



The Case for Λ CDM

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Abstract

The case is simple: there is no compelling theoretical argument against a cosmological constant and Λ CDM is the *only* CDM model that is consistent with all present observations. Λ CDM has two noteworthy features: it can be falsified in the near future (the prediction $q_0 \sim -\frac{1}{2}$ is an especially good test), and, if correct, it has important implications for fundamental physics.

1 Introduction

1.1 Motivation

Inflation is a bold and expansive idea that stands upon the tall shoulders of the hot big-bang cosmology [19, 25]. It holds the promise of extending the standard cosmology to times as early as 10^{-32} sec, of addressing almost all of the pressing issues in cosmology, and of shedding light on the unification of the forces and particles of Nature. Inflation is ripe for testing and the cold dark matter (CDM) theory of structure formation plays a central role.

While there is no standard model of inflation, there are three robust predictions: (i) spatially flat Universe ($\Omega_0 \equiv \sum_i \Omega_i = 1$ where $i =$ baryons, cold dark matter, hot dark matter, vacuum energy, radiation, etc.); (ii) nearly scale-invariant spectrum of gaussian density perturbations; and (iii) nearly scale-invariant spectrum of gravitational waves. (A few would dispute the prediction of a flat Universe [7, 18]; however, I believe there is ample reason for calling it a robust prediction [25].) These predictions, together with the big-bang nucleosynthesis determination of the baryon density, $\Omega_B h^2 = 0.008 - 0.024$ [9], and the failure of hot dark matter models to account for the structure observed today [44], make the cold dark matter theory of structure formation an important secondary prediction.

When the CDM scenario emerged more than a decade ago many referred to it as a “no-parameter theory” because it was so specific compared to previous models of structure formation [4]. However, this was enthusiasm speaking

as there are cosmological quantities that must be specified in any theory of structure formation. These parameters lead to families of CDM models. Fortunately, observations are becoming good enough to both decisively test inflation and discriminate between CDM models.

1.2 CDM parameters

Broadly speaking the parameters can be organized into two groups [12]. First are the cosmological parameters: the Hubble constant h ; the density of ordinary matter $\Omega_B h^2$; the power-law index n that describes the shape of the spectrum of density perturbations and the overall normalization of the power spectrum A [$P(k) = Ak^n$]; the level of gravitational waves, specified by the ratio of their contribution to the variance of the quadrupole anisotropy to that of density perturbations ($\equiv T/S$) and the tensor power-law index n_T .

The inflationary parameters are in this group because there is no standard model for inflation. They are related to the scalar-field potential $V(\phi)$ that drives inflation

$$S \equiv \frac{5\langle |a_{2m}^S|^2 \rangle}{4\pi} = \frac{2.2(V_*/m_{\text{Pl}}^4)}{(m_{\text{Pl}}V_*/V_*)^2} \quad (1)$$

$$T \equiv \frac{5\langle |a_{2m}^T|^2 \rangle}{4\pi} = 0.61(V_*/m_{\text{Pl}}^4) \quad (2)$$

$$n - 1 = -\frac{m_{\text{Pl}}}{8\pi} \left(\frac{V'}{V_*} \right)^2 + \frac{m_{\text{Pl}}^2}{4\pi} \left(\frac{V'}{V_*} \right)' \quad (3)$$

$$n_T = -\frac{m_{\text{Pl}}}{8\pi} \left(\frac{V'}{V_*} \right)^2 \quad (4)$$

where prime indicates derivative with respect to ϕ and ϕ_* is the value of the scalar field when scales of order $10^4 h^{-1}$ Mpc crossed outside the horizon during inflation. It should be noted that the scalar and tensor power spectra are not exact power laws; e.g., the variation of the scalar index n with scale (or running) is given by [26]:

$$\begin{aligned} \frac{dn}{d \ln k} = & - \frac{1}{32\pi^2} \left(\frac{m_{\text{Pl}}^3 V'''}{V} \right) \left(\frac{m_{\text{Pl}} V'}{V} \right) \\ & + \frac{1}{8\pi^2} \left(\frac{m_{\text{Pl}}^2 V''}{V} \right) \left(\frac{m_{\text{Pl}} V'}{V} \right)^2 - \frac{3}{32\pi^2} \left(m_{\text{Pl}} \frac{V'}{V} \right)^4 \end{aligned} \quad (5)$$

I know of no model for which $n = 1$ (exact scale invariance); for several interesting models $n \sim 0.94 - 0.96$; for natural or power-law inflation n can be as small as 0.7; for hybrid inflation n can be as large as 1.2, or as small as 0.7 [38, 30]. The level of gravitational radiation can be negligible or significant, and typically, $dn/d \ln k \simeq -10^{-3}$.

The second group of parameters specifies the composition of invisible matter in the Universe: radiation, dark matter, and cosmological constant. Radiation refers to relativistic particles: the photons in the CBR, three massless neutrino species and possibly other undetected relativistic particles (some particle-physics theories predict the existence of additional massless particle species [13, 23]). The level of radiation today is important as it determines when the transition from radiation domination to matter domination took place, and thereby determines the characteristic turnover scale in the present CDM power spectrum, k_{EQ} (Fig. 1).

Dark matter could include other particle relics in addition to CDM. For example, a neutrino species of mass 5 eV (or two or more neutrino species whose mass summed to 5 eV) would account for about 20% of the critical density ($\Omega_\nu = m_\nu/90h^2$ eV). Predictions for neutrino masses range from 10^{-12} eV to several MeV, and there is some experimental evidence that at least one of the neutrino species has mass [33, 1, 20, 21, 15, 2].

In the modern context a cosmological constant corresponds to an energy density associated with the quantum vacuum. Since there is no reliable calculation of the quantum vacuum energy [39], the existence of a cosmological constant must be regarded as a logical possibility.

1.3 CDM family of models

The original no-parameter CDM model, or standard CDM, is characterized by simple choices for the cosmological and the invisible matter parameters: precisely scale-invariant density perturbations ($n = 1$), $h = 0.5$, $\Omega_B = 0.05$, $\Omega_{\text{CDM}} = 0.95$; no radiation beyond the photons and the three massless neutrinos; no dark matter beyond CDM; no gravitational radiation and zero cosmological constant. The overall normalization of the power spectrum (i.e., A) was determined by setting the *rms* mass fluctuation in spheres of radius $8h^{-1}$ Mpc ($\equiv \sigma_8$) equal to the inverse of the bias parameter $b \sim 1-2$, allowing for the likely possibility that light (in the form of optically bright galaxies) is more clustered than mass.

The COBE detection of CBR anisotropy on angular scales of 10° to 90° changed the normalization procedure. By requiring the predicted level of CBR anisotropy to be consistent with COBE, the power spectrum is normalized on very-large scales ($10^3 h^{-1}$ Mpc) without regard to biasing. COBE also put the stake through the heart of standard CDM: COBE-normalized standard CDM predicts too much power on small scales [31, 29] (see Fig. 1). When normalizing CDM by large-angle CBR anisotropy, the level of gravitational radiation must be specified because some of the anisotropy on large angular scales could arise from gravity waves: a higher level of gravitational radiation leads to a lower level of density perturbations.

The standard CDM set of parameters is not sacred; it was simply a starting point. In making the case for Λ CDM I will discuss four “families” of CDM mod-

els. They are distinguished by their invisible matter content: standard invisible matter content (sCDM); extra radiation (τ CDM); small hot dark matter component (ν CDM); and a cosmological constant (Λ CDM). There are, of course, more complicated possibilities, e.g., $\nu\Lambda$ CDM, etc.

2 Evidence: observations favor Λ CDM

In reviewing the observations I will show that of these four models only Λ CDM is consistent with all present data.

2.1 CBR anisotropy

There are now more than ten independent detections of CBR anisotropy on angular scales from 0.5° to 90° and the angular power spectrum is beginning to show a Doppler peak at $l \sim 200$ (as expected for a flat Universe). However, the strongest constraints come from the COBE four-year data set [3, 17, 41] which implies: quadrupole anisotropy $Q = (18 \pm 2 \pm 1) \mu\text{K}$ (for $n = 1$), where the error is statistical + systematic arising from galaxy subtraction, and $n = 1.1 \pm 0.2$. CBR anisotropy serves to normalize the power spectrum and exclude models with $n < 0.7$.

2.2 Power spectrum

There are three robust constraints to the power spectrum from observations of the contemporary Universe: the shape derived from several redshift surveys [34] (Fig. 1); the value of σ_8 derived from the abundance of x-ray clusters [42], $\sigma_8 = (0.5 - 0.8)\Omega_{\text{Matter}}^{-0.56}$; the level of inhomogeneity on small scales ($\sim 0.2h^{-1}\text{Mpc}$) required to insure early structure formation (neutral gas in damped Ly- α systems at redshift four, $\Omega_{\text{DLy-}\alpha}h = 0.001 \pm 0.0002$ [28, 35]). For all four families, there are a variety of cosmological parameters for which the COBE-normalized power spectrum is consistent with these three constraints (Fig. 2).

2.3 Matter density

Determinations of Ω_0 and Ω_{Matter} provide powerful tests of inflation as well as discriminating between the different CDM models. At present, the best the strongest statements that can be made are: $\Omega_0 \geq \Omega_{\text{Matter}} > 0.3$, based upon the peculiar velocities of thousands of galaxies (including the Milky Way) [36, 11]; $\Omega_\Lambda < 0.7$, based upon the frequency of gravitational lensing of QSOs [24]. While a decisive determination of Ω_0 is lacking, there is little evidence to suggest that $\Omega_{\text{Matter}} = 1$ – as predicted in all but Λ CDM – and much evidence that $\Omega_{\text{Matter}} \sim 0.3$ – as predicted by Λ CDM.

2.4 Hubble constant/Age of Universe

Together, these two fundamental cosmological parameters have great leverage. Determinations of the Hubble constant based upon a variety of techniques (Type Ia and II supernovae, IR Tully-Fisher and fundamental plane methods) have converged on a value between $60 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $80 \text{ km s}^{-1} \text{ Mpc}^{-1}$. This corresponds to an expansion age of less than 11 Gyr for a flat, matter-dominated model; for Λ CDM, the expansion age can be significantly higher, as large as 16 Gyr for $\Omega_\Lambda = 0.6$ (Fig. 3). On the other hand, the ages of the oldest globular clusters indicate that the Universe is between 13 Gyr and 17 Gyr old; further, these age determinations, together with the those for the oldest white dwarfs and the long-lived radioactive elements, provide an ironclad case for a Universe that is at least 10 Gyr old [5, 8, 10]. *Unless the age of the Universe and the Hubble constant are near the lowest values consistent with current measurements, only Λ CDM model is viable.*

2.5 Cluster baryon fraction

Clusters are large enough that the baryon fraction should reflect its universal value, $\Omega_B/\Omega_{\text{Matter}} = (0.008 - 0.024)h^{-2}/(1 - \Omega_\Lambda)$. Most of the (observed) baryons in clusters are in the hot, intracluster x-ray emitting gas. From x-ray measurements of the flux and temperature of the gas, baryon fractions in the range $(0.04 - 0.10)h^{-3/2}$ have been inferred [43, 6, 40]; further, a recent detailed analysis and comparison to numerical models of clusters in CDM indicates an even smaller scatter, $(0.07 \pm 0.007)h^{-3/2}$ [14]. From the cluster baryon fraction and Ω_B , Ω_{Matter} can be inferred: $\Omega_{\text{Matter}} = (0.25 \pm 0.15)h^{-1/2}$, which for the lowest Hubble constant consistent with current determinations ($h = 0.6$) implies $\Omega_{\text{Matter}} = 0.32 \pm 0.2$. *Unless one of the assumptions underlying this analysis is wrong, only Λ CDM is viable.*

2.6 And the winner is ...

Since only Λ CDM is consistent with all the observations there can be little debate that it is the current strawman for structure formation. (Unless one is willing to dispute some of the observations or their interpretations – e.g., the Hubble constant, age of the Universe, or cluster baryon fraction.) Further, taken together, the constraints argue for $\Omega_\Lambda \sim 0.5 - 0.65$ and $h \sim 0.6 - 0.7$ as the best fit model (Fig. 4). Others have reached similar conclusions [27, 32].

3 Falsification of Λ CDM

At the moment, the case for Λ CDM hinges upon the cluster baryon fraction and measurements of the age and Hubble constant. However, in the near future there are a number of tests that can distinguish Λ CDM from its CDM siblings.

- Deceleration parameter. This is the most striking test: Λ CDM predicts $q_0 \equiv \frac{1}{2} - \frac{3}{2}\Omega_\Lambda \sim -\frac{1}{2}$, while the other CDM models predict $q_0 = \frac{1}{2}$. Two groups (The Supernova Cosmology Project and The High- z Supernova Team) are hoping to determine q_0 to a precision of ± 0.2 by using distant Type Ia supernovae ($z \sim 0.3 - 0.7$) as standard candles. Together, they discovered more than 40 high redshift supernovae last fall and winter and both groups should be announcing results soon.
- Hubble constant. Since the Universe is at least 10 Gyr old, a determination that the Hubble constant is $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ or greater would rule out all models but Λ CDM; on the other hand, a determination that the Hubble constant is below $55 \text{ km s}^{-1} \text{ Mpc}^{-1}$ would undermine much of the motivation for Λ CDM.
- Cluster baryon fraction. This strongly favors Λ CDM. Further evidence that x-ray measurements have correctly determined the total cluster mass (e.g., from weak gravitational lensing) and baryon mass (e.g., from AXAF) would strengthen the case for Λ CDM. On the other hand, discovery of a systematic effect that lowers the cluster baryon fraction by a factor of two (e.g., underestimation of cluster mass because gas is not supported by thermal pressure alone, or overestimation of cluster gas mass because the gas is clumped) would undermine the case for Λ CDM.
- Gravitational lensing. It has long been appreciated that Λ CDM predicts a much higher frequency of gravitationally lensed QSOs [37, 16]; however, modelling uncertainties have precluded setting a limit more stringent than $\Omega_\Lambda \lesssim 0.7$ [24]. With new QSO lensing surveys coming gravitational lensing should not be forgotten as a striking signature of Λ CDM.
- Early structure formation. Because Λ CDM is slightly antibiased (Fig. 1) structure formation commences earlier. The study of the Universe at high redshift by HST and Keck will test this prediction.
- Redshift surveys. The differences in the level of biasing, power spectrum and redshift space distortions between Λ CDM and the other models are significant. The two large redshift surveys coming on line (SDSS and 2dF) should be able to discriminate between the different CDM models.
- Theory. The theoretical underpinnings of Λ CDM could be changed by new arguments against or in favor of a cosmological constant.
- CBR anisotropy in the MAP/COBRAS/SAMBA era. The high-resolution CBR maps that will be made by these two satellite-borne experiments will settle the issue decisively (Fig. 5) – among other things, by determining both Ω_0 and Ω_Λ to better than 10% [22].

4 Final Remarks

Inflation is a bold, expansive and attractive extension of the standard cosmology, and the cold dark matter theory of structure formation provides a crucial test of it. Although I am not wedded to any CDM model – I will be happy to see any one proven correct – the only model consistent with all present observations is Λ CDM. To be fair, the case hinges upon the Hubble constant and cluster baryon fraction, neither one of which has been settled completely; however, new observations (especially q_0) should clarify matters soon.

To end, I summarize my view of the world models discussed by this panel. Λ CDM is the best fit to the present data; sCDM is the most elegant; ν CDM has the most striking signature – around 5 eV of neutrino mass; the defect models are the most interesting; and OCDM is my worst nightmare!

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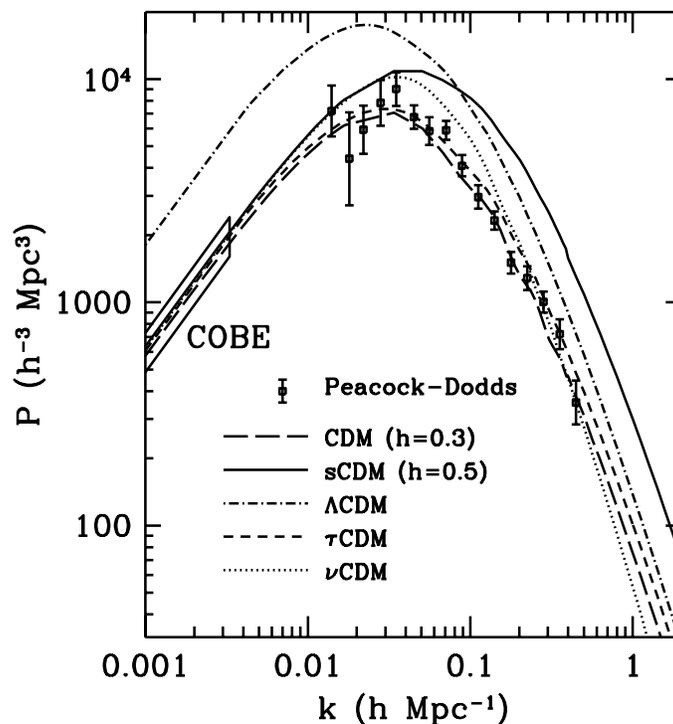


Figure 1: Measurements of the power spectrum, $P(k) = \langle |\delta_k|^2 \rangle = Ak^n$, and the predictions of different COBE-normalized CDM models. The points are from several redshift surveys compiled by Peacock and Dodds [34]; the models are: Λ CDM with $\Omega_\Lambda = 0.6$ and $h = 0.65$; standard CDM (sCDM), CDM with $h = 0.35$; τ CDM (with the energy equivalent of 12 massless neutrino species) and ν CDM with $\Omega_\nu = 0.2$ (unspecified parameters have their standard CDM values). The offset between a model and the points indicates the level of biasing. Note, Λ CDM does not pass through the COBE rectangle because a cosmological constant alters the relation between the power spectrum and CBR anisotropy (from Ref. [12]).

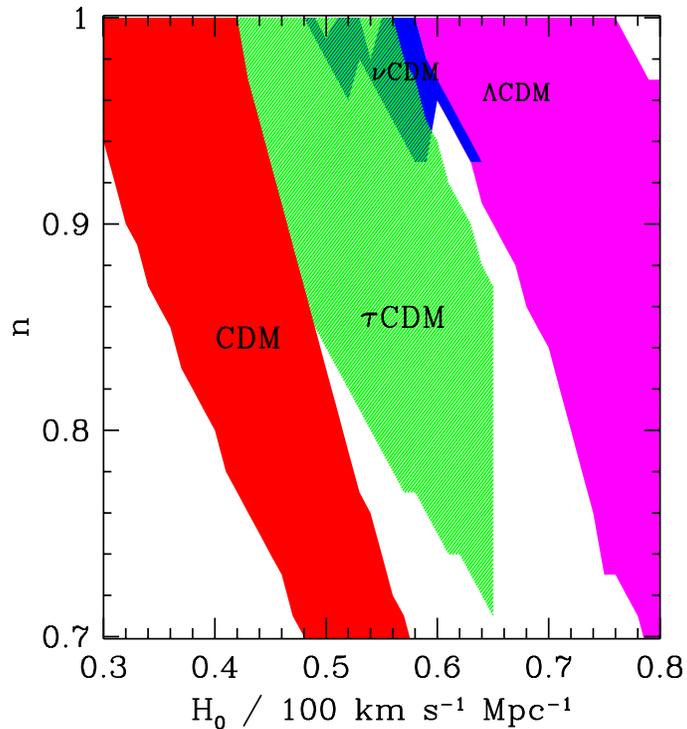


Figure 2: Acceptable values of the cosmological parameters n and h for CDM models with standard invisible-matter content (CDM), with 20% hot dark matter (νCDM), with additional relativistic particles (the energy equivalent of 12 massless neutrino species, denoted τCDM), and with a cosmological constant that accounts for 60% of the critical density (ΛCDM). The τCDM models have been truncated at a Hubble constant of $65 \text{ km s}^{-1} \text{ Mpc}^{-1}$ because a larger value would result in a Universe that is younger than 10 Gyr (from Ref. [12]).

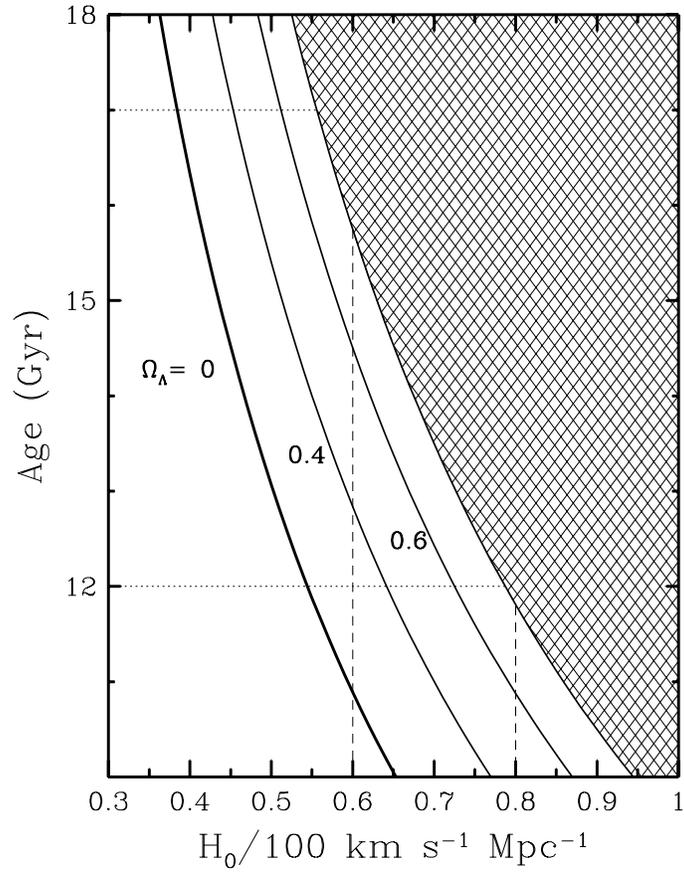


Figure 3: The relationship between age and H_0 for flat-universe models with $\Omega_{\text{Matter}} = 1 - \Omega_\Lambda$. The cross-hatched region is ruled out because $\Omega_{\text{Matter}} < 0.3$. The dotted lines indicate the favored range for H_0 and for the age of the Universe (from Ref. [12]).

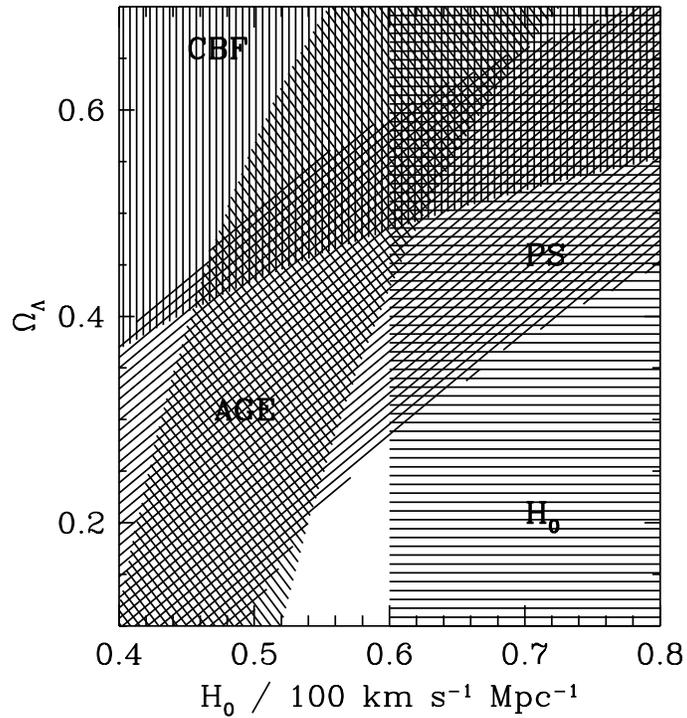


Figure 4: Summary of constraints projected onto the $H_0 - \Omega_{\text{Matter}}$ plane: (CBF) comes from combining the BBN limit to the baryon density with x-ray observations of clusters; (PS) arises from the power spectrum; (AGE) is based on age determinations of the Universe; (H_0) indicates the range currently favored for the Hubble constant. (Note the constraint $\Omega_\Lambda < 0.7$ has been implicitly taken into account since the Ω_Λ axis extends only to 0.7.) The darkest region indicates the parameters allowed by all constraints (from [27].)

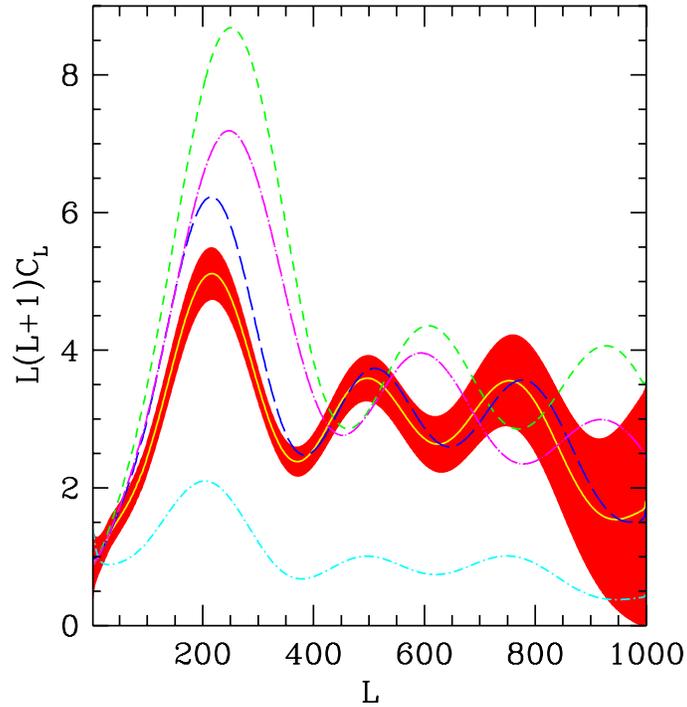


Figure 5: Angular power spectra of CBR anisotropy for several CDM models and the anticipated uncertainty (per multipole) from a CBR satellite experiment similar to MAP. From top to bottom the CDM models are: CDM with $h = 0.35$, τ CDM with the energy equivalent of 12 massless neutrino species, Λ CDM with $h = 0.65$ and $\Omega_\Lambda = 0.6$, ν CDM with $\Omega_\nu = 0.2$, and CDM with $n = 0.7$ (from Ref. [12]).