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Next to Leading Order Three Jet Production at Hadron Colliders[1]

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Abstract

I present results from a next-to-leading order event generator of purely gluonic jet production. This calculation is the first step in the construction of a full next-to-leading order calculation of three jet production at hadron colliders. Several jet algorithms commonly used in experiments are implemented and their numerical stability is investigated. A numerical instability is found in the iterative cone algorithm which makes it inappropriate for use in fixed order calculations beyond leading order.

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1 Introduction

In this talk I report the first step in constructing a Next-to-Leading Order (NLO) three jet event generator for hadron colliders [1], work that has been done in collaboration with Walter Giele. This first step involves the construction of the purely gluonic contribution to this cross section. It is expected that this calculation will have a number of phenomenological applications. In this talk, I will present results on the numerical stability of various jet algorithms and on their applicability to the comparison of theory to experiment. In particular, I will find that the iterative cone algorithms currently used by CDF and D0 have an infrared instability which precludes their use in fixed order calculations beyond leading order. In section 2, I will describe the methods and techniques used in the event generator, in section 3, I will describe the jet algorithms used, in section 4, I will present the results of our study and in section 5, I will summarize our findings.

2 The method

The calculation of gluonic three jet production at next-to-leading order combines the $gg \to ggg$ process computed to one loop [2] with the $gg \to gggg$ process computed at Born approximation [3, 4, 5, 6, 7, 8]. individually, each of these contributions contains infrared singularities. Only the sum of the two contributions is finite and therefore only the sum provides a meaningful calculation of the cross section. The one loop amplitudes are infrared singular because of massless partons going "on shell" within the loops. These singularities appear as double and single poles in the dimensional regulator ε multiplying the Born amplitude. The bremsstrahlung term ($gg \to gggg$) becomes singular when one final state parton becomes very soft or when one parton becomes highly collinear with another. In either case, the singularities appear when there is a loss of parton resolvability, that is when the four parton final state is indistinguishable from a three parton final state.

The cross section for any number of *resolved* partons is finite and well defined at each order in perturbation theory. All that is needed is a resolution criterion. This resolution criterion can take many forms, from a simple invariant mass cut to a full blown fragmentation function. For this study a simple invariant mass resolution criterion, $s_{\rm min}$, suffices [9, 10]. For two partons i and j, if $s_{ij} > s_{\rm min}$, the two partons are said to be resolved from one another, while if $s_{ij} < s_{\rm min}$, the partons are said to be unresolvable and the event falls into the soft or collinear region of phase space. Note that for massless partons $s_{ij} = 2E_iE_j(1-\cos\theta_{ij})$, so that the $s_{\rm min}$ criterion regulates both soft ($E_i \to 0$) and collinear ($\theta_{ij} \to 0$) configurations simultaneously.

If $s_{\rm min}$ is chosen to be sufficiently small, the concept of parton resolution should not interfere with that of jet resolution. The identification of a parton as "resolved" does not preclude it from being clustered with other partons by the jet algorithm. It simply organizes the partonic calculation and allows for the cancellation of the infrared singularities. This cancellation is accomplished by an improved version the phase space slicing method [11], in which unresolvable configurations are removed from the bremsstrahlung calculation, the true matrix elements and unresolved phase space are replaced by their asymptotic infrared forms, and are integrated analytically, yielding $2 \to 3$ parton configurations with exactly the right singularities needed to cancel those of the one loop terms as well as a residual logarithmic dependence on $s_{\rm min}$. When these terms are combined with the one loop contribution, the result is finite but explicitly $s_{\rm min}$ dependent. The resulting $2 \to 4$ contribution is also finite but is implicitly $s_{\rm min}$ dependent, since an $s_{\rm min}$ dependent volume of phase space has been sliced out or it. This $s_{\rm min}$ dependence cancels when the $2 \to 3$ and $2 \to 4$ contributions are combined, providing an important cross check that the rearrangement of terms has been handled properly.

The improvement on the slicing method introduced in this calculation, involves correcting for the approximation of using the infrared asymptotic forms of the matrix elements and phase space. We refer to this improved slicing algorithm as the subtraction method. If $s_{\min}$ is chosen to be very small, the asymptotic approximations will be very good, and the subtraction method will not offer a significant improvement over slicing. However, since each term depends quadratically on the logarithm of $s_{\min}$, the magnitude of the each term, and therefore the magnitude of the cancellation between the two terms grows as $s_{\min}$ becomes small. A large cancellation requires that each term be computed to very high precision which demands a heavy toll in computer resources. For practical reasons, one would therefore like to use the largest possible value of $s_{\min}$. It is expected that the subtraction method will allow us to use a larger value of $s_{\min}$ than is possible with the slicing method.

3 Jet Algorithms

The purpose of the jet algorithm is to quantify certain topological features of hadronic energy flow in scattering processes. By identifying high transverse momentum hadronic clusters in collisions we can make a connection with the underlying partonic scattering and apply perturbative QCD to predict the cross section. The NLO three jet calculation provides new sensitivity to the details of jet algorithms. Leading order calculations are not sensitive to the details of jet algorithms, they simply use the algorithms to identify hard scattering processes and eliminate configurations in which partons are clustered together. With minimal tuning, all jet algorithms can be made to give the same results. The only previous NLO jet calculation for hadron colliders is for two jet production [12, 10]. Since the relevant final states contain at most three partons, kinematics forbids the clustering of more than a single pair, so that this calculation places no greater demand on the jet algorithm than leading order calculations. In the NLO three jet calculation, there are as many as four partons in the final state, and it is possible to obtain double clusterings, resulting in only two jets in the final state. Such configurations belong to the NNLO two jet cross section and are excluded from the NLO three jet calculation. It is at the multiple clustering boundary that the differences in jet algorithms become apparent.

In this study, we consider four common jet clustering algorithms:

(a) The "fixed-cone" algorithm used by UA2 [13].

(b) The "iterative-cone" algorithm, used by CDF [14] and D0 [15].

(c) The "EKS" algorithm, used in NLO 1-jet and 2-jet inclusive calculations [16].

(d) The "K_T" algorithm [17], under study by CDF and D0 [18].

The fixed-cone algorithm is phenomenologically inspired; one simply draws a cone of radius R (in pseudorapidity η – azimuthal angle ϕ space) around the highest transverse energy (E_T) cluster and all hadronic energy within that cone is assigned to the jet. The final jet coordinates are determined by the E_T weighted center of the jet. The iterative cone algorithm seeks to find the best possible cone by starting as before but once the E_T weighted center is found, a new cone is drawn around that center and a new E_T weighted center is found. This process is iterated until a stable cone axis is found. The EKS algorithm is a modification of the "Snowmass Accord" [19], in which one computes the hypothetical E_T weighted axis between two partons, and if both partons are within radius R of the axis, they are clustered into a jet unless the partons are more than $R \times R_{\text{sep}}$ ($R_{\text{sep}} \leq 2$) apart. This simulates the iterative-cone algorithm by assuming that it always find the optimum jet-axis. The K_T algorithm is inspired by the parton shower dynamics of QCD. Jets are built up by merging those clusters which are "closest" to one another in $d_{ij} = \min\{E_{Ti}, E_{Tj}\}\Delta R_{ij}$. For a complete description of these algorithms and our implementation, see the references cited above as well as reference [1].

The numerical stability of the four jet algorithms is related to the degree to which the algorithm is sensitive to soft radiation, or in other words the infrared stability of the particular algorithm. For the method of resolved partons, as is used in this paper, infrared stability is related to the extent to which the results are independent of the the resolution parameter s_{min}. This dependence is shown in fig. 1 and will be discussed in the next section.

4 Numerical Results

For the numerical results in this section we used the CTEQ3M [20] parton distribution functions (PDF's), fixed renormalization and factorization scales of 100 GeV and a center of mass energy of the $p\bar{p}$-system equal to 1800 GeV. To select events we required one jet with $E_T > 50$ GeV and at least two other jets with $E_T > 20$ GeV, all in the rapidity region, $|\eta| < 4$.

The first consideration is the s_{min}-dependence of the cross section and the determination of the range of s_{min} for which the approximations made in the different numerical methods are valid. The results are shown in fig. 1 for both the slicing and subtraction methods and all four jet algorithms. The fixed cone, EKS and K_T algorithms behave as expected and it is clear how to choose s_{min} for them. For the slicing method one must choose s_{min} smaller than 1 GeV^2 in order to get the correct answer. As expected the subtraction method allows us to choose larger values of s_{min}, though the value should still not be larger than 10 GeV^2.

We now consider the iterative cone algorithm. As can be seen in the upper curve of fig. 1c, the cross section displays a clear logarithmic dependence on s_{min}. This means that the cancellation is failing and that the algorithm is not infrared safe in that we can change the jet multiplicity by adding a soft parton somewhere in the event. This behavior can be understood in terms of QCD dynamics and the iterative nature of the algorithm. QCD prefers, kinematics permitting, to radiate in a collinear fashion. The preferred three jet configuration, therefore, is to have a single hard jet balanced by two narrowly separated jets, rather than to have three equally separated jets. Thus, for any cone size R, the most preferred configuration is to have two jets separated by $R + \epsilon$. For the tree level and virtual contributions this is a three-jet event. The situation should not change if we create a four parton configuration by adding a soft parton in between the two adjacent hard partons (as preferred by QCD's propensity for collinear radiation), and in fact it does not change for any of the jet algorithms other than the iterative cone. The soft parton gets clustered with one of the hard partons, slightly changing the jet parameters, but not affecting the jet multiplicity.

In the case of the iterative cone however, one of the two hard partons will cluster with the soft parton thereby shifting its jet-axis to within R of the other parton. Because the algorithm is iterative, the two clusters will subsequently be merged further into a single jet resulting in a two jet final state. Thus, we have changed the jet multiplicity by adding an arbitrarily soft parton to the event and as a result, the algorithm is infrared unstable and cannot be used in higher fixed order perturbative QCD. This change in jet multiplicity comes about because this is a fixed order calculation involving a delicate balance of separately divergent pieces. Real jets in an experiment are sprays of hadronic energy, not single partons. Thus, soft radiation between jets would not cause all of the energy to become concentrated into a single jet. More likely, it would simply increase the overlap of the two jets and therefore slightly shift their energy distributions. Thus, the algorithm may appear to be experimentally stable, but it is

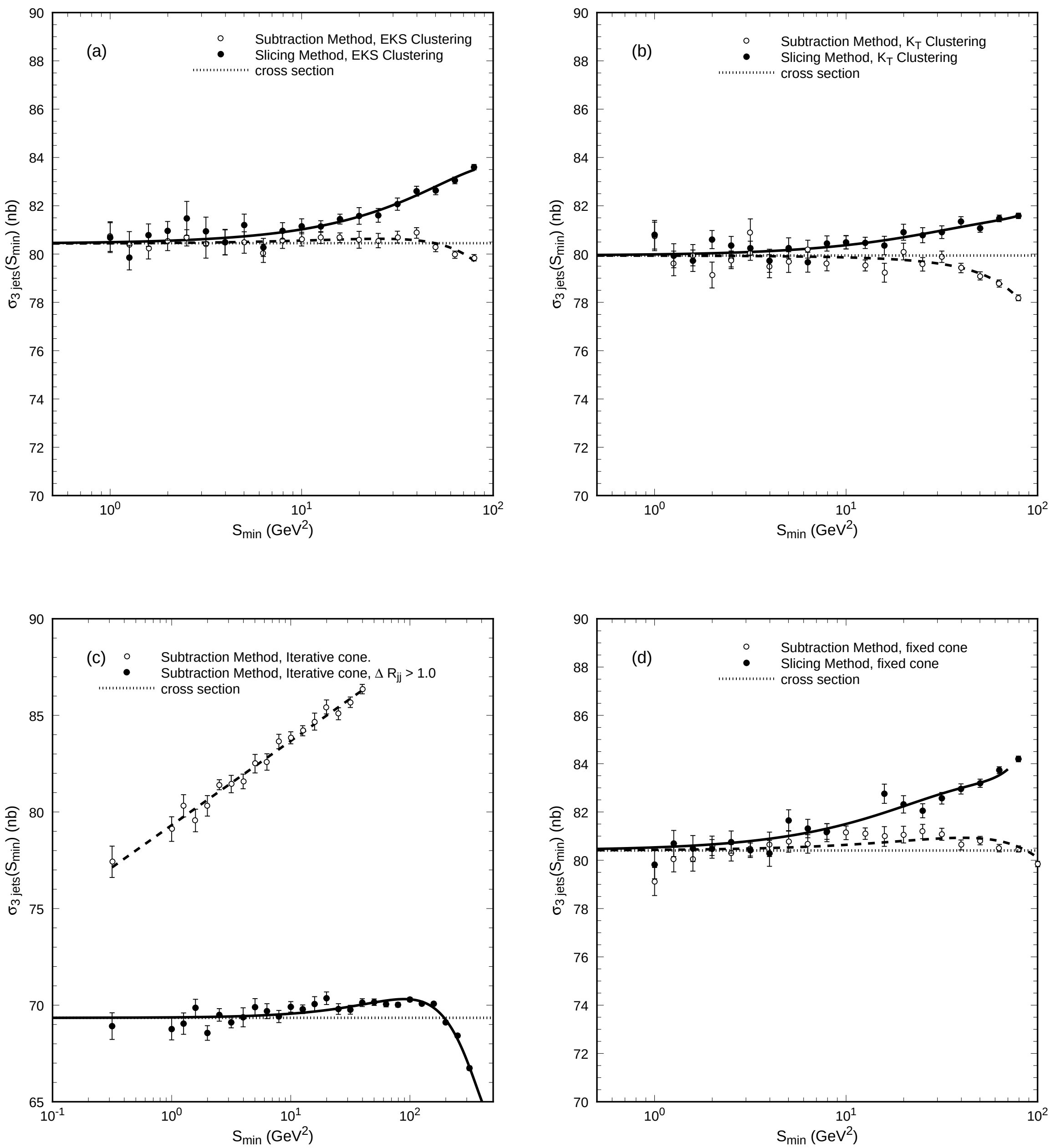

Figure 1: $\sigma_{3\mathrm{jets}}$ vs. s_{min} for different jet algorithms and numerical methods.

nonetheless clear that we cannot use it within the NLO calculation.

Note that this result does not make the one- and two-jet inclusive cross sections infrared unstable since in those cases we do not have to resolve three-jet configurations. Both CDF and D0 have compared their multi-jet data (i.e. more than two jets in the final state) with LO Monte Carlos [15, 21]. It is interesting to note that the experiments have in fact added an additional cut to their multi-jet cross section in order to make these comparisons. This cut requires all the jets in the event to be further apart then their cone-size of $R = 0.7$. For CDF this cut was $\Delta R_{jj} > 1.0$, while for D0 the requirement is $\Delta R_{jj} > 1.4$. This additional requirement in the jet algorithm changes the s_{min}-dependence of the cross section dramatically, as can be seen clearly in the lower curve of fig. 1c. In fact the behavior is now very similar to the other three algorithms. This is no surprise since with this additional selection cut the infrared instability is removed. This means that the iterative cone algorithm needs to be augmented with a jet separation cut in order to be an infrared safe jet algorithm.

5 Conclusions

In this talk I have presented results on the purely gluonic contribution to the NLO 3-jet cross section. All of the techniques used can be readily applied to the quark contributions. I have shown that the subtraction method significantly improves the numerical performance of the phase space slicing method.

All of the relevant experimental jet algorithms were implemented in the NLO 3-jet event generator and their radiative effects studied. For the iterative cone algorithm it was necessary to augment the algorithm with an additional jet separation cut in order to obtain infrared stability. Both CDF and D0 already apply such a cut in their multijet analysis, though the reason is the inefficiency of the cluster algorithm instead of the theoretically motivated removal of the infrared instability. The other jet algorithms behaved properly and no additional cuts were needed.

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