.3(M/5x10^{14} g) & (M\sim 5x10^{14} g) \end{cases} \qquad \text{(halo)} \qquad (2.7)$$

In Figure (1) we have interpolated between the values of M to obtain a more extended limit.

For $M>10^{15}g$ (disc) or $M>10^{14}g$ (halo), one also gets a limit from the fact that no interstellar comet has been observed in telescopic surveys (such as Messier's) over the last 300 y. Using 1AU rather $R_\oplus$ for the relevant cross-section in eqn (2.1) then gives a constraint

$$f(M) < \begin{cases} 1x10^{-6}(M/10^{15} g) & (M>10^{15} g) \quad \text{(disc)} \\ 6x10^{-7}(M/10^{14} g) & (M>10^{14} g) \quad \text{(halo)} \end{cases} \qquad (2.8)$$

so only objects larger than $10^{21}g$ (disc) or $2x10^{20}g$ (halo) could provide all the dark matter. Naked eye observations would have sufficed to detect $M>10^{17}g$ disc objects or $M>10^{16}g$ halo objects (which are as bright as Halley) out to several AU over the last 400y and this increases the lower limit to $5x10^{21}g$ (disc) or $10^{21}g$ (halo). This assumes that pure hydrogen snowballs produce tails when they approach the Sun, as seems likely.

Finally we must consider the possibility that snowballs destroy each other through collisions within the age of the Universe, as first discussed by Hegyi & Olive (1983). Since the collisional cross-section for snowballs is just their geometrical cross-section (πr^2), the timescale on which they collide is

$$t_{coll} = 4r\rho_o/(3f\rho V) = (16/9\pi)^{1/3}\rho_o^{2/3}M^{1/3}/(f\rho V) \qquad (2.9)$$

where ρ_o is the internal density of the snowballs (i.e. solid hydrogen density ~ 0.1 g cm^{-3}) and the appropriate velocity is $V_h \approx 270 kms^{-1}$ for halo objects and $V_d \approx 60 kms^{-1}$ for disc objects. In both cases therefore snowballs can survive for the age of the Galaxy t_g if

$$f < (M/M_{min})^{1/3}\ ,\quad M_{min} \approx \begin{cases} 1\times10^{6}(t_g/10^{10}y)^3 g & \text{(disc)} \\ 3\times10^{4}(t_g/10^{10}y)^3 g & \text{(halo)} \end{cases} \qquad (2.10)$$

Only snowballs larger than M_{min} could comprise all the dark matter. Where it overlaps, this limit is considerably weaker than the meteor and fireball limits but it extends to lower values of M. For smoothly distributed intergalactic snowballs, $\rho \approx 2\times10^{-29} f h^2 g cm^{-3}$ (where f is the fraction of the critical density in the snowballs) and we assume $V = 10^3 kms^{-1}$ (although this value is rather uncertain). At earlier epochs ρ scales with redshift as $(1+z)^3$ and t as $(1+z)^{-3/2}$, so we obtain a limit

$$f < (M/M_{min})^{1/3}\ ,\quad M_{min} \approx 4\times10^{-8}(t_o/10^{10}y)^3(1+z)^{9/2}h^6 g \qquad (2.11)$$

(The evolution of V is neglected because it is model-dependent.) Limit (2.11) is very sensitive to z but, provided the snowballs form after $z = 10^3$, it is always weaker than the limit (2.10).

It must be stressed that all the terrestrial encounter limits presume that the snowballs are not sublimated by the heat of the Sun when they enter the inner solar system. However, White (1996) argues that this may not apply for snowballs with radius less than 1 km (or $M < 10^{14}$g), in which case only the collisional limits (2.10) and (2.11) pertain. In this context, it is important to establish whether limit (2.5) can be extended to the outer planets (like Jupiter) since the sublimation radius scales as the square root of the distance from the Sun. One must also be cautious in applying the above limits to primordial black holes, the only other low mass dark matter candidate. In this case, one can still use eqn (2.1) to calculate the encounter rate but PBHs striking the Earth or its atmosphere would have very different signatures from comets or meteors. Small mass PBHs would pass straight through a solid body, merely leaving small accretion cylinders, although larger ones could have more dramatic effects. A full discussion of this goes beyond the scope of the present paper.

3. The Disruption of Clusters

In this section we will study the tidal disruption of a star cluster by a passing compact object. The tidal force of the object will increase the velocity dispersion within the cluster, leading to an increase in its total energy. The cluster will therefore expand and eventually become unbound. Depending on the circumstances, either a single encounter or a succession of encounters will be required for its disruption. Various people have analysed this process in particular astronomical contexts. For example, Spitzer (1958) has studied the disruption of open star clusters by passing interstellar clouds. Carr (1978), Sakellariadou (1984), Wielen (1987, 1988, 1991), Ostriker et al. (1989) and Moore (1993) have studied the disruption of globular clusters by giant black holes. Carr (1978) and Wielen (1985) have studied the disruption of open clusters by giant molecular clouds and black holes. Bahcall et al. (1985) have studied the disruption of binary systems by objects in the Galactic disc. Gerhard & Fall (1983) have studied the tidal interactions of spiral galaxies in clusters.

For present purposes we need a very general analysis which can be used in all of these situations. We need to go beyond the standard impulse approximation, in which the disrupting objects are assumed to be moving sufficiently fast that they pass the cluster in a time less than its internal dynamical timescale and move on nearly straight orbits (Spitzer 1958). We also need to allow for the effects of encounters in which the impact parameter is either more or less than the radius of the cluster. The full details of the analysis are produced elsewhere (Carr & Sakellariadou 1997). Here and in Appendix A we merely summarize the key results and then apply them in §4 to studying the disruption of binaries and open clusters by disc compact objects, the disruption of globular clusters by halo compact objects and the disruption of galaxies by dark objects in clusters. For specificness we assume that the compact objects are black holes. However, nothing depends too crucially on this; we only require that the objects be much smaller than the clusters they are disrupting.

For simplicity we assume that the cluster is spherically symmetric with its stars all having the same mass and an isotropic velocity distribution. The density distribution within the cluster will depend upon the context but, for a cluster of mass M_C and radius R_C, its total energy and the 3-dimensional stellar velocity dispersion will always have the form

$$E_C = -\gamma(GM_C^2/R_C), \quad v_C^2 = 2\gamma(GM_C/R_C) \qquad (3.1)$$

where the value of the constant γ depends upon the density profile. To keep our analysis as general as possible, we will usually leave γ unspecified. However, in later applications we will focus on three particular cases. We will model open clusters as *homogenous* spheres, in which case there is a sharp cut-off at radius R_C and γ=0.3. We will model globular clusters as *Plummer* spheres, in which case R_C is interpreted as the half-mass radius (since the sphere extends to infinity) and γ=0.15. We will model galaxies as *isothermal* spheres with a cut-off at some radius R_C, in which case γ=3/4. (See Appendix A).

If the black hole has mass M and, at closest approach, distance p and velocity V relative to the cluster centre, then - as shown in Appendix A - the total change in the energy of the stars in the cluster can approximated as

$$\Delta E = \begin{cases} (4\alpha^2/3)\, G^2M^2M_CR_C{}^2/(V^2p^4) & (p >> R_C) \\ (3\beta^2)\, G^2M^2M_C/(V^2R_C{}^2) & (p << R_C) \end{cases} \tag{3.2}$$

for an impulsive encounter. Here α is the ratio of the root-mean-square cluster radius to R_C and β is the ratio of the root-mean-square inverse cluster radius to $R_C{}^{-1}$:

$$\alpha \equiv \langle r^2\rangle^{1/2}/R_C \ , \quad \beta \equiv \langle r^{-2}\rangle^{1/2}R_C \tag{3.3}$$

These parameters have well-defined values for a uniform sphere ($\alpha=\sqrt{3/5}$, $\beta=\sqrt{3}$), a Plummer sphere ($\alpha\approx3$, $\beta\approx1$) and an isothermal sphere ($\alpha=1/\sqrt{3}$, $\beta\approx$2-3) but for now we leave them unspecified. We note that the expressions in eqn (3.2) do not match at $p=R_C$ since they only apply in the small and large p limits. The expression in the $p>>R_C$ regime is the same as that derived by Spitzer (1958). In the $p<<R_C$ regime, Wielen (1985) replaces p by R_C in the Spitzer formula. Moore (1993) covers both regimes with the fit $\Delta E=\Delta E(p=0)[1+(p/R_C)]^{-4}$. Clearly both approximations agree with eqn (3.2) to an order of magnitude. Note that the coefficient in eqn (3.2) in the $p<<R_C$ case is an overestimate if the black hole is replaced by a more extended compact object (see §7).

If the change in energy ΔE exceeds the cluster's gravitational binding energy E_C, then disruption will be a one-off event, requiring a single encounter. We can express the condition for this as an upper limit on the black hole speed V for given distance of closest approach p:

$$V < \begin{cases} (2\alpha^2/3\gamma^2)^{1/2} V_C (M/M_C)(R_C/p)^2 & (p \gg R_C) \\ (3\beta^2/2\gamma^2)^{1/2} V_C (M/M_C) & (p \ll R_C) \end{cases} \qquad (3.4)$$

If this condition is not satisfied, then the disruption of the cluster will be a cumulative effect, requiring many encounters. V/V_C is usually large and, in this case, one-off disruption can only occur for $M \gg M_C$. Indeed we show below that eqn (3.4) can never be satisfied for $M<M_C$.

Eqn (3.2) presupposes the validity of the impulse approximation and this breaks down if the traversal time of the hole exceeds the dynamical time of the cluster. The condition for this is

$$V < \begin{cases} V_C\,(p/R_C) & (p > R_C) \\ V_C & (p < R_C) \end{cases} \qquad (3.5)$$

and the value of ΔE is then modified by a factor which is tabulated by Spitzer (1958). If the parameter $\zeta \equiv (pV_C/R_C V)$ is small (corresponding to impulsive encounters), this factor is 1 but it decreases as $\exp(-\zeta)$ for large values of ζ. Since V is defined to be the speed of the black hole relative to the cluster centre, this necessarily exceeds the escape velocity at distance p, given roughly by

$$V_e(p)^2 \approx 2G(M+M_C)(p^2+R_C^2)^{-1/2} \qquad (3.6)$$

More precisely $V = \sqrt{V_e(p)^2+V_\infty^2}$ where V_∞ is the speed of the black hole at infinity (prior to the encounter).

We first consider the $M \gg M_C$ case, shown in Figure 2(a). *The* condition $V>V_e(p)$ now becomes

$$V > \begin{cases} \gamma^{-1/2} V_C (M/M_C)^{1/2} (R_C/p)^{1/2} & (p \gg R_C) \\ \gamma^{-1/2} V_C (M/M_C)^{1/2} & (p \ll R_C) \end{cases} \qquad (3.7)$$

and gravitational focussing is only important close to this limit. Conditions (3.4) and (3.5) intersect at

$$V = (2\alpha^2/3\gamma^2)^{1/6} V_C (M/M_C)^{1/3}\ , \quad p = (2\alpha^2/3\gamma^2)^{1/6} R_C (M/M_C)^{1/3} \qquad (3.8)$$

so we have disruption by impulsive multiple encounters if V exceeds V_{min} and if p lies between p_{dis} and p_{max} where

$$V_{min} = V_C(M/k_2M_C)^{1/3} \tag{3.9a}$$

$$p_{max} = R_C(V/V_C) \tag{3.9b}$$

$$p_{dis} = R_C(M/k_2M_C)^{1/2}(V_C/V)^{1/2}\ \theta[V_C(M/k_1M_C)-V] \tag{3.9c}$$

Here θ is the Heaviside function and the constants k_1, k_2 are given by

$$k_1 \equiv (\sqrt{2}\gamma/\sqrt{3}\beta), \quad k_2 \equiv (\sqrt{3}\gamma/\sqrt{2}\alpha) \tag{3.10}$$

from eqn (3.4). We have impulsive one-off disruption if V lies between V_{min} and V_{max} and p between p_{min} and p_{dis} where eqns (3.4) and (3.7) imply

$$V_{max} = V_C(M/k_1M_C) \tag{3.11a}$$

$$p_{min} = R_C(M/\gamma M_C)(V/V_C)^{-2}\ \theta[V_C(M/\gamma M_C)^{1/2}-V] \tag{3.11b}$$

In the $M \ll M_C$ case, shown in Figure 2(b), the (M/M_C) factors in condition (3.7) are omitted. Conditions (3.4) and (3.7) are now incompatible, so there is no one-off disruption in this case but one still has disruption by multiple encounters for $p<p_{max}$.

The possible consequences of encounters in the $M>M_C$ case are summarized in Figure (2a), which shows the (V,p) regime in which one has: (A) multiple-encounter disruption; (B) one-off impulsive disruption; and (C) non-impulsive encounters. Figure (2b) shows the $M<M_C$ situation: disruption can only occur via multiple encounters in this case, so region (B) does not exist.

In order to determine the rate at which clusters disappear due to the processes discussed above, we must calculate the likelihood of passing black holes having values of p and V in the relevant range. In this context, it is important to distinguish between V and p (defined at closest approach) and the asymptotic velocity V_∞ and impact parameter p_∞ of the black hole since it is the latter which determine the encounter rate. From energy and angular momentum conservation, these quantities are related by

$$V^2 - V_e(p)^2 = V_\infty^2 \quad , \quad pV = p_\infty V_\infty \tag{3.12}$$

For a given cluster, the number of encounters with black holes with parameters in the range (p,p+dp), (V,V+dV), ($\theta,\theta+d\theta$) and ($\phi,\phi+d\phi$) [where polar coordinates θ and ϕ specify the orientation of the black hole's velocity] in the time interval (t,t+dt) is

$$dN(p,V,\theta,\phi) = (2\pi p_\infty dp_\infty)(nV_\infty dt)(V_\infty{}^2F(V_\infty)dV_\infty \sin\theta\, d\theta\, d\phi) \qquad (3.13)$$

Here n is the number density of the black holes, assumed for simplicity to all have the same mass M, and $4\pi V_\infty{}^2F(V_\infty)dV_\infty$ is the fraction of them with velocities in the interval $(V_\infty, V_\infty + dV_\infty)$. We assume either a Maxwellian distribution with 1-dimensional velocity dispersion σ, in which case

$$F(V_\infty) = (2\pi^{1/2}\sigma)^{-3}\exp(-V_\infty{}^2/4\sigma^2) \qquad (3.14a)$$

or a discrete distribution with all the holes having the same velocity V_0, in which case

$$F(V_\infty) = \delta(V_\infty - V_0)/(4\pi V_0{}^2) \qquad (3.14b)$$

If we integrate over θ and ϕ assuming an isotropic velocity distribution and then use eqn (3.12) to derive the Jacobian $\partial(p_\infty,V_\infty)/\partial(p,V)$, the total encounter rate becomes

$$dN(p,V)/dt = 8\pi^2 n \int VdV \int pdp\ [V^2+(p/2)dV_e{}^2/dp]\ F([V^2-V_e(p)^2]^{1/2}) \quad (3.15)$$

where the integral is carried out over the relevant (V,p) region in Figure (2). So long as gravitational foussing can be neglected, the last two terms just give $V^2F(V)$.

Disruption by single encounters

We first consider one-off disruption (which only occurs for $M >> M_c$). If all the holes have the same velocity V_0, assumed to be less than $V_{max} = V_c(M/k_1M_c)$ but more than $V_{min} = V_c(M/k_2M_c)^{1/3}$, then eqn (3.15) implies that the timescale for one-off disruption is

$$t_{dis} = (dN/dt)^{-1} = [\pi n V p_{dis}{}^2]^{-1} \qquad (3.16)$$

and eqn (3.9c) gives

$$t_{dis} = (\sqrt{6}\gamma/2\alpha\pi)M_c/(nR_c^2MV_c) \text{ for } k_1(V/V_c)<M/M_c<k_2(V/V_c)^3 \quad (3.17)$$

The range of values of M arises because one only has one-off disruption for $M>k_1M_c(V/V_c)$ and the impulse approximation fails for $M>k_2M_c(V/V_c)^3$. For a discrete velocity distribution, one-off disruption cuts off abruptly at these mass limits. Note that the timescale (3.17) depends only on the density nM of the compact objects and not on their velocity. If the holes have a Maxwellian velocity distribution, eqn (3.17) is still a good approximation for $V_{max}>>\sigma>>V_{min}$. For $\sigma>>V_{max}$, the disruption time exceeds the value given by eqn (3.17) by a factor $\sim(\sigma/V_{max})^3$. For $\sigma<<V_{min}$, it exceeds it by a factor $\sim \exp\{(V_{min}/4\sigma)^2\}$. From eqns (3.9a) and (3.11a), we therefore have

$$t_{dis} = \{\text{eqn (3.17)}\} \times \begin{cases} [MV_c/k_1\sigma M_c]^{-3} & \text{for } M/M_c<<k_1(\sigma/V_c) \\ 1 & \text{for } k_1(\sigma/V_c)<<M/M_c<<k_2(\sigma/V_c)^3 \\ \exp\{[MV_c^3/8k_2M_c\sigma^3]^{2/3}\} & \text{for } M/M_c>>k_2(\sigma/V_c)^3 \end{cases} \quad (3.18)$$

With an extended velocity distribution one-off disruption does not cut off sharply at the upper and lower mass limits in eqn (3.17). However, we will find that disruption is anyway dominated by multiple-encounters in the $M<<k_1M_c(\sigma/V_c)$ regime.

Disruption by multiple encounters

For multiple disruption the rate of change of energy in the cluster is obtained by multiplying ΔE by the encounter rate $dN(p,V)/dt$ and then integrating over p and V. We first assume all the holes have the same velocity $V_o \approx V$. For $V>V_{max}$, we then obtain

$$dE/dt = (2\pi nG^2M^2M_c)/(3VR_c^2)\left[9\beta^2\int_0^{R_c} p\,dp + 4\alpha^2\int_{R_c}^{p_{max}} R_c^4p^{-3}dp\right]$$
$$\approx \pi\xi nG^2M^2M_c/(3V) \quad \text{with} \quad \xi = 4\alpha^2+9\beta^2 \quad (3.19)$$

For comparison, Wielen (1985) uses $\xi=8\alpha^2$ (i.e. he assumes the $p<R_c$ and $p>R_c$ contributions are the same). For $V_{max}>V>V_{min}$ the p integral does not extend below p_{dis} (which itself exceeds R_c), so eqn (3.9c) yields

$$dE/dt = (8\pi\alpha^2nG^2M^2M_c)/(3VR_c^2)\int_{p_{dis}}^{p_{max}} R_c^4p^{-3}dp = (4\pi\alpha\gamma/\sqrt{6})G^2nMM_c^2/V_c \quad (3.20)$$

For $V<V_{min}$, one is in the non-impulsive regime, so the energy transfer is reduced by a factor $\exp(pV_c/R_cV)$ and the lower limit in the p-integral is the value p_{min} given by eqn (3.11b). We therefore obtain

$$dE/dt = (8\pi\alpha^2 nG^2M^2M_c)/(3VR_c^2) \int_{p_{min}}^{\infty} R_c^4 p^{-3} \exp(-pV_c/R_cV)\, dp$$

$$\approx (4\pi\alpha^2\gamma^3/3)(nG^2M_c^3V^3/V_c^4)\ \exp[-(M/\gamma M_c)(V_c/V)^3] \qquad (3.21)$$

Disruption by multiple encounters, unlike one-off encounters, does not cut off abruptly once V falls below V_{min} even for a discrete velocity distribution.

We next consider the Maxwellian case. As shown in Appendix A, for $\sigma>V_{max}$ we obtain eqn (3.19) but with V replaced by $\sqrt{\pi}\sigma$; this is very close to the 3-dimensional velocity dispersion $\sqrt{3}\sigma$. For $V_{max}>\sigma>V_{min}$, we again obtain eqn (3.20). For $\sigma<V_{min}$, there is an exponentially damped contribution deriving from the impulsive encounters associated with the holes with velocity V_{min} on the exponential tail of the Maxwellian velocity distribution and this gives

$$dE/dt \sim (G^2nM^2M_cV_c^2/\sigma^3)\ \exp\{-[MV_c^3/8k_2M_c\sigma^3]^{2/3}\} \qquad (3.22)$$

where we have dropped a numerical coefficient and the same exponential factor appears as in eqn (3.18).

The timescale on which clusters are disrupted by multiple encounters will be taken to be $|E/(dE/dt)|$. This neglects the changes of ξ and R_c during the evolution of the cluster; both of these would tend to increase with time and this would reduce t_{dis}. However, Wielen (1985) argues that this reduction is balanced by the fact that the outermost stars would escape from the cluster before sharing their energy with the other stars. On this assumption, we obtain

$$t_{dis} = \begin{cases} (3\gamma/\pi\xi)VM_c/(GM^2nR_c) & \text{for } M/M_c<k_1(V/V_c) \\ (\sqrt{6}\gamma/2\alpha\pi)M_c/(nR_c^2MV_c) & \text{for } k_1(V/V_c)<M/M_c<k_2(V/V_c)^3 \\ (V_c^2/nR_c^2V^3)\ \exp[(M/\gamma M_c)(V_c/V)^3] & \text{for } M/M_c>k_2(V/V_c)^3 \end{cases} \qquad (3.23a)$$

for a discrete velocity distribution. For a Maxwellian velocity distribution, V is replaced by $\sqrt{\pi}\sigma$ in the first expression, the second expression is the same and the last expression becomes

$$t_{dis} \sim M_c^2\sigma^3/(nM^2R_c^2V_c^4)\exp\{[MV_c^3/4k_2M_c\sigma^3]^2\} \text{ for } M/M_c>k_2(\sigma/V_c)^3 \qquad (3.23b)$$

We note that the timescale in the second expression in eqn (3.23a) is exactly the same as indicated by eqn (3.17), so multiple and one-off encounters contribute equally to disruption. We therefore take the "effective" disruption time to be half the value indicated by eqn (3.23a) in this case.

Finally we consider the situation with $M<M_c$. As indicated in Figure 2(b), there is no one-off disruption in this case. However, disruption by multiple encounters still occurs and eqn (3.19) still applies for $V>>V_c$. Therefore the limit (3.23a) for $M<k_1M_c(V/V_c)$ can be immediately extended into the $M<M_c$ domain. One can also still have non-impulsive encounters with $V<V_{min}\approx V_c$ and in this case eqn (3.21) with $p_{min}\approx R_c(V_c/V)^2$ gives

$$t_{dis} \sim (M_c^2V_c^2/nM^2R_c^2V^3)\ \exp[(V_c/V)^3] \qquad (3.24)$$

In practice, encounters in this regime are negligible.

4. Limits on the Density of Compact Objects

We will now apply this analysis in some specific astronomical contexts, replacing the "black hole" with any compact object and the "cluster" with various types of star systems. Let $\rho_{CO}(M)$ be the local density of compact objects of mass M and let us assume that the average cluster is known to survive for some time t_L. The requirement that this exceeds the *disruption* timescales derived in §3 imposes the following upper limit on ρ_{CO}:

$$\rho_{CO} < \begin{cases} (3\gamma/\pi\xi) M_c V/(G M t_L R_c) & \text{for } M < k_1 M_c (V/V_c) \\ (\sqrt{6}\gamma/4\pi\alpha) M_c/(V_c t_L R_c^2) & \text{for } k_1 M_c (V/V_c) < M < k_2 M_c (V/V_c)^3 \end{cases} \tag{4.1a}$$

where k_1 and k_2 are given by eqn (3.10), ξ by eqn (3.19) and V is interpreted as the 3-dimensional velocity dispersion for a Maxwellian distribution. One also has

$$\rho_{CO} < \begin{cases} (M V_c^2/R_c^2 V^3 t_L) \exp[(M/\gamma M_c)(V_c/V)^3] & \text{for } M > k_2 M_c (V/V_c)^3 \\ M_c^2 \sigma^3/(M R_c^2 t_L V_c^4) \exp\{[M V_c^3/8 k_2 M_c \sigma^3]^{2/3}\} & \text{for } M > k_2 M_c (\sigma/V_c)^3 \end{cases} \tag{4.1b}$$

for a discrete and Maxwellian velocity distribution, respectively. The smallest and largest mass regimes correspond to disruption by multiple encounters, the intermediate one to disruption by both multiple and single *encounters*. [Note that Carr (1978) erroneously applies the $M < k_1 M_c (V/V_c)$ expression for values of M above $k_1 M_c (V/V_c)$; Carr (1994) also gives an erroneous expression in the $M > k_2 M_c (V/V_c)^3$ regime.] Any lower limit on t_L therefore places an upper limit on ρ_{CO}. The two expressions in eqn (4.1a) are discontinuous by a factor $6\alpha\beta/(4\alpha^2+9\beta^2)$ at $M = k_1 M_c (V/V_c)$ due to the approximations used. A more precise treatment would obviously produce a smooth transition.

For any particular type of cluster (with fixed M_c and R_c) the qualitative form of the limit on ρ_{CO} is shown in Figure (3). Substituting for V_c using eqn (3.1), we note that limit (4.1a) bottoms out at a density

$$\rho_{max} = (\sqrt{3}\gamma/4\pi\alpha)(M_c/G R_c^3 t_L^2)^{1/2} \tag{4.2}$$

If this exceeds the actual dark matter density ρ, one gets no useful limit on the compact objects at all. If it is less than the actual density, one requires that M either exceed M_{min} or be less than M_{max} where

$$M_{min} = (\sqrt{3}\gamma/\sqrt{2}\alpha)M_c(V/V_c)^3 \qquad (4.3a)$$

$$M_{max} = (3\gamma/\pi\xi)(M_cV)/(G\rho t_L R_c) \qquad (4.3b)$$

If ρ_{max} is comparable to the actual density, then it is possible that the compact objects actually determine the lifetime t_L. M_{max} is taken to be the mass at which the permitted density begins to rise exponentially; this is somewhat below the actual limit.

Limits (4.2) and (4.3) are sensitive to the parameters α, β and γ. For a uniform cluster, α=0.8, β=1.7 and γ=0.3, implying ξ=30. For a Plummer sphere α=3, β=1 and γ=0.15, implying ξ=45. For an isothermal sphere, α=0.6, $\beta\approx$2-3 and γ=0.8, implying ξ=40. We also need to specify the density ρ and velocity dispersion V in the different contexts. For the local halo we assume $\rho\approx0.01M_\odot pc^{-3}\approx7\times10^{-25}$ g cm^{-3} and $V\approx270kms^{-1}$. For the local disc we assume $\rho\approx0.15M_\odot pc^{-3}\approx1\times10^{-23}$ g cm^{-3} and $V\approx60kms^{-1}$. However, these velocities should be multiplied by $\sqrt{2}$ if the clusters have the same velocity dispersion as the compact objects. V should be compared to the 3-dimensional velocity dispersion within the cluster, which eqn (3.1) gives as

$$V_c \approx 50\ (\gamma/0.3)^{1/2}(M_c/10^6M_\odot)^{1/2}(R_c/pc)^{-1/2}\ km\ s^{-1} \qquad (4.4)$$

The limits on the fraction of dark matter in compact objects are summarized in Figure (4), with relevant parameters given in Table (2).

Disruption of globular clusters by halo objects

We will first determine the limit associated with the disruption of globular clusters (cf. Carr 1978, Sakellariadou 1984, Wielen 1987, 1988, 1991). We will treat these as Plummer spheres with typical parameters $M_c=10^5M_\odot$, R_c=10pc and $V_c=4kms^{-1}$; since most globular clusters are known to have survived for the age of the Galaxy, we will take $t_L=10^{10}$y. From eqn (4.2) the associated limit on the compact object density is

$$\rho_{max} \approx 2\times10^{-26}(\alpha/3)^{-1}(\gamma/0.15)^{1/2}(M_c/10^5M_\odot)^{1/2}(R_c/10pc)^{-3/2} \times (t_L/10^{10}y)^{-1}\ gcm^{-3} \qquad (4.5)$$

For the globular cluster parameters chosen, this is a factor of 30 below the actual halo density, so one gets an interesting constraint on the black hole mass range. Eqn (4.3) with a $\sqrt{2}$ factor in V gives

$$M_{max} \approx 3x10^4(\gamma/0.15)(\xi/45)^{-1}(M_C/10^5M_\odot)(R_C/10pc)^{-1}(t_L/10^{10}y)^{-1}M_\odot \tag{4.6}$$

$$M_{min} \approx 6x10^9(\alpha/3)^{-1}(\gamma/0.15)(M_C/10^5M_\odot)(V_C/4kms^{-1})^{-3}M_\odot$$

Between these two masses compact objects can be excluded from comprising all the halo dark matter.

Numerical calculations for the disruption of globular clusters by Moore (1993) confirm the general qualitative features indicated above, with gradual mass loss for small halo objects and sudden disruption for larger ones. However, using data for nine particular globular clusters, he infers an upper limit of $10^3M_\odot$ on the mass of the halo objects. This is considerably stronger than the limit implied by eqn (4.6). The difference derives from the fact that Moore's globular clusters are smaller and more diffuse than assumed above. For example, Pal 5 (which gives his strongest limit) has M_C=$1.4x10^4M_\odot$ and R_C=14pc, so that M_{max} is reduced by a factor of 10 from eqn (4.6). His clusters are also at a larger Galactocentric radius, although this weakens the limit.

Disruption of open clusters by halo and disc objects

The disruption of open clusters by compact objects has also been discussed by various authors (Carr 1978, Sakellariadou 1984, Wielen 1985). We will treat these as uniform spheres with typical parameters M_c=$10^2M_\odot$, R_c=1pc and V_C=0.5kms^{-1}. Specifying the appropriate value for t_L is problematic since open clusters (unlike globular clusters) are continually being created and destroyed: 50% have survived for $2x10^8$y, 10% for $5x10^8$y and 2% for more than 10^9y (Janes & Adler 1982). If we adopt the first figure as the representative lifetime, eqn (4.2) gives

$$\rho_{max} \approx 5x10^{-24}(\alpha/0.8)^{-1}(\gamma/0.3)^{1/2}(M_C/10^2M_\odot)^{1/2}(R_C/pc)^{-3/2} \\ x\ (t_L/2x10^8y)^{-1}gcm^{-3} \tag{4.7}$$

This is a factor of 7 above the halo density, so one gets no useful limit. However, it is less than the disc dark matter density by a factor of 2, so the disruption of open clusters gives an interesting limit in this context. Eqn (4.3) with no $\sqrt{2}$ factor in V gives

$$M_{max} \approx 500(\gamma/0.3)(\xi/30)^{-1}(M_C/10^2M_\odot)(R_C/pc)^{-1}(t_L/2\times10^8y)^{-1}M_\odot$$

$$M_{min} \approx 9\times10^7(\alpha/0.8)^{-1}(\gamma/0.3)(M_C/10^2M_\odot)(V_C/0.5kms^{-1})^{-3}\ M_\odot \qquad (4.8)$$

As discussed by Wielen (1985), other processes are already known to disrupt open clusters (eg. encounters with molecular clouds), so the mass range may be even more confined than indicated above.

Disruption of binaries by disc and halo objects

To an order of magnitude, one can also use eqns (4.1) to determine the constraint on the black hole density associated with binary systems in the Galactic disc. For comparison, if the black holes have number density n and mass M, a more precise calculation of binary disruption (Heggie 1975) gives a binary half-life of

$$t_{1/2} = \begin{cases} [\ln 2\ (3/2\pi)^{1/2}/40]\ (M_1+M_2)V/[GnM^2a] & (M<<M_{crit}) \quad (4.9a) \\ [15\ln 2/2^5\pi]\ (M_1+M_2)^{1/2}/[G^{1/2}nMa^{3/2}] & (M>>M_{crit}) \quad (4.9b) \end{cases}$$

where M_1 and M_2 are the masses of the components, a is their separation and

$$M_{crit} = 0.12(M_1+M_2)(V/V_C) \quad \text{with} \quad V_C = [G(M_1+M_2)/a]^{1/2} \qquad (4.10)$$

Eqns (4.9) are equivalent to the first two expressions in eqn (3.23a) with $M_C\equiv(M_1+M_2)$ and $R_C\equiv a$, while the mass M_{crit} just corresponds to the mass $k_1M_C(V/V_C)$. Indeed, even though the previous analysis is really only applicable for many bodies, one can even reproduce the coefficients in eqns (4.9a) and (4.9b), 0.01 and 0.1 respectively, by choosing $\alpha=1$, $\gamma=0.3$ and $\xi=30$. However, there is an important qualitative difference in that a binary (unlike a cluster) can be disrupted by an encounter which is close to just one component and it is this process rather than multiple encounters which determines $t_{1/2}$ in the $M<<M_{crit}$ case. Our choice of α, γ and ξ should therefore be regarded as purely empirical.

Bahcall et al. (1985) were motivated by the claim that the distribution of wide binary separations exhibits a cut-off above about 0.1 pc (Latham et al. 1984). If we take $M_C=2M_\odot$, $R_C=0.1$ pc, $V_C=0.2$ km s^{-1} and $t_L=10^{10}$y in eqn (4.1), we then obtain a limiting density

$$\rho_{max} \approx 4\times10^{-25}\alpha^{-1}(\gamma/0.3)^{1/2}(M_C/2M_\odot)^{1/2}(R_C/0.1pc)^{-3/2} \times (t_L/10^{10}y)^{-1} g cm^{-3} \quad (4.11)$$

For halo objects this is weaker than the globular cluster limit and therefore uninteresting. However, the limit is interesting for disc objects, eqn (4.3) with no $\sqrt{2}$ factor in V then giving

$$M_{max} \approx 2(\gamma/0.3)(\xi/30)^{-1}(M_C/2M_\odot)(R_C/0.1pc)^{-1}(t_L/10^{10}y)^{-1}M_\odot$$

$$M_{min} \approx 2\times10^{7}\alpha^{-1}(\gamma/0.3)(M_C/2M_\odot)(V_C/0.2kms^{-1})^{-3}M_\odot \quad (4.12)$$

This is an important constraint because, if correct, it would rule out disk dark matter comprising stellar black holes. However, the Bahcall et al. conclusion has been disputed by Wasserman & Weinberg (1987, 1991), who claim that the apparent cut-off at 0.1pc is a selection effect and that, in any case, one would not expect a *sharp cut-off in the* distribution of binary separations. Unless future observations dispel this criticism, the strongest limit on the mass of disc objects comes from the disruption of open clusters.

Disruption of galaxies by dark objects in clusters

Finally we apply eqn (4.1) to the *disruption of galaxies in* clusters of galaxies by cluster compact objects. In this case, the "clusters" of §3 are entire galaxies. If we take the galaxies to be isothermal spheres with characteristic parameters $M_g=10^{11}M_\odot$, $R_g=10$ kpc and $V_C=270kms^{-1}$ and assume that they *survive for a cosmological* timescale of $10^{10}h^{-1}y$, then

$$\rho_{max} \approx 7\times10^{-27}(\alpha/0.6)^{-1}(\gamma/0.8)^{1/2}h(M_g/10^{11}M_\odot)^{1/2}(R_g/10kpc)^{-3/2} \times (t_L/10^{10}h^{-1}y)^{-1} g\ cm^{-3} \quad (4.13)$$

A typical cluster of galaxies has a mass of $10^{15}M_\odot$ and radius of 3 Mpc, corresponding to a mean density of 7×10^{-28}g cm^{-3}. This is below limit (4.13), which suggests that the disruption constraint is uninteresting. However, the density within a cluster increases as one moves inwards, so the disruption limit should at least become interesting sufficiently close to the centre.

For a more precise analysis, we consider the specific case of the Coma cluster (White et al. 1994). This has a virial radius of $1.5h^{-1}$Mpc and a mass of $4\times10^{14}h^{-1}M_\odot$ within that radius, which corresponds to a

3-dimensional velocity dispersion of $V \approx 10^3 kms^{-1}$ and a mean density of $2 \times 10^{-27} h^2$ g cm^{-3}. The density at the virial radius itself is about three times smaller than this. The dark matter density appears to increase at least as fast as r^{-2} down to 0.2 Mpc and then as r^{-1}. This means that the density has risen to at least $7 \times 10^{-26} h^2$ gcm^{-3} at $R = 0.15 h^{-1}$Mpc, which is 10 times the density given by eqn (4.13) for h=1. At this distance eqn (4.3) together with the $\sqrt{2}$ factor in V gives

$$M_{max} = 7 \times 10^9 h^{-1} (\gamma/0.8)(\xi/40)^{-1} (M_g/10^{11} M_\odot)(R_g/10\mathrm{kpc})^{-1} \times (t_L/10^{10} h^{-1} y)^{-1} M_\odot$$

$$M_{min} \approx 3 \times 10^{13} (\alpha/0.6)^{-1} (\gamma/0.8)(M_c/10^{11} M_\odot)(V_c/270 \mathrm{kms}^{-1})^{-3} M_\odot \qquad (4.14)$$

There are various caveats in imposing this limit. Firstly, we know that many galaxies in the central regions anyway collide with each other in the age of the Universe; at the very centre they may also be destroyed by the tidal field of the cD galaxy (Moore *et al.* 1996). Secondly, if galaxies are on radial orbits, they may only spend a small fraction of the time in the central regions, which would reduce the effective value of t_L. However, there is evidence that orbits are fairly isotropic at small radiii, implying that galaxies may remain there for most of a Hubble time (Dubinski 1997). Thirdly, not all galaxies have the mass and radius assumed in eqn (4.13) and some of them may be disrupted on a shorter timescale.

Another limit on the dark objects in clusters comes from considering unexplained tidal distortions in the galaxies. If we assume that compact objects induce noticeable tidal distortions in any galaxies which lie within a factor $\lambda \sim 3$ times the tidal radius $R_T \approx R_g (M/M_g)^{1/3}$ (the factor λ representing the difference between distortion and disruption), then the fraction of *instantaneously* distorted galaxies should be

$$\Delta N_g/N_g = \lambda^3 (\rho_{co}/\rho_g) \qquad (4.15)$$

where ρ_g is the internal galaxy density and ρ_{co} is the density of the dark objects. This effect only operates for $M > \lambda^{-3} M_g = 3 \times 10^9 M_\odot$ since otherwise the tidal distortion radius is smaller than R_g. One infers a limit on the compact object density

$$\rho_{CO} < \lambda^{-3} (\Delta N_g/N_g) \rho_g \approx 2 \times 10^{-26} (\lambda/3)^{-3} (\Delta N_g/N_g) \text{ g cm}^{-3} \qquad (4.16)$$

This is less than the typical mean cluster density of 10^{-27}g cm^{-3}, so that one has an interesting limit, providing the fraction of distorted galaxies is less than about 5%. In general the fraction of the cluster dark matter in compact objects must satisfy $f<20(\Delta N_g/N_g)$. Towards the centre of the cluster, the limit is stronger. Of course, the number of instantaneously distorted galaxies will be less than the number which have *ever* been distorted. Eqn (4.16) assumes that the distortion only persists during the encounter itself, the tidal stretching being reversed as the compact object recedes after closest approach. The limit would be stronger if the distortion persisted longer than this.

Van den Bergh (1969) applied this argument to the Virgo cluster. He found that 10 of the 73 cluster members exhibit distortions which might be attributed to tidal interaction and 6 of these have companions which presumably cause it. Therefore at most 4 out of 73 galaxies (6%) have unexplained distortions and even these cases might be due to internal effects. He inferred that black holes binding the cluster could not have a mass in the range 10^8-$10^{13}M_\odot$. However, it is not clear where his upper mass limit comes from and the lower mass limit would seem to be rather low. According to our analysis, only can only infer that the dark mass cannot be in compact objects larger than $3\times10^9M_\odot$, although the fraction would be much more constrained near the centre. This limit is shown by the broken line in Figure (4).

Table (2): *Parameters assumed and constraints derived for the disruption of globular clusters (GC) by halo objects, open clusters (OC) and binaries (B) by disc objects, and galaxies (G) by cluster objects. The maximum density of the objects, the maximum mass for which they can contain all the dark mass and the minimum mass for which the limits can be obviated are indicated. The constraints assume the values for α, β and γ appropriate in each context.*

	M_C ($M_\odot$)	R_C (pc)	V_C (km s^{-1})	ρ_{max} (g cm^{-3})	M_{max} ($M_\odot$)	M_{min} ($M_\odot$)
GC	10^5	10	4	2×10^{-26}	3×10^4	6×10^9
OC	10^2	1	0.5	5×10^{-24}	5×10^2	9×10^7
B	2	0.1	0.2	4×10^{-25}	2	2×10^7
G	10^{11}	10^4	260	$7\times10^{-27}h$	$7\times10^9h^{-1}$	3×10^{13}

5 The Effect of Dynamical Friction on Halo Objects

Any objects in the Galactic halo will tend to lose energy to lighter objects via dynamical friction and consequently drift towards the Galactic nucleus (Chandrasekhar 1960). At a given Galactocentric radius r, this means that all objects larger than some mass $M_{df}(r)$ will have been dragged into the nucleus by now. [The radius r should not be confused with radius R used in the discussion of clusters.] In this section we calculate the form of the function $M_{df}(r)$ and then use upper limits on the dark mass in the Galactic nucleus to infer constraints on the fraction $f_h(M)$ of the halo contained in compact objects of mass M. More technical aspects of the calculation are relegated to Appendix B.

Various sources of drag will act upon the halo objects, so it is important to identify the dominant one at each Galactocentric radius. Within the central few kiloparsecs the dominant source will be the spheroid stars. Beyond that the effect of the disc stars will also be important, although this is complicated since it depends on the inclination of the orbit of the halo object relative to the plane of the Galaxy. At still larger radii the halo objects themselves may provide the dominant source of drag. [This assumes they have an extended mass spectrum, so that the smaller ones can provide drag for the larger ones; we will take this to be the case, even though - for simplicity - most of the considerations of this paper take the dark objects to have a discrete mass spectrum.] The dynamical effect of the spheroid stars has been discussed in detail by Carr & Lacey (1987; CL) and here we extend their analysis beyond the spheroid. A precise treatment of dynamical friction in this region is complicated. However, since the combined density of the spheroid, disc and halo is such as to produce a roughly flat rotation curve at large r, we can derive approximate results by using the dynamical friction formula appropriate for a single-component constant-velocity isothermal sphere (Tremaine et al. 1975). For convenience we will take the halo to be spherically symmetric.

We must first specify the density of each component. We assume that the spheroid has a density profile

$$\rho_s(r) = \begin{cases} \rho_1 (r_1/r)^{9/5} & (r < r_1) \\ \rho_1 (r_1/r)^3 & (r > r_1) \end{cases} \qquad (5.1)$$

where r_1=800pc and $\rho_1 = 1.8 M_\odot pc^{-3}$ and that the halo has an isothermal density profile

$$\rho_h(r) = \begin{cases} \rho_o(r_o/r_c)^2 & (r < r_c) \\ \rho_o(r_o/r)^2 & (r > r_c) \end{cases} \quad (5.2)$$

where r_o=8kpc is our own Galactocentric distance, r_c is the halo core radius (probably in the range 2-8 kpc) and $\rho_o=0.01 M_\odot pc^{-3}$ is the local halo density. Various alternatives to eqn (5.2) have been proposed: Navarro et al. (1995) argue for a profile which goes from r^{-1} to r^{-3} with increasing r; Hernquist (1990) for one which goes from r^{-1} to r^{-4}. The calculations below can easily be extended to cover these cases but we adopt eqn (5.2) since this case is the simplest to analyse. Both the Navarro et al. and Hernquist et al. profiles scale as r^{-2} over some range of radii and the precise form of the core profile is not crucial anyway since the disc has a large effect in this region.

Comparison of eqns (5.1) and (5.2) shows that the halo density dominates the spheroid density outside a radius

$$r_2 = 1.8(r_c/2\mathrm{kpc})^{2/3}\mathrm{kpc} \quad (5.3)$$

and this is always within the halo core for reasonable values of r_c. However, the mass m(r) within radius r will be dominated by the spheroid well beyond r_2. Indeed, eqns (A3) and (A16) of the Appendix show that the halo only dominates m(r) outside the radius r_3 determined by

$$r_3 = (2/3)r_c + [1.3 + 1.5\ln(r_3/r_1)]\ \mathrm{kpc} \quad (5.4)$$

This is always exceeds r_c and it may be close to r_o. For example, r_3=5kpc for r_c=2kpc, r_3=6kpc for r_c=3kpc and r_3=7kpc for r_c=4kpc. The disc will also be important in the region $r>r_2$ and this may increase the radius r_3 beyond which the halo dominates the mass. One can thus divide the Galaxy into four regimes: the inner spheroid ($r<r_1$) and the outer spheroid ($r_1<r<r_2$) [the density and mass being dominated by the spheroid in both of these regimes], the outer halo ($r>r_3$) [in which the density and mass are dominated by the halo], and a complicated intermediate region ($r_2<r<r_3$) [in which neither the spheroid nor the halo dominate and the effect of the disc must be included].

If we assume for simplicity that each halo object always moves on a circular orbit, then its orbital radius will shrink according to (Chandrasekhar 1960)

$$dr/dt = -4\pi G^2 M(\ln\Lambda)A(v_c/\sqrt{2}\sigma)r\rho(r)/[(1+d\ln v_c/d\ln r)v_c(r)^3] \qquad (5.5a)$$

$$\Lambda \approx r v_c^2/GM \qquad (5.5b)$$

$$A(x) \equiv \mathrm{erf}(x)-x(d/dx)\mathrm{erf}(x) \qquad (5.5c)$$

Here $v_c(r)$ and $\sigma(r)$ are the circular velocity and 1-dimensional velocity dispersion at radius r for the objects providing the drag. The result for a more realistic distribution of orbital eccentricities probably does not differ very much. A precise calculation of $v_c(r)$ and $\sigma(r)$, allowing for the effects of the spheroid, disc and halo, is difficult because these functions depend upon both the mass and density at radius r. However, we note that dr/dt depends primarily on $\rho(r)$ and $v_c(r)$, with the latter being determined entirely by m(r). The dependence on $\sigma(r)$, the only term which involves both $\rho(r)$ and m(r), is weak. For an approximate analytic calculation, we therefore adopt the spheroid expressions for all the terms in eqn (5.5a) if $r<r_2$ and the halo expression for all the terms if $r>r_3$. Solving for $v_c(r)$ and $\sigma(r)$ in these two radial regimes then gives expressions for dr/dt as a function of r and these are derived in Appendix B. In the intermediate regime $r_2<r<r_3$, we will merely invoke the observed constancy of $v_c(r)$ in eqn (5.5a) and not attempt to solve the problem self-consistently.

Since dr/dt always goes roughly like $r\rho/v_c^3$ with $v_c \sim (m/r)^{1/2}$, it scales as $\rho m^{-3/2} r^{5/2}$. In the regions for which the mass and density are dominated by the spheroid and halo alone, $m \sim \rho r^3$ and so dr/dt goes like $\rho^{-1/2} r^{-2}$. The power law dependences of ρ on r in the different regimes then imply that the drift rate scales as

$$dr/dt \propto \{r^{-11/10}, r^{-1/2}, r^{-1}\} \quad \{r<r_1, r_1<r<r_2, r>r_3\}$$

and the corresponding drift time is

$$t_{dr} = \{10/21, 2/3, 1/2\}\ r/(dr/dt) \quad \{r<r_1, r_1<r<r_2, r>r_3\} \qquad (5.6)$$

The form of t_{dr} as a function of r is shown in Figure 5(a). For $r<r_2$ and $r>r_3$ this is an increasing function of r, so t_{dr} is also nearly the timescale $t_{df}(r)$ on which the compact object drifts from the initial radius r to the origin. For $r_2<r<r_3$, the form of t_{dr} is difficult to calculate analytically. However, as discussed in Appendix B, an approximation in which one neglects the effect of the disc and assumes that m is dominated by the spheroid and ρ by the halo suggests that t_{dr} first decreases and then increases with increasing r. In this case, the time to drift to the origin is roughly $t_{dr}(r_2)$ for values of r such that $t_{dr}(r)$ is less than this. We can therefore identify a radius r_4 at which $t_{dr}=t_{dr}(r_2)$ and assume that $t_{df}(r)$ is flat between r_2 and r_4.

The expressions for dr/dt given by eqns (B10) and (B22) imply that t_{df} is less than the age of the Galaxy t_g providing r is less than

$$r_{df} = \begin{cases} 1.0(M/10^6M_\odot)^{10/21}(t_g/10^{10}y)^{10/21}\ \mathrm{kpc} & (r_{df}<r_1) \quad (5.7a) \\ 1.4(M/10^6M_\odot)^{2/3}(t_g/10^{10}y)^{2/3}f_1(r_{df})\ \mathrm{kpc} & (r_1<<r_{df}<r_2) \quad (5.7b) \\ 1.2(M/10^6M_\odot)^{1/2}(t_g/10^{10}y)^{1/2}f_2(r_{df})\ \mathrm{kpc} & (r_{df}>>r_4) \quad (5.7c) \end{cases}$$

where r_{df} increases discontinuously from r_2 to r_4. The first two expressions were derived by CL. The functions f_i have only a weak dependence on r_{df}:

$$f_1(r_{df}) = [1+(6/11)\ln(r_{df}/r_1)]^{-2/3}[1+(6/5)\ln(r_{df}/r_1)]^{-1/3} \quad (5.8a)$$

$$f_2(r_{df}) = [1-(r_c/3r_{df})]^{-1/2}[1-(2r_c/3r_{df})]^{-1/4} \quad (5.8b)$$

For example, if r_c=2kpc, f_1 varies between 1 and 0.6 as r_{df} goes from r_1 to r_2, while f_2 varies from 1.6 to 1.1 as r_{df} goes from r_c to r_o.

For each of the Galactocentric radii (r_1,r_2,r_o), eqn (5.7) specifies the mass for which the timescale t_{df} equals the age of the Galaxy:

$$M_1 = 6\times10^5(t_g/10^{10}y)^{-1}M_\odot$$

$$M_2 = 3\times10^6(t_g/10^{10}y)^{-1}(r_c/2\mathrm{kpc})[f_1(r_2)/0.6]^{-3/2}M_\odot \quad (5.9)$$

$$M_o = 4\times10^7(t_g/10^{10}y)^{-1}[f_2(r_o)/1.1]^{-2}M_\odot$$

Note that $f_1(r_2)$ and $f_2(r_o)$ themselves depend weakly on r_c. If the halo objects all have the same mass M, then comparison with these values immediately indicates the Galactocentric radius within which halo objects are dragged into the nucleus by now. In the Lacey-Ostriker scenario (see §6), $M=2\times10^6 M_\odot$ and so r_{df} lies between r_1 and r_2 but generally M could lie in any range. Although other dynamical limits may preclude *all* the halo being in objects as large as the mass-scales indicated by eqn (5.9), we must also consider the situation in which the halo fraction is small.

Eqn (5.7) has the feature that r_{df}, regarded as a function of either M or t_g, jumps discontinuously to r_4 once it reaches r_2. The value of r_4 comes from substituting $M=M_2$ in eqn (5.7c), so it is given implicitly by

$$r_4 = 3(r_c/2\text{kpc})^{1/2}[f_1(r_2)/0.6]^{-3/4}[f_2(r_4)/1.5] \qquad (5.10)$$

Generally r_4 lies between r_2 and r_3, so one should not strictly apply eqn (5.7c) (which assumes halo domination of the mass) all the way down to r_4. However, since the extra contribution of the disc ensures that the rotation curve remains flat well below r_3, the error involved in this extrapolation is only small. Note also that expressions (5.7a) and (5.7b) are discontinuous at $r=r_1$. This is because the expression for t_{dr} in the $r>r_1$ regime omits the time required to drift through the $r<r_1$ regime and this cannot be neglected just above r_1. Figure (5b), which shows the dependence of r_{df} upon M, smooths out this discontinuity.

The total mass dragged into the Galactic nucleus is just the halo mass contained within the radius r_{df}. From eqn (B16) this is

$$M_N = \begin{cases} (4\pi/3) f_h \rho_c r_{df}^3 \approx 7\times10^8 (r_c/2\text{kpc})^{-2} (r_{df}/\text{kpc})^3 M_\odot & (r_{df}<r_c) \\ 4\pi f_h \rho_c r_c^2 r_{df} \approx 8\times10^9 (r_{df}/\text{kpc})(1-2r_c/3r)\, M_\odot & (r_{df}>r_c) \end{cases} \qquad (5.11)$$

where $\rho_c \approx 0.16 (r_c/2\text{kpc})^{-2} M_\odot \text{pc}^{-3}$ is the core density and f_h is the initial fraction of the halo density in the compact objects (assumed to be independent of r). For the different radial regimes we therefore have

$$M_N = \begin{cases} 7\times10^8 f_h (M/10^6 M_\odot)^{10/7} (t_g/10^{10}\text{y})^{10/7} (r_c/2\text{kpc})^{-2} M_\odot & (M<M_1) \\ 2\times10^9 f_h (M/10^6 M_\odot)^2 f_1(M)^3 (t_g/10^{10}\text{y})^2 (r_c/2\text{kpc})^{-2} M_\odot & (M_1<<M<M_2) \\ 8\times10^9 f_h (M/10^6 M_\odot)^{1/2} f_2(M) (t_g/10^{10}\text{y})^{1/2} M_\odot & (M>>M_2) \end{cases} \qquad (5.12)$$

where we have put $(1-2r_c/3r_{df})\approx0.8$ in the last expression. M_N exceeds the upper observational limit of $3\times10^6 M_\odot$ on the central dark mass (Sellgren et al. 1990, Spaenhauer et al. 1992) unless

$$f_h < \begin{cases} (M/2\times10^4M_\odot)^{-10/7}(r_c/2\mathrm{kpc})^2(t_g/10^{10}y)^{-10/7} & (M<M_1) \\ (M/4\times10^4M_\odot)^{-2}f_1(M)^{-3}(r_c/2\mathrm{kpc})^2(t_g/10^{10}y)^{-2} & (M_1<<M<M_2) \\ (M/0.1M_\odot)^{-1/2}f_2(M)^{-1}(t_g/10^{10}y)^{-1/2} & (M>>M_2) \end{cases} \qquad (5.13)$$

These limits, which are shown in Figure (5c), are sensitive to the age of the Galaxy and the halo core radius but the dominant constituent of the halo must anyway be smaller than

$$M_{max}=2\times10^4(t_g/10^{10}y)^{-1}(r_c/2\mathrm{kpc})^{1.4}M_\odot \qquad (5.14)$$

Eqn (5.13) extends the CL limit and also modifies the dynamical friction limit shown in Carr (1994). Eqns (5.12) and (5.13) are discontinuous at the mass junctions for the same reason that eqn (5.7) is but these discontinuities are smoothed out in the figures.

Although this argument would seem to preclude supermassive black holes as halo objects, there is an important caveat in this conclusion (Hut & Rees 1992). One can use eqn (5.12) to determine the number of holes which have drifted into the Galactic nucleus by now (M_N/M) and hence the time between their arrivals (Mt_g/M_N). [In the Lacey-Ostriker scenario (discussed in §6) about 10^3 holes have drifted into the Galactic nucleus, corresponding to one arrival every 10^7y.] Once two black holes have reached the nucleus, they will form a binary system and this will eventually coalesce due to energy loss through gravitational radiation. If a third hole arrives before coalescence occurs, then the "slingshot" mechanism could eject one of the holes and the remaining pair might also escape due to the recoil (Saslaw et al. 1974). In the Lacey-Ostriker scenario, Hut & Rees estimate that the time for binary coalescence is *shorter* than the interval between infalls, suggesting that slingshot is ineffective, and this conclusion would apply for a wide range of M. However, there is another problem with applying eqn (5.5): since the energy lost by the infalling holes must be gained by the stars responsible for the drag, these stars will tend to drift outwards, so their number density in the central region will be depleted. Eventually this will suppress dynamical friction altogether unless there is an efficient mechanism to replenish the loss-cone (Begelman et al. 1980). If the halo objects are not black holes, the slingshot mechanism is probably too weak to eject them from the nucleus. Since limit (5.13) is not completely firm, it is only shown dotted in Figure (7).

6. Disc Heating by Halo Compact Objects

As halo objects traverse the Galactic disc, they will impart energy to the stars there. This will lead to a gradual puffing up of the disc, with older stars being heated more than younger ones. This problem was first analysed by Lacey (1984) and Lacey & Ostriker (1985), who argued that black holes of around $10^6 M_\odot$ could provide the best mechanism for generating the observed amount of disc-puffing (Wielen 1977). In particular, they claimed that this could explain: (1) why the velocity dispersion of disc stars σ scales with age as $t^{1/2}$; (2) the relative velocity dispersions in the radial, azimuthal and vertical directions; and (3) the existence of a high energy tail of stars with large velocity (cf. Ipser & Semenzato 1985). In order to normalize the $\sigma(t)$ relationship correctly, the number density of the holes n must satisfy $nM^2 \approx 2\times10^4 M_\odot{}^2 pc^{-3}$ and combining this with the local halo density $\rho_h = nM \approx 0.01 M_\odot pc^{-3}$ gives $M \approx 2\times10^6 M_\odot$.

This argument may no longer be compelling because more recent measurements give smaller velocity dispersions for older stars, so that σ may no longer rise as fast as $t^{1/2}$ and may even be flat for the oldest stars (Carlberg et al. 1985, Knude et al. 1987, Stromgren 1987, Gomez et al. 1990, Meusinger et al. 1991). Heating by a combination of spiral density waves, giant molecular clouds and infalling satellite galaxies may now give a better fit to the data (Lacey 1991). On the other hand, this conclusion is disputed by Wielen et al. (1992) and Fuchs et al. (1996) who argue that both the velocity dispersion and metallicity dispersion data indicate $\sigma \sim t^{1/2}$; they suggest that the conflict may result from the use of a sample which is unrepresentative in that it avoids old metal-weak stars. Wielen & Fuchs (1988) claim that black hole heating also explains the dependence of the velocity dispersion upon Galactocentric distance. In any case, one can still use the Lacey-Ostriker argument to place an upper limit on the density in halo objects of mass M, as emphasized by Carr et al. (1984). We briefly review and update the analysis here.

Lacey & Ostriker start with Chandrasekar's (1960) expressions for 2-body encounters and then use various approximations in applying them to study the interactions between disc stars and halo black holes. The holes are assumed to be much more massive than the stars and to have an isothermal distribution with an isotropic Maxwellian velocity distribution. Their 1-dimensional velocity dispersion σ_h is taken to be much larger than that of the stars and the halo rotation is presumed negligible. The three components of the velocities in the radial, tangential and vertical directions (u,v,w) have dispersions σ_u, σ_v and σ_w which evolve as

$$\sigma_u(t) = (\sigma_{uo}^2 + D_e t)^{1/2}$$

$$\sigma_v(t) = (\sigma_{vo}^2 + \beta^{-2} D_e t)^{1/2} \qquad (6.1)$$

$$\sigma_w(t) = (\sigma_{wo}^2 + D_z t)^{1/2}$$

where (σ_{uo}, σ_{vo}, σ_{wo}) are the initial dispersions and

$$\beta \equiv 2\Omega/\omega = [1+(1/2)(d\ln\Omega/d\ln r)]^{-1/2} \qquad (6.2)$$

with Ω and ω being the orbital and horizontal epicyclic *frequencies* respectively. The diffusion coefficients in eqn (6.1) are

$$D_e \equiv 2\pi G^2 n M^2 \ln\Lambda \; \{2\beta^2 A(v_c/\sqrt{2}\sigma_h) + B(v_c/\sqrt{2}\sigma_h)\}(\sigma_h^2/v_c^3)$$

$$D_z \equiv 2\pi G^2 n M^2 \ln\Lambda \; B(V_c/\sqrt{2}\sigma_h)/(\sigma_h^2/v_c^3) \qquad (6.3)$$

$$A(x) \equiv \mathrm{erf}(x) - x(d/dx)\mathrm{erf}(x), \qquad B(x) \equiv (2x^2-1)\mathrm{erf}(x) + x(d/dx)\mathrm{erf}(x)$$

where v_c is the rotational velocity of the stars and $\Lambda \approx m_{gal}/M$ from eqn (5.5b). The total velocity dispersion therefore *evolves* according to

$$\sigma(t) \equiv (\sigma_u{}^2 + \sigma_v{}^2 + \sigma_w{}^2)^{1/2} = (\sigma_o^2 + \alpha G^2 n M^2 t/v_c)^{1/2} \qquad (6.4)$$

where the dimensionless quantity α is given by

$$\alpha \equiv 4\pi \ln\Lambda \; \{(1+\beta^2)A(v_c/\sqrt{2}\sigma_h) + (1+1/2\beta^2)B(v_c/\sqrt{2}\sigma_h)\}(\sigma_h/v_c)^2 \qquad (6.5)$$

If the observed total velocity dispersion is σ_{obs}, this yields a limit

$$\rho M < \sigma_{obs}^2 v_c/(\alpha t_s G^2) \qquad (6.6)$$

where $\rho = nM$ is the local density of the holes and t_s is the time for which the stars have been heated (i.e. their age). Note that the form of eqn (6.6) can also be inferred from the first expression in eqn (4.1a) if we regard the Galactic disc as a "cluster" which is being heated by encounters. However, since the disc is only being puffed up and not completely disrupted, the limit is reduced by a factor $(\sigma/v_c)^2$.

To apply this result we need to evaluate the various numerical factors in eqn (6.5). Lacey uses $\ln\Lambda \approx 13$, $\beta \approx 1.5$ and $v_c/\sqrt{2}\sigma_h \approx 1$ [implying $A(v_c/\sqrt{2}\sigma_h) \approx 0.3$ and $B(v_c/\sqrt{2}\sigma_h) \approx 0.?$]. The value of σ_{obs} depends on which stars one considers: Ostriker & Lacey took it to vary from $20\,\mathrm{kms^{-1}}$ for A-F stars with an age of 10^9y to $60\,\mathrm{kms^{-1}}$ for K-M stars with an age of 10^{10}y (this being compatible with the $\sigma \sim t^{1/2}$ relation). The estimate for the older stars is now in dispute but these are the ones which give the strongest limit if σ grows more slowly than $t^{1/2}$. Assuming a local halo density of $\rho_h \approx 0.01 M_\odot \mathrm{pc^{-3}}$, eqn (6.6) gives a maximum mass for the black holes which dominate the halo of

$$M_{max} = 3\times10^6 (\rho_h/0.01 M_\odot \mathrm{pc^{-3}})^{-1} (\sigma_{obs}/60\,\mathrm{kms^{-1}})^2 (t_s/10^{10}\mathrm{y})^{-1} M_\odot \quad (6.7)$$

and the general limit on the halo fraction in black holes of mass M is

$$f_h < \min\,[1, (M/M_{max})^{-1}] \quad (6.8)$$

This limit is shown by the broken line in Figure (5c).

By applying the same analysis to the giant Sc galaxy NGC 3198, using the velocity dispersion determined by Bottema (1988) and the halo parameters derived by van Albada et al. (1985), Friese et al. (1995) obtain an upper limit of $2\times10^6 M_\odot$, which is about the same as that obtained for the Milky Way. Since M_{max} scales as $\rho_h^{-1}\sigma^2 t_s^{-1}$, one can also apply the disc-heating argument to galaxies with higher dark matter density or lower stellar velocity dispersion or larger age to obtain even stronger constraints (Fuchs & Wielen 1993). For the gas-rich dwarf galaxy DD0154 (which has $\sigma = 17\,\mathrm{kms^{-1}}$, an age of at least 1.5 Gyr and a central dark matter density of $0.009 M_\odot \mathrm{pc^{-3}}$), Rix & Lake (1993) find $M < 7\times10^5 M_\odot$. For the dwarf galaxy GR8 (which has $\sigma = 4\,\mathrm{kms^{-1}}$, an age of at least 1 Gyr and a central dark matter density of $0.07 M_o\,\mathrm{pc^{-3}}$), they find $M < 6\times10^3 M_\odot$. Friese et al. (1995) argue that the light from these galaxies is dominated by bright young stars which would not have experienced much heating anyway. On the other hand, Fuchs et al. (1996) show that the halo of DDO154, if made of $10^6 M_\odot$ black holes, would have become very diluted by now as a result of 2-body relaxation. In any case, unless the black holes form pregalactically, there is no reason for expecting halo objects to have the same mass in different galaxies, so these limits are not shown in Figure (7).

7 Dark Clusters

We have seen that both the cluster disruption and dynamical friction constraints may be incompatible with the proposal that the halo is populated with supermassive black holes. There is also the problem that halo black holes might generate too much radiation through accretion as they traverse the disc (Ipser & Price 1977, 1982, Carr 1981, McDowell 1985, Heckler & Kolb 1996). However, it is still possible that the halo is made of supermassive objects which are themselves *clusters* of black holes of more modest mass. The accretion luminosity is then reduced by a factor of order the number of objects per cluster (since the Bondi accretion rate and hence the luminosity scale as the square of the black hole mass) and the dynamical friction problem is avoided provided the clusters are disrupted by collisions before they are dragged into the Galactic nucleus by dynamical friction. Even if the clusters comprise objects other than black holes, so that the accretion problem is circumvented, it is still necessary to invoke collisional disruption to avoid the dynamical friction problem.

The dark cluster proposal was originally examined by Carr & Lacey (1987; CL) in an attempt to salvage the Lacey-Ostriker scenario for disc-heating by $2\times10^6 M_\odot$ black holes. We have seen that this scenario may no longer be plausible but the cluster proposal itself is still viable and indeed arises very naturally in many models for Population III formation (Ashman 1990). Kerins & Carr (1994; KC) therefore generalized the idea to a scenario in which the clusters have arbitrary mass and are made of brown dwarfs. Wasserman & Salpeter (1994) applied the same idea to study clusters of neutron stars. Moore & Silk (1995) extended these calculations, allowing for two extra dynamical effects: the disruption of globular clusters by the dark clusters (cf. §4) and the destruction of the dark clusters themselves by the Galactic tidal field. Further studies of the cluster scenario have been made by De Paolis et al. (1995) and Kerins (1997). We review and update these calculations here.

Let us assume that the dark clusters all have the same mass M_C and radius R_C. They may then be disrupted by essentially the same processes discussed in §3, except that the "black hole" in that analysis is replaced by another cluster of the same mass as the one being disrupted (i.e. $M=M_C$). This means that one-off disruption (which requires $M>>M_C$) never occurs but multiple encounters between clusters will still lead to their disruption. For simplicity we assume that the clusters have a uniform density. A necessary condition for disruption is that the 3-dimensional velocity dispersion within the clusters, which is given by eqn (4.4) with $\gamma=0.3$, be less than the relative 3-dimensional velocity dispersion ($\sqrt{2}V_h$) of the halo objects. Outside the halo core, $V_h \approx 270 kms^{-1}$ and so we require

$$R_c > 0.02(M_c/10^6 M_\odot) \text{ pc} \qquad (7.1)$$

[Eqn (B20) implies that V_h decreases at smaller Galactic radii, reaching a minimum which is smaller by a factor $\sqrt{5}/3 \approx 0.7$ at the core radius itself; although this would increase the coefficient in eqn (7.1) to 0.03, the effect of the disc and spheroid stars will counteract this, so we will always normalize V_h to $270 kms^{-1}$.] We will find that condition (7.1) is always satisfied for the clusters of interest. However, one also requires enough encounters within the age of the Galaxy for disruption to occur and this condition need not be. CL show that the disruption time can be approximated as

$$t_{dis} = (\sqrt{3}/20\sqrt{\pi}) V_h/(G f \rho_h R_c)$$

$$\approx 2\times10^{10} f_h^{-1}(V_h/270 kms^{-1})(\max[r,r_c]/2kpc)^2 (R_c/pc)^{-1} y \qquad (7.2)$$

where f_h is the fraction of the halo in the clusters and we have used eqn (5.2) for the halo density. [This is the same as the first expression in eqn (3.23a) with $M=M_c$ except that the numerical coefficient is changed because eqn (3.2), from which it derives, must be modified for clusters of equal mass.] The disruption timescale is minimized at the halo core radius and then increases with Galactocentric radius. Therefore, if disruption is to occur at all, it must do so at r_c and the condition for this is

$$R_c > 1.8 f_h^{-1}(r_c/2kpc)^2 (t_g/10^{10}y)^{-1}(V_h/270 kms^{-1}) \text{ pc} \qquad (7.3)$$

If this is satisfied, clusters will also be disrupted within a Galactocentric radius

$$r_{dis} \approx 1.5(R_c/pc)^{1/2}(V_h/270 kms^{-1})^{-1/2}(t_g/10^{10}y)^{1/2} f_h^{1/2} \text{ kpc} \qquad (7.4)$$

Indeed it is then a general feature of the cluster scenario, as emphasized by De Paolis et al. (1995), that there is some Galactocentric radius within which the halo is broken up into its cluster components.

The dynamical friction limit will be obviated providing r_{dis} exceeds the radius r_{df} derived in §4. CL assumed that the only source of dynamical friction is the spheroid stars. For general M, eqn (5.7b) with f_1=0.6 then implies that dynamical friction is inoperative providing

$$R_c > 0.3(M_c/10^6M_\odot)^{4/3}(V_h/270kms^{-1})(t_g/10^{10}y)^{1/3}f_h^{-1}pc \qquad (M_1<M_c<M_2) \qquad (7.5)$$

where M_1 and M_2 are given by eqn (5.9). However, we have seen that collisions - if important at all - necessarily persist beyond the halo core and, since r_c always exceeds the radius r_2, it is more appropriate to use eqns (5.7c). Instead of limit (7.5) we then obtain

$$R_c > 0.8(M_c/10^6M_\odot)(V_h/270kms^{-1})f_h^{-1}\,pc \qquad (M_c>M_4) \qquad (7.6)$$

where we have taken f_2=1.1 in eqn (5.7c). If this condition is not satisfied, then M_c must be less than the value indicated by eqn (5.14). In order to avoid the clusters evaporating as a result of 2-body relaxation within the age of the Galaxy, one requires another lower limit on the cluster radius:

$$R_c > 0.04(m_*/0.01M_\odot)^{2/3}(t_g/10^{10}y)^{2/3}(M_c/10^6M_\odot)^{-1/3}\,pc \qquad (7.7)$$

where m_* is the mass of the components. [Note that KC used the wrong exponent for the M_c term but this is corrected in Carr (1994).]

CL imposed an *upper* limit on R_c on the grounds that they wanted the clusters to provide disc-heating down to at least 4 kpc. This condition need not applied here. However, another upper limit comes from requiring that the clusters do not disrupt at our own Galactocentric radius r_o~8kpc and from eqn (7.4) this implies

$$R_c < 30(r_o/8kpc)^2(V_h/270kms^{-1})\,(t_g/10^{10}y)^{-1}pc \qquad (7.8)$$

If this condition were not satisfied, the disc-heating constraint would not apply since the clusters would not survive long enough to heat the disc even locally. Together with eqns (6.7) and (7.7), eqn (7.8) requires that the mass of the cluster components satisfy

$$m_* < 200(M_c/10^6M_\odot)^{1/2}(V_h/270kms^{-1})^{3/2}(t_g/10^{10}y)^{-5/2}M_\odot$$
$$< 300(V_h/270kms^{-1})^{3/2}(t_g/10^{10}y)^{-5/2}M_\odot \qquad (7.9)$$

This probably excludes their being VMO black holes but not ordinary stellar black holes or neutron stars.

Two other upper limits on R_C can be imposed. Clusters at radius r will be destroyed by the Galactic tidal field unless (CL)

$$R_C < \{M_C/m(r)[3-(d\ln m/d\ln r)]\}^{1/3} r \qquad (7.10)$$

where m(r) is the total mass within radius r. At our own Galactocentric radius [where the halo dominates the mass and one is outside the halo core, so that (dlnm/dlnr)=1], this gives [cf. Moore & Silk (1995)]

$$R_C < 100(M_C/10^6 M_\odot)^{1/3} pc \qquad (7.11)$$

Dark clusters will also be disrupted by tidal shocking as they traverse the Galactic disc on a timescale

$$t_{dis} \sim M_C/(\rho_d R_C^2 V_C) \sim M_C^{1/2}/(G^{1/2} R_C^{3/2} \rho_d) \qquad (7.12)$$

where ρ_d is the disc density; this is less than the age of the Galaxy for

$$R_C < 5(t_g/10^{10}y)^{-2/3}(M_C/10^6 M_\odot)^{1/3}\ pc \qquad (7.13)$$

Finally we must consider how the limits discussed in earlier sections are modified if the compact objects are replaced by dark clusters. The disc-heating limit still applies except that one may no longer get a high velocity tail of stars with $V > 100 kms^{-1}$. CL showed that the condition for this is just

$$R_C < 0.5(M_C/10^6 M_\odot)\ pc \qquad (7.14)$$

and this is incompatible with the disruption condition (7.6). The globular cluster disruption limit discussed in §4 must be modified if the radius of the dark cluster exceeds the radius of the globular cluster (here denoted by R_{GC}). In this case, only the second part of the p-integral in eqn (3.19) applies and the lower limit in the integral becomes R_C rather than R_{GC}. This means that the disruption time is increased by a factor $(R_C/R_{GC})^2$ and eqn (4.3b) becomes

$$M_{max} = (\sqrt{3}/20\sqrt{\pi})(M_C V_h R_C^2)/(G \alpha t_L R_{GC}^3) \quad (R_C > R_{GC}) \qquad (7.15)$$

Thus the upper limit on the dark cluster mass increases as R_c^2 if the dark cluster is larger than the globular cluster.

These dynamical limits are indicated in Figure (6). This shows that the values of M_c and R_c are constrained to a rather narrow wedge. The globular cluster limit is shown with a broken line since (as discussed in §4) the interpretation of this limit is not completely clear. The imposition of condition (7.6) only shaves off a small corner of the permitted region and (as anticipated) the impulsive condition (7.1) is always satisfied. There is some uncertainty in the position of the dynamical friction boundary since this is sensitive to the halo core radius: the limit is shown for r_c=2kpc and r_c=8kpc since this spans the range of likely values. The evaporation limit (7.7) also depends on m_*: Figure (6) assumes $m_*=0.02M_\odot$ (corresponding to the brown dwarf scenario) and $m_*=0.2M_\odot$ (corresponding to the white dwarf scenario).

8. Constraints on Intergalactic Dark Objects

The dynamical constraints on dark intergalactic objects are much weaker than those for objects in halos and clusters. The most interesting one comes from the fact that, if there were a population of huge intergalactic objects, each galaxy would have a peculiar velocity due to its gravitational interaction with the nearest one (Carr 1978). If the objects were smoothly distributed, with number density density n_D and density parameter Ω_D, the typical distance between them would be

$$d \approx n_D^{-1/3} \approx 30\Omega_D(M)^{-1/3}(M/10^{16}M_\odot)^{1/3}\, h^{-2/3}\ \mathrm{Mpc} \qquad (8.1)$$

where $h \equiv H_0/100$ is Hubble parameter. This would also be the expected distance of the nearest object to a typical galaxy like our own. If the objects have a characteristic peculiar velocity V, the distance they traverse over the age of the Universe ($t_0 \approx 10^{10}h^{-1}$y) would be about $10(V/10^3\mathrm{kms}^{-1})h^{-1}$Mpc. This is well below d for $M>10^{15}M_\odot$, so one can regard the distance to the nearest dark object as being essentially constant in this case. Over the age of the Universe the nearest one will therefore induce a peculiar velocity in the Milky Way of about

$$V_{pec} \approx GMt_0/d^2 \approx 500h^{1/3}\Omega_D(M)^{2/3}(M/10^{16}M_\odot)^{1/3}(t_0/10^{10}h^{-1}y)\ \mathrm{km\ s^{-1}} \qquad (8.2)$$

Since the microwave background dipole anisotropy shows that the peculiar velocity of our own Galaxy is only 400kms^{-1}, one infers

$$\Omega_D < (M/5\times10^{15}M_\odot)^{-1/2}(t_o/10^{10}h^{-1}y)^{-3/2}h^{-1/2} \qquad (8.3)$$

and this is shown in Figure (7). This updates the limit first given by Carr (1978). Note that nothing in this argument requires that the dark object be a black hole or even compact. The same analysis would apply for a dark supercluster or indeed the sort of "Great Attractor" invoked to explain large-scale streaming motions.

The requirement that there be at least one object of mass M within the current particle horizon implies a *lower* limit

$$\Omega_D > 3\times10^{-8}(M/10^{16}M_\odot)(t_o/10^{10}h^{-1}y)^{-3}\, h \qquad (8.4)$$

where we have used eqn (8.1) with $d=3ct_o\approx10h^{-1}$Gpc (the current horizon size if the Universe has the critical density). This intersects eqn (8.3) at a mass $8\times10^{20}(t_o/10^{10}y)M_\odot$, so this corresponds to the largest possible dark object within the visible Universe. Note that the gravitational lensing limits on Ω_D associated with multiple-imaging of distant quasars (Press & Gunn 1979) are not useful in the mass range considered here since the separation between the images would be too large for them to be identified.

9. Conclusions

The various dynamical limits discussed in this paper are brought together in Figure (7). This shows the upper limits on the fraction of the Galactic disc, the Galactic halo, clusters of galaxies and the background Universe in compact objects of mass M. In order to put the limits on one diagram, they are expressed in terms of the density parameter $\Omega_{CO}(M)$ associated with the compact objects, where the disc, halo and cluster dark matter are assumed to have density parameters 0.001, 0.1 and 0.2, repectively. However, this representation is merely used for convenience and the actual densities assumed for the different sites are not crucial. Anyway we cannot assume that the limits obtained for the Milky Way apply to all other galactic discs and halos. Figure (7) updates and - in some respects corrects - Figure (1) of Carr (1978) and Figure (4) of Carr (1994).

The limits on the bottom right of Figure (7) correspond to the requirement that there be at least one object of mass M within each site (i.e. within the Galactic disc, the Galactic halo, a typical cluster and the current particle horizon). We term these "incredulity" limits since one's belief in the compact dark object hypothesis is irrelevant beneath these lines. Although the incredulity limits are computationally trivial, they are useful since they imply absolute upper limits on the mass of compact objects in different locations. The limiting mass is $10^5 M_\odot$ for the disc, $10^9 M_\odot$ for the halo, $10^{12} M_\odot$ for clusters and (as discussed above) $10^{20} M_\odot$ for the Universe. Many other constraints - based on nucleosynthetic, background light, source count and gravitational lensing considerations - can be placed on the density of compact objects in different locations and these are reviewed by Carr (1994). In general the dynamical limits are most useful in the very high mass regime (where disruptive effects are important) and the very low mass regime (where encounter effects are important).

Providing the dark objects are not clustered, the various limits already identify the most plausible candidates (Carr 1997). If the disc matter is real, it is probably in the form of brown dwarfs. The halo dark matter could consist at least partly of Population III remnants and white or brown dwarfs are the favoured candidates from a theoretical point of view, although the microlensing data currently indicate a lens mass in the white dwarf range (Alcock et al. 1996). The background dark matter must be mainly non-baryonic if inflation requires a critical density but one cannot exclude primordial black holes, in which case these objects might also provide the halo dark matter. It has been claimed that microlensing of distant quasars already provides evidence for the primordial black hole proposal (Hawkins 1993, 1996) but this claim is controversial. Unless one invokes an unconventional cosmological nucleosynthesis scenario, the cluster dark matter must also be mainly non-baryonic, although there would need to be some baryonic fraction if halos are themselves baryonic. Although the favoured unclustered candidates are all in the mass range for which dynamical effects are unimportant, in many scenarios one would expect the dark objects to cluster and the dynamical limits are then crucial.

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APPENDIX A: DISRUPTION OF CLUSTERS

We first derive eqn (3.2) using an approach similar to that of Gerhard & Fall (1983). If the black hole has mass M and, at closest approach, position $\mathbf{p}=p\mathbf{e_p}$ and velocity $\mathbf{V}=V\mathbf{e_v}$ relative to the cluster centre (where $\mathbf{e_p}$ and $\mathbf{e_v}$ are orthogonal unit vectors), then the velocity change induced in a star at position $\mathbf{r}$ relative to the cluster centre can be approximated by

$$\delta\mathbf{u} \approx 2GMV^{-1}(p^2+2r^2/3)^{-1}\{2p^2(p^2+2r^2/3)^{-1}(\mathbf{r}.\mathbf{e_p})\mathbf{e_p}+(\mathbf{r}.\mathbf{e_v})\mathbf{e_v}-\mathbf{r}\} \qquad \text{(A1)}$$

this expression covering both the $p>R_C$ and $p<R_C$ situation. The mean-square value of $\delta\mathbf{u}$ for stars at a distance r from the cluster centre is therefore

$$<(\delta\mathbf{u})^2> \approx (8/3)(GMr/Vp^2)^2(1+4r^4/9p^4)(1+2r^2/3p^2)^{-4} \qquad \text{(A2)}$$

where in averaging over the direction of $\mathbf{r}$ we have used the approximation $(\mathbf{r}.\mathbf{e_v})^2=(\mathbf{r}.\mathbf{e_p})^2=r^2/3$. To obtain the total change in the energy of the stars in the cluster, ΔE, we must integrate $\rho<(\delta\mathbf{u})^2>/2$ over all values of r and this gives

$$\Delta E = (4/3)(GM/Vp^2)^2 \int_0^{R_C} d^3\mathbf{r}\, r^2\rho(r)(1+4r^4/9p^4)(1+2r^2/3p^2)^{-4} \qquad \text{(A3)}$$

For impact parameters much larger than R_C we can neglect the last two terms in eqn (A3). For p much smaller than R_C we can approximate these terms as $9p^4/4r^4$ in the $p<r<R_C$ part of the integral; this approximation fails for $0<r<p$ but this part of the integral is anyway negligible. Thus in general we can approximate ΔE by eqn (3.2) where

$$\alpha^2 \equiv R_C^{-2}\, M_C^{-1}\int_0^{R_C} d^3\mathbf{r}\, r^2\rho(r)\,, \qquad \beta^2 \equiv R_C^{2}M_C^{-1}\int_0^{R_C} d^3\mathbf{r}\, r^{-2}\rho(r) \qquad \text{(A4)}$$

For multiple disruption the rate of change of energy in the cluster is obtained by multiplying $<(\delta\mathbf{u})^2>/2$ [given by eqn (A2)] by $dN(p,V)/dt$ [given by eqn (3.15)] and then integrating over r, p and V. For a Maxwellian velocity distribution, one obtains

$$dE/dt = (4\pi^{1/2}nG^2M^2)/(3\sigma R_c^2)\int_{V_{min}}^{\infty}dz\ z\ \exp(-z^2/4)\ \int_{p_{dis}}^{p_{max}}dy\ \int d^3\mathbf{r}\ \rho(r)r^2H(y,z,\lambda,r)$$

where

$$H \equiv y^{-3}(1+4r^4/9R_c^4y^4)(1+2r^2/3R_c^2y^2)^{-4}$$
$$\times\ [1-\lambda y^2z^{-2}(1+y^2)^{-3/2}]\ \exp[\lambda(1+y^2)^{-1/2}/2] \qquad \text{(A5)}$$

Here $y \equiv p/R_c$, $z \equiv V/\sigma$, $\lambda \equiv G(M+M_c)/R_c\sigma^2$ and the (y,z) integral is carried over the range of values associated with region A in Figures (2). If all the holes have the same velocity V_o, one obtains

$$dE/dt = (8\pi nG^2M^2)/(3V_oR_c^2)\int_{p_{dis}}^{p_{max}}dy\ \int d^3\mathbf{r}\ \rho(r)r^2H'(y,r)$$

where

$$H' \equiv y^{-3}(1+4r^4/9R_c^4y^4)(1+2r^2/3R_c^2y^2)^{-4}$$
$$\times\ [1-\lambda' y^2(1+y^2)^{-3/2}]\ [1+2\lambda'(1+y^2)^{-1/2}] \qquad \text{(A6)}$$

Here $\lambda' \equiv G(M+M_c)/R_cV_o^2$ and the other symbols are defined as before.

A precise evaluation of the integrals in eqns (A5) and (A6) can only be done numerically but various approximations permit an analytical evaluation. Firstly, since the minimum velocity compatible with multiple-disruption well exceeds $V_e(p)$, we can neglect the difference between V and V_∞ and between p and p_∞, i.e. we can drop the last two terms in the expressions for H and H'. Secondly, to simplify the p integral, we can split it into two regimes, one with $p<R_c$ and the other with $p>R_c$, and then use approximation (3.2). For a discrete velocity distribution, eqn (A6) gives eqns (3.19)-(3.21). For Maxwellian velocity distribution eqn (A5) gives

$$dE/dt = (\pi^{1/2}nG^2M^2M_c)/(3\sigma R_c^2)\left[\int_{V_{max}}^{\infty}\left[9\beta^2\int_0^{R_c}pdp + 4\alpha^2\int_{R_c}^{p_{max}}R_c^4p^{-3}dp\right]\right.$$
$$\left. +\int_{V_{min}}^{V_{max}}\left[4\alpha^2\int_{p_{dis}}^{p_{max}}R_c^4p^{-3}dp\right]\right]z\ \exp(-z^2/4)\ dz \qquad \text{(A7)}$$

For $\sigma > V_{max}$, the second velocity integral is negligible and we can approximate the first velocity integral by putting $V_{min} \approx 0$. One then obtains eqn (3.19) but with V replaced by $\sqrt{\pi}\sigma$. For $V_{max} > \sigma > V_{min}$, the first velocity integral is negligible and the second one can be approximated by putting $V_{min} \approx 0$ and $V_{max} \approx \infty$. This yields eqn (3.20). For $\sigma < V_{min}$, there are two exponentially damped contributions: one [scaling as $\exp(-M^{2/3})$] deriving from the impulsive encounters associated with the holes with velocity V_{min} on the exponential tail of the Maxwell velocity distribution; the other [scaling as $\exp(-M)$] deriving from the the non-impulsive encounters. The first effect dominates and gives eqn (3.22).

In applying these formulae to particular astronomical situations, we need to estimate the parameters α, β and γ. We will model open clusters as *homogenous* spheres since they are very diffuse. In this case there is a sharp cut-off at radius R_C and we have $\alpha=0.8$, $\beta=1.7$ and $\gamma=0.3$. We will also model dark clusters in this way. This may not be a very realistic assumption but we adopt it for simplicity and also because it is not clear what is better. We will model globular clusters as *Plummer* spheres with total mass M_C and length-scale ε. In this case the density profile is

$$\rho(r) = 3M_C(4\pi\varepsilon^3)^{-1}[1+(r/\varepsilon)^2]^{-5/2} \qquad \text{(A8)}$$

where $R_C=1.3\varepsilon$ is interpreted as the half-mass radius since the sphere extends to infinity. This implies $\alpha=3$, $\beta=1$ and $\gamma=(3\pi/64)=0.15$. We will model galaxies as *isothermal* spheres with a constant density core of radius r_C and a cut off at some radius R_C :

$$\rho(r) = M_C(4\pi R_C)^{-1}(r^2+r_C{}^2)^{-1}\theta(R_C-r) \qquad \text{(A9)}$$

where θ is the Heaviside function. The cut-off must be specified since the mass increases indefinitely with radius. In this case, $\alpha=0.6$, $\beta=(R_C/r_C)^{1/2}=2\text{-}3$ and $\gamma=3/4$. Eqn (A9) is supposed to specify the density distribution of all the matter in galaxies (including their dark halos) and not just the visible stars.

APPENDIX B: DYNAMICAL FRICTION FROM SPHEROID AND HALO

Our aim here is to evaluate eqn (5.5) for the different sources of dynamical drag. Carr & Lacey (1987) have analysed the effect of spheroid stars on halo objects and, for completeness, we first repeat their analysis here. We introduce dimensionless variables

$$x=r/r_1,\ \tilde{\rho}=\rho_s/\rho_1,\ \tilde{m}=m/(4\pi\rho_1 r_1^3),\ \tilde{v}=v/(4\pi G\rho_1 r_1^2)^{1/2},\ \tilde{t}=t(4\pi G\rho_1)^{1/2} \quad \text{(B1)}$$

Eqn (5.1) implies for $\{x<1,\ x>1\}$

$$\tilde{\rho} = \{x^{-9/5},\ x^{-3}\} \quad \text{(B2)}$$

$$\tilde{m} = \{(5/6)x^{6/5},\ 5/6 + \ln x\} \quad \text{(B3)}$$

$$\tilde{v}_c^2 = \tilde{m}(x)/x = \{(5/6)x^{1/5},\ (5+6\ln x)/(6x)\} \quad \text{(B4)}$$

$$1 + (d\ln\tilde{v}_c/d\ln x) = \{11/10,\ (11+6\ln x)/2(5+6\ln x)\} \quad \text{(B5)}$$

$$\ln\Lambda = \ln(\tilde{m}/\tilde{M}) = \{(6/5)\ln x - \ln(6\tilde{M}/5),\ \ln[1+(6/5)\ln x] - \ln(6\tilde{M}/5)\} \quad \text{(B6)}$$

The 1-dimensional velocity dispersion is determined from

$$\tilde{\sigma}(x)^2 = \tilde{\rho}(x)^{-1}\int_x^{\infty} \tilde{\rho}(y)y^{-2}\tilde{m}(y)dy \quad \text{(B7)}$$

and this gives

$$\tilde{\sigma}(x)^2 = \{(25/48)x^{1/5}-x^{9/5}/4,\ \ln x/(4x) + 13/(48x)\} \quad \text{(B8)}$$

$$\tilde{v}_c/(\sqrt{2}\,\tilde{\sigma}) = \{[20/(25-12x^{8/5})]^{1/2},\ 2[(5+6\ln x)/(13+12\ln x)]^{1/2}\}$$

$$\approx \{2/\sqrt{5},\ \sqrt{2}\}\ \text{for}\ \{x<<1,\ x>>1\} \quad \text{(B9)}$$

The last two equations are slightly innacurate because they neglect the different density distribution outside the radius r_2 where the halo dominates the spheroid but the error is small. Eqn (5.5a) becomes

$$dr/dt = -\eta\ \ln\Lambda(x)\ \{(12\sqrt{6}/11\sqrt{5})A(2/\sqrt{5})x^{-11/10},$$

$$(12\sqrt{6})A(\sqrt{2})x^{-1/2}(11+6\ln x)^{-1}(5+6\ln x)^{-1/2}\} \quad \text{(B10)}$$

$$\eta = G^{1/2}M/(\sqrt{4\pi}\rho_1{}^{1/2}r_1{}^2) \approx 2\times10^{-8}(M/10^6M_\odot)\ \mathrm{pc\ y^{-1}} \qquad \text{(B11)}$$

where $A(x)$ is defined by eqn (5.5c) and we use $4\pi\rho_1 r_1{}^3=1.2\times10^{10}M_\odot$. For present purposes we neglect the dependence of $\ln\Lambda$ on x and write

$$\ln\Lambda \approx -\ln[3M/(10\pi\rho_1 r_1{}^3] \approx 9 - \ln(M/10^6M_\odot) \qquad \text{(B12)}$$

so this term is about 9. We also use $A(2/\sqrt{5})\approx0.3$ and $A(\sqrt{2})\approx0.7$. In a time t compact objects of mass M will therefore drift into the Galactic nucleus from a Galactocentric radius

$$r_{df} = \begin{cases} (6.8\eta t)^{10/21} r_1{}^{11/21} & (r_{df} < r_1) \qquad \text{(B13a)} \\ (11\eta t)^{2/3} r_1{}^{1/3}[1+(6/11)\ln(r_{df}/r_1)]^{-2/3}[1+(6/5)\ln(r_{df}/r_1)]^{-1/3} & (r_{df} > r_1) \qquad \text{(B13b)} \end{cases}$$

where we have used eqn (5.6) in determining the numerical coefficients and we regard the dependencies on r_{df} on the right-hand-side as weak. Inserting the values for η, t and r_1 then gives eqns (5.7a) and (5.7b).

We now carry out a similar analysis for the "halo" drag. We introduce dimensionless variables

$$x=r/r_c,\ \tilde{\rho}=\rho_h/\rho_c,\ \tilde{m}=m/(4\pi\rho_c r_c{}^3),\ \tilde{v}=v/(4\pi G\rho_c r_c{}^2)^{1/2},\ \tilde{t}=t(4\pi G\rho_c)^{1/2} \qquad \text{(B14)}$$

noting that these are different from the ones used in the spheroid case. Let us first neglect the effect of the extra mass within the spheroid. Eqn (5.2) then implies for $\{x<1,\ x>1\}$

$$\tilde{\rho} = \{1,\ x^{-2}\}\ , \qquad \text{(B15)}$$

$$\tilde{m} = \{x^3/3,\ \ x-2/3\} \qquad \text{(B16)}$$

$$\tilde{v}_c{}^2 = \tilde{m}(x)/x = \{x^2/3,\ \ 1-2/(3x)\} \qquad \text{(B17)}$$

$$1 + (d\ln\tilde{v}_c/d\ln x) = \{2,\ (3x-1)/(3x-2)\} \qquad \text{(B18)}$$

$$\ln\Lambda = \{3\ln x - \ln(3\tilde{M}_c),\ \ln(x-2/3) - \ln\tilde{M}_c\} \qquad \text{(B19)}$$

and eqn (B7) gives

$$\tilde{\sigma}(x)^2 = \{4/9-x^2/6,\ \ (1/2)-2/(9x)\} \qquad \text{(B20)}$$

$$\tilde{v}_C/(\sqrt{2}\ \sigma) = \{\sqrt{3}\ x/\sqrt{8-3x^2},\ \sqrt{3(3x-2)/(9x-4)}\}$$

$$\approx \{\sqrt{3/8}\ x,\ 1\} \text{ for } \{x<<1,\ x>>1\} \qquad \text{(B21)}$$

Eqn (5.5a) becomes

$$dr/dt = -\eta\ \ln\Lambda(x)\{A(\sqrt{3/8}x)(3\sqrt{3}/2)x^{-2}),\ \ A(1)x^{-1}(1-1/3x)^{-1}(1-2/3x)^{-1/2}\} \qquad \text{(B22)}$$

$$\eta = G^{1/2}M/(\sqrt{4\pi}\rho_C{}^{1/2}r_C{}^2) \approx 1.2\text{x}10^{-8}(r_C/2\text{kpc})^{-1}(M/10^6M_\odot)\ \text{pc y}^{-1} \qquad \text{(B23)}$$

where we use $4\pi\rho_C r_C{}^3 = 1.6\text{x}10^{10}(r_C/2\text{kpc})M_\odot$. As before we neglect the dependence of $\ln\Lambda$ on x and write

$$\ln\Lambda \approx -\ln[\{3,1\}M/(4\pi\rho_C r_C{}^3] \approx \{9,10\} - \ln(M/10^6M_\odot) + \ln(r_C/2\text{kpc}) \qquad \text{(B24)}$$

so this term is always about 10. We also use $A(1)\approx0.3$. In a time t compact objects of mass M will therefore drift into the Galactic nucleus from a Galactocentric radius

$$r_{df} = \begin{cases} (70A(\sqrt{3/8}x)\eta r_C{}^2t)^{1/3} & (r_{df} < r_C) \qquad \text{(B25a)} \\ (6\eta r_C t)^{1/2}[1-(r_C/3r_{df})]^{-1/2}[1-(2r_C/3r_{df})]^{-1/4} & (r_{df} > r_C) \qquad \text{(B25b)} \end{cases}$$

where we have used eqn (5.6) and we regard the dependency on r_{df} on the right-hand-side as weak. Evaluating the terms in eqn (B25b) yields eqn (5.7c).

We now consider how this analysis can be modified to allow for the excess mass from the spheroid. The problem is that, although the *density* is dominated by the halo beyond r_2, the *mass* is dominated by the spheroid up to the radius r_3 given by the eqn (5.4). Therefore, as jusified in §5, for $r_2<r<r_3$ we must use the halo expression for the ρ term in eqn (5.5a) but the spheroid expression for all the other terms. Instead of the first expression in eqn (B22), we then obtain

$$dr/dt = -\eta \ln\Lambda(x)\ (12\sqrt{6})B(\sqrt{2})(11+6\ln x)^{-1}(5+6\ln x)^{-1/2}$$

$$\times \{0.09(r_c/2\mathrm{kpc})^{-2}x^{5/2},\ 0.6x^{1/2}\} \text{ for } \{r_2<r<r_c, r_c<r<r_3\} \qquad \text{(B27)}$$

This shows that the drift timescale $r/(dr/dt)$ is a *decreasing* function of r for $r_c>r>r_2$ and so, as discussed in §5, the time to reach the origin is the drift timescale at r_2 for some range $r_2>r_{df}>r_4$. This means that the function $r_{df}(M)$ jumps discontinuously from r_2 to the value r_4 indicated by eqn (5.10).

FIGURE CAPTIONS

Figure (1). The constraints on the fraction of the Galactic disc (broken line) and the Galactic halo (solid line) in compact objects of mass M. The shaded regions are excluded by collisional disruption, by upper limits on the frequency of meteors, fireballs and interstellar comets, and by the number of terrestrial impact craters. The limits are mainly based on updated versions of the calculations of Hills (1986).

Figure (2). The regimes of velocity V and impact parameter p for which compact objects of mass M disrupt a cluster of mass M_C and velocity dispersion V_C through (A) multiple encounters and (B) one-off disruption. V necessarily exceeds the escape velocity, so one must be above the shaded region, but there is a small region (C) in which the impulse approximation fails (so that disruption is weak). (a) shows the $M>M_C$ case; (b) the $M<M_C$ case.

Figure (3). The constraint on the density ρ_{CO} of compact objects of mass M associated with the disruption of clusters mass M_C, radius R_C, internal velocity dispersion V_C and lifetime t_L. The shaded region is excluded by multiple-encounter disruption for $M<M_C(V/V_C)$ and by single-encounter disruption for $M_C(V/V_C)^3>M>M_C(V/V_C)$. The upper limit cuts off for $M>M_C(V/V_C)^{1/3}$ since the impulse approximation then fails. V is the velocity of the compact objects, interpreted as the 3-dimensional velocity dispersion for a Maxwellian distribution. One has an interesting constraint providing the density at which the limit bottoms out is less than the actual density.

Figure (4). The fraction of the Galactic disc, the Galactic halo and clusters of galaxies in compact objects of mass M. The shaded regions are excluded by the disruption of binaries, open clusters, globular clusters and galaxies.

Figure (5). (a) The dependence on Galactocentric distance r of the timescale t_{dr} on which halo objects are dragged into the Galactic nucleus through dynamical friction. The different regimes correspond to the drag being dominated by the inner spheroid, the outer spheroid and the outer halo. (b) The M-dependence of the Galactocentric radius r_{df} at which dynamical friction can drag halo objects into the nucleus within the age of the Universe. (c) The constraint on the fraction f_h of the halo in compact objects of mass M, assuming no slingshot mechanism operates. The disc-heating limit is also shown (dotted).

Figure (6). The domain of mass M_C and radius R_C which could be occupied by dark clusters. The boundaries are associated both with disruptive effects which would destroy the clusters within the age of the Universe (collisions, the Galactic tidal field, disc-shocking, 2-body relaxation) and dynamical consequences which would be inconsistent with observation (dynamical friction, globular cluster disruption, disc-heating). The globular cluster limit is shown broken because it is not completely secure. The dynamical friction limit depends on the halo core radius and is shown for values of 2kpc and 8kpc. The 2-body relaxation limit depends on the mass of the cluster components and is shown for values of $0.01 M_\odot$ and $0.5 M_\odot$.

Figure (7). The dynamical constraints on the density parameter Ω_{CO} for compact objects of mass M in the Galactic disc, the Galactic halo, clusters of galaxies and the intergalactic medium. The total density parameter associated with each of these sites is taken to be 0.001, 0.1, 0.2 and 1, respectively. This figure brings together the constraints in Figures 1, 4 and 5, as well as the large-scale streaming and incredulity limits.

Table (1). Possible locations of dark baryons and formation epochs of compact objects.

Table (2). Parameters assumed and constraints derived for the disruption of globular clusters (GC) by halo objects, open clusters (OC) and binaries (B) by disc objects, and galaxies (G) by cluster objects. The maximum density of the objects, the maximum mass for which they can contain all the dark mass and the minimum mass for which the limits can be obviated are indicated. The constraints assume the values for α, β and γ appropriate in each context.

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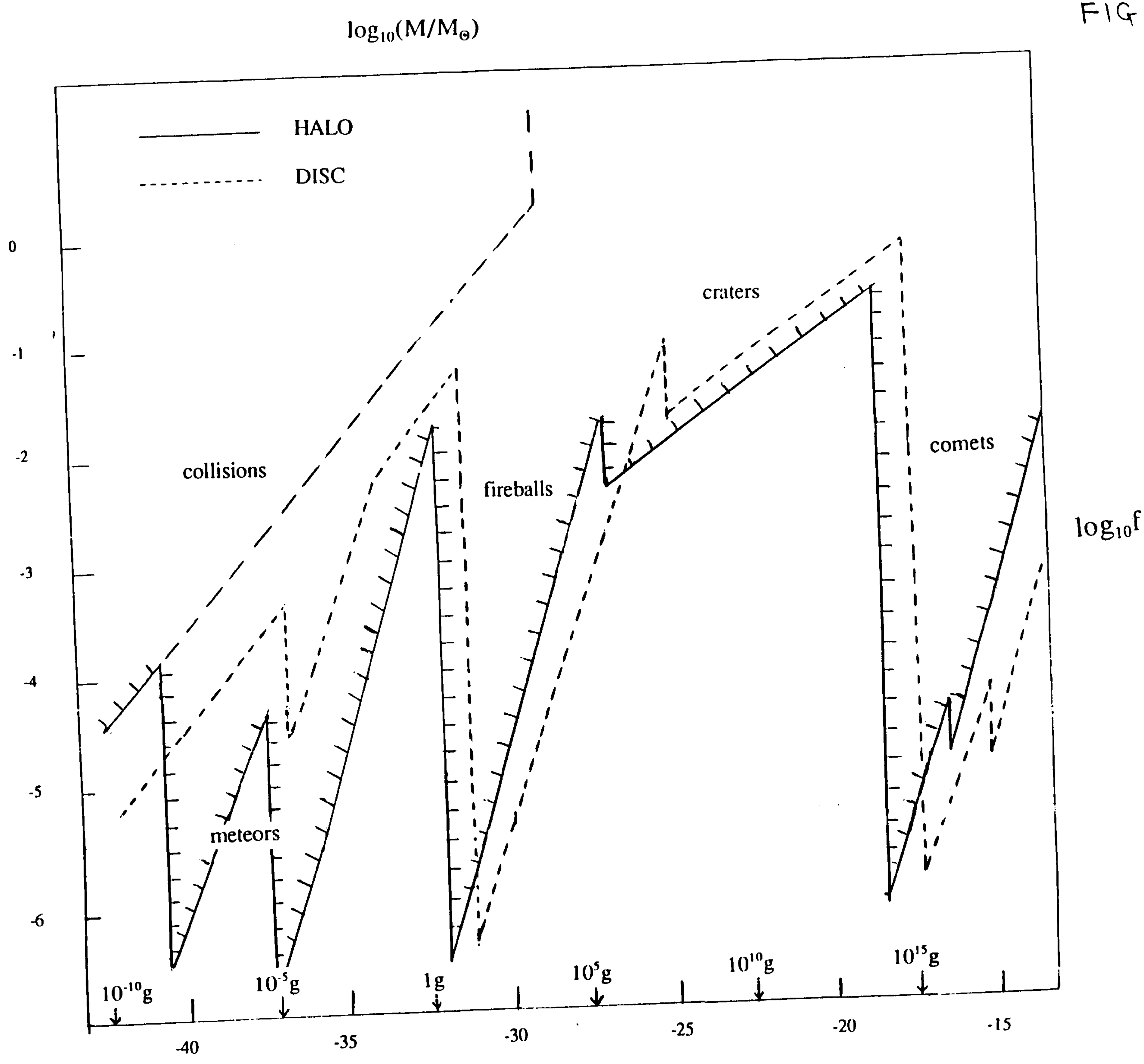
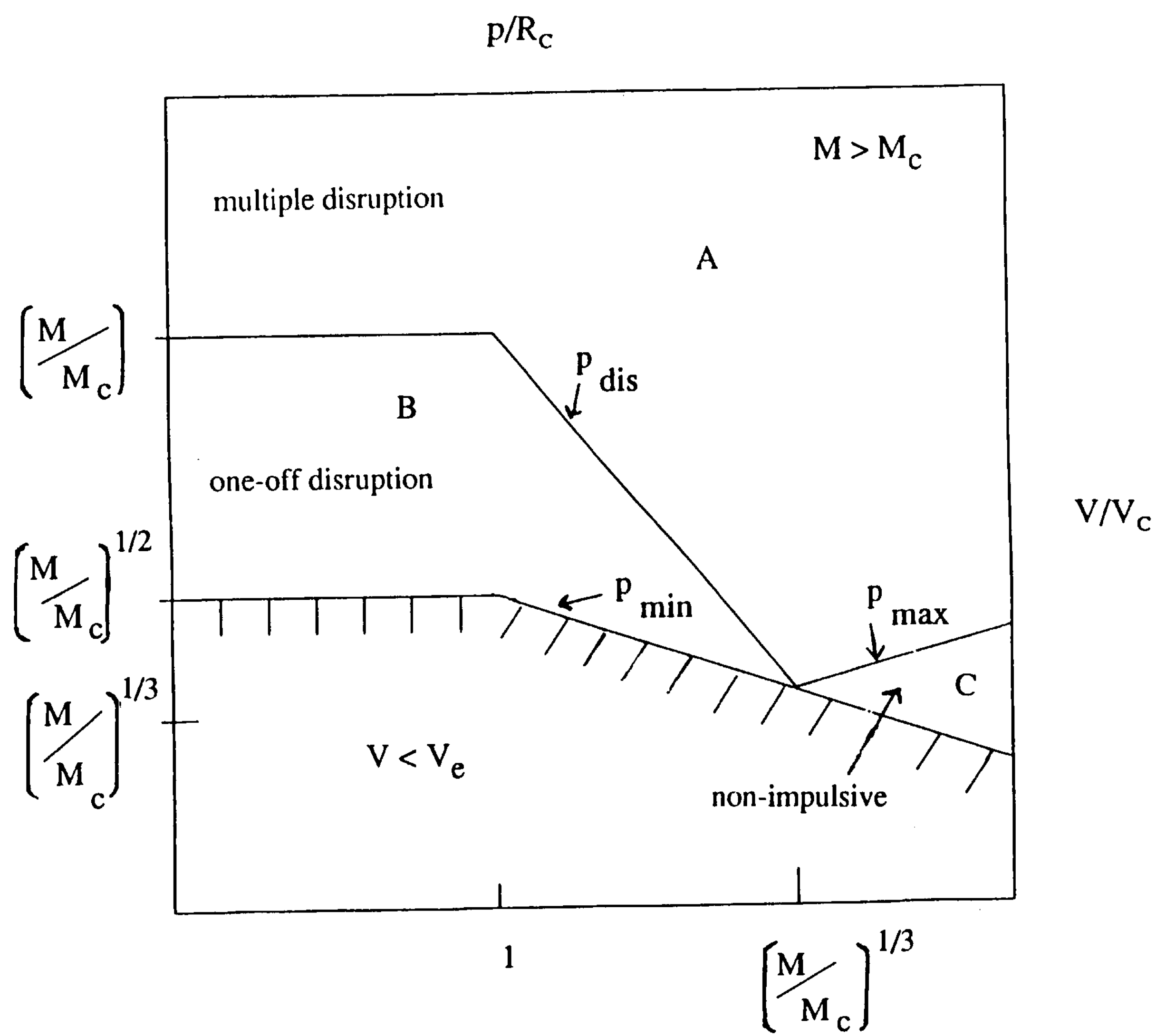
FIG (1)



log10(M/M⊙)
HALO
DISC
collisions
meteors
fireballs
craters
comets
log10f
0
-1
-2
-3
-4
-5
-6
-40
-35
-30
-25
-20
-15
10-10g
10-5g
1g
105g
1010g
1015g

FIG (2a)



p/R_c
M > M_c
multiple disruption
A
(M/M_c)
p dis
B
one-off disruption
V/V_c
(M/M_c)^1/2
p min
p max
C
(M/M_c)^1/3
V < V_e
non-impulsive
1
(M/M_c)^1/3

FIG (2b)

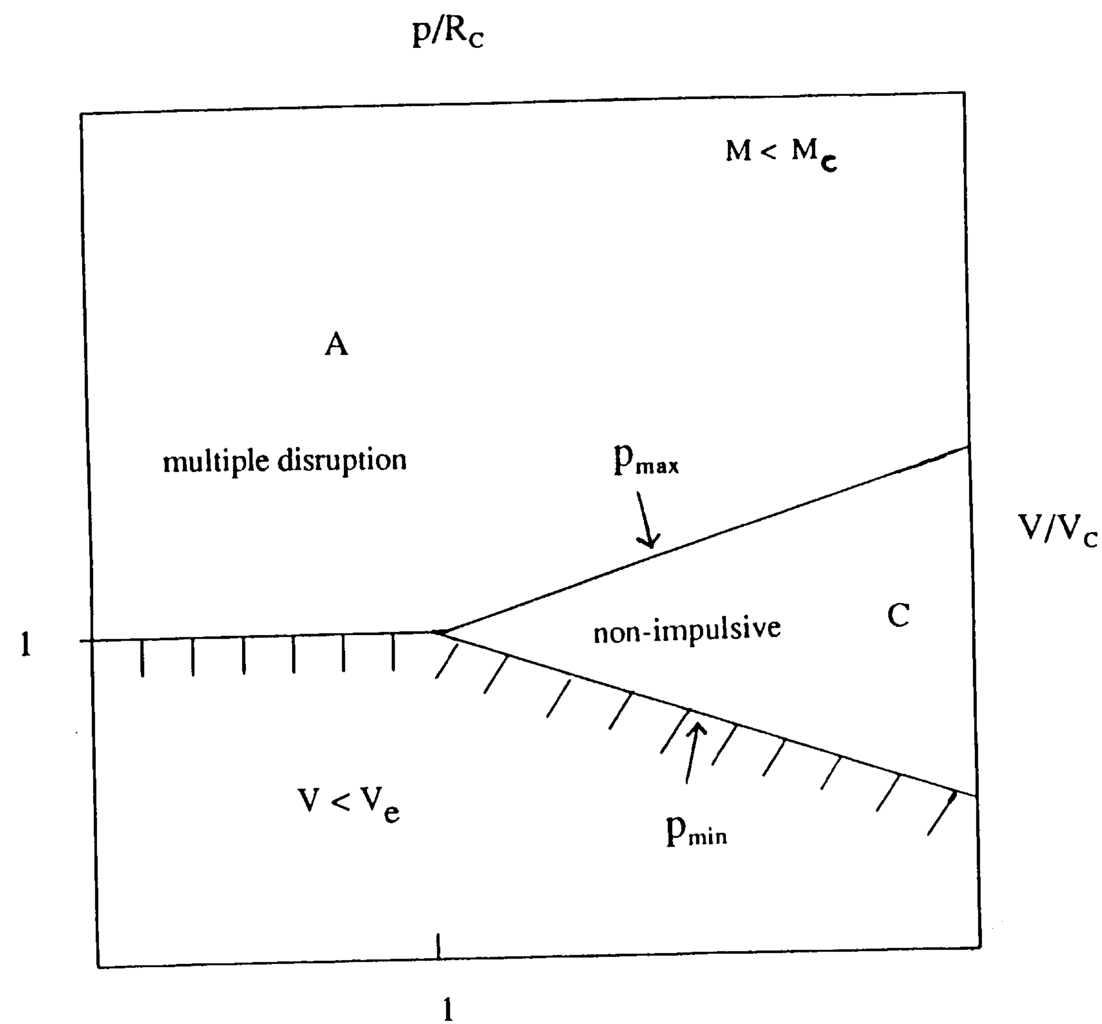

p/R_c
M < M_c
A
multiple disruption
p_max
V/V_c
1
non-impulsive
C
V < V_e
p_min
1

FIG (3)

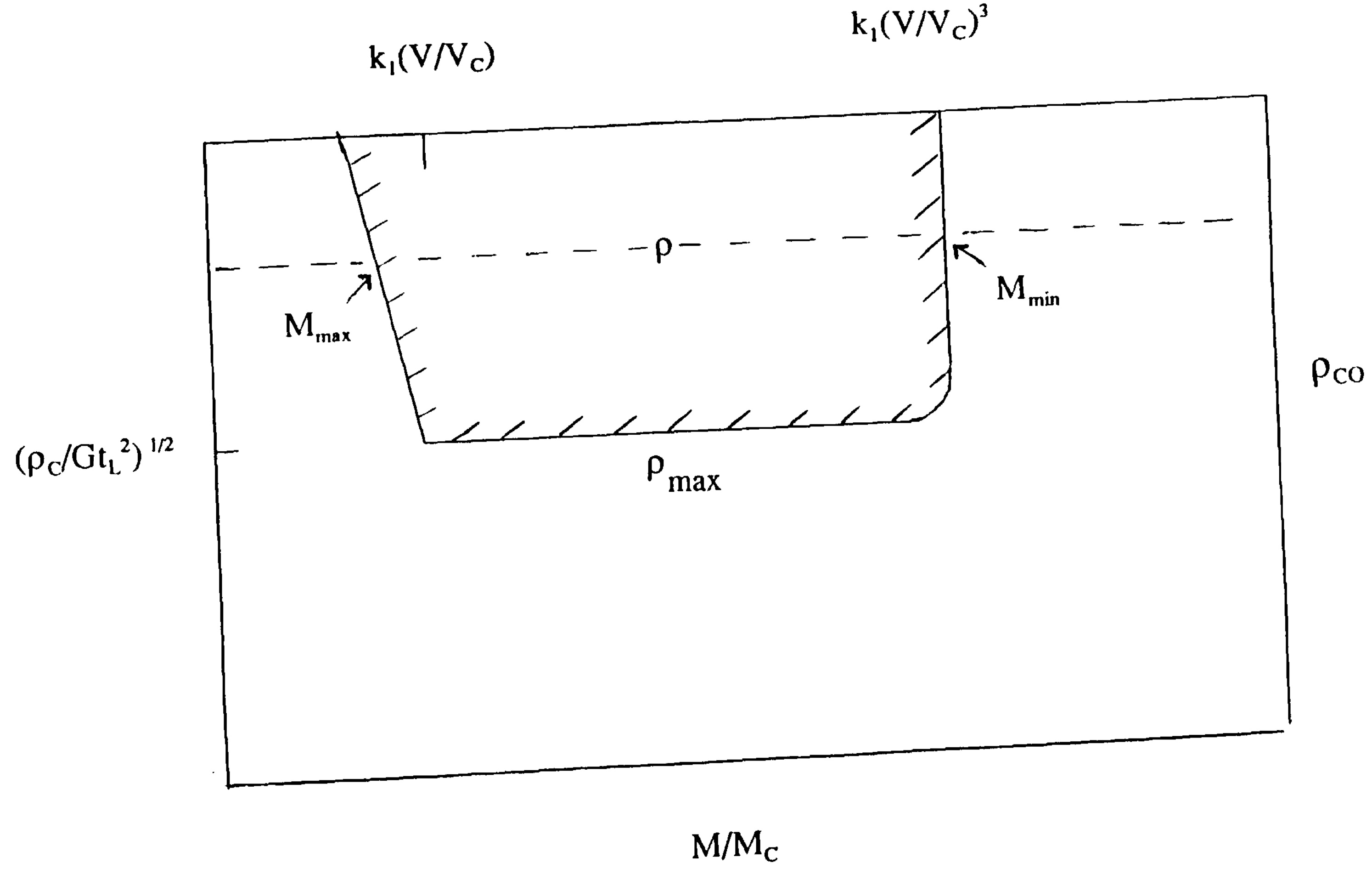


FIG (4)

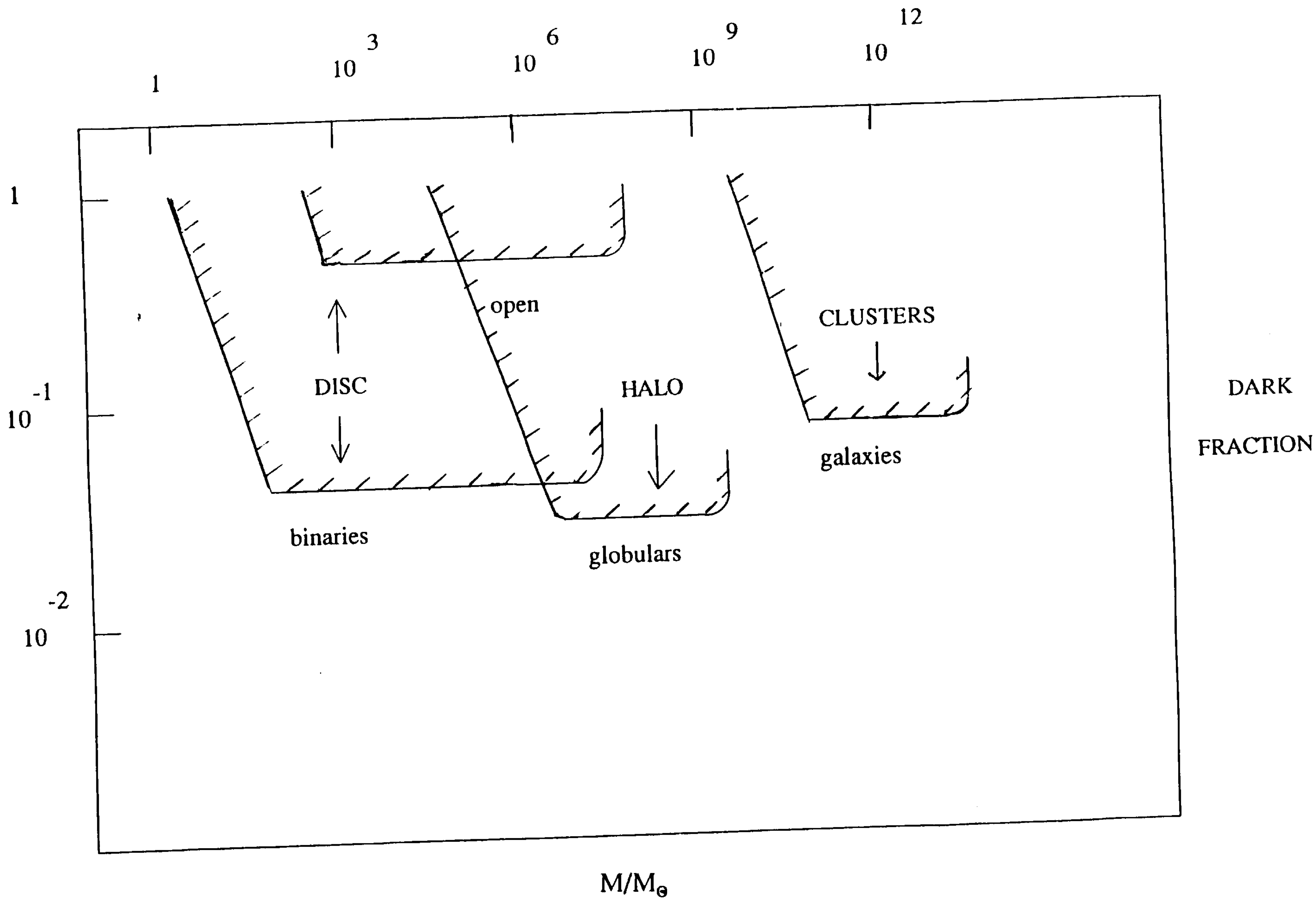
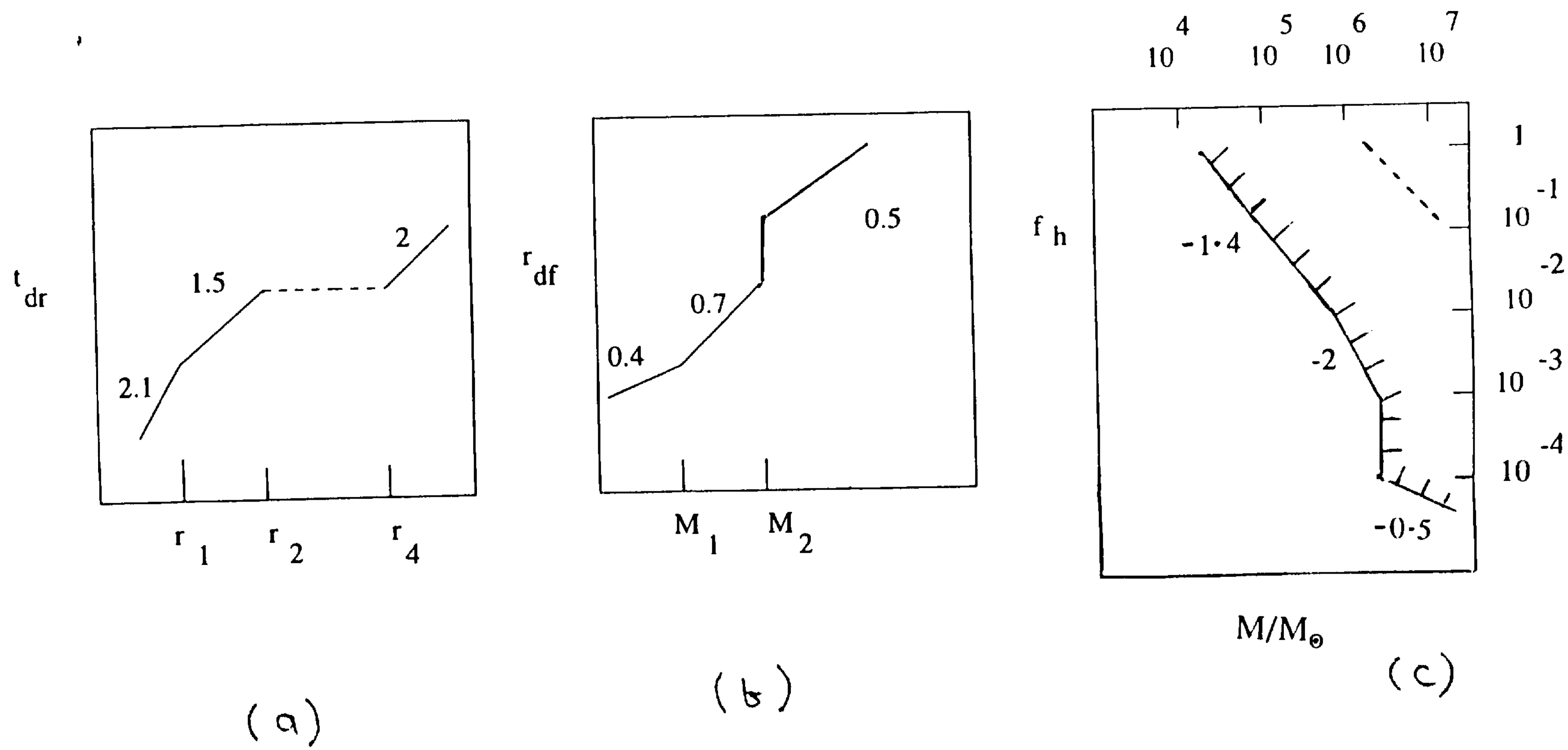
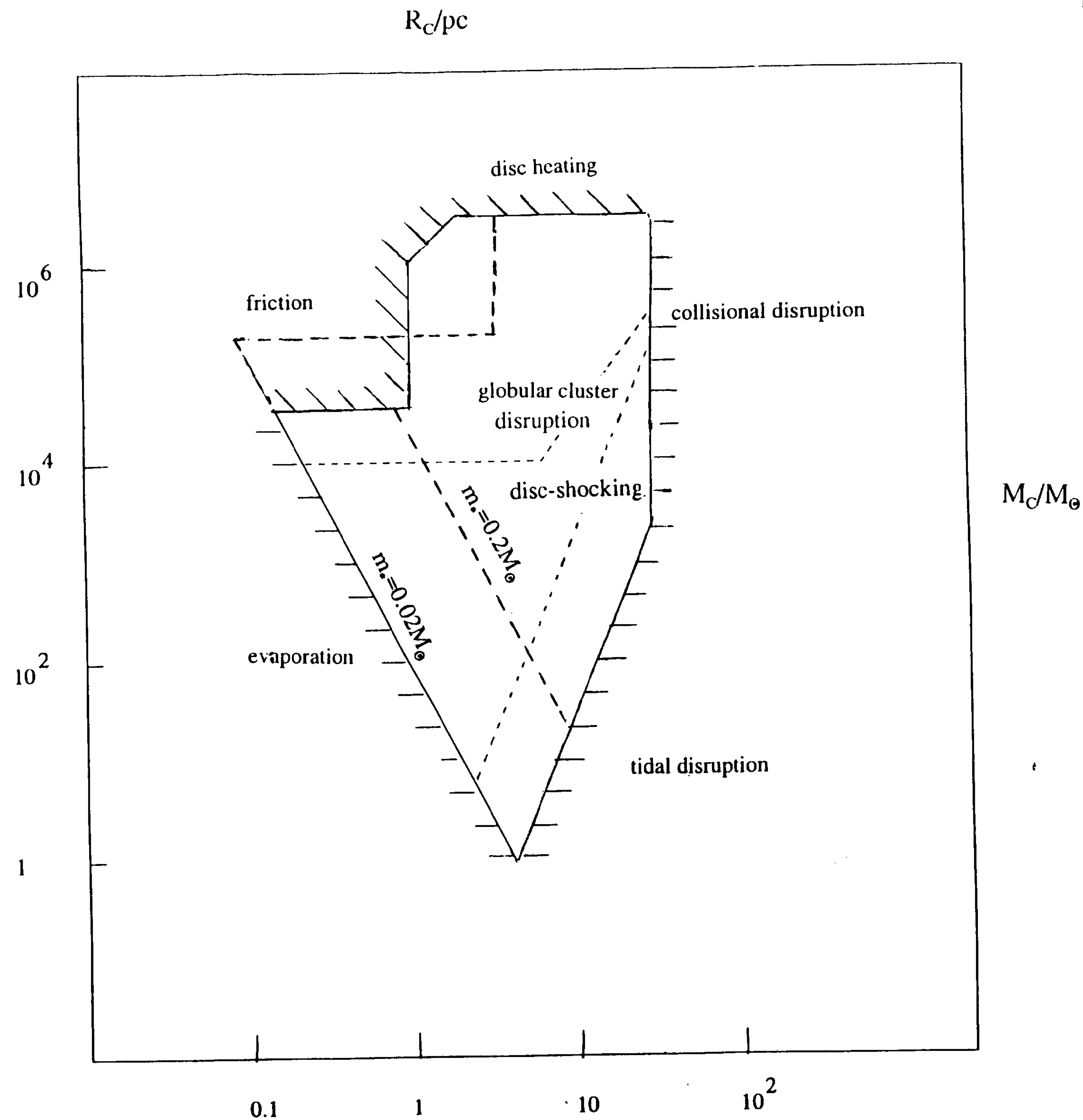

1
10 3
10 6
10 9
10 12
1
10 -1
10 -2
open
CLUSTERS
DISC
HALO
binaries
globulars
galaxies
DARK
FRACTION
M/M☉

FIG (5)



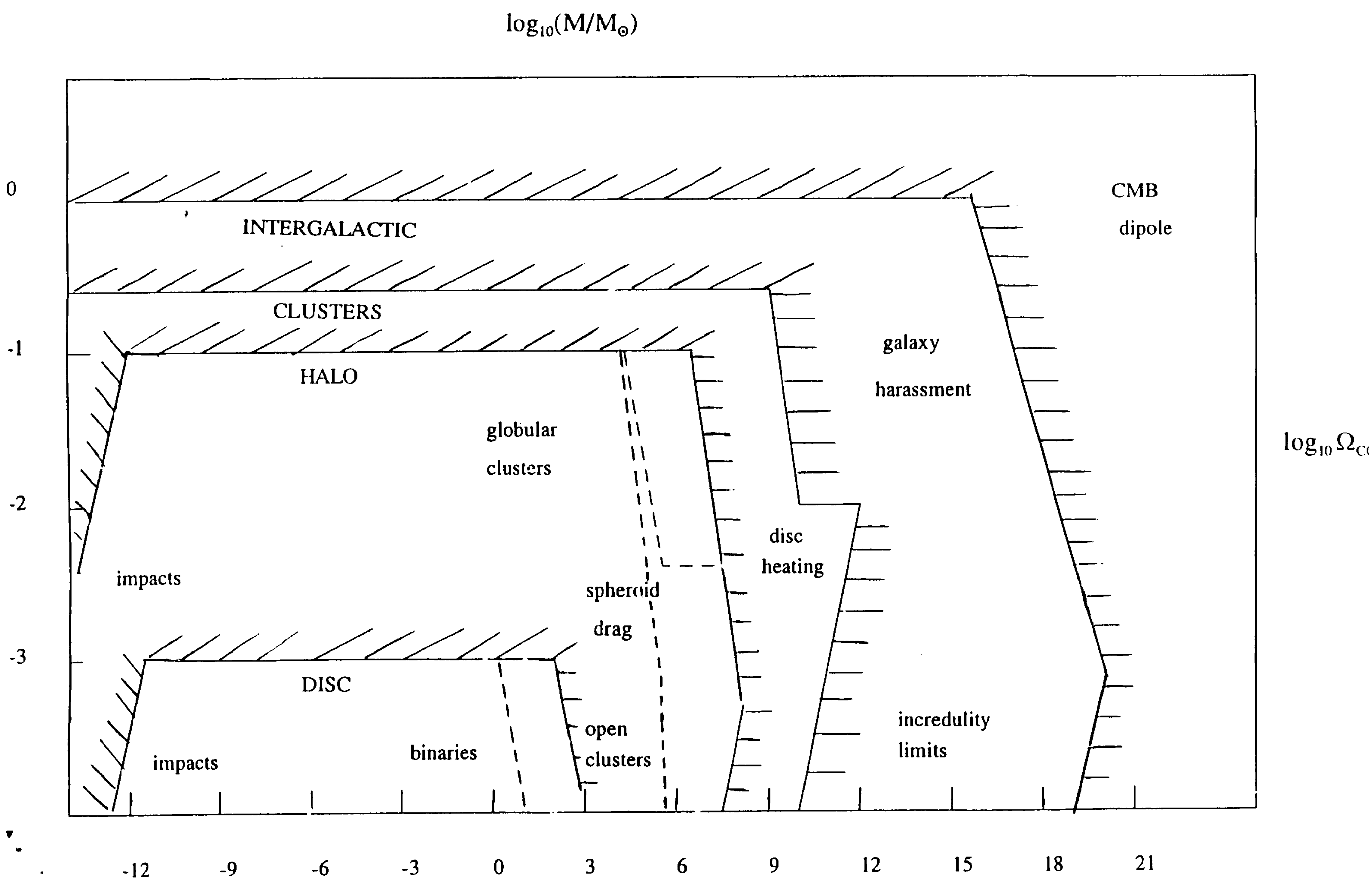
t_{dr}
2.1
1.5
2
r_1
r_2
r_4
(a)
r_{df}
0.4
0.7
0.5
M_1
M_2
(b)
10^4
10^5
10^6
10^7
f_h
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
-1·4
-2
-0·5
$M/M_\odot$
(c)

FIG



R_C/pc
disc heating
friction
collisional disruption
globular cluster
disruption
disc-shocking
$m_*=0.2M_\odot$
$m_*=0.02M_\odot$
evaporation
tidal disruption
$M_C/M_\odot$
10^6
10^4
10^2
1
0.1
1
10
10^2

FIG (7)



$\log_{10}(M/M_\odot)$
0
-1
-2
-3
INTERGALACTIC
CLUSTERS
HALO
DISC
globular
clusters
impacts
impacts
binaries
spheroid
drag
open
clusters
disc
heating
galaxy
harassment
incredulity
limits
CMB
dipole
-12
-9
-6
-3
0
3
6
9
12
15
18
21
$\log_{10}\Omega_{CO}$