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# Determination of $W$ -boson Properties at Hadron Colliders

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## Abstract

Methods for measuring the  $W$ -boson properties at hadron colliders are discussed. It is demonstrated that the ratio between the  $W$ - and  $Z$ -boson observables can be reliably calculated using perturbative QCD, even when the individual  $W$ - and  $Z$ -boson observables are not. Hence, by using a measured  $Z$ -boson observable and the perturbative calculation of the ratio of the  $W$ - over  $Z$ -boson observable, we can accurately predict the  $W$ -boson observable. The use of the ratio reduces both the experimental and theoretical systematic uncertainties substantially. Compared to the currently used methods it might, at high luminosity, result in a smaller overall uncertainty on the measured  $W$ -boson mass and width.

# 1 Introduction

The measurement of the  $W$ -boson mass and width at Hadron Colliders has a long history of slowly improving uncertainties. Now, with the expected high luminosity TEVATRON runs, there will be a dramatic increase in the accuracy of the  $W$ -boson mass and width measurements. This will require a better understanding of the theoretical uncertainties resulting from uncalculated QED [1, 2] and QCD corrections. The latter is the subject of this paper. There are several problems with the theoretical predictions/models currently used in the extraction of the  $W$ -boson properties.

First of all, the phenomenological models used by the experimenters combine certain measured aspects of vector boson production with the theoretical calculation. The D0 collaboration [3] uses the resummed calculation of Ref. [4]. The theoretical model used is based on Next-to-Leading Order (NLO) matrix elements [5] in which the large logarithms associated with the small values of the vector boson transverse momentum,  $P_T^V$ , are resummed [6]. This resummation technique [7] necessitates the input of experimental data to determine the non-perturbative phenomenological parameters. In Ref. [4], lower energy Drell-Yan data were used for this purpose. The D0 collaboration uses its own Drell-Yan data to determine the non-perturbative parameters. Once the non-perturbative parameters have been determined the resummed calculation is used to predict the leptonic observables from which the  $W$ -boson properties are extracted. The CDF collaboration [8] uses a leading-order calculation of the  $W$ -boson production and decay which is subsequently folded with the  $P_T^W$ -spectrum. The  $Z$ -boson transverse momentum distribution is used as a first guess for the  $P_T^W$ -spectrum and is subsequently scaled [9] in order to match the component of the recoil energy perpendicular to the direction of the charged lepton in  $W$ -boson events. In both the D0 and CDF procedure it is difficult to evaluate the theoretical uncertainty.

Secondly, certain observables like the transverse energy distribution of the charged lepton are not perturbatively calculable due to large radiative corrections. This forces the experimenters to rely more on the transverse mass distribution to extract the  $W$ -boson mass. This distribution does not suffer from large radiative corrections (i.e., is not very sensitive to the  $P_T^W$ -spectrum), but by definition depends on the neutrino transverse momentum. The reconstruction of the latter requires the measurement of the hadronic part of the event, resulting in substantial systematic uncertainties which ultimately will limit the precision on the  $W$ -boson mass and width determination. This problem of missing energy reconstruction will be compounded in the future TEVATRON runs because of the expected increase in the number of interactions per crossing [10].

The obvious solution to both of the above problems is not new, but with the planned high luminosity TEVATRON runs it can be brought to full maturity. The basic idea is to use the measured lepton distributions in  $Z$ -boson decay, along with the calculated ratio of the  $W$ - over  $Z$ -boson distribution, to predict the equivalent leptonic distribution in the  $W$ -boson case. By comparing this prediction to the measured leptonic distribution the  $W$ -boson mass and width can be extracted. Because of the use of the ratio, the  $W$ -boson mass and width will be given relative to the  $Z$ -boson mass and width. The  $Z$ -boson mass and width are accurately known from the LEP experiments [11]. This method reduces many of the experimental systematic uncertainties because they are largely correlated between the  $W$ - and  $Z$ -boson distributions. The drawback is that the leptonic  $Z$ -boson cross section is an order of magnitude smaller than the leptonic  $W$ -boson cross section, resulting in a larger statistical uncertainty. However, the exchange of the systematic uncertainties for statistical uncertainties is exactly what is needed for future high luminosity runs. This method also reduces the theoretical uncertainty as only the ratio of the distribution has to be calculated, and not the distribution themselves. As we will see, because  $W$ - and  $Z$ -boson productions properties are very similar, the large radiative corrections that might affect the individual distributions cancel in the ratio. In other word, the ratio can be reliably calculated using perturbative QCD even in regions of phase space where the individual distributions cannot. For example, all the large logarithms associated with small values of the transverse energy of the vector boson cancel in the ratio and we get a well behaved perturbative expansion which has a solid footing in the theory (see e.g. Ref. [12] for a comparison between the transverse momentum of the  $W$ - and  $Z$ -boson). There is no need for resummation in the calculation of the ratio. The theoretical

uncertainty associated with perturbative QCD can be estimated from the size of the next-to-leading order calculation compared to the leading order calculation. If necessary, the uncertainty could be further decreased by calculating the next-to-next-to-leading order corrections [13]. This gives a good understanding of the theoretical uncertainty on the  $W$ -boson mass and width measurements, which will be important in order to interpret the upcoming high luminosity results at the TEVATRON.

One remaining unknown at the moment is the uncertainty related to the parton distribution functions. Obviously, by taking the  $W$ - over  $Z$ -boson ratio these uncertainties will also be reduced. The ratio will strongly depend on the  $u$ -quark/ $d$ -quark ratio. However, with the current status of Parton Density Functions (PDF's) there is no way of quantifying this statement [14].

In section 2, we set up the theoretical framework for the comparison between the  $W$ - and  $Z$ -boson observables. In section 3 we consider explicitly the transverse momentum of the vector boson. The transverse mass and the transverse energy of the charged lepton distributions are discussed in sections 4 and 5, respectively. Finally, in section 6 we summarize our findings and draw some conclusions.

## 2 The theoretical framework

The basic idea is to use the  $Z$ -boson observables combined with perturbative QCD to predict the  $W$ -boson observables. The main difference between the  $W$ - and  $Z$ -boson production is due to the difference in the mass. To minimize this difference we consider the ratio of the observable  $\mathcal{O}$ , which has the dimension of mass, and the vector boson mass  $M_V$ :

$$X_{\mathcal{O}}^V = \frac{\mathcal{O}^V}{M_V}. \quad (1)$$

Next, we define the ratio

$$R_{\mathcal{O}}(X_{\mathcal{O}}) = \frac{A_W(X_{\mathcal{O}}^W = X_{\mathcal{O}})}{A_Z(X_{\mathcal{O}}^Z = X_{\mathcal{O}})}, \quad (2)$$

where  $A_V$  is the differential cross section with respect to the scaled variable:

$$A_V(X_{\mathcal{O}}^V) = \frac{d\sigma_V}{dX_{\mathcal{O}}^V}. \quad (3)$$

The measured  $Z$ -boson differential cross section,  $A_Z(X_{\mathcal{O}})|_{\text{measured}}$ , can be used along with the calculated ratio  $R_{\mathcal{O}}(X_{\mathcal{O}})$  to predict the  $W$ -boson differential cross section,  $A_W(X_{\mathcal{O}})|_{\text{predicted}}$ :

$$A_W(X_{\mathcal{O}})|_{\text{predicted}} = R_{\mathcal{O}}(X_{\mathcal{O}}) \times A_Z(X_{\mathcal{O}})|_{\text{measured}}. \quad (4)$$

This expression can be used to relate the differential cross section with respect to the non-scaled variables:

$$\left. \frac{d\sigma^W}{d\mathcal{O}^W} \right|_{\text{predicted}} = \frac{M_Z}{M_W} \times R_{\mathcal{O}} \left( \frac{\mathcal{O}^W}{M_W} \right) \times \left. \frac{d\sigma^Z}{d\mathcal{O}^Z} \right|_{\text{measured}}^{\mathcal{O}^Z = \frac{M_Z}{M_W} \mathcal{O}^W}. \quad (5)$$

Finally we define the “ $K$ -factor” as the ratio of the Next-to-Leading Order (NLO) over Leading Order (LO) calculation of  $R_{\mathcal{O}}(X_{\mathcal{O}})$ :

$$K_{\mathcal{O}}(X_{\mathcal{O}}) = \frac{R_{\mathcal{O}}^{NLO}(X_{\mathcal{O}})}{R_{\mathcal{O}}^{LO}(X_{\mathcal{O}})}. \quad (6)$$

To demonstrate that the ratio can be calculated perturbatively, we will show that for the observables of interest the  $K$ -factor is close to unity over the relevant  $X_{\mathcal{O}}$  range. In other words we will show that the scaled  $W$ - and  $Z$ -boson distributions have very similar radiative corrections.

In the explicit calculation of the ratio we use the DYRAD<sup>1</sup> program [19]. For the renormalization and factorization scales we choose the average of the  $W$ - and  $Z$ -boson masses. Other choices can be made for the scales, but the dependence of the ratio on that choice is small. Similar cuts should be imposed on the  $Z$ - and  $W$ -boson decay leptons. The transverse energy lepton cuts should be applied on the mass scaled variables to avoid large radiative corrections in the region close to the cuts. The only cut that cannot be matched between the  $W$ - and  $Z$ -boson case is the rapidity cut on the second charged lepton in the  $Z$ -boson decay. This is because the longitudinal momentum of the neutrino in the  $W$ -boson decay cannot be reconstructed. However, this difference is geometrical in nature and any difference will be modeled accurately by the perturbative calculation. On the other hand, the rapidity cut on the second charged lepton in the  $Z$ -boson decay should be as weak as possible in order to increase the  $Z$ -boson sample. In this paper we will not impose cuts on the leptons as they have no bearing on the argumentation or the conclusions reached. It is, however, trivial to include these cuts using the DYRAD program. The leptonic branching ratio's were not included in any of the numerical results.

### 3 The transverse momentum of the vector boson

The transverse momentum of the vector boson,  $P_T^V$ , is an interesting test of the ratio method for several reasons. First of all, for both the  $W$ - and  $Z$ -boson there are published measurements of the transverse vector boson momentum. This means we can actually apply the method described in section 2 and predict the  $W$ -boson  $P_T$ -distribution using the  $Z$ -boson  $P_T$ -distribution over the entire  $P_T$ -spectrum with a small theoretical uncertainty. The current data sets of both CDF and D0 contain an integrated luminosity of well over  $100 \text{ pb}^{-1}$ . Unfortunately, the vector boson transverse momentum results using this high statistics data have not been published yet. We will use the CDF  $W$ -boson data containing an integrated luminosity of  $4.1 \text{ pb}^{-1}$  [15] and the D0  $Z$ -boson data using an integrated luminosity of  $12.8 \text{ pb}^{-1}$  [16] to demonstrate the potential of the ratio method. Secondly, the transverse  $P_T$ -distribution has an infrared unstable region as the transverse momentum goes to zero and it is interesting to study the behavior of the ratio in that region.

The general form of the  $n$ -th order scaled transverse momentum differential cross section is [6]

$$A_V^{(n)}(X_{P_T}) = \frac{\sigma_V^{(0)}}{X_{P_T}} \sum_{k=1}^n \sum_{m=0}^{2k-1} a_{k,m}^V \alpha_S^k \log^m(X_{P_T}) + R(\alpha_S), \quad (7)$$

where  $X_{P_T} = P_T^V/M_V$ ,  $R(\alpha_S)$  represent the remaining terms for which the divergence is weaker than  $1/X_{P_T}$ ,  $\sigma_V^{(0)}$  is the born cross section and  $\alpha_S$  is the strong coupling constant. In the limit that  $X_{P_T} \rightarrow 0$  all terms in the expansion can be neglected with respect to the leading logarithmic term:

$$\lim_{X_{P_T} \rightarrow 0} A_V^{(n)}(X_{P_T}) = \frac{a_{n,2n-1}}{X_{P_T}} \alpha_S^n \log^{2n-1}(X_{P_T}) \sigma_V^{(0)}, \quad (8)$$

where the constants  $a_{n,2n-1}$  cannot depend on the vector boson type because of the universality of the leading logarithms, such that

$$\lim_{X_{P_T} \rightarrow 0} R_{P_T}^{(n)} = \frac{\sigma_W^{(0)}}{\sigma_Z^{(0)}}. \quad (9)$$

At any order in  $\alpha_S$  the ratio converges toward the leading order result in the infrared unstable region, i.e. the radiative corrections to the ratio are very small in that region. There is no need for resummation of large logarithms, the perturbation series is well behaved and the theoretical uncertainty is small. This despite the fact that the individual differential cross section in the numerator and denominator of the ratio are unreliable at small  $X_T$ .

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<sup>1</sup>The program can be obtained from the WWW-page: "WWW-theory.fnal.gov/people/giele/dyrad.html".

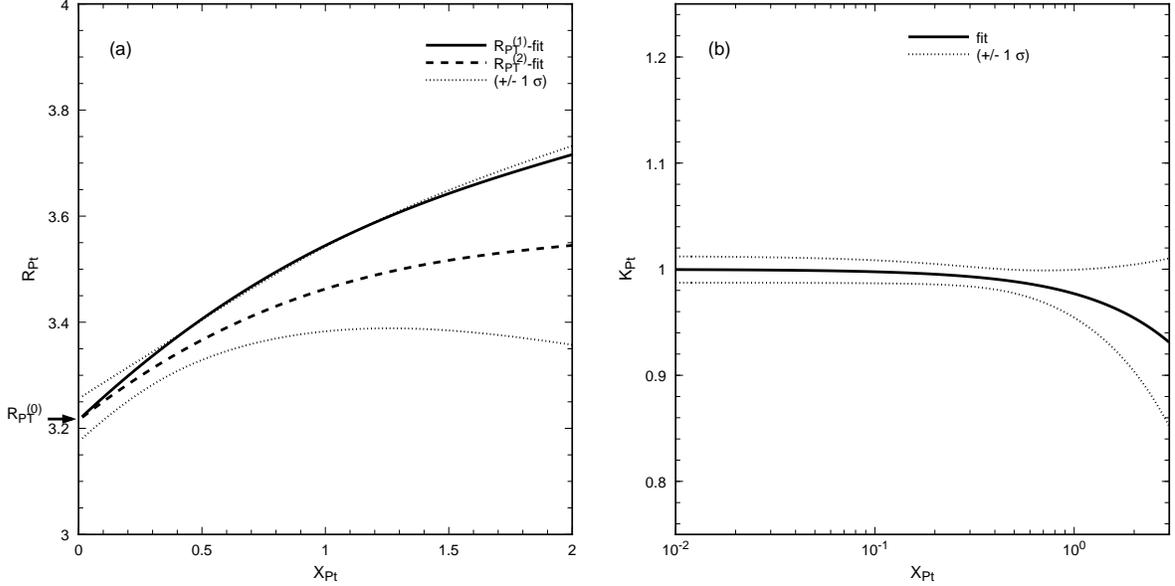


Figure 1: a) The ratio  $R_{P_T}$  as a function of the scaled vector boson transverse momentum,  $X_{P_T} = P_T^V/M_V$ , at different orders in  $\alpha_S$ . The arrow on the left side represent the LO value of the ratio. The solid line and the dashed lines are the result of fits to the order  $\alpha_S$  and  $\alpha_S^2$  Monte-Carlo calculations. The dotted lines represent the one sigma uncertainty due to the Monte-Carlo integration for the order  $\alpha_S^2$  calculation. The uncertainty for the order  $\alpha_S$  is smaller and not shown. b) The  $K$ -factor,  $K_{P_T} = R_{P_T}^{(2)}/R_{P_T}^{(1)}$ , as a function of  $X_{P_T}$  together with the uncertainty range associated with the Monte-Carlo integration.

In fig. 1a we show the vector boson  $P_T$ -ratio,  $R_{P_T}(X_T)$ , at three orders in  $\alpha_S$ <sup>2</sup>. The lowest order ratio  $R_{P_T}^{(0)}$  is quite trivial, because  $P_T^V$  is always zero due to the lack of any initial state radiation. The value of the ratio is simply given by the ratio of born cross sections

$$R_{P_T}^{(0)} = \frac{\sigma_W^{(0)}}{\sigma_Z^{(0)}} = (3.216 \pm 0.001) . \quad (10)$$

The uncertainty is due to the Monte-Carlo (M.C.) integration. The NLO ratio  $R_{P_T}^{(1)}(X_T)$  at order  $\alpha_S$  can have non-zero  $P_T^V$  due to initial state radiation. However, for the virtual corrections the  $P_T^V$ -value remains identical to zero. This causes the low  $P_T^V$ -region to become infrared unstable and a large logarithm is generated, see eq. (7). As given in eq. 9, for the ratio  $R_{P_T}(X_T)$  this means that the  $X_{P_T} \rightarrow 0$  limit is determined by the leading order ratio  $R_{P_T}^{(0)}(X_T)$ :

$$\lim_{X_{P_T} \rightarrow 0} R_{P_T}^{(n)}(X_T) = R_{P_T}^{(0)} . \quad (11)$$

This can be seen numerically in fig. 1a. Away from  $X_{P_T} = 0$  there is a slow rise in the ratio as a function of  $X_{P_T}$ . This is simply a consequence of the combination of phase space effects and parton distribution

<sup>2</sup>For the order  $\alpha_S^2$  the two loops virtual corrections are not included in the calculation, but it is enough to realize that these contributions factorize.

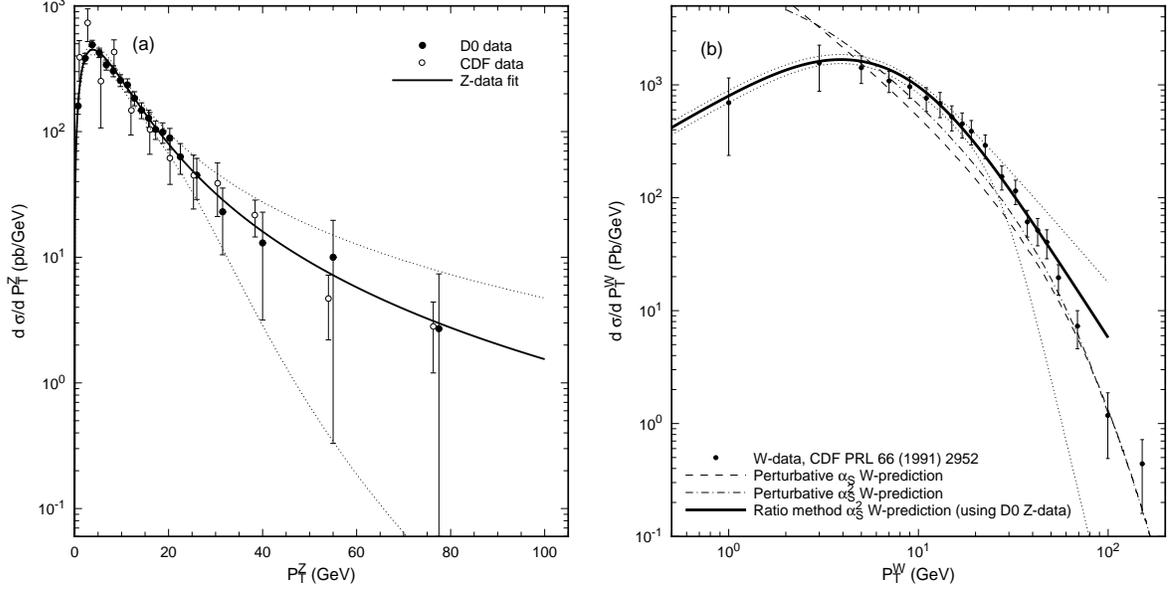


Figure 2: a) The  $P_T^Z$ -distributions for both the CDF [20] and D0 [16] data. The solid line represent the fit to the D0-data as given in ref. [16]. The dotted lines represent a fit to the D0-data uncertainties. b) The  $W$ -boson CDF-data, along with the perturbative order  $\alpha_S$  and  $\alpha_S^2$  calculation and the order  $\alpha_S^2$  prediction using the ratio method with the D0  $Z$ -boson data. The dotted lines represent the uncertainties stemming from the D0-data.

functions. The order  $\alpha_S^2$  is important so we can estimate the theoretical uncertainty in the ratio prediction. Again, close to the infrared unstable point the behavior of  $R_{P_T}^{(2)}(X_T)$  is given by eq. 11. As can be seen in fig. 1b the radiative effects to the ratio are very small and, within the numerical accuracy of the M.C., consistent with zero, i.e.  $|1 - K_{P_T}(X_T)| < 0.05$ <sup>3</sup> in the relevant  $X_T$ -region. Using this information we can make our prediction for the ratio. The central value is  $R_{P_T}^{(2)}(X_T)$  and as an conservative estimate of the uncertainty we take  $|R_{P_T}^{(2)}(X_T) - R_{P_T}^{(1)}(X_T)|/R_{P_T}^{(2)}(X_T) = |1 - K_{P_T}(X_T)| < 5\%$  over the entire  $X_{P_T}$  region of interest. Note that for small values of  $X_{P_T}$  the uncertainty is substantially smaller ( $< 2\%$ ) and the uncertainty is certainly correlated to some degree between different  $X_{P_T}$ -values. At the moment the corrections are very small compared to the experimental uncertainties and can, for all practical purposes, be neglected.

In fig. 2a we show the published CDF and D0  $Z$ -boson  $P_T^Z$ -data together with a fit to the D0 data (the solid line) and its uncertainties (the dotted lines). All the data we are using were corrected for the effect of leptonic cuts. Using the calculated ratio shown in fig. 1a we can predict the CDF  $W$ -boson  $P_T$ -spectrum using the measured D0  $Z$ -boson  $P_T$ -spectrum, see eq. 5. The results are given in fig. 2b with the order  $\alpha_S^2$  ratio-prediction. The prediction agree very well with the  $W$ -boson  $P_T$ -spectrum for all measured points (as low as 1 GeV). However, as expected the order  $\alpha_S$  and  $\alpha_S^2$  prediction for the  $W$ -boson distribution itself fails below 20 GeV due to the presence of large logarithms. Note that the experimental uncertainties in this particular data set is large and we do not have to worry about the theoretical uncertainty on the ratio. It would be very interesting to repeat this analysis for the more recent data sets of both CDF and D0 with each

<sup>3</sup>This number can be improved by simply running the Monte-Carlo integration with a larger number of events.

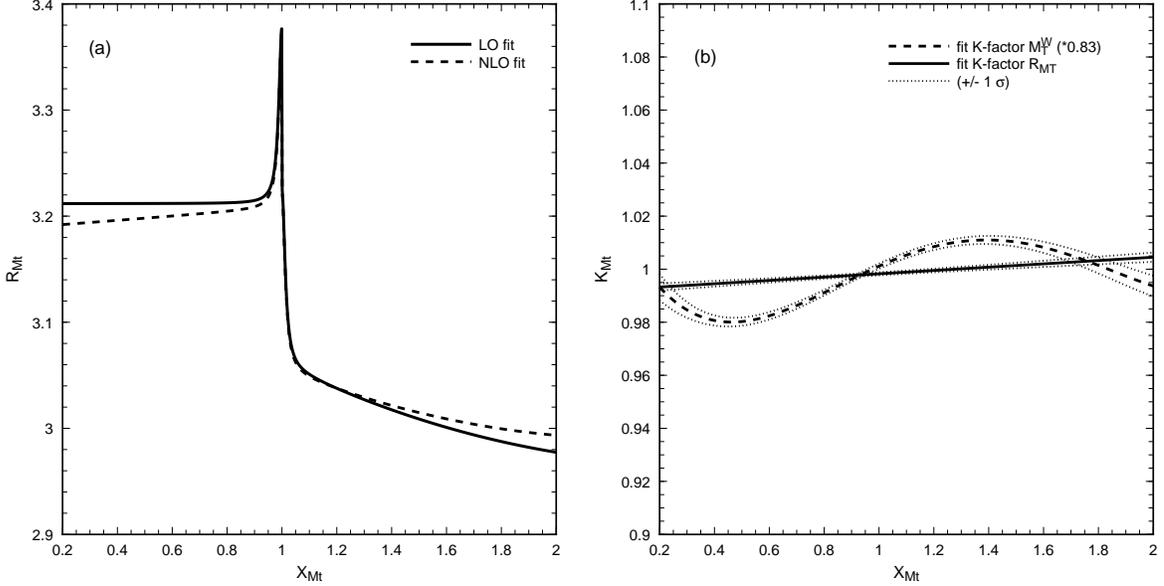


Figure 3: a) The LO (solid line) and NLO (dashed line) ratio  $R_{M_T}$  as a function of the scaled transverse mass  $X_{M_T} = M_T^V/M_V$ . The leptonic branching fractions are not included. b) The  $K$ -factor  $K_{M_T} = R_{M_T}^{(1)}/R_{M_T}^{(0)}$  as a function of  $X_{M_T}$  (solid line), and the  $K$ -factor for the  $W$  transverse mass (dashed line), normalized to 1 at  $X_{M_T} = 1$ . In both case we have included the one sigma uncertainty range associated with the Monte-Carlo integration.

have an integrated luminosity over  $100 \text{ pb}^{-1}$ . This would reduce the experimental uncertainties substantially and possibly probe regions of smaller transverse momentum [21].

In some respect the method that we are suggesting here is equivalent to the resummed calculation. Although the latter qualitatively describes the  $P_T^W$ -spectrum, before it can make a quantitative prediction it needs data (e.g. the  $P_T^Z$ -spectrum) to determine phenomenological non-perturbative parameters. Therefore, both the ratio method and the resummed calculation can use the  $Z$ -boson data to predict the  $W$ -boson data. However our method only uses perturbative QCD, we do not need to resum large logarithms in the low  $P_T^V$ -region. In fact, it is in that region that the uncertainty is the smallest. As the transverse momentum of the vector boson becomes smaller the QCD radiation becomes more and more independent of the hard process, i.e. it factorizes. In the ratio the radiative corrections start to cancel more and more and the ratio converges to the leading order ratio leaving no theoretical uncertainty. If deviations are observed from the predicted behavior by this method in the low  $P_T$ -region they can only be ascribed to either PDF uncertainties or to higher twist (non perturbative) effects. Because in the ratio method only perturbative QCD is used, the estimate of the uncertainty in the theoretical prediction is well understood.

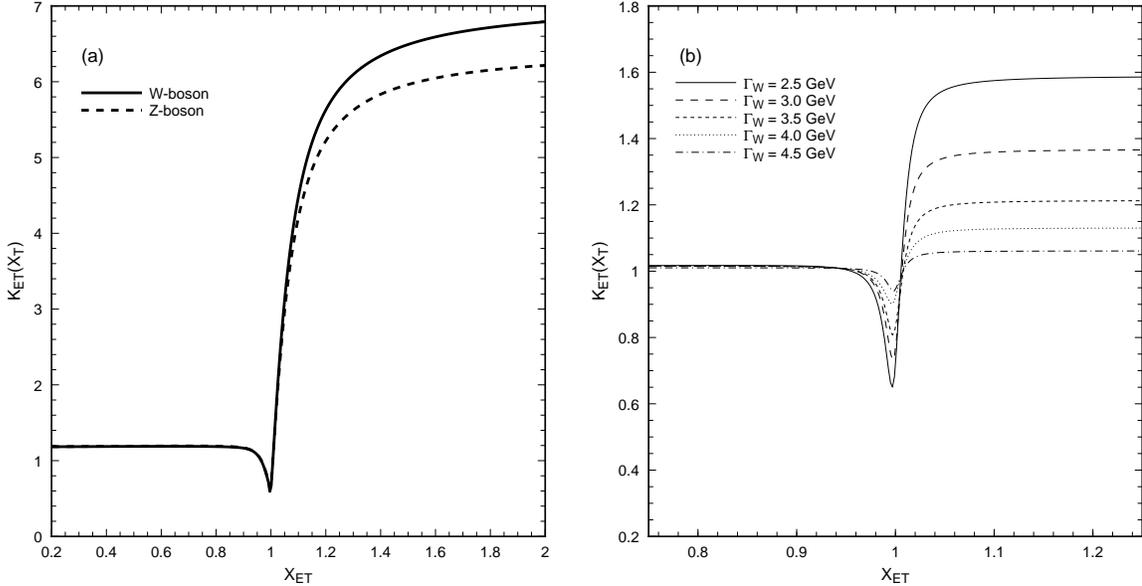


Figure 4: a) The  $K$ -factor of the lepton transverse energy distribution for both the  $W$ -boson (solid line) and  $Z$ -boson (dashed line). b) The  $K$ -factor (NLO/LO) for the ratio  $(d\sigma^W/dX_{E_T}(\Gamma_W)) / (d\sigma^W/dX_{E_T}(\Gamma_W = 5 \text{ GeV}))$  for different  $\Gamma_W$  in the numerator.

## 4 The transverse mass of the lepton pair

The transverse mass,  $M_T$ , distribution is currently used to determine the  $W$ -boson mass [8] and width [17]. As already mentioned, it is expected that for this method at high luminosity the systematic uncertainty will be larger than the statistical uncertainty [10]. This is mainly due to the increase in the number of interactions per crossing and the corresponding degradation of the neutrino transverse momentum reconstruction. The method described in this paper will therefore be very useful at high luminosity, as it trade systematic for statistical uncertainties.

In fig. 3a we show both the LO and NLO ratio  $R_{M_T}$  as a function of the scaled transverse mass  $X_{M_T} = M_T/M_V$ . As can be seen there is a remnant of the Breit-Wigner resonance at  $X_{M_T} = 1$ . This is due to the fact that the scaled  $W$ -boson width (i.e.  $\Gamma_W/M_W$ ) is about 10% smaller than the scaled  $Z$ -boson width, making the  $W$ -boson scaled distribution slightly narrower than the  $Z$ -boson scaled distribution. Below  $X_{M_T} = 1$  the LO ratio tends rapidly to the LO cross section ratio of 3.216 and is for all practical purpose a constant. Above the resonance region the LO ratio is slowly falling, again due to the larger scaled  $Z$ -boson width. The radiative corrections are small, this can also be seen in fig. 3b where  $K_{M_T}$ , the  $K$ -factor of the ratio  $R_{M_T}$ , is plotted. The range corresponds to the Monte-Carlo integration uncertainty. In the region most relevant for the  $W$ -boson mass determination ( $0.9 < X_{M_T} < 1.1$ ) the corrections are smaller than 0.3% and can almost be neglected altogether. This means that the theoretical uncertainty in this region is conservatively less than 0.3%. In the tail region above  $X_{M_T} > 1.1$ , the corrections become slightly larger and positive. The radiative corrections affect the narrower  $W$ -boson transverse mass distribution more. This region is of interest because it can be used to determine the  $W$ -boson width [17]. At high luminosity [10] the

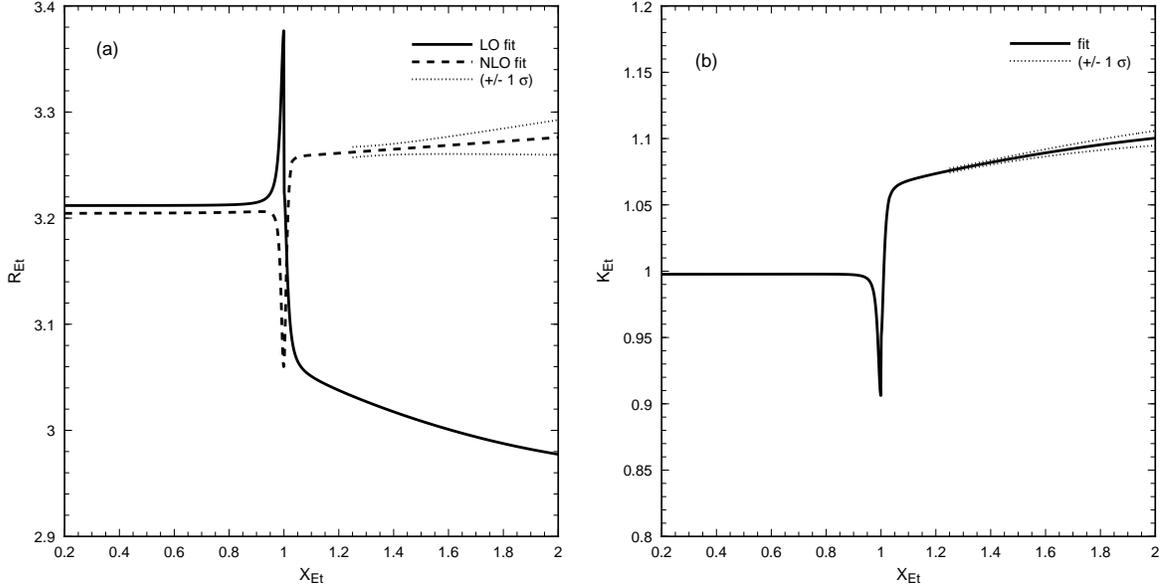


Figure 5: a) The LO (solid line) and NLO (dashed line) ratio  $R_{E_T}$  as a function of the lepton scaled transverse energy  $X_{E_T} = 2 \times E_T^V/M_V$ . b) The  $K$ -factor  $K_{E_T} = R_{E_T}^{(1)}/R_{E_T}^{(0)}$  as a function of  $X_{E_T}$ . The dotted lines represent the one sigma uncertainty associated with the Monte Carlo integration.

method described in ref. [17] might result in a smaller uncertainty on the width than the more traditional method using the ratio of the inclusive  $Z$ - and  $W$ -boson cross sections [18]. The method described in this paper should give the best constraint on the width as it combines both the shape of the transverse mass distribution and the total cross section ratio between  $W$ - and  $Z$ -boson production. Even for large  $X_{M_T}$  (up to  $X_{M_T} = 2$ ) the radiative corrections to the ratio are less than 1%. This gives us a NLO prediction for the radiative tail region with a conservative theoretical uncertainty of less than 1%. Clearly there is no need for resummation in the perturbative calculation of this observable.

The fact that the corrections to the ratio are very small is not surprising. It is well known that the transverse mass distribution itself has small radiative corrections, i.e. is not sensitive to the  $P_T^W$ -spectrum. This was the very reason, the authors of ref. [22] suggested this particular distribution as a sensible observable to measure the  $W$ -boson mass. In fig. 3b, we have also plotted the  $K$ -factor of the  $W$ -boson transverse mass distribution itself, normalized to 1 at  $X_{M_T} = 1$ . As can be seen, the corrections to the shape of the distribution are of the order of a few percents, even though the overall size of the corrections are of the order of 20%. The advantage of using the ratio method is noticeable.

## 5 The leptonic transverse energy distribution

The transverse energy distribution of the charged lepton is subject to large radiative corrections. To use this distribution to extract the  $W$ -boson mass must involve a rigorous understanding of the large corrections and the correlation of these corrections with the  $W$ -boson mass and width. The large NLO corrections

are illustrated in fig. 4a where the  $K$ -factor is presented for both the  $W$ - and  $Z$ -boson cases as a function of  $X_{E_T} = 2E_T/M_V$ <sup>4</sup>. The radiative corrections are about 40% at  $X_{E_T} = 1$  and rise to over 600% for  $X_{E_T} > 1.1$ . Clearly, the perturbative prediction of the shape of this distribution is unreliable. This is unfortunate, because for this distribution it is obviously not necessary to reconstruct the transverse energy of the neutrino. However, the ratio method significantly reduces the theoretical uncertainty and will make this distribution useful for extraction of the  $W$ -boson mass and width.

The LO and NLO ratio  $R_{E_T}$  are presented in fig. 5a as a function of  $X_{E_T}$ . At LO the ratio is identical to the LO transverse mass ratio (fig. 3a). Below the jacobian peak region ( $X_{E_T} = 1$ ), the NLO ratio is very close to the LO ratio. However, around and above the resonance region the shape of the NLO ratio is inverted compared to the LO ratio. The radiative corrections increase the width of the  $W$ - and  $Z$ -boson distribution by about the same amount. This result into a scaled width of the distribution which is bigger in the  $W$ -boson case than for the  $Z$ -boson case, as  $M_W < M_Z$ , resulting in the inverted shape. In the transverse mass case (and for that matter the invariant mass case) the radiative corrections do not change the width of the distribution because of the correlation between transverse momenta of the two vector boson decay leptons.

Even though the corrections are larger than in the transverse mass case, by taking the ratio we have achieved a cancelation of the radiative corrections by about an order of magnitude. Around (above) the resonance region, there is a shape (normalization) uncertainty of at most 10% as can be seen in fig. 5b. As the experiments become more accurate it might be necessary to extend the calculation of the ratio one order higher in  $\alpha_S$  and thereby reducing the theoretical uncertainty substantially. All the virtual contributions needed are already known in the literature [13] and such a calculation is certainly within the realm of possibilities. Using the ratio method for the lepton transverse energy with the currently available CDF and D0 data containing over 100 pb<sup>-1</sup> will tell us if such a calculation is necessary.

It is interesting to note that the cancellation of the large corrections is due in part to the closeness of the scaled widths. To demonstrate the relation between the width and the size of the corrections we show in fig. 4b the NLO  $K$ -factor for the ratio  $(d\sigma^W/dX_{E_T}(\Gamma_W)) / (d\sigma^W/dX_{E_T}(\Gamma_W = 5 \text{ GeV}))$  for different  $\Gamma_W$  in the numerator. Clearly if the scaled widths were not close, the corrections to the ratio would still be large.

## 6 Conclusions

In this paper we have presented an alternative method to calculate  $W$ -boson observables by using the experimentally measured  $Z$ -boson observable and a perturbative calculation of the ratio of the  $W$ - over  $Z$ -boson observables. The radiative corrections, which can be large for these observables, tend to cancel in the ratio. Using this method, the predicted transverse momentum distribution and the transverse mass distribution of the  $W$ -boson have small radiative corrections over the whole range of relevant values. For the transverse energy distribution of the lepton a NNLO prediction might be necessary for the main injector data. With the current data sets the method described in this paper can be used to estimate the future expected uncertainties. One can then decide on the necessity of higher order predictions. It is worthwhile to investigate the method outlined in this paper as an alternative way to measure the  $W$ -boson parameters. Possibly, this method could augment/replace the current methods at high luminosity runs. It reduces both theoretical and experimental systematic uncertainties at the price of increasing the statistical uncertainty. Such a trade-off becomes more and more interesting as the integrated luminosity increases at the TEVATRON.

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<sup>4</sup>We normalize this scaled variable such that the remnant of the jacobian peak is at  $X_{E_T} = 1$ .

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