



# Fermi National Accelerator Laboratory

FERMILAB-Pub-96/435-T  
hep-ph/9612265  
December 1996

## A Way to Measure Very Large $\Delta m$ for $B_s$ Mesons

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### Abstract

While present vertex technology cannot measure  $x_s$  much beyond 20, the Standard Model accomodates significantly larger  $x_s$  values. This note presents a method to determine very large  $x_s$  with present technology. The determination is based upon subtle coherence effects between initial  $B_s$  mesons and daughter neutral kaons, discovered by one of us several years ago. The method may be useful also for measuring very small width or mass differences in mixed neutral  $B$  or  $D$  mesons.

Pacs numbers: 14.40.Nd, 13.25.Hw, 13.25.Es



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The physics of  $B$ -mesons is an intensively developing area of particle physics. It is strongly stimulated by the hope to understand the mechanism(s) of  $CP$ -violation. In this respect especially interesting are neutral  $B_d$  and  $B_s$  mesons [1]. But to use them as a testing ground for  $CP$ -studies one should know, to good precision and in great detail, the basic properties of neutral  $B$ -mesons themselves. Ways to attain this goal are rather well investigated for  $B_d$ . The situation for  $B_s$  appears more confused.

The difficulty is mainly a consequence of the expected large mass difference between the two eigenstates of  $B_s$  (analogous to  $K_S$  and  $K_L$  for kaons). The most direct traditional method [2] to measure this mass difference  $\Delta m_s$  is to find and trace the  $(\Delta m_s t)$ -oscillations of flavor-specific  $B_s$ -decays which, in addition, must be flavor-tagged. Ref. [3] pointed out that the  $B_s$ -decay modes need not be flavor-specific and that the value of  $\Delta m_s$  could be extracted from time-dependences of such flavor-tagged decays as, e.g.,  $B_s \rightarrow J/\psi \phi$ . Both methods require superb vertex detectors to resolve very rapid oscillations.

At present, LEP experiments [2] report a limit on the  $B_s$  mass difference

$$|x_s| = \frac{|\Delta m_s|}{\Gamma(s)} \gtrsim 15.$$

Current vertex technology is able to resolve only marginally larger values of  $x_s$  of, say, up to about 20 [4], while theory accomodates values of up to 40 or even higher [5–7]. The present note suggests a novel method which allows the measurement of very large  $x_s$ .

If the traditional method is being integrated over time to overcome insufficient resolution, the oscillations still give a non-vanishing contribution sensitive to  $|x_s|$ , but the sensitivity drops as  $\mathcal{O}(1/x_s^2)$  (for discussion on this point see, e.g., ref. [8]).

In this brief note we show that there exists a triggerable, though rare, decay of  $B_s$  that is more sensitive to  $x_s$ , even at large values, and, moreover, sensitive to its sign. As we will show this allows the experimental identification of whether the heavy eigenstate  $B_s^H$  is approximately  $CP$ -odd (as predicted within the Standard Model) or  $CP$ -even.

To be concrete, we mean the meson decay

$$B_s(\bar{B}_s) \rightarrow J/\psi \bar{K}^0(K^0), \tag{1}$$

induced by the quark decay  $b \rightarrow c\bar{c}d$ . As demonstrated in refs. [9–11], decays of neutral  $B$ -mesons producing neutral kaons have some unique properties. They are based on the coherence of oscillations of the two flavors: beauty before the  $B$ -meson decay, and strangeness after it. When studying decays of this daughter kaon, such double coherence generates various unusual effects and gives a better insight into details of mixing and  $CP$ -violation [9–12].

For simplicity we begin with a comparison of the decay (1) and the similar decay

$$B_d(\bar{B}_d) \rightarrow J/\psi K^0(\bar{K}^0). \tag{2}$$

induced by the more intensive quark decay  $b \rightarrow c\bar{c}s$ . Of course, they differ in intensity due to different CKM matrix elements (the ratio of amplitudes is approximately  $\tan\theta_C$ ). But there is another difference as well. Decay (2) admits only the transitions

$$B_d \rightarrow K^0, \quad \bar{B}_d \rightarrow \bar{K}^0, \quad (3)$$

while the only transitions in the decay (1) are

$$B_s \rightarrow \bar{K}^0, \quad \bar{B}_s \rightarrow K^0. \quad (4)$$

So, in the decay (2), transitions (3) provide equality of final strangeness  $S_f$  and initial beauty  $B_i$ . Transitions (4) for the decay (1) make  $S_f$  and  $B_i$  have opposite signs. But in both cases  $S_f$  and  $B_i$  are related to each other unambiguously. It is just this fact that makes the final state (before the kaon decay) be coherent to the initial one for both decays (1) and (2). As a result, evolution of the produced kaon retains “memory” of the initial  $B$ -evolution. And after the daughter kaon also decays, distributions of this secondary decay reveal information on evolution and decay properties of the initial  $B$ -meson.

Neutral kaons evolve in time orders of magnitude less rapidly than  $B_s$ -mesons. Their time-evolution can thus be easily traced at present, thereby providing a tool to measure the  $B_s - \bar{B}_s$  mixing parameters through the above-mentioned coherence effect.

Now we can compare decays (1) and (2) in more detail. We consider both of them as cascade decays assuming some particular mode for the neutral kaons. Most convenient are either semileptonic decays

$$K^0(\bar{K}^0) \rightarrow l^+\nu\pi^-(l^-\bar{\nu}\pi^+), \quad (5)$$

or 2-pion decays

$$K^0(\bar{K}^0) \rightarrow \pi^+\pi^-. \quad (6)$$

So, every cascade consists of two stages. The first one is the evolution and decay (1) or (2) of the initial  $B$ -meson state. The second stage is the evolution and decay (5) or (6) of the daughter kaon state. For each stage we use the relevant proper time in the rest-frame of the  $B$  or  $K$  respectively. Distributions over time  $t_1$  of the first stage of the cascade (decay time of  $B$ ) and time  $t_2$  of the second stage (decay time of kaon) are strongly correlated in both cases. The form of the correlation depends on the kaon decay mode and on one more parameter. For the decay (2) it is

$$\lambda_d = \frac{1 - \epsilon_d}{1 + \epsilon_d} \cdot \frac{1 + \epsilon_K}{1 - \epsilon_K} \cdot \frac{\bar{a}_d}{a_d} \quad (7)$$

(see ref. [11], with minor differences in notations, e.g.,  $\lambda$  instead of  $\lambda_d$ ; in what follows, for particular expressions we refer to the paper [11], which supersedes papers [9, 10]). Amplitudes  $a_d$  and  $\bar{a}_d$  correspond to the transitions

$$B_d \rightarrow J/\psi K^0, \quad \bar{B}_d \rightarrow J/\psi \bar{K}^0.$$

Similar calculations for the decay (1) produce the analogous parameter

$$\lambda_s = \frac{1 - \epsilon_s}{1 + \epsilon_s} \cdot \frac{1 - \epsilon_K}{1 + \epsilon_K} \cdot \frac{\bar{a}_s}{a_s}, \quad (8)$$

where  $a_s$ ,  $\bar{a}_s$  are amplitudes for the transitions

$$B_s \rightarrow J/\psi \bar{K}^0, \quad \bar{B}_s \rightarrow J/\psi K^0.$$

Note that  $B_s(\bar{B}_s)$  mesons in the decay (1) generate the same final state as  $\bar{B}_d(B_d)$  in the decay (2). As a result, time distributions for a cascade initiated by the decay (1) may be related to distributions for a similar cascade beginning with the decay (2) by the simple changes  $\lambda_d \rightarrow 1/\lambda_s$ ,  $a_d \rightarrow \bar{a}_s$ ,  $\bar{a}_d \rightarrow a_s$ , which are analogous to changes relating initial  $B_d$  and  $\bar{B}_d$  states in the decay (2) (see ref. [11]). In such a way we can easily write all the necessary expressions for  $B_s$ -mesons using the corresponding formulas of ref. [11].

Thus, for cascades (1), (5), produced by the initially pure  $B_s$  or  $\bar{B}_s$  states we obtain

$$W_s^+(t_1, t_2) = \left| a_s \frac{1 + \epsilon_K}{1 - \epsilon_K} \right|^2 \cdot F(t_1, t_2; \lambda_s, -1), \quad W_s^-(t_1, t_2) = |a_s|^2 \cdot F(t_1, t_2; \lambda_s, 1); \quad (9)$$

$$\bar{W}_s^-(t_1, t_2) = \left| \bar{a}_s \frac{1 - \epsilon_K}{1 + \epsilon_K} \right|^2 \cdot F(t_1, t_2; \lambda_s^{-1}, -1), \quad \bar{W}_s^+(t_1, t_2) = |\bar{a}_s|^2 \cdot F(t_1, t_2; \lambda_s^{-1}, 1). \quad (10)$$

The time distributions for the cascades (1), (6) are

$$W_s^{\pi\pi}(t_1, t_2) = \frac{|a_s|^2}{2} \cdot \frac{1 + |\epsilon_K|^2}{|1 - \epsilon_K|^2} \cdot F(t_1, t_2; \lambda_s, -\eta),$$

$$\bar{W}_s^{\pi\pi}(t_1, t_2) = \frac{|\bar{a}_s|^2}{2} \cdot \frac{1 + |\epsilon_K|^2}{|1 + \epsilon_K|^2} \cdot F(t_1, t_2; \lambda_s^{-1}, \eta). \quad (11)$$

Here  $\eta = \eta_{\pi\pi}$  is the standard parameter describing the contribution of  $K_L$  to the decay (6). The function  $F$  has a rather complicated structure. If we are mainly interested in the primary-beauty decay distribution, it may be presented as

$$F(t_1, t_2; \lambda_s, c) = \exp(-\Gamma_+^{(s)} t_1) \cdot A(t_2; \lambda_s, c) + \exp(-\Gamma_-^{(s)} t_1) \cdot A(t_2; -\lambda_s, c) \\ + 2 \exp(-\Gamma^{(s)} t_1) \cdot \text{Re} [\exp(-i\Delta m_s t_1) \cdot B(t_2; \lambda_s, c)] \quad (12)$$

with  $\Gamma^{(s)} = (\Gamma_+^{(s)} + \Gamma_-^{(s)})/2$ ,  $\Delta m_s = m_-^{(s)} - m_+^{(s)}$ ;  $m_\pm^{(s)}$  and  $\Gamma_\pm^{(s)}$  are the masses and widths of two beauty-strange meson eigenstates. We define  $B_+^{(s)}$  as the approximately  $CP$ -even state which is the main source of  $J/\psi K_L$ , while the approximately  $CP$ -odd state  $B_-^{(s)}$  is the main source of  $J/\psi K_S$  (see refs. [11, 13–15] for more detailed discussions related to this point; see also the concluding discussion below).  $\Delta m_s$  is defined here so as to have a positive kaon

analog  $\Delta m_K = m_L - m_S$ . The coefficients  $A$  and  $B$  themselves have a similar three-term structure:

$$A(t_2; \lambda_s, c) = \exp(-\Gamma_S t_2) \cdot \left| \frac{1 + \lambda_s}{4} \right|^2 + \exp(-\Gamma_L t_2) \cdot \left| c \cdot \frac{1 - \lambda_s}{4} \right|^2 + 2 \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right) \cdot \text{Re} \left[ \exp(-i\Delta m_K t_2) \cdot c \cdot \frac{1 + \lambda_s^*}{4} \cdot \frac{1 - \lambda_s}{4} \right]; \quad (13)$$

$$B(t_2; \lambda_s, c) = \exp(-\Gamma_S t_2) \cdot \frac{1 + \lambda_s^*}{4} \cdot \frac{1 - \lambda_s}{4} + |c|^2 \cdot \exp(-\Gamma_L t_2) \cdot \frac{1 - \lambda_s^*}{4} \cdot \frac{1 + \lambda_s}{4} + \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right) \cdot \left[ c \cdot \exp(-i\Delta m_K t_2) \left| \frac{1 + \lambda_s}{4} \right|^2 + c^* \cdot \exp(i\Delta m_K t_2) \left| \frac{1 - \lambda_s}{4} \right|^2 \right], \quad (14)$$

where  $\Gamma_{S,L}$  are the widths of  $K_{S,L}$ .

Expressions (12)–(14) show that time-distributions of the primary and secondary decays are essentially correlated. Information about the properties of decays and mixing of  $B_s$  and  $\bar{B}_s$  continue to be encoded in the time-dependence of the daughter kaon decays (on  $t_2$ ), even when all  $B_s$ -decay times (i.e.,  $t_1$ ) have been integrated over (e.g., because of insufficient resolution). The resulting time distributions of secondary decays can be easily expressed through an integral consisting again of three terms with different  $t_2$ -dependences:

$$I(t_2; \lambda_s, c) = \int_0^\infty dt_1 F(t_1, t_2; \lambda_s, c) = \exp(-\Gamma_S t_2) \cdot C(\lambda_s) + |c|^2 \cdot \exp(-\Gamma_L t_2) \cdot C(-\lambda_s) + 2 \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right) \cdot \text{Re} [c \cdot \exp(-i\Delta m_K t_2) \cdot D(\lambda_s)]; \quad (15)$$

$$4\Gamma^{(s)} C(\lambda_s) = [(1 + |\lambda_s|^2)/2 - y_s \text{Re}\lambda_s](1 - y_s^2)^{-1} + [(1 - |\lambda_s|^2)/2 - x_s \text{Im}\lambda_s](1 + x_s^2)^{-1}, \quad (16)$$

$$4\Gamma^{(s)} D(\lambda_s) = [(1 - |\lambda_s|^2)/2 + iy_s \text{Im}\lambda_s](1 - y_s^2)^{-1} + [(1 + |\lambda_s|^2)/2 - ix_s \text{Re}\lambda_s](1 + x_s^2)^{-1}. \quad (17)$$

$$y_s = (\Gamma_+^{(s)} - \Gamma_-^{(s)})/2\Gamma^{(s)}, \quad x_s = \Delta m_s/\Gamma^{(s)}. \quad (18)$$

If the main goal is to extract  $\Delta m_s$ , then in the frame of the Standard Model one may neglect  $CP$ -violation and to a good approximation use  $\lambda_s = -1$ , thus strongly simplifying the above expressions. The expectations

$$|y_s| \ll 1 \quad \text{and} \quad |x_s| \gg 1$$

give further simplifications; e.g., time distributions of secondary leptons with charge  $\pm 1$  [or of cascading  $K^0(\bar{K}^0) \rightarrow \pi^+\pi^-$  decays] for initially pure  $B_s$  are determined by  $I(t_2; -1, \mp 1)$  [or by  $I(t_2; -1, -\eta)$ ] and take the form

$$W_s^\pm(t_2) \propto (1 + y_s) \exp(-\Gamma_S t_2) + (1 - y_s) \exp(-\Gamma_L t_2) \mp \frac{2}{x_s} \sin(\Delta m_K t_2) \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right); \quad (19)$$

$$\begin{aligned}
W_s^{\pi\pi}(t_2) \propto & (1 + y_s) \exp(-\Gamma_S t_2) + (1 - y_s) |\eta|^2 \exp(-\Gamma_L t_2) \\
& - \frac{2}{x_s} |\eta| \sin(\Delta m_K t_2 - \varphi) \exp\left(-\frac{\Gamma_S + \Gamma_L}{2} t_2\right)
\end{aligned} \tag{20}$$

with  $\varphi = \arg \eta$ .

These distributions look like distorted time distributions for the corresponding modes of the usual kaon decays. The distortion of non-oscillating terms reveals the width difference (for  $B_s$  !), while the amplitude of oscillations directly depends on the  $B_s$  mass difference. We see that the amplitude decreases with increasing  $\Delta m_s$ , but slower than other integrated effects. The sign of the oscillating term is directly related to the sign of the mass difference. In this respect eqs. (19), (20) are similar to the corresponding results for cascading decays of  $B_d$  [16].

In conclusion we briefly discuss the identification method for  $B$ -meson eigenstates applied above (as well as in refs. [9–11, 16]) and compare it to other methods used in the literature. The starting point is that detailed studies of an unstable particle (especially, short-living) are possible only through observation of its decay final states. So if  $CP$ -conservation were exact, our definition of  $CP$ -parity for the eigenstates and the method of their identification by  $CP$ -parity of their decay final states would be exact and quite adequate. In the case of really small  $CP$ -violation our definition of the  $CP$ -parities becomes approximate, but still unambiguous and independent of a decay mode. When the intrinsic  $CP$ -violation increases, some decay modes may give indefinite (or even reversed)  $CP$ -parities to the eigenstates. So, our definition of  $CP$ -parities of the eigenstates may appear mode-dependent. Nevertheless, for most decay final states our separation of the eigenstates continues to be unambiguous for each particular decay mode (even if the eigenstates'  $CP$ -parities become reversed). Necessary comparisons of various decay modes are possible by comparing the signs of  $\Delta m$  and  $\Delta\Gamma$  as measured in the corresponding decay modes. This problem is very interesting by itself. It is even more so if  $CP$ -violation is generated by the CKM-mechanism, since the known information on the CKM-matrix makes very probable such large intrinsic violation (we mean that, according to experimental data, at least two angles of the unitarity triangle for the  $b$ -quark tend to be large, and some final states could produce inverted  $CP$ -parity in comparison to others, if  $\pi/4 < |\gamma| < 3\pi/4$ ; see discussion in ref. [14]).

Identification of nearly degenerate states by their heavier or lighter masses, denoted usually by  $H$  or  $L$  indices, is universal (i.e., definitely mode-independent) and seems at first sight more natural and convenient. But experimentally it is inapplicable directly, even to neutral kaons. Classification of kaon eigenstates by their masses became meaningful only as a result of complicated interference experiments where the sign of  $\Delta m_K$  was measured.

Identification of eigenstates by their longer ( $L$ ) or shorter ( $S$ ) lifetimes is good for kaons since their lifetimes differ very strongly (more than 500 times) and at least  $K_L$  (but not  $K_S$ ) may be easily separated. However, such a procedure is definitely useless experimentally for  $D$  and  $B_d$  mesons (and hard enough for  $B_s$ ).

Contrary to these two, the above procedure is well-defined for all particular experimental conditions and looks natural in theoretical calculations for any particular decay mode. After measuring the signs of  $\Delta m$  and  $\Delta\Gamma$  all the procedures should become equivalent.

The technique outlined here for measuring very large  $B_s - \bar{B}_s$  mixing effects does not require fine resolution in the  $B_s$ -decay time. Instead it requires copious production of  $B_s$ -mesons that should be flavor-tagged. The mode  $B_s \rightarrow J/\psi \bar{K}^0$ , although CKM suppressed, is triggerable at hadron accelerators. If  $x_s$  remains out of reach for existing vertex technology, we recommend detailed feasibility studies of the cascading processes  $B_s(t) \rightarrow J/\psi K^0 [\rightarrow \pi^+ \pi^-, \pi l \nu]$ . Depending on the detector configuration, one may wish to cut on  $B_s$  decay times that are very close to the primary interaction vertex. Although the detector may not be able to resolve the rapid  $(\Delta m_s t)$ -oscillations, it may be able to give very interesting information, e.g., to distinguish the two different  $B_s$  lifetimes. Because integrating over all  $t_1$  removes such information, we presented in detail (see Eqs.(9)–(14)) the complete time distributions of the cascade processes with which further detailed analyses can be conducted.

This note addressed only one aspect of coherence effects between heavy neutral mesons and their decay final states containing a single neutral kaon. Detailed investigations of such coherence effects may prove useful also in measuring a small lifetime difference for the two  $B_d$  eigenstates, and in measuring small  $D^0 - \bar{D}^0$  mixing parameters<sup>1</sup>. Furthermore, they will shed important light upon  $CP$ -violation and extraction of CKM parameters. We hope to return to those intriguing issues in the future.

## Acknowledgements

We thank D.Richards for reading the manuscript. Ya.Azimov appreciates the hospitality of the Theory Group at Fermilab where this work was done. This work was supported in part by Universities Research Association Inc., under Contract No. DE-AC02-76CH03000 with the United States Department of Energy.

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<sup>1</sup>The complication due to doubly Cabibbo suppressed modes can be incorporated in a straightforward fashion. It will be presented elsewhere.

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