

Propagation of Majorana fermions in hot plasma

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Abstract

The properties of Majorana fermions in hot plasma are studied. One-loop resummed propagator, dispersion relations and their interpretation are discussed. It is shown that particle and hole -like solutions appear as in Dirac/chiral fermion case. The dispersion relations are, however, crucially different. We find that, in presence of a large zero temperature bare mass, hole -like excitations possess a negligible effective mass. As an example of real application, we consider the neutralinos in the minimal supersymmetric extension of the standard model and argue that for realistic values of the soft supersymmetry breaking masses the existence of practically massless hole -like excitations have a considerable effect on the thermal properties, *e.g.* the thermalization rate, of particles interacting with these Majorana excitations.

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The knowledge of the behaviour of a high temperature plasma is crucial to explain many puzzles in cosmology, *e.g.* the generation of the presently observed baryon asymmetry in the Universe during the electroweak phase transition [1]. It is well-known that the interaction of a fermion with a plasma in thermal equilibrium at temperature T modifies the fermionic dispersion relation and the poles of the fermion propagator with respect to the zero temperature case [2, 3, 4]. For instance, for an exactly conserving parity gauge theory at finite temperature (like QCD or QED), it has been shown that dispersion relations for a Dirac fermion are characterized by two possible solutions of positive energy. The addition of a quark to the equilibrium plasma, described by the incoherent superposition of many states $|\Phi\rangle$, produces a fermionic excitation (particle) $b^\dagger(\mathbf{p}, \lambda)|\Phi\rangle$ with momentum \mathbf{p} and helicity λ , while the operator $d(-\mathbf{p}, \lambda)$ does not annihilate the ground state as it does at zero temperature. On the contrary, the removal of an antiquark $d(-\mathbf{p}, \lambda)|\Phi\rangle$ produces a state with all the same quantum numbers as the particle and is referred to as a hole state (or, more precisely, antiparticle hole state). The energies of particles and holes is not the same since there is no combination of parity, charge conjugation and time reversal able to relate them. Moreover, in the limit of vanishing bare mass m (or $|\mathbf{p}| \gg m$), particles have the helicity equal to their chirality, while holes have the helicity opposite to their chirality. Consequently the hole solution propagates with the wrong correlation between chirality and helicity.

Many other different cases have been analyzed in the literature, but, to our knowledge, attention has been devoted only to the study of the properties of Dirac/chiral fermions propagating in a thermal background. However, in many attractive extensions of the Standard Model (SM) there may appear Majorana fermions with a non-negligible mass at zero temperature. In some cases these Majorana fermions have only chiral interactions. A striking example characterized by these features is provided by the Minimal Supersymmetric extension of the Standard Model (MSSM) [5] where the neutrally charged fermionic superpartners of the boson fields present in the SM, called neutralinos, possess a Majorana nature and may have a nonvanishing bare mass due to soft supersymmetry breaking interactions even in the presence of unbroken gauge symmetry. The purpose of this Letter is to study the properties of this kind particles, their resummed propagator, dispersion relation and interpretation. This study is strongly motivated by the recent observation that light stops, charginos and neutralinos may play a crucial role in generating the baryon asymmetry during the electroweak phase transition [6]. Since a detailed calculation of the final baryon asymmetry must incorporate the effects of the incoherent nature of plasma physics on CP -violating observables [7], a careful computation of the thermalization rate of supersymmetric particles in the thermal bath by making use of improved propagators and including resummation of hard thermal loops is called for. In this paper we will confine ourselves

to the inspection of the properties of improved propagators for Majorana fermions having nonvanishing bare mass and chiral interactions and the full computation of the thermalization rate of supersymmetric particles will be presented elsewhere [8].

In the MSSM chiral interactions with the surrounding thermal bath produce corrections to the inverse propagators of Majorana fermions of the general form¹

$$S^{-1}(p) = p_\mu \gamma^\mu + f_\mu \gamma^\mu \gamma^5 - m, \quad (1)$$

where m is (renormalized) chiral mass. Notice, the absence of the term $f'_\mu \gamma^\mu$ which may be traced back to the nature of Majorana particles.

The general form given above is easily shown to be valid by computing, for instance, the corrections to the inverse propagators of the \tilde{W}^3 - and \tilde{B} -neutralinos which are mass eigenstates in the unbroken gauge symmetry case². Corrections come from the quark-scalar quark-neutralino interactions

$$\begin{aligned} & - \sqrt{2} g_2 \sum_i \bar{q}_i P_R [T_{3i} \tilde{W}^3 - \tan\theta_W (T_{3i} - e_i) \tilde{B}] \tilde{q}_{iL} \\ & + \sqrt{2} g_2 \tan\theta_W \sum_i e_i \bar{q}_i P_L \tilde{B} \tilde{q}_{iR} + \text{h.c.}, \end{aligned} \quad (2)$$

and the chargino-neutralino- W^\pm interaction

$$g W_\mu^- \tilde{W}^3 \gamma^\mu \tilde{W}. \quad (3)$$

At finite temperature the one loop correction function f_μ is of the form

$$f_\mu = a(\omega, p)p_\mu + \delta_{0\mu} b(\omega, p) \quad (4)$$

with

$$a(\omega, p) = \frac{m^2(T)}{p^2} \left(1 - \frac{\omega}{p} \ln \left| \frac{\omega + p}{\omega - p} \right| \right) \quad (5)$$

and

$$b(\omega, p) = \frac{m^2(T)}{p} \left[-\frac{\omega}{p} + \left(\frac{\omega^2}{p^2} - 1 \right) \frac{1}{2} \ln \left| \frac{\omega + p}{\omega - p} \right| \right]. \quad (6)$$

Here $\omega = p_0$, $p = |\mathbf{p}|$ and $m(T) \propto T$ is the finite temperature plasma mass, $m_{\tilde{W}^3}^2 = (3/16)g^2 T^2$ and $m_B^2 = (2/9)g_1^2 T^2$ for the example sketched above. The normal fermion mass m is generated in MSSM by soft supersymmetry breaking terms, usually denoted

¹In this paper we will confine ourself to the case of unbroken gauge symmetry since the computation of the baryon asymmetry is usually made by making an expansion of the propagating Higgs bubble configuration $H(z)$ around $H(z) = 0$, see M. Carena *et al.* in [6].

²The case of Majorana higgsinos \tilde{H}_1^0 and \tilde{H}_2^0 is more involved since they mix even in the unbroken gauge symmetry case due to the presence of the term $\mu H_1 H_2$ in the superpotential. This case will be extensively considered in [8].

by M_2 and M_1 for \widetilde{W}^3 and \widetilde{B} , respectively. Note that possible nonchiral interaction corrections would be easily taken into account by replacing p_μ with $p_\mu + f'_\mu$ where f' would be of the same form than f but with different thermal mass $m(T)$.

To obtain the propagator one has simply to invert eq. (1). The propagator can in general be given in the form

$$S(p) = F + \widetilde{F}\gamma^5 + F_\mu\gamma^\mu + \widetilde{F}_\mu\gamma^\mu\gamma^5 + F_{\mu\nu}\sigma^{\mu\nu}, \quad (7)$$

where the sixteen parameters F , \widetilde{F} , F_μ , \widetilde{F}_μ , $F_{\mu\nu}$ are to be determined. The calculation of propagator is now somehow more tedious than in the cases of massless Dirac fermions and/or fermions without chiral interactions because in these cases some special properties may be used. In the present case, however, by virtue of the Lorentz structure of the system the functions F_μ , \widetilde{F}_μ , $F_{\mu\nu}$ are possible to write in the form³

$$F_\mu = F_p p_\mu + F_f f_\mu, \quad (8)$$

$$\widetilde{F}_\mu = \widetilde{F}_p p_\mu + \widetilde{F}_f f_\mu, \quad (9)$$

$$F_{\mu\nu} = F_{pf}(p_\mu f_\nu - f_\mu p_\nu) + F_\epsilon \epsilon_{\mu\nu\alpha\beta} p^\alpha f^\beta, \quad (10)$$

where $\epsilon_{\mu\nu\alpha\beta}$ is the usual completely antisymmetric tensor and coefficients F_p , F_f , \widetilde{F}_p , \widetilde{F}_f , \widetilde{F}_{pf} and F_ϵ are now (pseudo)scalars like F and \widetilde{F} . After some long, but straightforward algebraic manipulations the coefficients read

$$F = -\frac{m(f^2 - m^2 - p^2)}{(p-f)^2(p+f)^2 - m^4}, \quad (11)$$

$$\widetilde{F} = 0, \quad (12)$$

$$F_p = \frac{f^2 + m^2 + p^2}{(p-f)^2(p+f)^2 - m^4}, \quad (13)$$

$$F_f = -\frac{2(p \cdot f)}{(p-f)^2(p+f)^2 - m^4}, \quad (14)$$

$$\widetilde{F}_p = \frac{2(p \cdot f)}{(p-f)^2(p+f)^2 - m^4}, \quad (15)$$

$$\widetilde{F}_f = -\frac{f^2 - m^2 + p^2}{(p-f)^2(p+f)^2 - m^4}, \quad (16)$$

$$F_{pf} = 0, \quad (17)$$

$$F_\epsilon = \frac{m}{(p-f)^2(p+f)^2 - m^4}. \quad (18)$$

One can establish that in the special cases $m = 0$ and/or $f_\mu = 0$ the propagator coincides with the known ones.

³Note that inverting S^{-1} one has no need to take care of the fact that at finite temperature the Lorentz symmetry is actually broken to simple $O(3)$ symmetry.

From the denominator of the propagator one can read out the dispersion relation (naturally coinciding with $\det S^{-1} = 0$)

$$(p - f)^2(p + f)^2 - m^4 = 0. \quad (19)$$

This equation appears to have two different positive energy solutions $\omega_{\pm}(p)$. Even in massless limit $m \rightarrow 0$ only two different positive energy solutions exist, because it is easy to show that $(p - f)^2 > 0$ for $\omega > 0$. Note that only the positive energy solutions are physically independent, because for Majorana particle the antiparticle, corresponding the negative energy solution coincides with the particle itself.

The zero momentum limit $p \rightarrow 0$, however, can be solved exactly. It appears that the effective masses are given by ($m_{\pm} > 0$)

$$m_{\pm}^2 \equiv \omega_{\pm}^2(0) = \frac{1}{2} \left(\sqrt{m^4 + 4m^4(T)} \pm m^2 \right). \quad (20)$$

Here we make the crucial observation that in the limit of large bare mass, $m \gg m(T)$, the hole excitation effective mass m_- becomes much smaller than $m(T)$ and the temperature, $m_- \ll m(T) < T$. Expression (20) could be compared to the corresponding one for the effective masses of pure Dirac case (with nonvanishing bare mass m) where $f_{\mu} = 0$, but with a $\tilde{f}_{\mu}\gamma^{\mu}$ -term: effective masses are given by $(\omega_{\pm}^{\text{Dirac}}(0))^2 = \frac{1}{2}[m^2 + 2m^2(T) \pm \sqrt{m^4 + 4m^2m^2(T)}]$. The reader should remember that SM Dirac fermions do not possess a bare mass in the case in which the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is unbroken and their effective mass at finite temperature reduces to the plasma mass $m(T)$. On the contrary, supersymmetric Majorana fermions receive a bare mass from supersymmetry breaking even in the case of unbroken gauge symmetry and are characterized by the novel feature that the hole-like excitation may possess a very small effective mass. This property will necessarily affect the kinematics involved in the computation of the thermalization rate of the degrees of freedom interacting with these hole-like excitations.

It can be also proved that for $m \neq 0$ the derivative of ω with respect to spatial momentum is zero, *i.e.* $\frac{\partial \omega_{\pm}(p)}{\partial p} = 0$. This is in contrast with the knowledge about the massless Dirac and chiral fermions, where it is possible to show that $\frac{\partial \omega(p)}{\partial p} = \pm \frac{1}{3}$. It can be, however, shown that this peculiar property is not general but is closely related to vanishing of mass $m = 0$. Thus Majorana case represents no exemption in this sense. Indeed, at the massless limit $m = 0$ the solutions of Eq. (19) coincides to the known ones with $\frac{\partial \omega_{\pm}(p)}{\partial p} = \pm \frac{1}{3}$.

Although no simple formula for the solutions can be given, asymptotic formulas for small and large p are possible to calculate. For small momenta $p \sim 0$ they read

$$\omega_{\pm}(p) \simeq m_{\pm} + K_{\pm} p^2, \quad (21)$$

where

$$K_{\pm} \equiv \left. \frac{1}{2} \frac{\partial^2 \omega_{\pm}}{\partial p^2} \right|_{p=0} = \frac{1 + \frac{4}{9} \left(\frac{m(T)}{m_{\pm}} \right)^4 - \frac{5}{9} \left(\frac{m(T)}{m_{\pm}} \right)^8}{2m_{\pm} \left[1 - \left(\frac{m(T)}{m_{\pm}} \right)^8 \right]}. \quad (22)$$

From this we can read out that there exists a critical value for the ratio $m/m(T)$,

$$\left. \frac{m}{m(T)} \right|_{\text{crit}} = \left(\frac{4}{3\sqrt{5}} \right)^{1/2} \simeq 0.772 \quad (23)$$

such that for values smaller than it, $\omega_{-}(p)$ has a minimum for some value of p . This phenomenon is similar to what occurs in the Dirac field case and can be understood because at the limit $m \rightarrow 0$ the solution of the Majorana case tends smoothly towards the Dirac one. However, the derivative of ω_{\pm} at $p = 0$ is not continuous at the same limit. This reflects the fact that free field theory contains naturally the mass term, too. So from the point of view of naturalness $m = 0$ is a kind of pathological case. In Figs. 1 and 2 the dispersion relations are schematically given in two cases $m/m(T) = 1 > \left. \frac{m}{m(T)} \right|_{\text{crit}}$ (Fig. 1) and $m/m(T) = 1/4 < \left. \frac{m}{m(T)} \right|_{\text{crit}}$ (Fig. 2).

For large p values the functions ω_{\pm} are asymptotically given by

$$\omega_{-}(p) \sim p \left(1 + 2 e^{-1 - \frac{p^2}{m^2(T)} \sqrt{\left(\frac{m}{m(T)} \right)^4 + 4}} \right) \quad (24)$$

and

$$\omega_{+}(p) \sim p + \frac{\sqrt{m^4 + 4 m^4(T)}}{2p} - \frac{M^4}{2p^3} \ln \frac{4 p^2}{\sqrt{m^4 + 4 m^4(T)}}. \quad (25)$$

These relations coincide with the ones known in the literature for the Dirac fermion case [2] at the limit $m \rightarrow 0$. It should be noted that, for non-zero m , ω_{-} tends towards the asymptote $\omega = p$ even faster than the massless case, and therefore these modes also disappear from the spectrum very fast when momentum increases.

Let us now discuss the nature of the thermal excitations for the Majorana fermions. To do so, let us first remind the reader what happens for a Dirac fermion. At zero temperature, the ground state is the vacuum. Only four different excitations may be created from the vacuum: $b^{\dagger}(\mathbf{p}, \lambda)|\text{vac}$ (fermion) and $d^{\dagger}(\mathbf{p}, \lambda)|\text{vac}$ (antifermion) each with two values of helicity λ . At finite temperature, the ground state of the plasma contains a large number of fermion-antifermion pairs, which are not virtually present at $T = 0$ because of quantum fluctuations, but real [2]. In such a case there are eight different operators that are able to create elementary excitations: $B_p^{\dagger}(\mathbf{p}, \lambda)$ and $B_h^{\dagger}(\mathbf{p}, \lambda)$, which are linear combinations of the usual $T = 0$ fermion creation operator $b^{\dagger}(\mathbf{p}, \lambda)$ and antifermion annihilation operator $d(-\mathbf{p}, \lambda)$, and $D_p^{\dagger}(-\mathbf{p}, \lambda)$ and $D_h^{\dagger}(-\mathbf{p}, \lambda)$, which are linear combinations of the usual $T = 0$ antifermion creation

operator $d^\dagger(-\mathbf{p}, \lambda)$ and fermion annihilation operator $b(\mathbf{p}, \lambda)$. The features of the hole states become clear in the massless limit when, for instance, $B_h^\dagger(\mathbf{p}, \lambda) \equiv d(-\mathbf{p}, \lambda)$. Consider an antifermion in the plasma with momentum $-\mathbf{p}$ and spin polarized along the direction $-\lambda\hat{\mathbf{p}}$. It has chirality λ . When applied to the $T \neq 0$ ground state $d(-\mathbf{p}, \lambda)$ produces the removal of this antifermion and the creation of a hole with momentum \mathbf{p} and spin along $\lambda\hat{\mathbf{p}}$. The hole has, therefore, chirality $-\lambda$, but helicity λ . The hole has the wrong correlation between chirality and helicity [2] and it is referred to as an antifermion hole.

In the Majorana case, since at $T = 0$ particles and antiparticles coincide, the operators $b^\dagger(\mathbf{p}, \lambda)$ and $d^\dagger(-\mathbf{p}, \lambda)$ have to be identified and there is no longer any distinction between a fermion and an antifermion. In other words, when $B_p^\dagger(\mathbf{p}, \lambda) \equiv D_p^\dagger(-\mathbf{p}, \lambda)$ is applied to the ground state, it generates a Majorana fermion with spin along $\lambda\hat{\mathbf{p}}$ and helicity λ , while $B_h^\dagger(\mathbf{p}, \lambda) \equiv D_h^\dagger(-\mathbf{p}, \lambda)$, when applied to the ground state, it generates a Majorana hole (or Majorana fermion hole) with spin along $-\lambda\hat{\mathbf{p}}$ and the wrong helicity λ . Only the positive energy solutions associated to the operators $B_{p,h}^\dagger(\mathbf{p}, \lambda)$ are physically independent, because for Majorana particle the antiparticle, corresponding the negative energy solution coincides with the particle itself.

In conclusion, in this letter we have studied the structure of Majorana fermions with chiral interactions in hot plasma. An example of such theory is provided by the MSSM where neutralinos are such fields. For them, the value of the ratio $m/m(T)$ is usually larger than the critical value $\left. \frac{m}{m(T)} \right|_{\text{crit}} \simeq 0.772$, even though smaller values are not excluded. We have also found that for large m the Majorana hole excitation effective mass m_- may be very small, $m_- \ll T$, and therefore it may have remarkable effect on, *e.g.*, the thermalization rate (and thus on the coherence length) of particles interacting with them. In MSSM the usually adopted values of the soft supersymmetry breaking masses are larger than the corresponding thermal masses and therefore the smallness of m_- may drastically affect the properties of the other degrees of freedom present in the hot plasma [8].

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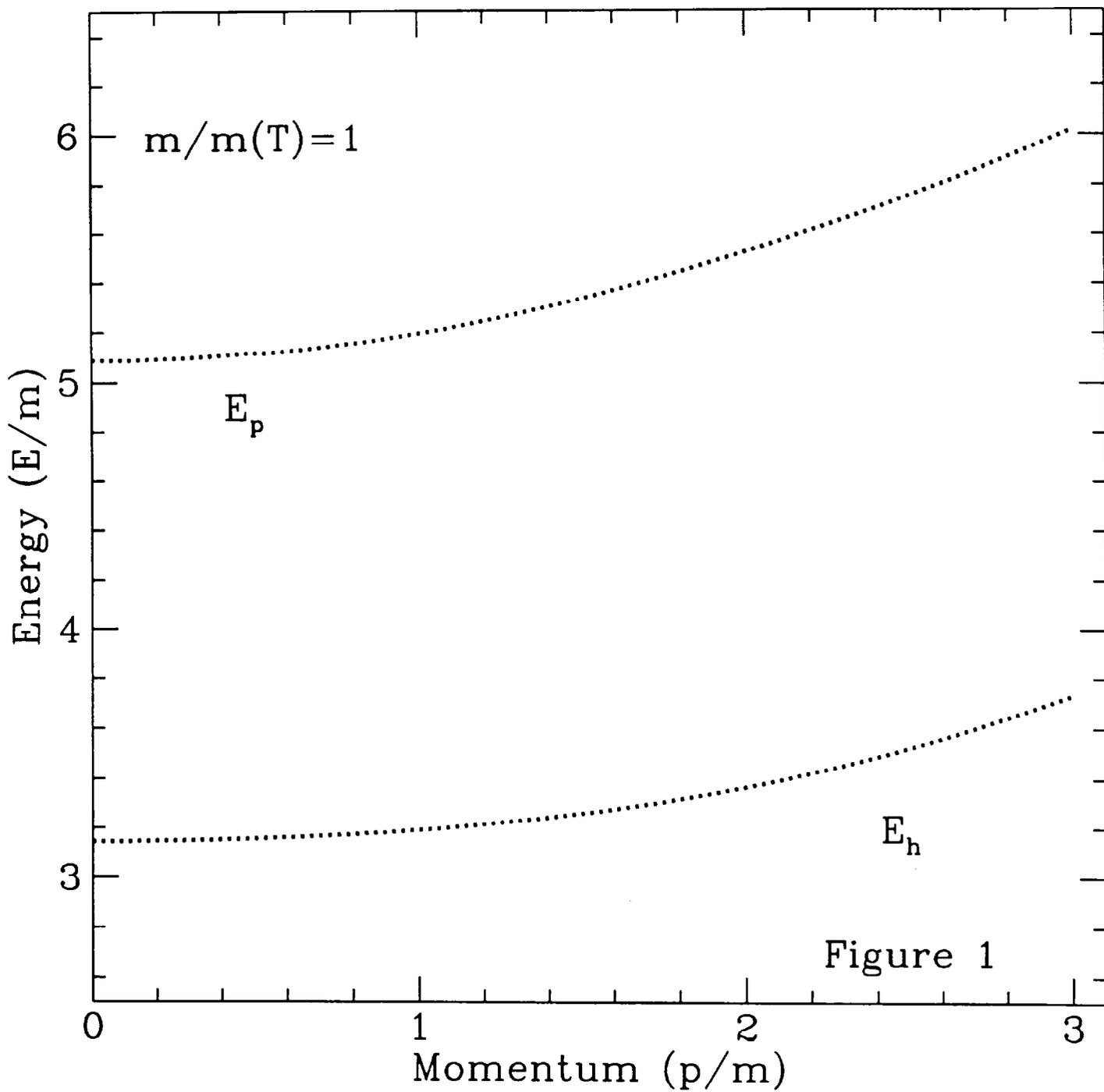
References

- [1] A.G. Cohen, D.B. Kaplan and A.E. Nelson, *Ann. Rev. Nucl. Part. Sci* **43** (1993) 27; V.A. Rubakov and M.E. Shaposhnikov, hep-ph/9603208, submitted to *Usp. Fiz. Nauk*.
- [2] H.A. Weldon, *Phys. Rev.* **D26** (1982) 2789; H.A. Weldon, *Physica* **A158** (1989) 169; H.A. Weldon, *Phys. Rev.* **D40** (1989) 2410.
- [3] R. Pisarski, *Nucl. Phys.* **A498** (1989) 423c; G. Gattoff and J. Kapusta, *Phys. Rev.* **D41** (1990) 611; E. Petitgirard, *Z. Phys.* **C54** (1992) 673; J.P. Blaizot and J.Y. Ollitrault, *Phys. Rev.* **D48** (1993) 1390.
- [4] K. Enqvist, P. Elmfors and I. Vilja, *Nucl. Phys.* **B412** (1994) 459; C. Quimbay and S. Vargas-Castrillon; *Nucl. Phys.* **B451** (1995) 265.
- [5] For a review, see, for instance, H.E. Haber and G.L. Kane, *Phys. Rep.* **117** (1985) 75.
- [6] P. Huet and A.E. Nelson. *Phys. Rev.* **D53** (1996) 4578; M. Carena, M. Quiros, A. Riotto, I. Vilja and C.E.M. Wagner, FERMILAB-PUB-96-271-A, to appear.
- [7] P. Huet and A.E. Nelson. *Phys. Lett.* **B355** (1995) 229; A. Riotto, *Phys. Rev.* **D53** (1996) 5834.
- [8] K. Enqvist, A. Riotto and I. Vilja, in preparation.

Figure captions

Figure 1. Dispersion relations of Majorana particles with $m/m(T) = 1$.

Figure 2. Dispersion relations of Majorana particles with $m/m(T) = 1/4$.



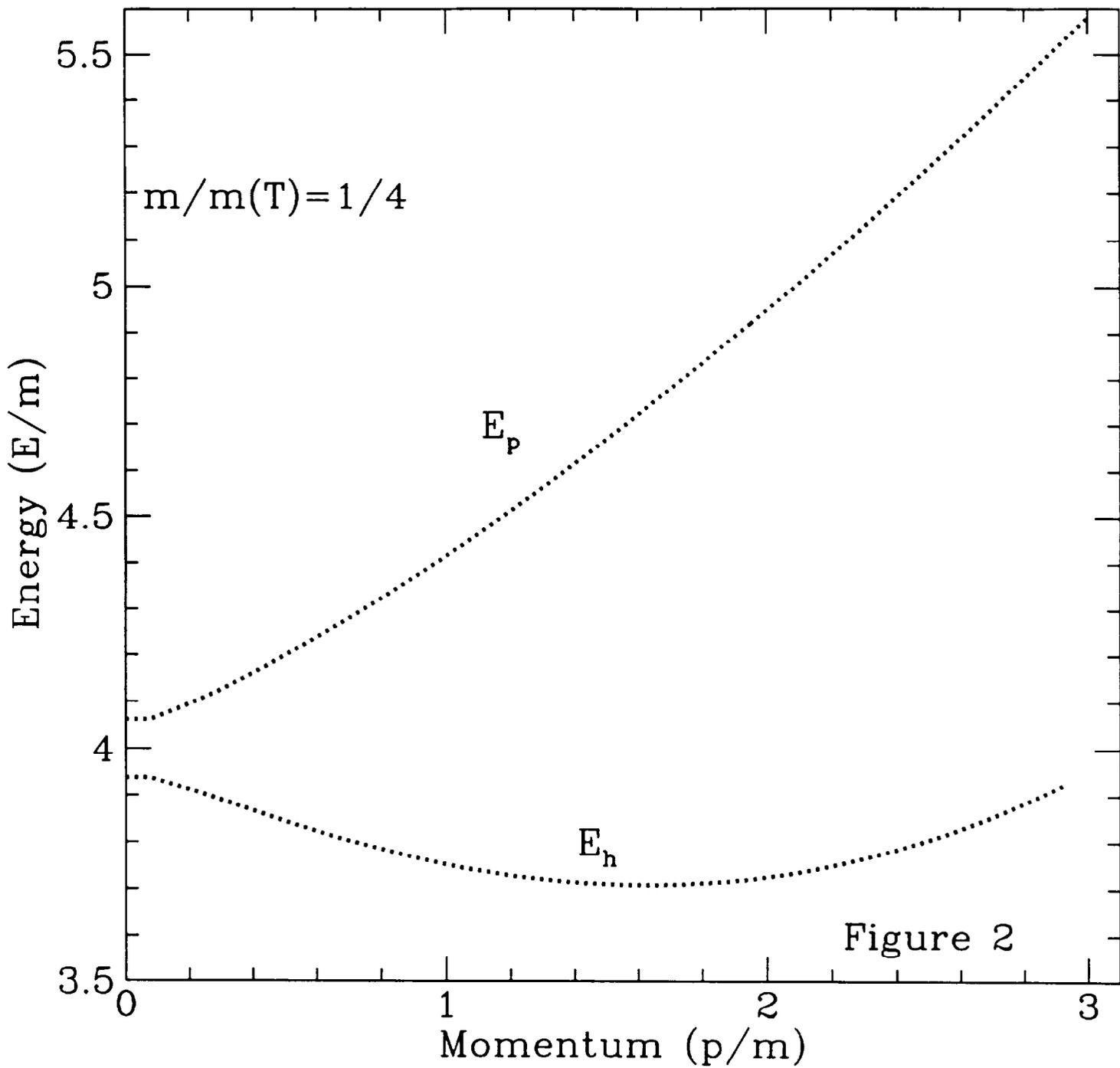


Figure 2