Fermi National Accelerator Laboratory

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Jet Energy Resolution due to Calorimetric Resolution

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Introduction

The LHC will operate at 7 TeV/beam rather than the 1 TeV/beam of the Fermilab Tevatron. Thus, the jet energy range will extend, at the LHC, beyond the kinematic range which we are accustomed to. It is then appropriate to see if our understanding of the jet energy resolution extends naturally. To begin, one attempts a simple analytic analysis of the energy resolution of a jet as an ensemble of particles.

Initial Calculations

Consider a jet to be an ensemble of n objects labeled by index i. The energy E of the jet is the sum of the energies of the particles making up that ensemble. Error propagation, assuming no correlation, allows one to relate the jet energy error to the single particle error.

$$E = E_1 + E_2 + \ \ldots \ = \Sigma E_i \tag{1}$$

$$E_i = z_i E$$

$$(dE)^2 = \Sigma (dE_i)^2$$

Assume, for simplicity, that all particles are measured by a calorimetric device which can be characterized by a "stochastic",a, and a "constant" ,b, term in the energy resolution. The jet energy resolution is then related to the single particle response as follows.

$$(dE_i/E_i)^2 = (a^2/E_i) + b^2 \tag{2}$$

$$(dE)^2 = \Sigma (E_i)^2 (a^2/E_i + b^2)$$

$$(dE/E)^2 = a^2/E + b^2 \Sigma z_i^2$$

Clearly, tentative conclusions can immediately be drawn. For low energy jets the energy resolution is dominated by the stochastic part and the fractional energy error of a jet is the same as that for a single particle. This is roughly the experience for jets as observed at Tevatron energies and below. However, there is a different behavior in the high energy regime. As E gets very large, $(dE/E)^2 \dashrightarrow b^2 \Sigma z_i^2$, and the jet energy resolution is dominated by the constant terms. Note that, the weight is given to the highest energy particles in the jet, i.e. the largest z_i. Thus, the energy resolution for very energetic jets is driven by the constant term in the single particle resolution and by the highest energy fragments of the jet. The low energy fragments and the magnitude of the stochastic term are simply not relevant.

One can also make a modest extrapolation. Suppose that the jet is composed of two different types of particles. If one type, the gammas, are measured arbitrarily well, while the other type is measured with standard calorimetric error, then low energy jets have an energy resolution, $(dE/E)^2 \sim a^2(1-\langle zo \rangle)/E$, where $\langle zo \rangle$ is the mean energy fraction of the jet which goes into neutral particles. For very high energy jets one expects that $(dE/E)^2 \sim b^2(1-\langle zo \rangle^2)\Sigma z_i^2$.

Simple Monte Carlo Results

In order to go one step beyond the simple calculations, a lowest order model was written to simulate the fragmentation of a jet into final state particles. A power law fragmentation function, D(z), was used which is normalized such that the energy sum rule (Eq.1) is satisfied. A low energy cut off is adopted for the fragments, leading to a mean jet multiplicity which goes as ln(E). This simple model is, however, in accord with measured jet fragmentation properties observed at the Tevatron.

$$zD(z) = (a + 1)(1 - z)^a \qquad (3)$$

$$\int zD(z)dz = 1$$

$$\int D(z)dz = \langle n \rangle \sim \ln(E)$$

$$z_{min} = m_\pi/E$$

This model was used to pick the z value of the fragments. Transverse momentum, for this study, was ignored. The charge was randomly chosen to be 1/3 neutral and 2/3 probability to be charged. The standard deviation of the observed means is large. For charged and neutral fragments, $\langle zc \rangle = 0.69 \pm 0.19$ and $\langle zo \rangle = 0.30 \pm 0.20$. The most energetic fragment carries, on average, a fraction $\langle z_1 \rangle = 0.24 \pm 0.09$, of the jet energy for E = 200 GeV jets. The correlation , event by event, of zo and zc is shown in Fig.1. The sum which appear in the jet constant term, $\langle \mathrm{sqrt}(\Sigma z_i^2) \rangle$, is observed to be 0.357 +- 0.083.

The energy of the individual particles was smeared according to the CMS baseline. For neutral energy a 4% stochastic coefficient and a 1% constant term was chosen. The exact values of these parameters is not relevant, as the neutral energy is effectively measured with perfect accuracy with respect to the charged hadrons.

For hadrons, an e^{-6} probability was assigned to non interacting punch through which exits the calorimetry before interacting. The energy was assigned a 30% rms, consistent with H2 test beam results. For hadrons showering inside the CMS HCAL, a 5% chance to have a high side tail due to a weighting procedure error leading to a twice value rms was assigned consistent with test beam data. For this study various values of the parameters a and b were assigned. The test beam data are consistent with a = 100% and b = 5%. Leakage and dead material were assigned a low energy tail which was consistent with the H2 test beam data gathered in 1996.

Finally a π/e value was assigned indicative of the residual non compensation after all longitudinal segmentation was utilized. The ratio was observed to be 96% at 30 GeV, rising as ln(E) to 99% at 300 GeV. Note that the relative calibration of the ECAL, H1 and H2 compartments were adjusted to make this ratio close to one. The departure from one represents a residual non linearity in the CMS calorimetric system.

The resulting distributions for E = 200 GeV jets are shown in Fig.2. The z1 distribution has a mean of 0.241 while the jet energy measurement has $\langle dE/E \rangle = 0.037$ with a standard deviation of 0.049. The shift of the mean from zero is due to the assumed p/e residual non linearity. The rms is entirely consistent with the analytic estimates developed above.

Results on jet rms values for 200 and 2000 GeV jets are shown in Fig.3. At the top of Fig.3 is shown the rms of dE/E for E = 200 GeV jets with b = 5% and for a = 50, 100 and 150%. At the bottom of Fig.3 is shown the rms of dE/E for E = 2000 GeV jets with a = 100% and with b = 0, 5 and 10%. The results can be roughly understood as being the sum in quadrature of Eq.2 suitably modified by the existence of charged and neutral fragments, the latter effectively being measured with no error.

$$(dE/E)^2 \sim a^2(1-\langle zo \rangle)/E + b^2(1-\langle zo \rangle^2)\Sigma z_i^2 \qquad (4)$$

Thus, for the 200 GeV jets, one expects, applying the values given above, a 1.7% constant term in quadrature with 2.9%, 5.8% and 8.65% stochastic error. This means errors of 3.3%, 6.0% and 8.8% are expected for 200 GeV jets with hadron calorimetry having 5% constant error and 50%, 100% and 150% stochastic coefficients. A glance at Fig.3 shows that these expectations a borne out in the model. Similarly, a 2000 GeV jet with 100% stochastic coefficient and 0%, 5% and 10% constant term is expected to have a jet resolution with a stochastic contribution of 2.2% folded in quadrature with a constant term of 0%, 1.7% and 3.4% respectively. The full resolution expected is 2.2%, 2.8% and 4.0% which is in good agreement with the model results shown in Fig.3.

Summary

In summary a simple set of analytic expressions was developed relating jet and single particle energy resolutions. These were checked against a simple Monte Carlo model which embodies many of the properties of real jets. The agreement was found to be excellent. One can conclude that at low energies the jet and single particle resolutions are roughly equal. Note that in scattering and out scattering losses due to fixed cone algorithms used in the presence of underlying event particles is not addressed here. At high energies, one can conclude that jets may have better resolutions than single particles. This fact may make searches for a possible composite structure of quarks, which will utilize multi TeV jets at the LHC, somewhat easier than might have been anticipated.

References

Figure Captions

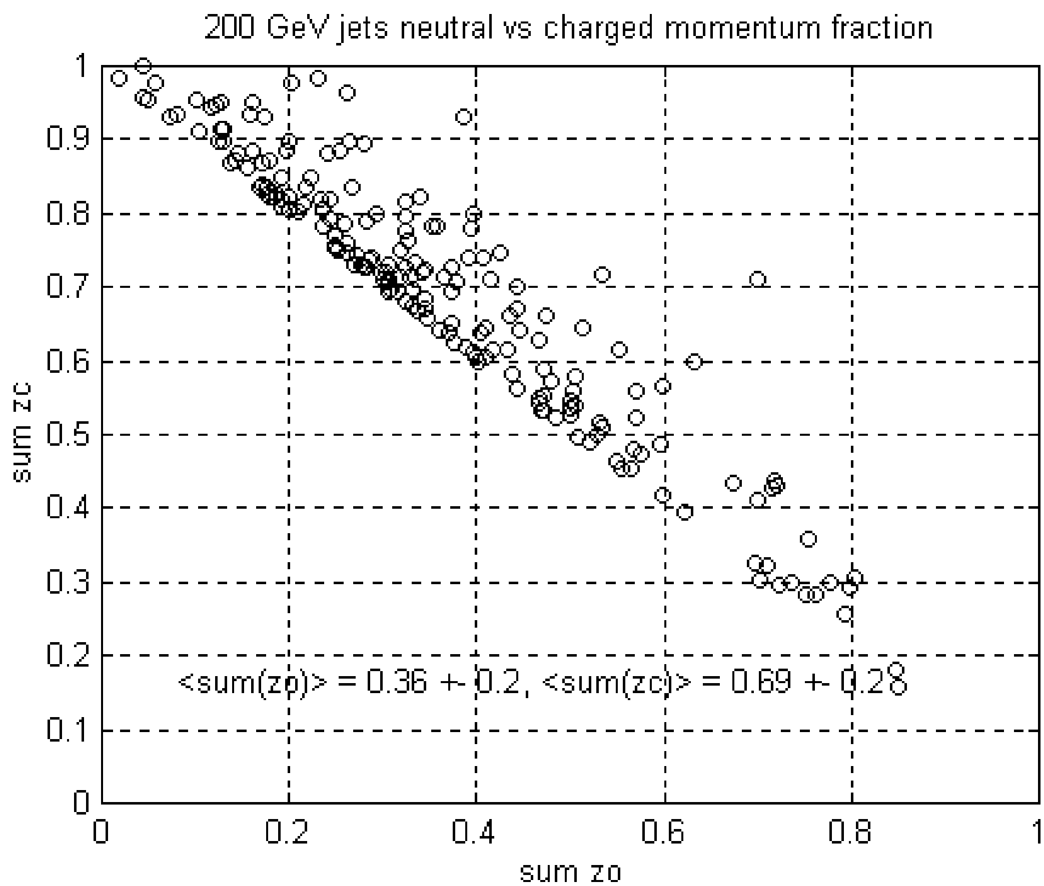

Figure 1. Scatter plot of the summed neutral momentum fraction in the jet vs the summed charged momentum fraction. On average the model requires 1/3 neutral and 2/3 charged.

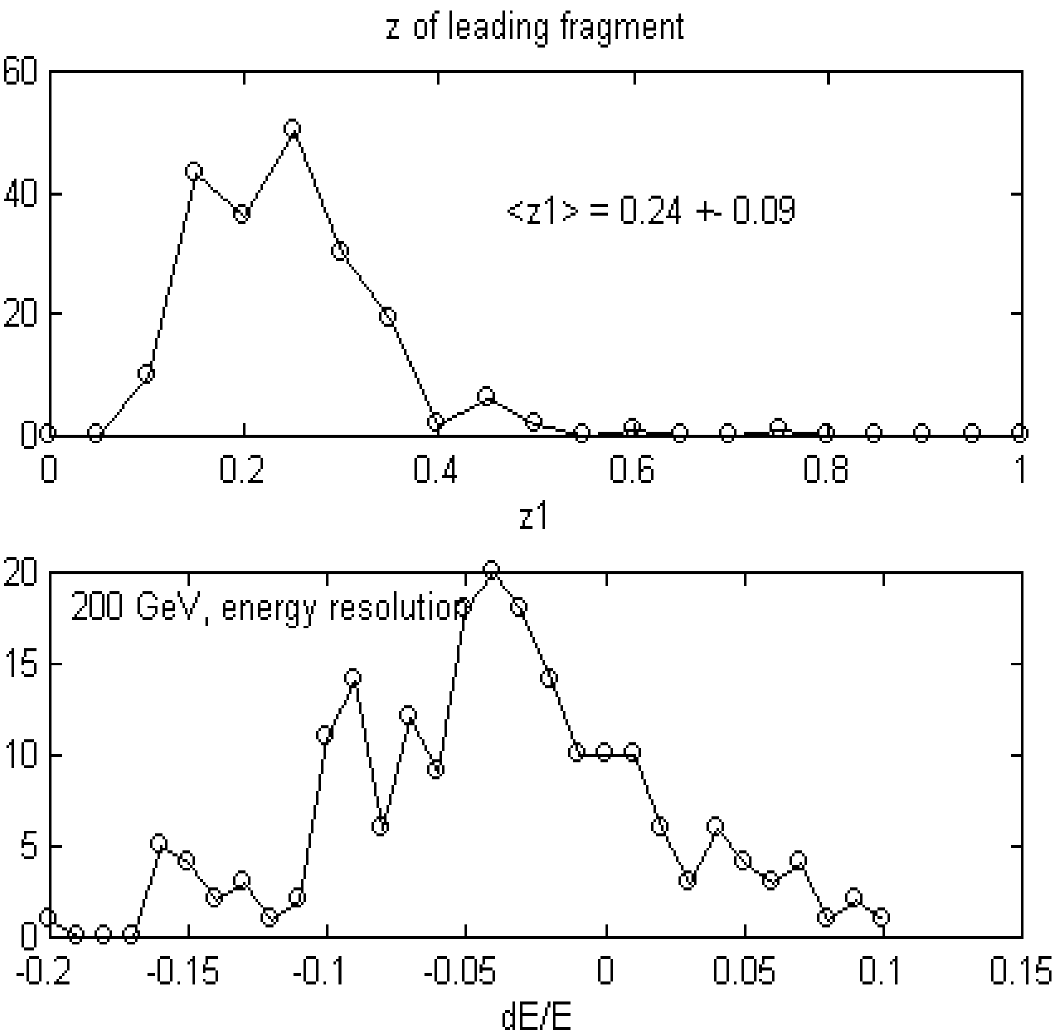

Figure 2 a Histogram of momentum fraction of the fastest particle in a 200 GeV jet. The mean over 200 jets is <z1> = 0.24.
b Histogram of dE/E for 200 GeV jets with charged particle resolutions with 100% stochastic coefficient and 5% constant term.

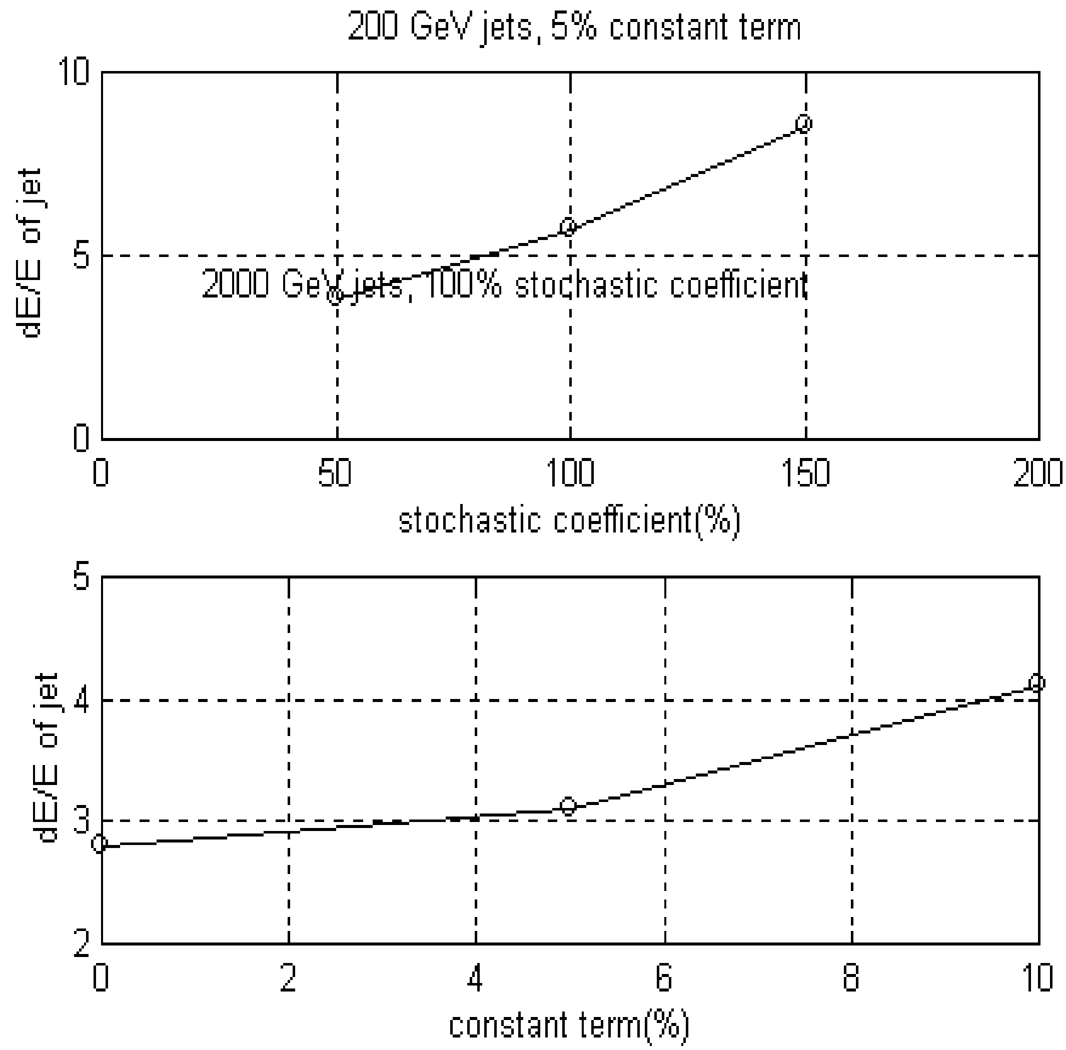

Figure.3 a Energy resolution for 200 GeV jets with 5% constant term and stochastic coefficients of 50%, 100% and 150%.
b Energy resolutions for 2000 GeV jets with 100% stochastic coefficeint and 0%, 5% and 10% constant term for the single particle resolution.