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Abstract The underlying connection between channeling radiation and laser x-ray sources based on Thomson scattering has been established using a recently developed virtual quanta technique employing the Correspondence Principle. It is shown that these two types of radiation are very similar to each other but channeling radiation represents a more general case. For electron beam energies in the $10 \div 100 MeV$ regime the x-ray energies are about the same. Recent experimental results are analyzed and the advantages and the disadvantages of both types of x-ray sources probed. The virtual quanta approach can be used to apply well-developed channeling radiation theories to non-linear problems in laser-electron scattering.

Much attention has been given recently to the generation of x-ray sources using Thomson scattering of laser photons on relativistic electrons [1, 2, 3]. Intense, sub-picosecond x-ray sources offer new potentials for several areas of science including radiation chemistry and observations of lattice vibrations [1, 4]. This development has been driven by the invention of chirped terawatt lasers [5]. Terawatt (TW) lasers have also made possible intense picosecond electron beams based on RF photo-injectors [6].

While the theory of interaction of laser light with electrons was proposed in the sixties [7, 8, 9] the development of lasers with beam intensities $10^{18} \div 10^{19}$ W/m^2 make laser x-ray sources (LXS) attractive for applications. The recent demonstration of sub-picosececond x-ray pulse generation using 90° Thomson scattering [2, 10] has sharpened interest in this possibility.

To appreciate the importance of Thomson scattering the capabilities and limitations of other x-ray producing processes such as x-ray lasers must be considered. One possibility that has been largely overlooked is channeling radiation (CR), proposed originally by Kumakhov [11]. Channeling radiation appears during electron and positron motion close to the planes and axes of a single crystal. This process is well understood both experimentally and theoretically [12, 13].

For a typical 30 μm diameter TW laser beam the electrostatic energy density is of the same order as the energy density in a crystal. In the electron energy range $20 \div 200$ MeV the maxima of the x-ray spectra for both CR and LXS ($\lambda_0 \sim 1\,\mu$m) lie in the energy region $6 \div 600$ KeV [13]. It is therefore interesting to compare the efficiencies of LXS with CR as possible sources of intense x-rays.

The relation of LXS and CR is echoed by the similarity of undulator radiation to radiation from electrons moving in a laser field[14, 15]. However, undulator wavelengths are $10^4 \div 10^5$ times longer than the laser wavelengths, whereas CR electron oscillation wavelengths in this energy regime are similar to laser photon wavelengths.

Below we mainly consider a conventional LXS with electrons interacting with a counterstreaming laser field [15]. For the LXS case the x-ray frequency is twice doppler shifted upward from the incident laser frequency ω_0, so that $\omega \sim 4\omega_0\gamma^2$ where $\gamma = (1-\beta^2)^{-1/2}$ is the Lorentz factor.

Thomson scattering and channeling radiation (indeed, radiation for any static field) appear to be rather different processes. For CR electrons scatter coherently from strings or planes of crystal atoms due to scattering with virtual quanta from the lattice. For Thomson scattering real coherent photons scatter from the electrons. It is possible to increase laser power while crystal properties are fixed. In spite of these differences, the basic electromagnetic processes are quite similar.

A useful framework to compare these apparently distinct processes is a virtual quanta approach by Lindhard [16] that exploits the Correspondence Principle. Using this approach one can explore how Thomson laser scattering can be viewed as virtual channeling radiation or, conversely, channeling radiation can be considered as virtual Thomson scattering. A corollary is the possibility of exploiting sophisticated treatments developed for CR to treat complex laser problems. The remainder of this article uses Lindhard's technique to compare the two processes and then explores the relative energy dependence and photon yields. The comparison is first treated using a harmonic expression for the electron motion and then generalized to include anharmonic terms for channeling radiation. Theoretical estimates are also compared to laser and channeling radiation experiments. More comprehensive treatments of these and related developments are in preparation.

The interaction of electrons with intense wave fields has led to the introduction of the Lorentz invariant field strength dimensionless parameter ν_0:

$$\nu_0^2 = \frac{e^2 \mathcal{E}_0^2}{2m^2 \omega_0^2 c^2} \tag{1}$$

where $\mathcal{E}_0$ is the electric field amplitude, ω_0 is the frequency of the wave, c is the speed of light, and e and m are the charge and mass of the electron. For relativistic electrons moving in external fields radiation properties are defined by the so-called non-dipole parameter $\nu_{nd} = \gamma\theta_e$, where θ_e is the electron's deviation due to the external field. In channeling $\theta_e \sim \theta_L$, where θ_L, the critical Lindhard angle [17], is

$$\theta_L = \sqrt{2U_m/E} \tag{2}$$

Here U_m is the depth of the channel potential well and E is the electron energy. Typical values of U_m are $U_m \sim 20$ eV for planar channeling and $U_m = 2Ze^2/d$ for axial channeling, where Z is the atomic number and d is the distance between atoms in the crystal atomic string.

The connection between the radiation processes for an electron moving in a static external field and in the field of a plane wave comes from the fact that the nondipole parameter $\nu_{nd} \sim \beta_\perp \gamma$ is an invariant [18] (here $\beta_\perp$ is the transverse velocity of the electron), and coincides with ν_0.

For an electron moving opposite to a linear polarized plane wave

$$\nu_0^2 = <\beta_\perp^2> \gamma^2 = \frac{x_m^2 \Omega_0^2 \gamma^2}{2c^2} \tag{3}$$

where $<\beta_\perp^2>$ is the averaged transverse velocity squared over $T = 2\pi/\Omega_0$ and x_m is the transverse amplitude where $x(t) = x_m \sin(\Omega_0 t)$ both for CR and LXS.

The transverse motion frequency is

$$\Omega_0 = \begin{cases} (1+\beta)\omega_0 \approx 2\omega_0\,; & \text{for LXS} \\ (2c/d_p)\sqrt{2U_m/E}\,; & \text{for channeling} \end{cases} \tag{4}$$

For 20 MeV electrons $(\hbar\Omega_0)_{chan} \sim 2$ eV (for planar silicon) to give peak energies of 6.4 keV. Laser photons with a wavelength 8000 Å or $(\hbar\omega_0)_{laser} \sim$ 1.5 eV give rise to x-ray peak energies of ~9.5 keV for 180° and ~4.25 keV for 90° Thomson scattering.

For channeling radiation ω goes as $E^{3/2}$ while for LXS it goes as E^2. Thus as the electron beam energy rises the LXS x-ray energy will rise relative to CR as $E^{1/2}$. In the 100-200 MeV electron energy regime they will be equal, while by 10 GeV the Thomson scattering x-ray energy in the first harmonic will be 10 times the CR first harmonic. However, high harmonics dominate for multi-GeV CR electrons. The Lindhard program proceeds by describing the interaction of an electron with an external field in the electron's rest frame. This gives both the CR and LXS spectral and angular distribution

$$\frac{dN}{d\hbar\omega d\Omega dt} = \frac{\alpha\nu_0^2\Omega_0^2}{4\pi\hbar\omega\gamma^2}\,\frac{1}{k^2}\Big(1 - \frac{\sin^2\theta}{2\gamma^2 k^2}\Big)\delta(\omega k - \Omega_0) \tag{5}$$

where we have summed over final polarizations and averaged over the azimuthal angle. Here θ is the photon scattering angle, $k = 1 - \beta\cos\theta$, α is the fine structure constant, and $\delta(x)$ is a delta function over frequencies. Equation (5) represents a dipole-like radiation spectrum leading to an interesting point: the dipole approximation corresponds to one photon Thomson scattering in the electron's rest frame. This is valid both for an electron accelerated by an external field (e.g. a channeling crystal) and for an electron moving in a laser field.

The non-dipole parameter ν_{nd} for LXS (ν_0) is defined by the laser intensity and does not depend on E. In channeling (as well as for electrons in other external static fields), the quantity $\nu_{nd} = \theta_L \gamma \sim E^{1/2}$. The dipole approximation $\nu_{nd} \ll 1$ means that θ_e for an electron in an external field is small compared with the effective radiation angle, that is $\theta_e \ll 1/\gamma$. The frequency spectrum of the radiation is then mainly due to the first harmonic. The higher harmonics contribute significantly only if $\nu_{nd} \geq 1$.

For CR the dipole approximation is violated when $E \geq 1$ GeV [13]. For modern lasers $\nu_0 \leq 1$. That is why for LXS even at multi-GeV electron energies the high harmonics give relatively weak contributions to the photon spectrum, whereas CR is essentially non-dipole in nature. With intense laser fields weak high harmonic radiation has been observed. Bula et al. [19] observe four harmonics for 46.6 GeV electrons. This is similar to early measurements of GeV planar positron CR [20] where the first few intense harmonics were observed. The difference is that in the laser case high harmonic radiation corresponds to the absorption of several real laser photons accompanied by the emission of a single photon of higher frequency while in channeling an electron absorbs several virtual quanta.

Integrating Eq.(5) over angle and frequency gives the total power radiated

$$P = \frac{2}{3} c r_0^2 A \gamma^2 \tag{6}$$

where $r_0 = e^2/mc^2$ is the classical electron radius and the quantity A is:

$$A = \begin{cases} (1+\beta)^2 \mathcal{E}_{laser}^2 \, ; & \text{for LXS} \\ \mathcal{E}_{cryst}^2 \, ; & \text{for channeling} \end{cases} \tag{7}$$

where $\mathcal{E}_{laser}^2 = \mathcal{E}_0^2/2$ is the mean square field of the laser, $\mathcal{E}_{cryst}$ is the electrostatic field of the crystal such that $e\mathcal{E}_{cryst} = |\nabla U|$ where U is the continuous Lindhard potential [17], and $\beta \sim 1$. For the case of LXS in the non-relativistic limit ($\beta \to 0$) Eq. (6) corresponds to classical Thomson scattering where $\sigma_0 = 8\pi r_0^2/3$.

The total number of photons emitted per unit time is

$$\frac{dN}{dt} = \frac{2}{3} \alpha \nu_0^2 \Omega_0 \tag{8}$$

For planar positron CR Eq. (8) should be averaged over the transverse energies $E_\perp$ in the channel [13]. For normal positron incidence the number of photons emitted per unit length z is

$$\frac{dN}{dz} = \frac{2}{9}\alpha\nu_L^2\theta_L/d_p \tag{9}$$

where $\nu_L = \theta_L\gamma$.

The relative yield of LXS and CR is determined by $\mathcal{E}^2_{laser}$ and $\mathcal{E}^2_{cryst}$ and by the interaction lengths of electrons with these fields, L_{laser} and L_{chan}

$$\xi = \frac{\text{LXS}}{\text{channeling}} = 4\,\frac{\mathcal{E}^2_{laser}L_{laser}}{\mathcal{E}^2_{cryst}L_{chan}} \tag{10}$$

The quantity L_{chan} is determined approximately by the effective channeling length [21] which is several times larger then the dechanneling length $\chi_{1/2}$ [21]. For 1 GeV electrons $L_{chan} \sim 10\,\mu m$ in 100 μm silicon, and can be much larger for a 1 cm thick crystal.

For LXS L_{laser} is determined by the system geometry and the beam parameters. For an arbitrary orientation the coefficient 4 in Eq. (10) is replaced by a somewhat smaller value.

For axially oriented crystals $\mathcal{E}^2_{cryst} \approx 520\ \ eV/\AA^3\ \ (k = 4.7)$ for $Si < 110 >$ and $\mathcal{E}^2_{cryst} \approx 580\ \ eV/\AA^3\ \ (k = 8)$ in diamond $< 100 >$. These values correspond to the experimental CR results [22, 23] for radiation energy losses. In heavy crystals $\mathcal{E}^2_{cryst}$ can substantially exceed the values given above so that $\mathcal{E}^2_{cryst} \sim 10^4\,eV/\AA^3$ for $W < 111 >$. The electric field at the focus of a TW laser is of the same order of magnitude as that in a crystal. For example, for a 10 TW laser and a beam radius of 50 μm one can get $\mathcal{E}^2_{laser} \approx 700$ eV$/\AA^3$ corresponding to $\nu_0 \approx 0.3$.

Although the LXS and CR formulas have similar forms, these two types of radiation have important differences. As noted earlier, the frequency dependence with energy for CR is different than for LXS. The number of emitted photons per unit time in channeling increases with E as $dN/dt \sim \gamma^{1/2}$, whereas in LXS dN/dt does not depend on energy. For both cases radiated power goes as γ^2. Unlike LXS, channeled electrons possess an additional mechanism for generating high harmonics – the anharmonism of the transverse motion. What distinguishes LXS is that the motion of an electron in the field of a plane wave is essentially harmonic. In channeling a much wider class of transverse trajectories exists, including an infinite transverse motion (quasichanneling). Specifically, the motion of an electron in the field of a linearly polarized wave corresponds to planar channeling of positrons [20] while a circularly polarized laser corresponds to axial electron channeling with circular transverse trajectories (see [13], §3.4).

To generalize beyond Eqs.(8) and (9) and properly describe axial and planar electron channeling assume the electron moves such that its transverse coordinate is a periodic function of time $\mathbf{r}_\perp(t+T) = \mathbf{r}_\perp(t)$. If the dipole approximation ($\nu_0 < 1$) holds the number of photons emitted per unit time is

$$\frac{dN}{dt} = \frac{4}{3}\frac{e^2\gamma^2}{\hbar\Omega_0 c^3}\sum_{k=1}^{\infty} k^{-1}|\mathbf{w}_k|^2 \tag{11}$$

where $\mathbf{w}_k$ is the Fourier component of acceleration $\mathbf{w}(t) = d^2\mathbf{r}_\perp/dt^2$

$$\mathbf{w}_k = \frac{1}{T}\int_0^T \mathbf{w}(t)\exp(i\Omega_0 kt)dt \tag{12}$$

and $T = 2\pi/\Omega_0$. In planar positron channeling the continuous potential has a parabolic character. This leads to the same type of transverse motion for the positron as for an electron in a plane wave. In this special case $\mathbf{w}_k$ is non-zero only if $k = 1$ and is equal to $w_k = -i\Omega_0^2 x_m/2$ so that Eq.(11) gives the result (8).

These radiation formulas do not include secondary factors which destructively affect CR and LXS and determine L_{chan} and L_{laser} in Eq. (10). An important factor diminishing L_{chan} is the multiple incoherent scattering of channeled electrons (positrons) [13]. Multiple scattering can also destroy the interference of radiation coming from the different periods of the transverse motion [24]. For LXS destructive factors include mutual refraction of electrons and photons [25]. These processes can substantially decrease L_{laser} when $\nu_0 \geq 1$. The scattering of an electron by an intense laser beam can lead to a change of the spectrum in the low frequency part [26].

Practically, it is possible to generate x-rays with LXS that have either high peak (HP) or moderate average power (MAP). In HP systems picosecond pulses are generated by picosecond electron micropulses. This requires electron-laser pulse synchronization in both time and space.

As noted earlier, a TW laser beam can be focused such that the energy density is $\sim 10^3$ eV/$\mathring{A}^3$ resulting in a photon yield of about ~ 1 photon/e$^-$

[15, 19]. A possible HP device with a power of 10 TW and a laser beam radius 50 μm has been discussed by Sprangle et al. [15]. This gives $\nu_0 \approx 0.3$ for a laser wavelength $\lambda = 1\,\mu m$. For this case where $L_{laser} \approx 300\,\mu m$ Eq. (8) gives $N \sim 1.8$ photon/e$^-$. This simple theoretical estimate also agrees with the recent SLAC experimental result [19] at 46.6 GeV. (As noted earlier, the total number of photons in LXS does not depend on E.) The energy of the SLAC laser pulse was 400 mJ with a transverse radius of 60 μm^2 resulting in $\mathcal{E}^2_{laser} \sim 1200\,eV/\AA^3$. This exceeds the corresponding values for CR in silicon and diamond crystals and gives $\nu_0 \approx 0.4$. The interaction length for the SLAC experimental geometry was $\sim\ 50\,\mu m$ (the electron-laser crossing angle was 17°). The number of photons emitted was $N \sim 0.6$ photon/e$^-$.

The emitted photons per electron for HP type LXS exceed the same quantities for CR for 50 – 300 MeV electrons (positrons). However, it should be noted that Sprangle et al. consider an ideal situation with a very powerful laser and perfect synchronization in time and space with no secondary destructive effects. At SLAC the electron beam was significantly larger than the laser focal area so that only a small fraction of the electrons ($\sim 10^{-3}$) crossed through the peak field region.

Sprangle et al. also considered devices with good moderate average power (MAP). MAP devices could reach values of L_{laser} up to several centimeters. For a current of 100 A and a 5 kW laser in a 50 μm radius at $\lambda \sim 1\,\mu m$ one can get $N \sim 4 \times 10^{-8}$ photon/e$^-$for $L_{laser} = 1.5$ cm. (In this case $\mathcal{E}^2_{laser} \approx 7 \times 10^{-7}\,eV/\AA^3$, $\nu_0 \approx 7 \times 10^{-7}$). However, this can be done much better with channeling. For 100 MeV positrons in 100 μm Si(110) ($d_p = 1.92\,\AA$) $N \sim 4 \times 10^{-3}$ photon/e$^-$(assuming dechanneling decreases the values in Eq. (10) by a factor of two).

Consider for comparison some CR results. With 350 MeV electrons on 170/mm Si Fujimoto and his collaborators [27] observed $\sim 4.6 \times 10^{-3}$ photon/e$^-$ for planar channeling and $\sim 2.7 \times 10^{-2}$ photon/e$^-$ for axial channeling. The photon spectra had a maxima in the region $\hbar\omega \sim 0.8$ MeV.

To obtain x-rays via CR the electron energy should be below ~ 100 MeV. For example, electrons in this energy range channeled in 105 μm Si(110) gave a yield of $N \sim 2.6 \times 10^{-3}$ photon/e$^-$ [28]. Theoretical analysis of this experiment [21] indicate this yield could be significantly increased by using a thicker crystal.

Channeling radiation effectiveness increases with electron (positron) energy. Experiments at Yerevan [23] with 4.5 GeV electrons on 1 cm thick diamond $< 111 >$ obtained $N \sim 5 \div 6$ photon/e$^-$. The photon energies in this case lie in the hard gamma region.

In summary, the following conclusions can be made. 1. A fundamental underlying connection between CR and LXS has been identified. 2. One can use an invariant field strength parameter for channeling as well as a non-dipole invariant parameter for LXS since both invariants turn out to represent the same quantity. 3. Well developed CR theories [13, 16] can be applied to complicated non-linear problems in LXS. 4. CR is an interesting x-ray source with relatively high mean values of emitted quanta per electron.

The advantages of LXS compared to CR are the higher degree of monochromaticity due to the pure harmonic electron transverse motion in the laser field and the absence of the incoherent background which is present in CR. Moreover, femtosecond x-ray pulses can be obtained directly with LXS. On the other hand, a DESY FEL design has considered sub-picosecond electron bunch lengths that could be used for CR. CR also has the virtue that a chirped terawatt laser is not needed. A significant limitation of CR we have not discussed is the problem of crystal survivability in an intense electron beam.

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