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The Charm Quark's Mass

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The charm quark's mass is determined from Monte Carlo calculations of the $\bar{c}c$ spectrum. The main sources of uncertainty are perturbation theory (for conversion to $\overline{\mathrm{MS}}$), the continuum-limit extrapolation, Monte Carlo statistics, and the effects of quenching. The (preliminary) result for the $\overline{\mathrm{MS}}$ mass is $\bar{m}_{\mathrm{ch}}(m_{\mathrm{ch}}) = 1.33 \pm 0.08$ GeV.

1. INTRODUCTION

Nonperturbative lattice calculations of the hadron spectrum provide a connection between experimentally measured masses and the couplings of the (lattice) QCD Lagrangian. By convention, however, the $\overline{\mathrm{MS}}$ couplings $\bar{\alpha}(M_Z)$ and $\bar{m}(\mu)$, used in phenomenology, are usually quoted. The two sets of definitions can be related to each other in perturbation theory. For example,

$$\bar{m}(\mu) = M(a)\left[1 + \frac{\alpha(q^*)}{4\pi}\left(C_0 + \gamma_0 \ln \mu^2 a^2\right)\right], \quad (1)$$

where $\gamma_0 = 4$. The lattice mass M and, by implication, $C_0 = C_0(M)$ are specified below. Eq. (1) omits higher orders in the gauge coupling α and power-law artifacts.

This paper determines the charm quark's mass, $\bar{m}_{\mathrm{ch}}$, from quenched calculations of the $\bar{c}c$ spectrum. To anticipate the main sources of uncertainty, let us recall recent determinations the average of the up and down quarks' masses [1,2]. There the three largest uncertainties [1] stem from, in descending order, the quenched approximation, the extrapolation to the continuum limit (even with the clover action), and perturbation theory. Each of these takes on a different guise for charm, however.

The error in a coupling from quenching can be partly explained by noting that couplings run differently in the quenched approximation [3,4]. One can account for this effect by running the couplings down to typical mesonic momenta with $n_f = 0$ and then back up to a high scale with $n_f \neq 0$. But $\bar{m}(\mu)$ does not run for $\mu < m$, so quenching should not affect $\bar{m}(m)$ much [5].

One might expect lattice spacing errors to be worse for charmonium than for light mesons, since $0.4 < m_{0,\mathrm{ch}}a < 1$ on our lattices. Our spectrum calculations are of mass splittings and the so-called kinetic mass of the meson, for which the cutoff effects are powers of $|\boldsymbol{p}a|$, not $m_0 a$ [6]. Indeed, we exploit two methods for determining $\bar{m}_{\mathrm{ch}}$, with opposite cutoff dependence. The two continuum limits agree, so cutoff effects are under *better* control.

That leaves perturbation theory as the source of the largest uncertainty. To make the most of the one-loop approximation, the only order available, we use results for $m_0 a \neq 0$ [7,8]. Furthermore, we try to reduce the effect of truncating at one loop by choosing $\alpha(q^*)$ in Eq. (1) to absorb logarithms from higher orders [9,10].

2. CUTOFF EFFECTS

In a heavy-quark system, such as charmonium, typical three-momenta are only a few hundred MeV, suggesting that worrisome lattice artifacts are of order $(m_0 a)^s$. On the other hand, it is well-known that actions for Wilson fermions approach the static limit as $m_0 a \to \infty$, showing that higher-dimension operators are suppressed by a factor of order $1/(m_0 a)^r$. The lattice Hamiltonian (defined by the transfer matrix) clarifies the middle ground, $m_0 a \approx 1$. One finds [6]

$$\hat{H}_{\mathrm{lat}} = \hat{H}_{\mathrm{cont}} + \delta\hat{H}. \quad (2)$$

Contributions to the artifact $\delta\hat{H}$ take the form

$$\langle a\delta\hat{H}_n^{[l]}\rangle \sim g^{2l} b_n^{[l]}(m_0 a)|\boldsymbol{p}a|^{s_n+1}, \quad (3)$$

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where $\boldsymbol{p}$ is a few hundred MeV, and $s_n > 0$. The function $b_n^{[l]}$ is bounded [6]. It is safe to replace it by a number of order unity, and thus the effect is about the same size for splittings of charmonium as for masses of light-quark hadrons.

Eq. (3) applies only if the hopping parameter κ is adjusted until the meson's kinetic mass

$$M_2 := (\partial^2 E/\partial p_i^2)^{-1}_{p_i=0} \tag{4}$$

equals the meson's physical mass. When $Ma \neq 0$, the rest mass $M_1 := E(\mathbf{0})$ is smaller. Nevertheless, the *splittings* of meson rest masses are accurate up to Eq. (3). In particular, the spin-averaged binding energy

$$B_1 a := (M_{1\bar{Q}Q}a)^{\mathrm{MC}} - 2(M_{1Q}a)^{\mathrm{PT}}, \tag{5}$$

where $M_{1\bar{Q}Q}$ is the spin average of mesons' rest masses, has relative errors of order $\min(\boldsymbol{p}^2a^2, v^2)$ [11]. (When the quark's rest mass M_{1Q} is computed to finite order in perturbation theory, $B_1 a$ suffers perturbative errors as well.)

To determine $\bar{m}_{\mathrm{ch}}$ we rely, therefore, on the following Monte Carlo calculations: We define the lattice spacing a in physical units from $\Delta M = M_{h_c} - \frac{3}{4}(M_{\eta_c} + 3M_{J/\psi})$ [3]. We then obtain the quark mass either from the spin-averaged binding energy B_1 of the 1S states, or from their spin-averaged kinetic mass $M_{2\bar{Q}Q}$.

3. PERTURBATION THEORY

3.1. When $m_0 a \neq 0$

If the Monte Carlo has $m_0 a \neq 0$ it is necessary to take $m_0 a \neq 0$ when deriving Eq. (1). Although C_0 remains bounded [6–8], its value can change significantly for nonzero $m_0 a$.

Eq. (1) is obtained by computing the quark's pole mass in lattice and in $\overline{\mathrm{MS}}$ perturbation theory. Because the lattice breaks Euclidean invariance, several "masses" (M_1, M_2, etc) describe the pole. One would like to pick a pole mass without dire lattice artifacts. We use two methods. In the first, we take the binding energy and set

$$m_{\mathrm{pole}} = \tfrac{1}{2}(M_{\bar{Q}Q}^{\mathrm{expt}} - B_1) \tag{6}$$

with $B_1 a$ from Eq. (5) and a from ΔM. In the second method, we use the quark's kinetic mass, but reduce uncertainty in tuning κ by taking a from the meson's kinetic mass:

$$m_{\mathrm{pole}} = (M_{2Q}a)^{\mathrm{PT}} \frac{M_{\bar{Q}Q}^{\mathrm{expt}}}{(M_{2\bar{Q}Q}a)^{\mathrm{MC}}}. \tag{7}$$

When $B_1 a$ and $M_{2Q}a$ are expanded in perturbation theory, Eqs. (6) and (7) can be matched to the expansion of m_{pole} in $\overline{\mathrm{MS}}$. The manipulations at one loop define M and C_0 in Eq. (1).

One needs, therefore, the loop corrections to the quark's rest and kinetic masses. From formulas [7,8] for M_1 and M_2, to all orders in g_0^2 and in $m_0 a$, one can expand

$$M_1 = \sum_{l=0} g_0^{2l} M_1^{[l]}. \tag{8}$$

One finds $M_1^{[0]} = \log(1+M_0)$, where $M_0 = 1/2\kappa - 1/2\kappa_{\mathrm{crit}}$. Refs. [7,8] show results for $M_1^{[1]}$.

The kinetic mass has further loop corrections, so let $Z_{M_2}(M_1) = M_2/m_2(M_1)$. The function $m_2(M)$ is chosen so that $Z_{M_2}^{[0]} = 1$, but it is evaluated at the all-orders M_1. With this definition $Z_{M_2}(0) = 1$, to all orders in g_0^2. Also, $Z_{M_2}^{[1]}$ is tadpole-free. It is small ($0 \geq Z_{M_2}^{[1]} > -0.1$) and hardly depends on the clover coupling c_{SW} [8].

3.2. Choosing $\alpha(q^*)$

With only the one-loop approximation at hand, the right-hand side of Eq. (1) is sensitive to the choice of scheme for α and its scale q^*. Since Eq. (1) is the combination of lattice and $\overline{\mathrm{MS}}$ perturbation theory, the original series must be expressed in a common schem and the scales must be run to a common one. Here we use the scales suggested in Refs. [9,10], primarily for α_V, but also for $\bar{\alpha}$.

For dimensional regulators Ref. [9] prescribes

$$\ln q_{\mathrm{BLM}}^2/\mu^2 = I^*/I, \tag{9}$$

where I^* is derived from the Feynman diagram for I by replacing gluon propagators by

$$q^{-2} \mapsto q^{-2}\left[\ln(q^2/\mu^2) - b_1^f/\beta_0^f\right]. \tag{10}$$

The constant depends on the scheme: for $\bar{\alpha}$, $b_1^f/\beta_0^f = 5/3$; for α_V, $b_1^f/\beta_0^f = 0$.

Similarly, for the lattice Ref. [10] prescribes

$$\ln q_{\mathrm{LM}}^2 a^2 = I^*/I, \tag{11}$$

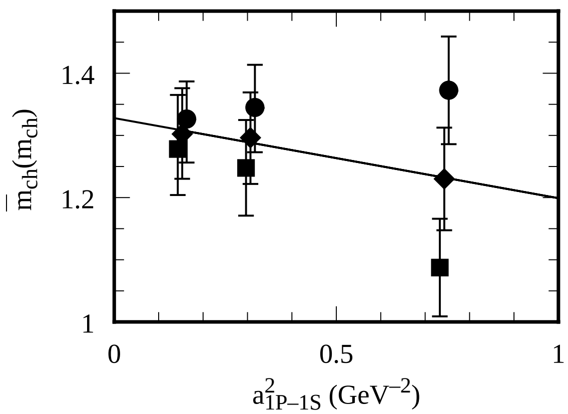

Figure 1. The charm quark's $\overline{\rm MS}$ mass vs. lattice spacing squared. Circles (squares) denote Method 1 (2). The curve is a linear fit of the average (diamonds). Offsets in a^2 are for clarity.

where I^* now comes from the replacement

$$\hat{q}^{-2} \mapsto \hat{q}^{-2}\ln(\hat{q}^2a^2). \tag{12}$$

With no constant, this prescription is for the coupling defined in Ref. [10], which coincides with α_V through next-to-leading order.

When combining the series to form Eq. (1), one can combine $q^2_{\rm BLM}$ and $q^2_{\rm LM}$ in the usual way,

$$\ln(q^{*2}a^2) = (I^*_{\rm lat} - I^*_{\rm cont})/(I_{\rm lat} - I_{\rm cont}) \tag{13}$$

provided the constants used to define the I^*s are compatible. (Otherwise the final q^* has problems as ma, $m/\mu \to 0$.) Most straightforward, we find, is to use α_V and to extract $\bar{m}_{\rm ch}(m_{\rm ch})$ directly from Eq. (1). The resulting q^*s are a few GeV but somewhat a dependent.

4. RESULTS

We have computed the charmonium spectrum for $(\beta,\ c_{\rm SW}) = (5.5,\ 1.69)$, $(5.7,\ 1.57)$, $(5.9,\ 1.50)$, and $(6.1,\ 1.40)$ [12]. Our (preliminary) results for $\bar{m}_{\rm ch}(m_{\rm ch})$ with tadpole-improved perturbation theory are plotted against a^2 in Fig. 1. The error bar is dominated by the unknown two-loop correction to Eq. (1), estimated to be twice the square of the one-loop term. When the analysis is repeated without tadpole improvement, but still choosing $\alpha(q^*)$ as in Sect. 3.2, the data change negligibly. The subdominant uncertainty is from the Monte Carlo statistics of $M_{2\bar{Q}Q}$.

Extrapolating the average of the two methods linearly in a^2 yields

$$\bar{m}_{\rm ch}(m_{\rm ch}) = 1.33 \pm 0.08 \text{ GeV}. \tag{14}$$

The error bar now incorporates uncertainty in the extrapolation, e.g., extrapolating linearly in a. Note that the quoted result neither explicitly corrects for, nor assigns an error to, quenching, because $\bar{m}(\mu)$ does not run when $\mu < m$ [5].

A 6% uncertainty for the charm quark's mass is twice the 3% quoted for the top quark's mass from collider experiments. Alas, without two-loop (or nonperturbative) matching for $m_0a \neq 0$, top standards will be impossible to achieve.

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