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Hara's Theorem and W -exchange in Hyperon Weak Radiative Decays

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Abstract

We reconsider Hara's theorem in its relation to the well-known properties of β -decay. All assumptions necessary for the theorem to be true are explicitly formulated. Further, we study the W -exchange contribution to weak radiative decays and show that it does not violate Hara's theorem. However, this contribution reveals the essential role of particle mixing in symmetry considerations and some peculiar features of gauge-invariant amplitudes under perturbative expansion. Together they explain an effect, which was treated as contradicting Hara's theorem, without any violation. The properties of W -exchange we describe here may have more general importance and should be taken into account in further detailed calculations of weak processes.

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1 Introduction

It is a common belief that weak and electromagnetic interactions may always be treated perturbatively, generally in contrast to strong interactions (SI). This makes many weak or electromagnetic processes involving hadrons a good arena for analyzing details of (nonperturbative) SI. They continue to provide a lot of phenomenological information on SI and present a vast testing ground for their theoretical description.

A special role could be played here by weak radiative decays (WRD) of hadrons. They exemplify rather rare hadron processes which are clearly of higher order in perturbations. Therefore, their interplay with SI might give further insight into details of nonperturbative SI. Moreover, experimentally available WRD have quite simple two-body kinematics, very similar to weak nonleptonic decays. It is therefore surprising that, up to now, there is no generally acceptable theoretical description of WRD (see the recent [1] and older [2] reviews and references therein; see also recent talks [3], [4]).

The last decade has witnessed significant experimental progress that has produced a large set of data on WRD, mainly from CERN and FNAL hyperon beams (see, e.g., summary table in ref. [5]). Though still incomplete, the data give good evidence for an essential role of W -exchange in inducing WRD. Those decays in which a W -boson can be exchanged between valence quarks are at least an order of magnitude more copious than decays that do not allow such an exchange.

This fact revived the old problem of the so-called Hara's theorem [6]. According to the theorem, the exact $SU(3)$ symmetry of SI would make some decays (in particular, $\Sigma^+ \rightarrow p\gamma$) have vanishing parity-violating amplitudes and, thus, vanishing decay asymmetries. Of course, the $SU(3)$ symmetry is violated in nature, so the deviation of experimental asymmetry [5] for $\Sigma^+ \rightarrow p\gamma$ from the prediction of Hara's theorem could arise quite naturally. But the large value of the asymmetry and its unexpected sign continue to be a hard problem of WRD theory.

The absence of a satisfactory simultaneous description for the $\Sigma^+ \rightarrow p\gamma$ decay width and asymmetry, together with the rather complicated character of the original proof, has, again and again, stimulated skepticism as to the correctness of Hara's theorem. As an example, we recall ref. [7], that was opposed by Gaillard [8]. Now that experiment suggests the important role of W -exchange, the essential contribution to such skepticism comes from the paper of Kamal and Riazuddin [9]. They argue that explicit calculations of W -exchange at the quark level directly contradict Hara's theorem, even for exact $SU(3)$ symmetry. So the question as to why the asymmetry for $\Sigma^+ \rightarrow p\gamma$ strongly deviates from the simple Hara's prediction becomes again topical [3], [4]. Its solution has been even assumed to be closely related to the general problem of correspondence between the quark and hadron levels [4].

In the present note we reconsider the above problems. First we repeat in more detail Gailard's arguments [8] for correctness of Hara's theorem and its similarity to usual β -decay. We formulate explicitly all the assumptions necessary to prove the theorem.

Then, in the case of exact U -symmetry, we consider structure and properties of quark- and hadron-level W -exchange contributions, both by themselves and accompanied by a photonic transition. On one side, we again prove Hara's theorem, for this particular case. On the other side, we demonstrate some simple, but unfamiliar properties of gauge-invariant amplitudes being expanded as perturbation series in weak interactions. These properties provide us the basis to understand the effect observed by Kamal and Riazuddin [9] without invoking a violation of Hara's theorem. An interesting point of our consideration is the essential role of quark and hadron mixing which may be also important for more detailed study of all WRD's or, even more generally, of higher order corrections in weak interactions.

2 β -decay and $\Sigma^+ \rightarrow p\gamma$

We begin with the usual β -decay $n \rightarrow p e \bar{\nu}$. Its Lagrangian is proportional to

$$L_\beta \propto J_\mu^{(3)} j^\mu + J_\mu^{(3)\dagger} j^{\mu\dagger}. \quad (1)$$

where j_μ is the lepton weak charged current and $J_\mu^{(3)}$ is the nucleon weak charged current; note that here we are not interested in the exact form of constant factors. It is well known that $J_\mu^{(3)}$ consists of two parts, the vector current $J_\mu^{(3)V}$ and the axial one $J_\mu^{(3)A}$. Their matrix elements between neutron and proton have the additional structure

$$\langle p | J_\mu^{(3)V} | n \rangle = \bar{v}_p \left[f_1(k^2) \gamma_\mu + f_2(k^2) \sigma_{\mu\nu} k^\nu + f_3(k^2) k_\mu \right] \psi_n, \quad (2)$$

$$\langle p | J_\mu^{(3)A} | n \rangle = \bar{v}_p \left[g_1(k^2) \gamma_\mu \gamma_5 + g_2(k^2) i \sigma_{\mu\nu} k^\nu \gamma_5 + g_3(k^2) i \gamma_5 k_\mu \right] \psi_n. \quad (3)$$

Here k is the momentum transfer and functions f_i, g_i are real at k^2 below hadron production thresholds, e.g., at space-like k . For the Dirac matrices, we use the notations of ref. [10].

Let us assume that isotopic symmetry is exact. Then every term of eqs. (2),(3) is a component of an isotopic vector. Due to the fact that p and n are members of the same isotopic doublet, all isotopic vectors involved in eqs. (2),(3) transform into themselves under G -transformations [11] and have some definite G -parity. Terms with f_1, f_2, g_2 are G -even, and ones with g_1, g_3, f_3 are G -odd. Weinberg [12] suggested further to separate hadronic weak charged currents in strangeness-conserving processes into two classes. Then terms with f_1, f_2, g_1, g_3 belong to the first class, and ones with f_3, g_2 are of the second class. From G -parity conservation by strong interactions, Weinberg concluded [12] that first-class bare interactions can induce only first-class phenomenological currents. As we know today strangeness-conserving hadron weak currents in the Standard Model are indeed of the first

class. Therefore, axial weak magnetism (the term with $\sigma_{\mu\nu}\gamma_5$) is forbidden in β -decay (together with the induced scalar term $f_3 k_\mu$) while vector weak magnetism (the term $f_2 \sigma_{\mu\nu} k^\nu$) is permitted [13] (together with the induced pseudoscalar term $g_3 i\gamma_5 k_\mu$) and observable.

Note that in β -decay, all the first-class currents have the same, and negative, GP -parity, while the GP -parity of the second-class terms would be positive. This allows one to relate absence of axial magnetism directly to CP -conservation in β -decay, without an independent assumption of absence of the second-class currents. Indeed, the lepton weak currents j_μ and j_μ^\dagger are known to transform into each other by CP transformation. So, if the Lagrangian (1) is to be CP -conserving the hadron weak currents J_μ and J_μ^\dagger should also be related to each other by CP -transformation. An essential additional feature which comes from exact isospin symmetry is that the two currents are various combinations of components of the same isotopic vector. Moreover, they may be transformed into each other by isospin rotation around the 2nd axis, without changing coefficient functions in eqs.(2),(3). These two properties together lead just to the necessity of negative GP -parity. The opposite sign between the CP -parity of the Lagrangian (1) and the GP -parity of the currents arises from the rotation of isotopic-vector components involved into G -transformation.

Of course, all the above is true only up to electromagnetic radiative corrections (or, more exactly, up to violation of isotopic symmetry and, therefore, G -parity conservation). Thus, the axial magnetism is not forbidden at the level of radiative corrections.

Now we turn to the $\Sigma^+ p\gamma$ interaction. We will follow the same logic as briefly presented by Gaillard [8]. In particular, we will use not the whole $SU(3)$ symmetry but a more narrow group, U -spin symmetry (the analog of isotopic I -spin symmetry which mixes d, s quarks instead of u, d), which is known [14] to be sufficient for Hara's theorem.

The effective Lagrangian responsible for the $\Sigma^+ p\gamma$ interaction has the form

$$L_S^{(\gamma)} \propto \left(J_\mu^{(S)} + J_\mu^{(S)\dagger} \right) A^\mu \quad (4)$$

which recalls eq. (1). Here A_μ is the photon field. Currents $J_\mu^{(S)}$ and $J_\mu^{(S)\dagger}$ satisfy strangeness selection rules $\Delta S = \pm 1$ respectively.

If the U -spin symmetry of strong and electromagnetic interactions is exact we may introduce the G_u -transformation analogous to the G -transformation of the usual isotopic group. Now we can simply modify the previous consideration of β -decay and apply it to WRD's.

Again, the current $J_\mu^{(S)}$ has vector and axial parts. Their matrix elements are

$$\langle p | J_\mu^{(S)V} | \Sigma^+ \rangle = \bar{v}_p \left[f_1^{(S)}(k^2) \gamma_\mu + f_2^{(S)}(k^2) \sigma_{\mu\nu} k^\nu + f_3^{(S)}(k^2) k_\mu \right] v_{\Sigma^+}, \quad (5)$$

$$\langle p | J_\mu^{(S)A} | \Sigma^+ \rangle = \bar{v}_p \left[g_1^{(S)}(k^2) \gamma_\mu \gamma_5 + g_2^{(S)}(k^2) i \sigma_{\mu\nu} \gamma_5 k^\nu + g_3^{(S)}(k^2) i \gamma_5 k_\mu \right] v_{\Sigma^+}. \quad (6)$$

An important point is that p and Σ^+ are members of the same U -spin doublet, just as p and n belong to the same I -spin doublet. In full similarity to the above β -decay considerations, currents $J_\mu^{(S)}$ and $J_\mu^{(S)\dagger}$ are related to each other by both Hermitian conjugation and CP -transformation. Exact U -spin symmetry gives possibility to connect CP -transformation of $J_\mu^{(S)}$ into $J_\mu^{(S)\dagger}$ with G_uP -transformation of $J_\mu^{(S)}$ into itself. Then we arrive at the result of Hara [6] which arises exactly as the above statements on β -decay.

Now we can generalize and collect together necessary requirements for Hara's theorem:

1. Exact U -spin symmetry for strong and electromagnetic interactions (violated, of course, by weak interactions).
2. Hermiticity of the effective Lagrangian for WRD.
3. CP -conservation.
4. Initial and final hadrons in a particular WRD being members of the same U -spin multiplet.
5. Weak interactions having a structure that produces a nonvanishing transition amplitude for the WRD, with a unique U -spin structure (vector in the case of $\Sigma^+ \rightarrow p\gamma$).

The meaning of the first 3 conditions is obvious. The last two conditions guarantee a definite G_uP -parity for all parts of the current $J_\mu^{(S)}$. They can be further generalized so to admit the transition amplitude being a mixture of various U -spin representations. The only problem is that all the involved representations should have odd integer U -spin to provide the same, negative, G_uP -parity.

Let us consider some particular cases. The pair $(p\Sigma^+)$ surely satisfies conditions 4 and 5 as they are. Its both members belong to the same U -spin doublet, and the transition amplitude can be only an U -vector. The same is true for another pair, $(\Sigma^-\Xi^-)$. Thus, these pairs correspond to two decays where Hara's theorem could be applicable.

On the other side, transition amplitude, e.g., in the decay $\Lambda \rightarrow n\gamma$, have more complicated U -spin structure. Even Λ itself is the mixture of U -singlet and U -vector components, while n is a pure U -vector. So the current can have several parts, with various properties under U -spin rotations and various G_uP -parities. We see that conditions 4 and 5 are violated (even in a more generalized form: there are U -vector and U -tensor parts in the amplitude). As a result, Hara's theorem is definitely inapplicable here.

The above conditions are simple and very general. Except U -symmetry, they are surely respected by many approaches that have been used to describe WRD. In particular, these assumptions are true for the W -exchange considered in ref. [9].

Up to now our consideration has been rather formal and could not be applied to experiment. The necessary next step would be using gauge invariance. For the real photon ($k^2 = 0$) it should eliminate the first terms in expressions (5), (6). The last terms give no contribution when multiplied by the real-photon polarization vector. Then the vector and axial magnetic terms become the only physical terms, and Hara's theorem becomes operative for experiment. But here we postpone this step. We return to it after the study of W -exchange.

3 Structure of the W -exchange contribution

To understand in more detail some specific features of W -exchange in WRD, we begin by considering this exchange by itself, without any photon emission.

The W -exchange for the transition $\Sigma^+ \rightarrow p$ corresponds to an interaction of the form

$$(\bar{u}_i O s_i)[W](\bar{d}_j O u_j)$$

with summation over color indices i, j . Here we explicitly show which quark pairs exchange the W -boson just to emphasize the color structure of their interaction. The coefficient contains product of the corresponding elements of the CKM-matrix. Also possible are interactions

$$(\bar{u}_i O d_i)[W](\bar{d}_j O u_j) \text{ and } (\bar{u}_i O s_i)[W](\bar{d}_j O s_j),$$

where the hadrons may stay untransformed.

If U -symmetry is exact the masses of d and s quarks coincide. The same is true for p and Σ^+ . Hence, we may mix these degenerate quarks (and hadrons). It is convenient for our purpose to choose the mixed quarks d' and s' so as to eliminate the transition $s' \rightarrow u$ in the CKM-matrix (note that in the case of 3 or more generations this does not mean vanishing of the relative transition $d' \rightarrow c$). Then the interaction

$$(\bar{u}_i O s'_i)[W](\bar{d}'_j O u_j)$$

becomes impossible and Σ'^+ can not be transformed to p' by single W -exchange.

Now the only possible interaction between light quarks due to W -exchange is

$$L_W \propto (\bar{u}_i O d'_i)[W](\bar{d}'_j O u_j). \tag{7}$$

It interchanges quark flavors without permutation of colors. So the question arises whether interaction (7) is capable of transforming p' into itself without any excitation. The answer is yes, due to colorlessness of hadrons. Indeed, the color wave function of baryons is totally

antisymmetric, and color permutation may only reverse the sign of transition amplitudes. For the standard $(V - A)$ vertices in the limit $m_W \rightarrow \infty$ this sign reversal may be eliminated by Fierz transformation. Then the W -exchange interaction (7) efficiently becomes totally symmetric under transposition of u and d' (or \bar{u} and \bar{d}'). This point-like 4-quark interaction can be effectively rewritten in the diagonal form

$$(\bar{d}'_i O d'_i)(\bar{u}_j O u_j),$$

where initial quarks retain their quantum numbers. Such interaction is surely able to provide diagonal transition $p' \rightarrow p'$.

Thus, we have achieved diagonalization of the degenerate hadronic states p' and Σ'^+ , when no transition between them (through W -exchange) is possible, though they both can transform into heavier states which contain, e.g., c or b quarks. Moreover, the interaction

$$(\bar{u}_i O s'_i)[W](\bar{s}'_j O u_j)$$

also becomes impossible, so that W -exchange can not transform Σ'^+ into itself, while being able to transform p' into itself.

Consider, for comparison, neutral members of the baryon octet n , Λ , Σ^0 , Ξ^0 . If the U -spin symmetry were initially exact, it would be useful to consider states $n' = (d'd'u)$, $\Xi'^0 = (s's'u)$ and two states Σ'^0 , Λ' which have the same quark content $(s'd'u)$, but, respectively, symmetrized or antisymmetrized spin and U -spin wave functions. W -exchange would not influence Ξ'^0 . The other three states would be influenced, but differently. n' can not go into any other of those states, while Λ' and Σ'^0 mix to each other since the interaction (7) can violate symmetrization of d' and s' .

Therefore, if the U -spin symmetry of strong and electromagnetic interactions were exact, W -exchange would eliminate degeneracy of states and separate definite combinations of them. One of the corresponding physical states, Σ'^+ , would be unchanged by single W -exchange, whereas another, p' , would be changed. In particular, the mass of p' would shift from the unchanged mass of Σ'^+ . From now on we can discuss problems related to Hara's theorem without invoking U -spin symmetry. Its only role has been to make possible the mixing of d and s quarks.

Now we are ready to consider the photon emission vertex. Since the photon has been assumed from the beginning to be U -spin invariant, and W -exchange itself can not transform Σ'^+ to p' or vice versa, they can not do it together as well. So photon emission (or absorption) can transform each of these two states either to itself or to heavier quark hadrons, but not to the other. Here we are interested in diagonal vertices. It is purely electromagnetic for Σ'^+ (remember that we account only for the lowest order, i.e. single W -exchange). The photonic vertex of p' has an additional contribution due to W -exchange that violates P - and C -parity. However we assume it to conserve combined CP -parity. Then the axial part

of the vertex should be C -even and, therefore, can not contain any axial magnetic term (it would be axial, but C -odd, just as the usual magnetic term).

This fact proves Hara's theorem specifically for W^- -exchange since it is just this additional vertex contribution that produces photonic transition between Σ^+ and p . Such an approach does not work for the transition between Ξ^- and Σ^- since W^- -exchange is impossible for this pair, because of absence of u -quarks.

4 Photonic vertex and gauge invariance

Now we discuss how gauge invariance manifests itself in the vertex $p'p'\gamma$. Remember that we take into account the W^- -exchange contribution that violates space and charge parities separately. Therefore, the effective interaction takes the form

$$L_\gamma = eJ'_\mu A^\mu \quad (8)$$

with current J'_μ having both vector and axial parts. Their matrix elements for p' are

$$\langle p'|J'_\mu{}^V|p'\rangle = \bar{v}_{p'} \left[f'_1(k^2)\gamma_\mu + f'_2(k^2)\sigma_{\mu\nu}k^\nu + f'_3(k^2)k_\mu \right] \psi_{p'}, \quad (9)$$

$$\langle p'|J'_\mu{}^A|p'\rangle = \bar{v}_{p'} \left[g'_1(k^2)\gamma_\mu\gamma_5 + g'_2(k^2)i\sigma_{\mu\nu}\gamma_5k^\nu + g'_3(k^2)i\gamma_5k_\mu \right] \psi_{p'}. \quad (10)$$

When $k \rightarrow 0$ only terms $f'_1(0)\gamma_\mu$ and $g'_1(0)\gamma_\mu\gamma_5$ can survive. Standard application of the gauge-invariance condition leads to the conclusion

$$g'_1(0) = 0, \quad (11)$$

thus leaving us with only one term. Normalization to the usual electric charge gives the further relation

$$f'_1(0) = 1. \quad (12)$$

These relations are quite usual and familiar, and should not raise any questions. However, explicit calculations in perturbation theory directly violate them both. To see this, one can repeat the calculations of ref. [9], applying them to diagonal transitions. The calculations are straightforward, and we will not describe them here. Instead we consider what is the reason for and meaning of such results.

Denote the "bare" propagator of p' (i.e., with strong and electromagnetic interactions taken into account but without any weak interactions) with 4-momentum q as $S_0(q)$. It may be written as

$$S_0^{-1}(q) = a_0(q^2)\hat{q} - M_0(q^2). \quad (13)$$

On the mass-shell, $q^2 = m_0^2$. Near the mass-shell

$$S_0 \approx (\hat{q} - m_0)^{-1}. \quad (14)$$

i.e., $a_0(m_0^2) = 1$ and $M_0(m_0^2) = m_0$.

Weak interactions (in particular, W -exchange) produce an additional self-energy part $\Sigma(q)$. So the total p' -propagator $S(q)$ is determined by the expression

$$S^{-1}(q) = S_0^{-1}(q) - \Sigma(q). \quad (15)$$

Analogously, we denote the "bare" $p'p'\gamma$ -vertex as $\Gamma_\mu^{(0)}(q_1, q_2, k)$ where q_1 and q_2 are the momenta of the initial and final p' respectively, and k is the photon momentum. $\Gamma_\mu^{(0)}$ is purely vector. When both q_1 and q_2 are on the mass-shell it takes the standard form with Dirac and Pauli form-factors. Weak interactions produce an additional contribution $\delta\Gamma_\mu$ to the total $p'p'\gamma$ -vertex.

It is time now to recall that weak interactions (and W -exchange, in particular) violate parity. Therefore, $\Sigma(q)$ may be written as

$$\Sigma(q) = -a_+(q^2)\hat{q} - a_-(q^2)\hat{q}\gamma_5 + M_+(q^2) + M_-(q^2)i\gamma_5. \quad (16)$$

Similarly, $\delta\Gamma_\mu$ contains both vector and axial parts.

With weak interactions switched on, the mass-shell is determined by the physical mass m . Near it we may write

$$S^{-1}(q) \approx (a_0 + a_+)\hat{q} + a_-\hat{q}\gamma_5 - (M_0 + M_+) - M_-i\gamma_5, \quad (17)$$

where all functions are taken at $q^2 = m^2$. Therefore, the Dirac equation for a free physical p' (i.e., without any explicitly applied external field, but with complete account for the self-interaction) should be written as

$$[(a_0 + a_+)\hat{q} + a_-\hat{q}\gamma_5 - (M_0 + M_+) - M_-i\gamma_5]\psi_{p'} = 0. \quad (18)$$

The physical mass m is certainly related to parameters entering this equation, but we consider the relation somewhat later: for now we discuss the structure of the electromagnetic vertex.

We emphasize here that the photonic vertex is not gauge invariant by itself. Instead, gauge invariance gives a general relation between propagator and vertex:

$$\Gamma_\mu(q, q, 0) = \frac{\partial S^{-1}(q)}{\partial q^\mu} \quad (19)$$

at $q^2 = m^2$. So we have

$$f_1'(0) = a_0(m^2) + a_+(m^2), \quad g_1'(0) = a_-(m^2). \quad (20)$$

The same result appears from eq. (18) for the minimal electromagnetic interaction. Note that generally $a_0(m^2) \neq 1$, since $a_0(m_0^2) = 1$, and the strong interactions make $a_0(q^2) \neq \text{const}$

(compare eqs. (13),(14)); by the same reasoning $M_0(m^2) \neq m_0$. Thus, we see that the violation of the usual relations (11),(12) is directly traced to the non-canonical form of the Dirac equation (18) and to the corresponding changes in applying gauge invariance. If parity were conserved, eq. (11) would be satisfied, but eq. (12) could nevertheless be violated. This situation is really well known, since eq. (12) should work only after renormalization of both propagator and vertex. Eq. (18) can also be transformed to the canonical form by a transformation of the wave function that may be considered as a generalized renormalization. Correspondingly, the propagator of p' admits a transformation giving it the standard form near the physical mass-shell.

We define

$$\psi' = Z^{-1/2}\psi, \quad \bar{\psi}' = \bar{\psi} \bar{Z}^{-1/2}, \quad S'(q) = Z^{-1/2}S(q)\bar{Z}^{-1/2}. \quad (21)$$

Here Z is a constant matrix and $\bar{Z} = \gamma^0 Z^\dagger \gamma^0$. To arrive at the canonical form for the propagator and Dirac equation we take Z as a product of three factors

$$Z = \frac{1}{[(a_0 + a_+)^2 - a_-^2]^{1/2}} \cdot \frac{(a_0 + a_+) - a_- \gamma_5}{[(a_0 + a_+)^2 - a_-^2]^{1/2}} \cdot \frac{M_0 + M_+ - M_- i \gamma_5}{[(M_0 + M_+)^2 + M_-^2]^{1/2}}. \quad (22)$$

Correspondingly,

$$\bar{Z} = \frac{1}{[(a_0 + a_+)^2 - a_-^2]^{1/2}} \cdot \frac{(a_0 + a_+) + a_- \gamma_5}{[(a_0 + a_+)^2 - a_-^2]^{1/2}} \cdot \frac{M_0 + M_+ + M_- i \gamma_5}{[(M_0 + M_+)^2 + M_-^2]^{1/2}}. \quad (23)$$

We consider the operator part (in square brackets) of eq. (18) as $S'^{-1}(q)$ at $q^2 = m^2$ and transform it in accordance with eq. (21). Then the above factors work as follows. The factor containing parameters M takes γ_5 away from the mass terms of eq. (18). The matrix factor with parameters a does the same for the coefficient of \hat{q} . And the purely numerical factor normalizes this coefficient to unity. After that the Dirac equation for ψ' takes its familiar form with the physical mass

$$m^2 = \frac{(M_0 + M_+)^2 + M_-^2}{(a_0 + a_+)^2 - a_-^2}. \quad (24)$$

In accordance with eq. (19) the vertex function should also transform as

$$\Gamma'_\mu(q_1, q_2, k) = \bar{Z}^{1/2} \Gamma_\mu(q_1, q_2, k) Z^{1/2}. \quad (25)$$

Consider how the matrix renormalization influences the vertex. The factors of expressions (22), (23), containing parameters M , mix to each other the vector and axial magnetic terms, as well as the induced scalar and pseudoscalar terms, and change their relative intensity. But the transformation may not remove any of them totally. The matrix factors with parameters a mix vector and axial terms of the vertex and provide the relation (11). After that the remaining numerical part of the renormalization makes the relation (12) be true as well. So the new vertex Γ'_μ has just the conventional limit as $k \rightarrow 0$. Note that $g'_1(k^2)$ at $k^2 \neq 0$ may be non-vanishing even after the total renormalization.

Note also that all the above arguments which should lead to the vanishing coefficient function g'_2 for the axial magnetic term in eq. (10) can be applied only to the renormalized vertex Γ'_μ but not to Γ_μ . The reason is that the non-canonical form of the Dirac equation (18) generates a non-canonical expression for the charge-conjugation transformation. Therefore, familiar notions on charge-conjugation behavior of various vertices become distorted.

Formally, the renormalization by the matrices (22), (23) is true only at

$$|a_-| < |a_0 + a_+|,$$

which is satisfied since $a_0 \sim \mathcal{O}(1)$, while a_+ and a_- are induced by weak interactions. Eq. (24) shows that the opposite case would lead to the tachyon ($m^2 < 0$).

The matrix renormalization can also be formulated in terms of the unmixed states p and Σ^+ . In such a form the renormalizing matrix would have even more complicated structure to account for both parity violation and particle mixing.

Now we are ready to understand the origin and meaning of the results obtained by Kamal and Riazuddin [9]. They studied amplitudes for the transition $\Sigma^+ \rightarrow p$ which is closely related to our amplitudes for the diagonal transition $p' \rightarrow p'$. So we will discuss their results in terms of our amplitudes.

Consider $\langle p' | \Gamma_\mu \epsilon^\mu | p' \rangle$ where the vector ϵ^μ may be thought of as a photon polarization vector. Note that we do not make any additional renormalization (just as in ref. [9]). If we study only terms of zeroth order in k (i.e. take the limit $k \rightarrow 0$, again as in ref. [9]) we obtain

$$\langle p' | \Gamma_\mu(q, q, 0) \epsilon^\mu | p' \rangle = \bar{v}_{p'}(q) [f'_1(0) \hat{e} + g'_1(0) \hat{e} \gamma_5] v_{p'}(q). \quad (26)$$

It is simplest to calculate the right-hand side in the rest frame. Then, due to properties of the Dirac matrices, only the time component ϵ^0 contributes to the first term in r.h.s., while the second term contains $(\mathbf{e} \cdot \boldsymbol{\sigma})$ with only space components contributing. If we take $\epsilon^0 = 0$ (again, as in ref. [9]; it looks only natural for the photon polarization vector) then we see an axial contribution without any vector one.

It is just this result that was promoted [9] as contradicting Hara's theorem, according to which one would expect to find only a vector contribution, without an axial one. We see, however, that the result has really no relation to terms for which we should apply Hara's theorem. Those terms may appear only in calculations which completely account for the first order in k . Moreover, the correct relation between the physical vector and axial magnetic terms arises only after matrix renormalization (25) of the full photonic vertex. This means that only after such renormalization one may compare results of perturbative calculations and Hara's theorem predictions.

Note, ironically, that in the case of initially exact U -spin symmetry the decay $\Sigma^+ \rightarrow p\gamma$ would be impossible at all (again, to lowest order in W -exchange). Indeed, as we have seen,

real physical states in that case would be not Σ^+ and p , but Σ'^+ and p' which are incapable of transforming into one another by photon emission, though their differing masses allow the decay kinematically.

It is instructive, nevertheless, to see how the transition amplitude $\Sigma^+ \rightarrow p\gamma$ would look in the case of the initially exact U -symmetry. To do this we reverse the mixing transformation and express Σ^+ and p through Σ'^+ and p' . Then we may substitute the vertices $p'p'\gamma$ and $\Sigma'^+\Sigma'^+\gamma$ (which is pure electromagnetic; remember that transitions between Σ'^+ and p' are absent) and obtain the desired amplitude. If we apply the inverse mixing transformation to the non-renormalized p' we arrive at the amplitude having a structure that violates canonical expectations of both gauge invariance and Hara's theorem. It contains non-vanishing vector and axial terms at $k^2 = 0$ as well as axial magnetic term. Only if we apply the transformation to the renormalized p' (Σ'^+ is not influenced by W -exchange) the resulting amplitude looks as expected. Hence, renormalization of p' touches both p and Σ^+ .

This demonstrates importance of matrix renormalization even for the case of exact U -spin symmetry. If symmetry violation is more intensive than the influence of W -exchange, the states p' and Σ'^+ do not arise. But then we should renormalize both p and Σ^+ taking into account their transitions to each other through the W -exchange.

5 Conclusion and discussion

Let us briefly summarize the above discussion. Here we have reconsidered Hara's theorem and explicitly formulated its assumptions. Results of the theorem are in a very close relation to the well-known properties of usual β -decay, as was noted earlier by Gaillard [8]. Assumptions of the theorem are rather simple and clear-cut. When they are satisfied the theorem is surely true. And they are satisfied in many approaches used in the literature.

Of course, one of the assumptions for Hara's theorem, U -spin symmetry, is violated in nature and in calculations. However, in many applications its violation may be considered as small. That a similar possibility does not work for WRD's has caused a long-standing problem of their description which is still unsolved.

We have demonstrated, in particular, that for W -exchange Hara's theorem should also be true. More detailed study of W -exchange contributions shows that an effect stated some years ago as manifesting violation of Hara's theorem for W -exchange at the quark level does not really necessitate such violation. Instead, it can be explained as revealing insufficiency of standard purely numerical renormalization in perturbation theory for weak interactions. If parity is violated the fermion renormalization "constants" should be taken as combination of the unit matrix with γ_5 .

The "violation" of gauge invariance described in the preceding sections may look strange and even mysterious. But its reason is really quite clear. Conservation of the current J'_μ means that its matrix elements (9) and (10) taken over real physical states should vanish when being multiplied by k^μ . But each r.h.s. of eqs. (9), (10) consists of three elements. The square brackets contain the effective vertex with various matrix terms and corresponding coefficient functions. Two other elements are wave functions, initial and final. When expanding into perturbation series, all the three elements should be expanded. Meanwhile, standard application of gauge invariance implicitly assumes that wave functions have simple free structure and need not be expanded. It is true, but only after renormalization.

This familiar and trivial fact becomes not so trivial, when the mixing of various parities or even various particles is involved. Admixture of p to Σ^+ (and vice versa) induces admixture of "bare" electromagnetic vertices to the transition vertex. "Ideologically" the situation is reminiscent of the so-called pole approach to WRD, but the formulas may look unlike.

The present consideration shows that previous calculations of WRD amplitudes may need some revision since they have not taken into account necessity of a non-standard renormalization procedure. The possibility of particle mixing in weak interactions makes this procedure even more complicated.

In this regard, we would like to emphasize the large role of particle mixing in the present discussion and its possible role in future calculations. Such mixing is essentially nonperturbative, in the sense that while its coefficients depend on symmetry properties of the perturbation, they are independent of its intensity. Moreover, the mixing may complicate the apparent consequences of gauge invariance. Therefore, an accurate account for the mixing might open new ways to describe large symmetry-violation effects observed in WRD.

One more lesson which may also be of general character concerns properties of renormalization constants. It is clear that they need to be Lorentz-invariant. But if parity is violated there are no arguments why the renormalization of fermionic propagators and vertices could not use the matrix γ_5 , instead of being purely numerical. And as the above discussion, together with calculations of Kamal and Riazuddin [9], shows, at least in that particular case one should apply matrix renormalization (i.e., include γ_5 into renormalization constants) to have canonical expressions for the Dirac equation, gauge and charge-conjugation transformations, and so on. The same question arises, therefore, for radiative corrections in any weak processes.

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