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Supersymmetric Dynamical Generation of the Grand Unification Scale

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Abstract

The grand unification scale $M_{\rm GUT} \sim 10^{16}$GeV may arise from dynamical effects. With the advances in understanding of supersymmetric dynamics, we can break the grand unified group by introducing a strong gauge group which generates the grand unification scale. We also show how this mechanism can be combined with solutions to the doublet-triplet splitting problem. The same method can also be used for other symmetry breakings at intermediate scales as well.

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One of the outstanding questions in particle physics is the "hierarchy problem": why is the mass scale of ordinary particle physics so small compared to the gravitational scale? In supersymmetric (SUSY) theories, the electroweak scale is related to the supersymmetry breaking scale, the problem is then translated into why supersymmetry is broken at a scale much below the Planck mass $M_{\rm pl}$. As Witten pointed out[1], if supersymmetry is dynamically broken by non-perturbative effects, a large hierarchy between the SUSY-breaking scale and the Planck scale can be naturally generated. Recently, Seiberg and his collaborators have made great progress in understanding non-perturbative SUSY dynamics[2]. Many more models of dynamical supersymmetry breaking have been found, providing new hope to understand how supersymmetry is broken in nature and how the electroweak scale is generated.

In many extensions of the standard model (SM), there are additional intermediate scales between the electroweak scale and the Planck scale. One example is the grand unification scale. The fact that the gauge couplings of $\mathrm{SU}(3)_{\rm C}$, $\mathrm{SU}(2)_{\rm W}$ and $\mathrm{U}(1)_{\rm Y}$ of the minimal SUSY extension of the standard model meet together at about 10^{16}GeV[3] gives a non-trivial indication that a SUSY GUT[4] may be realized in nature and the GUT gauge group is broken at $\sim 10^{16}$GeV, two orders of magnitude beneath $M_{\rm pl}$.[1] In flavor theories which try to understand the fermion mass hierarchy, the Froggatt-Nielsen mechanism[6] is often used to generate small numbers from the ratios of different mass scales. This also requires flavor symmetry breaking at intermediate scales. Similar to the SUSY-breaking scale, these intermediate symmetry breaking scales may also arise from dynamical effects. We will try to use the advances in our understanding of SUSY dynamics to generate these symmetry breaking scales dynamically. Our philosophy is that no explicit mass parameter (except maybe $M_{\rm pl}$) should appear in the fundamental Lagrangian. There are other ways of generating intermediate scales, for example, soft SUSY-breaking scalar mass squareds can be driven negative by Yukawa couplings[7]. However, understanding the origin of symmetry breaking is one of the deepest problems we face, and it is important to explore various possibilities.

Taking a simple example, consider the superpotential

$$W = \lambda X(\bar{\phi}\phi - \mu^2), \tag{1}$$

where $\bar{\phi}, \phi$ transform under some symmetry group, X is a singlet, and μ is some mass parameter. The equation $\partial W/\partial X = 0$ will force $\bar{\phi}, \phi$ to get vacuum expectation values (VEVs) and break the symmetry at the scale μ. We would like to have the scale μ generated dynamically instead of being put in by hand. The simplest thing one may try to do is to replace μ^2 by

[1]The two scales are so close that there may be ways to reconcile the discrepancy, e.g. in strong coupled string theories[5].

$\bar{Q}Q$, where $\bar{Q}, Q$ transform under some strong gauge group and form a condensate. However, the extra coupling $X\bar{Q}Q$ often disturbs the original vacuum and may generate a runaway direction in which $\langle X\rangle \to \infty$ and $\langle \bar{Q}Q\rangle \to 0$. Adding X^3 interaction removes the runaway direction but also forces $\langle\phi\rangle = \langle\bar{\phi}\rangle = 0$. One can cure this by introducing an additional singlet. Consider a strong gauge group SU(N) with one flavor $\bar{Q}$, Q and a singlet S. The tree level superpotential is given by

$$W_{\rm tree} = \lambda_1 S\bar{Q}Q + \frac{\lambda_2}{3}S^3. \tag{2}$$

Because $N > 1$, a nonperturbative superpotential is generated dynamically[8],

$$W_{\rm dyn} = (N-1)\left(\frac{\Lambda^{3N-1}}{\bar{Q}Q}\right)^{\frac{1}{N-1}}. \tag{3}$$

Integrating out $\bar{Q}, Q$, (they get mass from the VEV of S, which is justified below,) the first term in $W_{\rm tree}$ together with $W_{\rm dyn}$ will generate a runaway superpotential for the singlet S,

$$N(\lambda_1 S\Lambda^{3N-1})^{\frac{1}{N}} \tag{4}$$

It will be stablized by the second term in $W_{\rm tree}$. Solving the equation of motion we find that S gets a VEV of the order of the strong SU(N) scale,

$$\langle S\rangle = \Lambda\left(-\frac{\lambda_1}{\lambda_2^N}\right)^{\frac{1}{3N-1}}. \tag{5}$$

Now we can replace μ^2 in (1) by S^2, and the coupling XS^2 will not destroy the original vacuum. In this way, we can break symmetry groups dynamically and generate the symmetry breaking scale from strong dynamics.

The superpotential in the above discussion is not the most general one which can be written down. The most general superpotential without any mass parameter is[2]

$$W = \lambda_1 S\bar{Q}Q + \frac{\lambda_2}{3}S^3 + \frac{\lambda_3}{2}S^2X + \frac{\lambda_4}{2}SX^2 + \frac{\lambda_5}{3}X^3 + (\lambda_6 S + \lambda_7 X)\bar{\phi}\phi. \tag{6}$$

(The $X\bar{Q}Q$ coupling can be removed by redefining X and S.) Including the nonperturbative superpotential and solving the equations of motion, one finds that vacua with $\langle\bar{\phi}\rangle, \langle\phi\rangle = 0$ and $\langle\bar{\phi}\rangle, \langle\phi\rangle \neq 0$ both exist. Multiple vacua are a generic feature of the supersymmetric theories. If we sit on the $\langle\bar{\phi}\rangle, \langle\phi\rangle \neq 0$ vacuum, the symmetry is broken dynamically.

[2]Planck scale mass terms are assumed to be absent or may be forbidden by some discrete symmetry. Non-renormalizable terms suppressed by $M_{\rm pl}$ are smaller and should have little effect on the vacua.

In the rest of this letter, we will concentrate on breaking the GUT symmetry dynamically. The apparent unification of the standard model gauge couplings at $M_{\rm GUT} \approx 10^{16}$GeV makes it the most possible intermediate scale to exist between $M_{\rm pl}$ and M_W. However, the method discussed here can be applied to other symmetry breakings as well.

Consider a GUT model based on the gauge group SU(N)×SU(5)$_{\rm GUT}$ with $N > 5$, where SU(5)$_{\rm GUT}$ is the ordinary grand unified group, and SU(N) is strongly coupled with scale $\Lambda \sim M_{\rm GUT}$. To break SU(5)$_{\rm GUT}$, the model contains the fields Q^i_α, $\bar{Q}^\alpha_i$, which transform like $(\mathbf{N}, \mathbf{5}^*)$ and $(\mathbf{N}^*, \mathbf{5})$ under SU(N)×SU(5)$_{\rm GUT}$, and Σ^j_i, which transform like $(\mathbf{1}, \mathbf{24})$, where $\alpha = 1, \cdots, N$ and $i, j = 1, \cdots, 5$ are SU(N) and SU(5)$_{\rm GUT}$ indices respectively. The tree level superpotential is given by

$$W_{\rm tree} = \lambda_1 Q \Sigma \bar{Q} + \frac{\lambda_2}{3} {\rm tr}(\Sigma^3). \tag{7}$$

These are the only terms one can write down without any mass parameter. For the SU(N) gauge group, the number of flavors $N_f = 5$ is less than the number of colors, so a nonperturbative superpotential is dynamically generated[8],

$$W_{\rm dyn} = (N-5) \left(\frac{\Lambda^{3N-5}}{\det Q\bar{Q}} \right)^{\frac{1}{N-5}}, \tag{8}$$

where the determinant is taken on the flavor (SU(5)$_{\rm GUT}$) indices. Combining it with the tree-level superpotential we can look for the vacua. There is one runaway vacuum which preserve SU(5)[3]: $\langle \Sigma \rangle = 0$, $\langle \bar{Q} Q \rangle \propto \mathbf{1} \to \infty$. We are interested in vacua in which $\langle \Sigma \rangle \neq 0$ and SU(5) is broken. To study them we can integrate out $\bar{Q}, Q$ and obtain an effective superpotential depending only on Σ,

$$W_{\rm eff} = N \Lambda^{\frac{3N-5}{N}} (\lambda_1^5 \det \Sigma)^{\frac{1}{N}} + \frac{\lambda_2}{3} {\rm tr} \Sigma^3. \tag{9}$$

Solving $\partial W_{\rm eff} / \partial \Sigma^i_j = 0$, we find only two inequivalent solutions,

$$\langle \Sigma \rangle = \frac{1}{\sqrt{60}} \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & -3 \end{pmatrix} v, \quad v = \frac{\Lambda}{\lambda_1} \left(\frac{10\sqrt{60}\lambda_1^3}{\lambda_2} \right)^{\frac{N}{3N-5}} \left(\frac{1}{50\sqrt{60}} \right)^{\frac{1}{3N-5}}, \tag{10}$$

[3]In the Higgs picture, the unbroken SU(5) may be thought as a combination of SU(5)$_{\rm GUT}$ and a subgroup of SU(N)

$$\text{and} \quad \langle \Sigma \rangle = \frac{1}{\sqrt{40}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4 \end{pmatrix} u, \quad u = \frac{\Lambda}{\lambda_1}\left(-\frac{10\sqrt{40}\lambda_1^3}{3\lambda_2}\right)^{\frac{N}{3N-5}}\left(\frac{1}{400\sqrt{40}}\right)^{\frac{1}{3N-5}}. \tag{11}$$

In these vacua, $SU(5)_{GUT}$ is dynamically broken down to $SU(3)\times SU(2)\times U(1)$ and $SU(4)\times U(1)$ respectively. The GUT scale is generated by the strong SU(N) group. If all Yukawa couplings are $\mathcal{O}(1)$, then we have $M_{GUT} \sim \mathcal{O}(\Lambda)$.[4]

If the fields get some soft SUSY-breaking masses after supersymmetry is broken, the degeneracy among these vacua will be lifted. One expect that the runaway vacuum will be disfavored if the soft SUSY-breaking mass squares are positive. In the runaway direction, the supersymmetric contribution to the potential scales as

$$V_{SUSY} \sim \left(\frac{\Lambda^{3N-5}}{v^{10}}\right)^{\frac{2}{N-5}} \frac{1}{v^2}, \tag{12}$$

where v is the VEV of Q and $\bar{Q}$. It is stablized by the soft breaking terms,

$$V_{soft} \sim m_s^2 v^2, \tag{13}$$

where m_s is the soft breaking mass. The minimum occurs when these two terms are balanced,

$$\left(\frac{\Lambda^{3N-5}}{v^{10}}\right)^{\frac{2}{N-5}} \frac{1}{v^2} \sim m_s^2 v^2. \tag{14}$$

Then, the VEVs of Q and $\bar{Q}$ scale as $v \sim \lambda(\Lambda/m_s)^{\frac{N-5}{2N}}$ and the vacuum energy scale as $m_s^{\frac{N+5}{N}}\Lambda^{\frac{3N-5}{N}}$.[5] Compared with the SU(5) breaking minima, which have energy $V \sim m_s^2\Lambda^2$, the runaway vacuum clearly has higher energy for $N > 5$ (and $\Lambda > m_s$). The Kähler potential is unknown so we can not compare the vacuum energies at the $SU(3)\times SU(2)\times U(1)$ and $SU(4)\times U(1)$ minima. We assume that the $SU(3)\times SU(2)\times U(1)$ minimum is somehow chosen by nature.

[4] A worry is that when incorporated into supergravity, the strong gauge dynamics may break supersymmetry[9], then the supersymmetry breaking scale will be too high. However, how to correctly incorporate supergravity is a complicated issue and worth further investigation. Without knowing it exactly, we will assume that supersymmetry is broken by some other sector, not by this strong SU(N) gauge group.

[5] For some N and m_s, v can be larger than M_{pl}, and then one may not trust this result. However, we just use these results to get a rough idea of the energy of the runaway vacuum and we will ignore this problem.

We can modify this model by adding a singlet S, coupled to $\bar{Q}, Q$, to lift the runaway direction. The most general tree-level superpotential without any mass parameter is

$$W_{\rm tree} = \lambda_1 Q\Sigma\bar{Q} + \frac{\lambda_2}{3}{\rm tr}\Sigma^3 + \lambda_3 SQ\bar{Q} + \frac{\lambda_4}{3}S^3 + \frac{\lambda_5}{2}S\,{\rm tr}\Sigma^2. \tag{15}$$

Again, with the nonperturbative superpotential (8), there are several discrete vacua, among them the desirable one breaking SU(5) down to SU(3)×SU(2)×U(1). There are other vacua which preserve SU(5) or break it down to SU(4)×U(1), SU(3)×U(1)^2, and SU(2)^2×U(1). Both S and Σ get VEVs of the order Λ except in the SU(5) preserving vacuum in which $\langle\Sigma\rangle = 0$.

One of the most serious problem of the grand unified theories is the "doublet-triplet splitting problem". Higgs doublets are responsible for the electroweak symmetry breaking and hence their masses are of the order of the weak scale. On the other hand, their color-triplet partners must have GUT scale masses in order to achieve successful gauge coupling unification and/or to avoid rapid proton decay. In the minimal SUSY SU(5), this hierarchy is obtained by an extreme fine tune of the parameters in the superpotential, which is obviously unsatisfactory. In the following we will combine the dynamical GUT breaking model with some known solutions to the doublet-triplet splitting problem.

The first solution we consider is the pseudo-Goldstone boson mechanism[10]. It is based on the gauge group SU(6). The SU(6) is broken down to the standard model gauge group by two kinds of Higgs representations, an adjoint Σ with the VEV,

$$\langle\Sigma\rangle = {\rm diag}(1,1,1,1,-2,-2)v, \tag{16}$$

and a fundamental-antifundamental pair H and $\bar{H}$ with the VEVs

$$\langle H\rangle = \langle\bar{H}\rangle = (a,0,0,0,0,0). \tag{17}$$

If there is no cross coupling between Σ and $H, \bar{H}$ in the superpotential,

$$W = W(\Sigma) + W(H,\bar{H}), \tag{18}$$

then there is an effective SU(6)$_\Sigma$×SU(6)$_H$ symmetry. These two SU(6)'s are broken down to SU(4)×SU(2)×U(1) and SU(5) respectively. By a simple counting of the Goldstone modes and the broken gauge generators, one can find there are two electroweak doublets not eaten by the gauge boson and hence left massless. They are linear combinations of the Σ and $H, \bar{H}$ fields,

$$\frac{aH_\Sigma - 3vH_H}{\sqrt{a^2+9v^2}}, \quad \frac{a\bar{H}_\Sigma - 3v\bar{H}_{\bar{H}}}{\sqrt{a^2+9v^2}}, \tag{19}$$

and they will get weak scale masses from radiative corrections.

The simplest way to generate v and a dynamically is to use the dynamical model with the singlet (15), replacing SU(5) by SU(6). The field content contains $Q, \bar{Q}$, S, Σ, $H, \bar{H}$ discussed above, and a additional singlet X. The superpotential is given by

$$W = \lambda_1 Q\Sigma\bar{Q} + \frac{\lambda_2}{3}\mathrm{tr}\Sigma^3 + \lambda_3 SQ\bar{Q} + \frac{\lambda_4}{3}S^3 + \frac{\lambda_5}{2}S\,\mathrm{tr}\Sigma^2 + \lambda_6 X\bar{H}H - \lambda_7 XS^2. \tag{20}$$

Similar to what we discussed before, there is a vacuum in which both $\langle S\rangle, \langle\Sigma\rangle \sim \mathcal{O}(\Lambda)$ and $\langle\Sigma\rangle$ takes the form

$$\langle\Sigma\rangle = \mathrm{diag}(1,1,1,1,-2,-2)v. \tag{21}$$

The last two terms in the superpotential will force $H, \bar{H}$ to get $\mathcal{O}(\Lambda)$ VEVs. Therefore, the pseudo-Goldstone boson mechanism can be achieved with both Σ and $H, \bar{H}$ VEVs generated dynamically, and their scales naturally tied together.

The problem with this model is that there is no explanation for the absence of the $\bar{H}\Sigma H$ coupling. This coupling, if it exists, destroys the pseudo-Goldstone boson mechanism. Although it is technically natural to omit this coupling in supersymmetric theories, one may prefer to having some symmetry reason to forbid this coupling. There are no such symmetries in this model. One possibility is to generate $H, \bar{H}$ VEVs from another sector. (Then we lose the natural link between the two scales.) For example, we can use the method discussed before: Introducing another gauge group SU(M) with one flavor of fundamental and anti-fundamental fields and generating H and $\bar{H}$ VEVs through the superpotential (6). In this case, we can assign separate Z_3 symmetries to the two sectors which generate the Σ and $H, \bar{H}$ VEVs. The lowest order nonrenormalizable coupling suppressed by $M_{\rm pl}$ between Σ and $H, \bar{H}$ allowed will be

$$\frac{(\bar{H}\Sigma H)^3}{M_{\rm pl}^6}. \tag{22}$$

The induced masses for the light Higgs doublets will be $M_{\rm GUT}(M_{\rm GUT}/M_{\rm pl})^6$ and no bigger than the weak scale if $M_{\rm GUT}/M_{\rm pl} \lesssim 1/200$. Another way is to use the anomalous U(1) symmetry to generate $H, \bar{H}$ VEVs and forbid $\bar{H}\Sigma H$ coupling, as discussed in [11].

Another solution to the doublet-triplet splitting problem which we consider is proposed by Yanagida et al. [12, 13, 14]. Let us first review the idea using the model given in [13]. The model is based on the gauge group $\mathrm{SU}(5)_{\rm GUT}\times\mathrm{SU}(3)_{\rm H}\times\mathrm{U}(1)_{\rm H}$, with the following fields, $R(\mathbf{5}^*, \mathbf{3}, 1)$, $\bar{R}(\mathbf{5}, \mathbf{3}^*, -1)$, $q(\mathbf{1}, \mathbf{3}, 1)$, $\bar{q}(\mathbf{1}, \mathbf{3}^*, -1)$, $\Sigma(\mathbf{24}, \mathbf{1}, 0)$, $H(\mathbf{5}, \mathbf{1}, 0)$, and $\bar{H}(\mathbf{5}^*, \mathbf{1}, 0)$, where the numbers in brackets denote the transformation properties under $\mathrm{SU}(5)_{\rm GUT}$, $\mathrm{SU}(3)_{\rm H}$ and $\mathrm{U}(1)_{\rm H}$, respectively. One can write down a superpotential,

$$W = m_R\,\mathrm{tr}(R\bar{R}) + \lambda R\Sigma\bar{R} + \frac{1}{2}m_\Sigma\,\mathrm{tr}(\Sigma^2) + hHR\bar{q} + h'\bar{H}\bar{R}q, \tag{23}$$

which has a vacuum as follows,

$$\langle R\rangle = \langle \bar{R}^{\rm T}\rangle = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \sqrt{\frac{5m_R m_\Sigma}{\lambda^2}},$$
$$\langle \Sigma\rangle = \frac{m_R}{2\lambda}{\rm diag}(2,2,2,-3,-3). \tag{24}$$

In this vacuum the gauge symmetry $\mathrm{SU}(5)_{\rm GUT}\times\mathrm{SU}(3)_{\rm H}\times\mathrm{U}(1)_{\rm H}$ is broken down to the standard model gauge group $\mathrm{SU}(3)_{\rm C}\times\mathrm{SU}(2)_{\rm W}\times\mathrm{U}(1)_{\rm Y}$. The terms $HR\bar{q}$ and $\bar{H}\bar{R}q$ in the superpotential marry the color-triplet Higgses in $H,\bar{H}$ to $\bar{q}$ and q so that they obtain the GUT scale masses while the doublet Higgses remain massless. The doublet-triplet splitting is achieved by the missing partner mechanism with small matter representations.

The low energy $\mathrm{SU}(3)_{\rm C}$ and $\mathrm{U}(1)_{\rm Y}$ are diagonal subgroups of the SU(3), U(1) in $\mathrm{SU}(5)_{\rm GUT}$ and the $\mathrm{SU}(3)_{\rm H}$, $\mathrm{U}(1)_{\rm H}$ groups in this model. The gauge coupling unification is not spoiled if the gauge couplings of the $\mathrm{SU}(3)_{\rm H}$ and the $\mathrm{U}(1)_{\rm H}$ are big enough. In fact, the corrections from the $\mathrm{SU}(3)_{\rm H}$ and $\mathrm{U}(1)_{\rm H}$ couplings lower the prediction for the strong coupling constant α_S in SUSY GUT and therefore move it in the right direction[15].

To combine it with the dynamical GUT breaking model, we consider the gauge group $\mathrm{SU(N)}\times\mathrm{SU}(5)_{\rm GUT}\times\mathrm{SU}(3)_{\rm H}\times\mathrm{U}(1)_{\rm H}$, with the following field content: $Q(\mathbf{N},\mathbf{5}^*,\mathbf{1},0)$, $\bar{Q}(\mathbf{N}^*,\mathbf{5},\mathbf{1},0)$, $\Sigma(\mathbf{1},\mathbf{24},\mathbf{1},0)$, $R(\mathbf{1},\mathbf{5}^*,\mathbf{3},1)$, $\bar{R}(\mathbf{1},\mathbf{5},\mathbf{3}^*,-1)$, $q(\mathbf{1},\mathbf{1},\mathbf{3},1)$, $\bar{q}(\mathbf{1},\mathbf{1},\mathbf{3}^*,-1)$, $H(\mathbf{1},\mathbf{5},\mathbf{1},0)$, $\bar{H}(\mathbf{1},\mathbf{5}^*,\mathbf{1},0)$, and $S(\mathbf{1},\mathbf{1},\mathbf{1},0)$. The tree-level superpotential is given by

$$W = \lambda_1 Q\Sigma\bar{Q} + \frac{\lambda_2}{3}{\rm tr}(\Sigma^3) + \lambda_3 R\Sigma\bar{R} - \lambda_4 S\,{\rm tr}(R\bar{R}) + \frac{\lambda_5}{3}S^3 + hHR\bar{q} + h'\bar{H}\bar{R}q. \tag{25}$$

We are interested in the vacuum in which $\langle\Sigma\rangle, \langle R\rangle, \langle\bar{R}\rangle \neq 0$. Integrating out $Q,\bar{Q}$, we obtain

$$W_{\rm eff} = N(\lambda_1^5 \det\Sigma)^{\frac{1}{N}}\Lambda^{\frac{3N-5}{N}} + \frac{\lambda_2}{3}{\rm tr}(\Sigma^3) + \lambda_3 R\Sigma\bar{R} - \lambda_4 S\,{\rm tr}(R\bar{R}) + \frac{\lambda_5}{3}S^3 + hHR\bar{q} + h'\bar{H}\bar{R}q. \tag{26}$$

Solving the equations of motion,

$$\frac{\partial W_{\rm eff}}{\partial S} = -\lambda_4{\rm tr}(R\bar{R}) + \lambda_5 S^2 = 0, \tag{27}$$
$$\frac{\partial W_{\rm eff}}{\partial R} = \lambda_3\Sigma\bar{R} - \lambda_4 S\bar{R} = 0, \tag{28}$$
$$\frac{\partial W_{\rm eff}}{\partial \Sigma} = \frac{\Lambda^{\frac{3N-5}{N}}}{\lambda_1}(\lambda_1^5\det\Sigma)^{\frac{1}{N}}(\Sigma^{-1} - \frac{1}{5}{\rm tr}(\Sigma^{-1})) + \lambda_2(\Sigma^2 - \frac{1}{5}{\rm tr}(\Sigma^2)) + \lambda_3(R\bar{R} - \frac{1}{5}{\rm tr}(R\bar{R})) = 0, \tag{29}$$

one can find a vacuum with

$$\begin{aligned}
\langle\Sigma\rangle &= \mathrm{diag}(2,2,2,-3,-3)v, \\
\langle R\rangle = \langle \bar{R}^{\mathrm{T}}\rangle &= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix} \times \frac{2\lambda_3}{\lambda_4}\sqrt{\frac{\lambda_5}{3\lambda_4}}v, \\
\langle S\rangle &= \frac{2\lambda_3}{\lambda_4}v,
\end{aligned} \tag{30}$$

where

$$v = 72^{\frac{1}{3N-5}}\lambda_1^{\frac{5-N}{3N-5}}\left(6\lambda_2 - \frac{8\lambda_3^3\lambda_5}{5\lambda_4^3}\right)^{-\frac{N}{3N-5}}\Lambda.$$

Thus, we obtain the desirable vacuum in the form of (24) and the missing partner mechanism for the doublet-triplet splitting problem can be implemented.

In summary, we have shown that how the grand unified gauge group can be dynamically broken down to the standard model gauge group without inputting the GUT scale explicitly by hand. We also showed that this mechanism can be combined with solutions to the doublet-triplet splitting problem. Although we concentrated our discussion on GUT symmetry breaking, the same method can also be useful for other symmetry breakings at intermediate scales as well.

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