In order to generate the observed baryon asymmetry, parameters $\lambda \approx \bar{\lambda}$, and of the order of the electroweak scale, are necessary. We conclude that a phase $\sin \theta \not< 0.1$ and Higgsino and gaugino mass parameter associated with the left-right stop mixing term and the Higgsino mass $\mu$, assuming the presence of non-trivial CP violating phases in the parameters of the LEP2 collider. In this article, we compute the baryon asymmetry of the electroweak production of the Higgs boson must be light at the weak scale, opening the window for electroweak baryogenesis in the MSSM. For one of light stops, the electroweak phase transition can be strongly first order, making physics beyond the Standard Model at energy scales of the order of the weak boson masses. This has been recently emphasized that, in the case of the weak scale, SUSY would be an interesting theoretical scenario, which de-}

**Abstract**

Electroweak baryogenesis is an interesting theoretical scenario, which de-

**Supergravity?**

Electroweak Baryogenesis and Low Energy
1. The origin of the observed baryon asymmetry is one of the most fundamental open questions in particle physics. The Standard Model (SM) fulfills all the requirements for a successful generation of baryon number, due to the presence of anomalous processes, which also put relevant constraints in models in which the baryon asymmetry is generated at a very high energy scale. However, the electroweak phase transition is too weakly first order to assure the preservation of the generated baryon asymmetry at the electroweak phase transition, as perturbative and non-perturbative analyses have shown. On the other hand, CP-violating processes are suppressed by powers of $m_f/M_W$, where $m_f$ are the light-quark masses. These suppression factors are sufficiently strong to severely restrict the possible baryon number generation. Therefore, if the baryon asymmetry is generated at the electroweak phase transition, it will require the presence of new physics at the electroweak scale.

Low energy supersymmetry is a well motivated possibility, and it is hence highly interesting to test under which conditions there is room for electroweak baryogenesis in this scenario. It was recently shown that the phase transition can be sufficiently strongly first order only in a restricted region of parameter space: The lightest stop must be lighter than the top quark, the ratio of vacuum expectation values $\tan \beta < 3$, while the lightest Higgs must be at the reach of LEP2. Similar results were independently obtained by the authors of Ref. These results have been confirmed by explicit sphaleron calculations in the Minimal Supersymmetric Standard Model (MSSM).

On the other hand, the Minimal Supersymmetric Standard Model contains, on top of the Cabbibo-Kobayashi-Maskawa (CKM) matrix phase, additional sources of CP-violation and can account for the observed baryon asymmetry. New CP-violating phases can arise from the soft supersymmetry breaking parameters associated with the stop mixing angle. Large values of the mixing angle are, however, strongly restricted in order to preserve a sufficiently strong first order electroweak phase transition. Therefore, an acceptable baryon asymmetry may only be generated through a delicate balance between the values of the different soft supersymmetry breaking parameters contributing to the stop mixing parameter, and their associated CP-violating phases. The value of the Higgsino and gaugino masses also play a very relevant role in the determination of the baryon asymmetry. Indeed, the final baryon asymmetry depends strongly on the relative value of the Higgsino and gaugino mass parameters with respect to the critical temperature $T_c = O(100 \, \text{GeV})$.

In this letter we compute the baryon asymmetry and the strength of the first order phase transition in the MSSM. We identify the region in the supersymmetric parameter space where baryon asymmetry is consistent with the observed value $n_B/s \sim 10^{-10}$ and, furthermore, it is not washed out inside the bubbles after the phase transition. We obtain that the second effect is guaranteed provided the light stop mass is in the range $M_Z \lesssim m_{\tilde{t}} \lesssim m_t$, the lightest Higgs boson mass is bounded by $m_H \lesssim 80 \, \text{GeV}$ and the CP-odd boson has a mass $m_A \gtrsim 150 \, \text{GeV}$, independently of the chargino and neutralino masses. On the other hand baryon asymmetry is generated mainly by charginos and neutralinos which are not much heavier than the critical temperature, independently of

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1. Explicit two-loop calculations have the general tendency to strengthen the phase transition thus making the previous bounds as very conservative ones.
the mass of the lightest stop.

2. As discussed above, a strongly first order electroweak phase transition can be achieved in the presence of a top squark lighter than the top quark [14]. In order to naturally suppress its contribution to the parameter $\Delta \rho$ and hence preserve a good agreement with the precision measurements at LEP, it should be mainly right handed. This can be achieved if the left handed stop soft supersymmetry breaking mass $m_{Q}$ is much larger than $M_{Z}$. For moderate mixing, the lightest stop mass is then approximately given by

$$
m_{i}^{2} \simeq m_{U}^{2} + D_{R}^{2} + m_{i}^{2}(\phi) \left( 1 - \frac{|\Delta_{i}|^{2}}{m_{Q}^{2}} \right),
$$

where $\Delta_{i} = A_{i} - \mu^{*} / \tan \beta$ is the particular combination appearing in the off-diagonal terms of the left-right stop squared mass matrix and $m_{U}^{2}$ is the soft supersymmetry breaking squared mass parameter of the right handed stop. Observe that the stop mass eigenvalues and hence, the stop contribution to the Higgs effective potential must be computed by taking into account the non-vanishing complex phases for the parameters $A_{i}$ and $\mu$. For most practical purposes, however, unless the phases are of order one, we can identify the absolute value of the parameters $A_{i}$ and $\mu$ with their real components and hence, we shall not distinguish them in our notation.

We shall work within the framework of the improved one-loop effective potential. Hence two loop corrections [15, 16] will not be included in our analysis. As stated above, since these corrections tend to increase the strength of the first order phase transition [17], our bounds should be taken as conservative ones. The preservation of the baryon number asymmetry requires the order parameter $v(T_{c})/T_{c}$ to be larger than one, where $v$ denotes the Higgs vacuum expectation value (VEV) and we have normalized it such that $v(0) = v = 246.22$ GeV. Following the analysis of Refs. [17, 18, 19], the order parameter $v(T_{c})/T_{c}$ is bounded to be below the maximum value obtained for $m_{A} \gg T_{c}$,

$$
\frac{v(T_{c})}{T_{c}} < \left( \frac{v(T_{c})}{T_{c}} \right)_{\text{SM}} + \frac{2 m_{i}^{3} \left( 1 - \frac{|\Delta_{i}|^{2}}{m_{Q}^{2}} \right)^{3/2}}{\pi m_{H}^{2}},
$$

where $m_{i} = \overline{m}_{i}(m_{i})$ is the on-shell running top quark mass in the MS scheme. The first term on the right hand side of Eq. (2) is the Standard Model contribution

$$
\left( \frac{v}{T} \right)_{\text{SM}} \simeq \left( \frac{40}{m_{H}[\text{GeV}]} \right)^{2},
$$

and the second term is the contribution that would be obtained if the right handed stop plasma mass vanished at the critical temperature (see Eq. (1)). The lower the value of this mass at the critical temperature, the larger the value of $v(T_{c})/T_{c}$ [20]. For $m_{A} \gtrsim 150$ GeV, preferred to enhance the strength of the phase transition [17, 21], the present experimental bound on the Higgs mass reads $m_{H} \gtrsim 65$ GeV, meaning that the SM contribution leads to a value of $v(T_{c})/T_{c}$ at most of order 1/3.
In order to overcome the Standard Model constraints, the stop contribution must be large. The stop contribution strongly depends on the value of \( m_{	ilde{t}_i} \), which must be small in magnitude, and negative, in order to induce a sufficiently strong first order phase transition. Indeed, large stop contributions are always associated with small values of the right handed stop plasma mass

\[
m_{\text{eff}}^{\tilde{t}} = -\tilde{m}_{\tilde{t}}^2 + \Pi_R(T)
\]

where \( \tilde{m}_{\tilde{t}}^2 = -m_{\tilde{t}}^2, \) \( \Pi_R(T) \approx 4 g_3^2 T^2 / 9 + h_t^2 / 6 [2 - \frac{\tilde{A}_t}{m_Q^2}] T^2 \) \[4, 12\] is the finite temperature self-energy contribution to the right-handed squarks \[4\] and \( h_t \) and \( g_3 \) are the top quark Yukawa and strong gauge couplings, respectively. Moreover, the trilinear mass term, \( \tilde{A}_t \), must be \( \tilde{A}_t^2 \ll m_Q^2 \) in order to avoid the suppression of the stop contribution to \( v(T_c)/T_c \).

Although large values of \( \tilde{m}_{\tilde{t}} \), of order of the critical temperature, are useful to get a strongly first order phase transition, they may also induce charge and color breaking minima. Indeed, if the effective plasma mass at the critical temperature vanished, the universe would be driven to a charge and color breaking minimum at \( T \geq T_c \) \[4, 12\]. Hence, the upper bound on \( v(T_c)/T_c \) Eq. (2) cannot be reached in realistic scenarios. A conservative bound on \( \tilde{m}_{\tilde{t}} \) may be obtained by demanding that the electroweak symmetry breaking minimum should be lower than any color-breaking minima induced by the presence of \( \tilde{m}_{\tilde{t}} \) at zero temperature, which yields the condition

\[
\tilde{m}_{\tilde{t}} \leq \left( \frac{m_{\tilde{t}}^2 g_3^2}{4 \cdot 2} \right)^{1/4}.
\]

It can be shown that this condition is sufficient to prevent dangerous color breaking minima at zero and finite temperature for any value of the mixing parameter \( \tilde{A}_t \) \[4, 12\]. In this work, we shall use this conservative bound.

In order to obtain values of \( v(T_c)/T_c \) larger than one, the Higgs mass must take small values, close to the present experimental bound. Numerically, an upper bound, of order 80 GeV, can be derived. For small mixing, the one-loop Higgs mass has a very simple form

\[
m_H^2 = M_Z^2 \cos^2 2\beta + \frac{3}{4\pi^2} \tilde{m}_{\tilde{t}}^4 \log \left( \frac{m_{\tilde{t}}^2 m_{\tilde{t}}^2}{\tilde{m}_{\tilde{t}}^4} \right) \left[ 1 + O \left( \frac{\tilde{A}_t}{m_Q} \right) \right],
\]

where \( m_{\tilde{t}}^2 \sim m_{\tilde{t}}^2 + m_{\tilde{t}}^2 \), is the heaviest stop squared mass. Hence, \( \tan \beta \) must take values close to one. The larger the left handed stop mass, the closer to one \( \tan \beta \) must be. In this work we shall take \( m_{\tilde{t}} \geq 500 \) GeV. This implies that the left handed stop effects decouple at the critical temperature and hence, different values of \( m_Q \) mainly affect the baryon asymmetry through the resulting Higgs mass.

3. Concerning the baryon asymmetry generation, the new source of CP-violation, beyond the one contained in the CKM matrix, may be either explicit \[20\] or spontaneous \[21\] in the Higgs sector (which requires at least two Higgs doublets). In both

\[\text{We are considering heavy (decoupled from the thermal bath) gluinos. For light gluinos, their contribution to the squark self-energies, } 2 g_3^2 T^2 / 9, \text{ should be added to } \Pi_R(T) \[4, 23\].\]
cases, particle mass matrices acquire a nontrivial space-time dependence when bubbles of the broken phase nucleate and expand during a first-order electroweak phase transition. The crucial observation is that this space-time dependence cannot be rotated away at two adjacent points by the same unitary transformation. This provides sufficiently fast nonequilibrium CP-violating effects inside the wall of a bubble of broken phase expanding in the plasma and may give rise to a nonvanishing baryon asymmetry through the anomalous \( \langle B + L \rangle \) -violating transitions \(^{22}\) when particles diffuse to the exterior of the advancing bubble.

Baryogenesis is therefore fueled by CP-violating sources which are locally induced by the passage of the wall \(^{23, 24}\). These sources should be inserted into a set of classical Boltzmann equations describing particle distribution densities and permitting to take into account Debye screening of induced gauge charges \(^{25}\), particle number changing reactions \(^{26}\) and to trace the crucial role played by diffusion \(^{27}\). Indeed, transport effects allow CP-violating charges to efficiently diffuse in front of the advancing bubble wall where anomalous electroweak baryon violating processes are unsuppressed. This amounts to greatly enhancing the final baryon asymmetry.

We shall make use of a method recently proposed by one of the authors \(^{28}\) to compute the effect of CP-violation coming from extensions of the Standard Model on the mechanism of electroweak baryogenesis. This method is entirely based on a nonequilibrium quantum field theory diagrammatic approach and has two main virtues: it may be applied for all wall shapes and sizes of the bubble wall and it naturally incorporates the effects of the incoherent nature of plasma physics on CP-violating observables. The method relies on the closed-time path (CPT) formalism, which is a powerful Green’s function formulation for describing nonequilibrium phenomena in field theory \(^{29, 30}\), as the ones we are interested to occur in the neighbourhood of the advancing bubble walls. Indeed, what we need is to compute the temporal evolution of classical order parameters, namely CP-violating particle currents, with definite initial conditions. In this respect, the ordinary equilibrium quantum field theory at finite temperature may not be applied, since it mainly deals with transition amplitudes in particle reactions.

It is well-known that self-energy corrections at one- or two-loops to the propagator modify the dispersion relations and introduce nontrivial effects (e.g. damping) due to the imaginary contributions to the self-energy \(^{17}\). This is because particles propagating in the plasma experience interactions with the surrounding particles of the thermal bath, and the correct way to describe this phenomenon is to substitute particles by quasiparticles and to adopt dressed propagators. The self-energy takes the form \( \Sigma(k) = \text{Re } \Sigma(k) + i \text{ Im } \Sigma(k) \). Due to the nonvanishing \( \text{Im } \Sigma \), the spectral function \( \rho(k, k') \) acquires in the weak limit a finite width

\[
\Gamma(k) = -\frac{\text{Im } \Sigma(k, \omega)}{2\omega(k)},
\]

\[
\omega^2(k) = k^2 + m^2 + \text{Re } \Sigma(k, \omega),
\]

\(^3\)This effect will be ignored since the impact on the final result is \( O(1) \) \(^{23}\).
and is expressed by

$$\rho(k, k^0) = i \left[ \frac{1}{(k^0 + i \varepsilon + i \Gamma)^2 - \omega^2(k)} - \frac{1}{(k^0 - i \varepsilon - i \Gamma)^2 - \omega^2(k)} \right]. \quad (8)$$

where $m$ indicates the tree-level mass. It is easy to show that the free propagator is obtained when taking the limit $\Gamma \to 0$.

The advantage of the method is that very general formulae can be obtained for the temporal evolution of CP-violating observables without any particular assumption on the relative magnitude of the mean free paths and the thickness of the wall. The standard thin wall and thick wall limits are recovered in the limit $\Gamma L_\omega \to 0$ and $\infty$, respectively. Moreover, because of the presence of a finite damping rate, the Green’s functions of the particles involved in the processes are damped for times $t \gtrsim \Gamma^{-1}$, reflecting the effects of the incoherent nature of the plasma on CP-violating observables.

Following [22, 23], we are interested in the generation of charges which are approximately conserved in the symmetric phase, so that they can efficiently diffuse in front of the bubble where baryon number violation is fast, and non-orthogonal to baryon number, so that the generation of a non-zero baryon charge is energetically favoured. Charges with these characteristics are the axial stop charge and the Higgsino charge, which may be produced from the interactions of squarks and charginos and/or neutralinos with the bubble wall, provided a source of CP-violation is present in these sectors. CP-violating sources $\gamma_Q(z)$ (per unit volume and unit time) of a generic charge density $J^0$ associated with the current $J^\mu(z)$ and accumulated by the moving wall at a point $z^\mu$ of the plasma can then be constructed from $J^\mu(z)$ [22, 23]

$$\gamma_Q(z) = \partial_0 J^0(z). \quad (9)$$

The definition of the CP-violating source $\gamma_Q(z)$ is appropriate to describe the damping effects originated by the plasma interactions, but does not incorporate any relaxation time scale arising when diffusion and particle changing interactions are included. However, one can leave aside diffusion and particle changing interactions and account for them independently in the rate equations [23, 24].

3.1 The right-handed stop current $J_R^\mu$ associated to the right-handed stop $\tilde{t}_R$ is given by,

$$J_R^\mu = i \left( \tilde{t}_R^\dagger \partial^\mu \tilde{t}_R - \partial^\mu \tilde{t}_R^\dagger \tilde{t}_R \right). \quad (10)$$

$\langle J_R^\mu(z) \rangle$ gets contributions from eight different one-loop triangle Feynman diagrams, symbolically depicted in Fig. 1a.

In the calculation of $\langle J_R^\mu(z) \rangle$ at the lowest order in a “Higgs insertion expansion”, and more precisely in the integrand of $\int dx dy$ ($x$ and $y$ being the time components of the four-vectors where the two insertions of the diagram in Fig. 1a have been defined), the function $H_2(x+z)H_1(y+z) - H_1(x+z)H_2(y+z)$ appears, where $H_i$, $i = 1, 2$, denote the two neutral Higgs components. In order to deal with analytic expressions,

4Hereafter the generic notation for the time component of the four-vector $z^\mu$ will be adopted as $z$, e.g. $z^\mu \equiv (z, \tilde{z})$. 

5
we can work out the thick wall limit and simplify the expressions obtained above by performing a derivative expansion

\[ H_i(x + z) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{\partial^n}{\partial z^n} H_i(z) \, x^n. \]  

(11)

The contribution to the current can then be expanded in a power series as:

\[
H_2(x + z) H_1(y + z) - H_1(x + z) H_2(y + z) = \sum_{n=0}^{\infty} \frac{\partial^n}{\partial z^n} \left[ H_1(z) \partial_z H_2(z) - H_2(z) \partial_z H_1(z) \right] f_n(x, y)
\]

(12)

where the function \( f_n \) is provided by the power expansion, e.g. \( f_0 = x - y \), \( f_1 = (x^2 - y^2)/2 \), … Notice that the term with no derivatives vanishes in the expansion (12), e.g. \( H_2(z) H_1(z) - H_1(z) H_2(z) \equiv 0 \), which means that the static term in the derivative expansion (11) does not contribute to the vacuum expectation value of the currents. Furthermore, the approximation of neglecting terms in (11) with \( n > 1 \) amounts to neglecting terms in (12) with \( n > 0 \). Since for a smooth Higgs profile, the subsequent derivatives with respect to the time coordinate are associated with higher powers of \( v_\omega/L_\omega \), and the integration over \( x \) and \( y \) of a higher order term in the expansion of the currents leads to a higher power of \( 1/\Gamma_t \), this is a good approximation for values of \( L_\omega \Gamma_t/v_\omega \gg 1 \). In other words, this expansion is valid only when the mean free path \( \tau_t \simeq \Gamma_t^{-1} \) is smaller than the scale of variation of the Higgs background determined by the wall thickness, \( L_\omega \), and the wall velocity \( v_\omega \). An estimate of the stop damping rate (in the low momentum limit) is \( \Gamma_t \sim 10^{-1} T \), approximated using the calculation made in Ref. [24]. With such value, our derivative expansion is perfectly justified since the wall thickness can span the range \((10 - 100)/T\).

Therefore, the currents are proportional to the function (the coefficient of \( f_0 \) in Eq. (12))

\[ H_1(z) \partial_z H_2(z) - H_2(z) \partial_z H_1(z) \equiv H^2(z) \partial_z \beta(z), \]

(13)

which should vanish smoothly for values of \( z \) outside the bubble wall. Here \( H^2 \equiv H_1^2 + H_2^2 \).

Since the time variation of the Higgs fields is due to the expansion of the bubble wall through the thermal bath, \( \langle J_R^\mu(z) \rangle \) will be linear in \( v_\omega \). This result explicitly shows that we need out of equilibrium conditions to generate \( \langle J_R^\mu(z) \rangle \) and that we have to call for the CTP formalism to deal with time-dependent phenomena. To work out exactly \( \langle J_R^\mu(z) \rangle \) one should know the exact form of the distribution functions which, in ultimate analysis, are provided by solving the Boltzmann equations. However, any departure from thermal equilibrium distribution functions is caused at a given point by the passage of the wall and, therefore, is \( O(v_\omega) \). Since \( \langle J_R^\mu(z) \rangle \) is already linear in \( v_\omega \), working with thermal equilibrium distribution functions amounts to ignoring terms of higher order in \( v_\omega \) [23], which is as accurate as the bubble wall is moving slowly in the plasma and we shall adopt this approximation from now on.

One can show that

\[ \langle J_R^\mu(z) \rangle = h_t^2 \text{Im} \left( A_{\mu} \right) \left[ H_1(z) \partial_z H_2(z) - H_2(z) \partial_z H_1(z) \right] G_R^\mu \]

(14)
\[
    G_R^Q = \frac{1}{4\pi^2\Gamma_R} \int_0^\infty dk \frac{k^2}{\omega_R \omega_{RQ}} \left[ (1 + 2\text{Re}(n_Q)) I_1(\omega_R, \Gamma_R, \omega_Q, \Gamma_Q) + (1 + 2\text{Re}(n_R)) I_1(\omega_Q, \Gamma_Q, \omega_R, \Gamma_R) \right] - \frac{2}{\gamma_0} \left( \text{Im}(n_R) + \text{Im}(n_Q) \right) I_2(\omega_R, \Gamma_R, \omega_Q, \Gamma_Q) \right] \]
\]

and \( \omega_R^{2(\ell)} = k^2 + m_V^2 \omega_Q^2(\ell), \) \( n_R^{(Q)} = \frac{1}{\exp \left( \omega_R^{(Q)} / T + i\Gamma_R^{(Q)} / T \right) - 1}, \) while \( \Gamma_R \sim \Gamma_Q \sim \Gamma_\gamma. \)

The functions \( I_1 \) and \( I_2 \) are given by

\[
    I_1(a, b, c, d) = \frac{r_1}{(r_1^2 + 1) [(a + c)^2 + (b + d)^2]} + \frac{r_2}{(r_2^2 + 1) [(a - c)^2 + (b + d)^2]},
\]

\[
    I_2(a, b, c, d) = \frac{r_1^2 - 1}{2(r_1^2 + 1) [(a + c)^2 + (b + d)^2]} + \frac{r_2^2 - 1}{2(r_2^2 + 1) [(a - c)^2 + (b + d)^2]},
\]

where \( r_1 = (a + c)/(b + d) \) and \( r_2 = (a - c)/(b + d). \)

**3.2** Similarly, the Higgsino current associated with neutral and charged Higgsinos can be written as

\[
    J_H^\mu = \overline{H} \gamma^\mu H
\]

where \( \overline{H} \) is the Dirac spinor

\[
    \overline{H} = \begin{pmatrix} \overline{H}_2 \\ \overline{H}_1 \end{pmatrix}
\]

and \( \overline{H}_2 = \overline{H}_0^0 (\overline{H}_2^+), \overline{H}_1 = \overline{H}_0^0 (\overline{H}_1^-) \) for neutral (charged) Higgsinos.

The Higgsino current \( \overline{H} \) gets its contribution from the typical triangle diagram of Fig. 1b and may be computed along the same lines described for the axial stop number in the previous subsection. Analogously to the case of right-handed stops, the dispersion relations of charginos and neutralinos are changed by high temperature corrections \[^{[57]}\]. Even though fermionic dispersion relations are highly nontrivial, relatively simple expressions for the fermionic spectral functions may be given in the limit in which the damping rate is smaller than the typical self-energy of the fermionic excitation (this limit is certainly satisfied in our case) \[^{[40]}\]. For instance, the spectral function of Higgsinos \( \overline{H} \) (and analogously for gauginos \( W, B) \) may be written as

\[
    \rho_H(k, k^0) = i \left( k + m_H \right) \frac{1}{(k^0 + i\varepsilon + i\Gamma_H)^2 - \omega_H^2(k)} - \frac{1}{(k^0 - i\varepsilon - i\Gamma_H)^2 - \omega_H^2(k)},
\]

where \( \omega_H^2(k) = k^2 + m_H^2(T) \) and \( m_H^2(T) \) is the Higgsino effective plasma squared mass in the thermal bath which may be well approximated by its value in the present vacuum,
\( m_H^2(T) \propto |\mu|^2 \). Similarly, \(|\mu|\) should be replaced by \( M_2 \) for \( \rho_w^{-}(\bf{k}, k^0) \), and by \( M_1 \) for \( \rho_\tilde{H}(\bf{k}, k^0) \).

In analogy to the stop case, in the region of parameters defined above, we can perform a Higgs insertion expansion of the CP-violating current. The vacuum expectation value of the (zero component of the) Higgsino current is then found to be

\[
\langle J^0_H(z) \rangle = \text{Im}(\mu) \left[ H_2(z) \partial_z H_2(z) - H_2(z) \partial_z H_2(z) \right] \left[ 3 M_2 \ g_2^2 \ G_H^W + M_1 \ g_1^2 \ G_H^B \right], \tag{21}
\]

where

\[
G_H^W = \int_0^\infty \frac{k^2}{dk} \frac{1}{4\pi^2 \Gamma_H \omega_H \omega_W} \left[ \left( 1 - 2 \text{Re}(n_H^-) \right) I_1(\omega_H^-, \Gamma_H^-, \omega_W^-, \Gamma_W^-) + \left( 1 - 2 \text{Re}(n_H^-) \right) I_1(\omega_W^-, \Gamma_W^-, \omega_H^-, \Gamma_H^-) \right] + 2 \left( \text{Im}(n_H^-) + \text{Im}(n_W^-) \right) I_2(\omega_H^-, \Gamma_H^-, \omega_W^-, \Gamma_W^-) \right] \tag{22}
\]

and \( \omega_H^2 H_W \) = \( k^2 + |\mu|^2 (M_2^2) \) while \( n_H^- W^1 \) = \( 1 / \left[ \exp \left( \omega_H^- / |W^1| / T + i \Gamma_H^- W / T \right) + 1 \right] \). The damping rate of charged and neutral Higgsinos is dominated by weak interaction and we take \( \Gamma_H^- \sim \Gamma_W^- \) to be of order of 5 \times 10^{-2} T. The Bino contribution may be obtained from the above expressions by replacing \( M_2 \) by \( M_1 \).

4. We can now solve the set of coupled differential equations describing the effects of diffusion, particle number changing reactions and CP-violating source terms. We will closely follow the approach taken in Ref. [43] where the reader is referred to for more details. If the system is near thermal equilibrium and particles interact weakly, the particle number densities \( n_i \) may be expressed as \( n_i = k_i \mu_i T^2 / 6 \) where \( \mu_i \) is the local chemical potential, and \( k_i \) are statistical factors of order of 2 (1) for light bosons (fermions) in thermal equilibrium, and Boltzmann suppressed for particles heavier than \( T \).

The particle densities we need to include are the left-handed top doublet \( q_L \equiv (t_L + b_L) \), the right-handed top quark \( t_R \), the Higgs particle \( h \equiv (H^0_1, H^0_2, H^-_1, H^+_2) \), and the superpartners \( \tilde{q}_L, \tilde{t}_R \) and \( \tilde{h} \). The interactions able to change the particle numbers are the top Yukawa interaction with rate \( \Gamma_t \), the top quark mass interaction with rate \( \Gamma_m \), the Higgs self-interactions in the broken phase with rate \( \Gamma_H \), the strong sphaleron interactions with rate \( \Gamma_{s\text{s}} \), the weak anomalous interactions with rate \( \Gamma_{w\text{s}} \) and the gauge interactions. We shall assume that the supergauge interactions are in equilibrium. Under these assumptions the system may be described by the densities \( Q = q_L + \tilde{q}_L, \ T = t_R + \tilde{t}_R \) and \( H = h + \tilde{h} \). CP-violating interactions with the advancing bubble wall produce source terms \( \gamma_H = \partial_z \langle J^0_H(z) \rangle \) for Higgsinos and \( \gamma_R = \partial_z \langle J^0_R(z) \rangle \) for right-handed stops, which tend to push the system out of equilibrium. Ignoring the curvature of the bubble wall, any quantity becomes a function of the coordinate \( z = z_3 + v_\omega z \), the coordinate normal to the wall surface, where we assume the bubble wall is moving along the \( z_3 \)-axis.
Assuming that the rates $\Gamma_i$ and $\Gamma_{ss}$ are fast so that $Q/k_i - \mathcal{H}/k_H - \mathcal{T}/k_T = \mathcal{O}(1/\Gamma_i)$ and $2Q/k_i - \mathcal{T}/k_T + 9(Q + \mathcal{T})/k_i = \mathcal{O}(1/\Gamma_{ss})$, one can find the equation governing the Higgs density
\[ v_\omega \mathcal{H}' - \mathcal{T}\mathcal{H}'' + \Gamma \mathcal{H} - \bar{\gamma} = 0, \quad (23) \]
where the derivatives are now with respect to $z$, $\mathcal{T}$ is the effective diffusion constant, $\bar{\gamma}$ is an effective source term in the frame of the bubble wall and $\Gamma$ is the effective decay constant \[ \boxed{\Gamma} \]. An analytical solution to Eq. (23) satisfying the boundary conditions $\mathcal{H}(\pm \infty) = 0$ may be found in the symmetric phase (defined by $z < 0$) using a $z$-independent effective diffusion constant (in agreement with our approximation in Eq. (11)) and a step function for the effective decay rate $\Gamma = \Gamma \theta(z)$. A more realistic form of $\Gamma$ would interpolate smoothly between the symmetric and the broken phase values. We have checked, however, that the result is insensitive to the specific position of the step function inside the bubble wall. The values of $\mathcal{T}$ and $\Gamma$ in (23) of course depend on the particular values of supersymmetric parameters. For the considered range we typically find $\mathcal{T} \sim 0.8 \text{ GeV}^{-1}$, $\Gamma \sim 1.7 \text{ GeV}$.

The dependence on $\bar{\gamma}$ is much more subtle, and a good approximation for the Higgs profiles is necessary in order to obtain a reliable approximation to the currents. As seen in Eq. (13), the current is proportional to the variation of the ratio of vacuum expectation values along the wall. The Higgs profiles along the wall are likely to follow the path of minimal energy connecting the false and the true vacua in the Higgs potential. Close to the symmetric phase, and at the temperature $T_0$, defined as the one at which the curvature at the origin vanishes, the ratio of vacuum expectation values is given by $\tan^2 \beta(z = 0) = m_i^2(T_0)/m_j^2(T_0)$, a quantity which may be computed numerically from the Higgs finite temperature effective potential as $m_i^2(T) = 1/2 \left( \partial^2 V_{\text{eff}} / \partial H_i^2 \right) |_{H_1 = H_2 = 0}$, $(i = 1, 2)$. Hence, the variation of $\beta$ is given by
\[ \Delta \beta = \beta(T_0) - \arctan(m_1(T_0)/m_2(T_0)) \quad (24) \]
This quantity tend to zero for large values of $m_A$, and takes small values, of order 0.015 for values of $m_A = 150$–200 GeV, giving rise to a further suppression of the final baryon number.

One would be tempted to adopt a linear ansatz already for the function (13) in the current, as done in 13. However, this approximation has the drawback that it may not be obtained starting from smooth Higgs profiles and can therefore be viewed as unphysical. Also, since the source in the diffusion equation depends on the derivatives of the current, linear walls (or, generally, all non-differentiable wall shapes) lead to the appearance of $\delta$-functions in the CP-violating source. A more realistic option is to specify the shapes of the Higgs fields and $\beta$ and to consequently compute the source. Realistic shapes for $H(z)$ (and probably for $\beta(z)$) would be given by functional kinks along the bubble wall. This provides a smooth function for the source $\bar{\gamma}$. Moreover, this way of proceeding leads to a strong suppression with respect to the result that one would have obtained assuming a linear approximation for the quantity $H_1(z) \partial_z H_2(z) - H_2(z) \partial_z H_1(z)$. This is due to the fact that, since $H_1(z) \partial_z H_2(z) - H_2(z) \partial_z H_1(z)$ goes to zero for $z \to \pm \infty$, the source $\bar{\gamma}$ must change sign inside the bubble wall, giving rise to some cancellation for the final expression of the baryon number, see Eqs. (29) and (32).
We consider here the following semirealistic approximation:

\[
H(z) = \frac{v(L_\omega)}{2} \left[ 1 - \cos \left( \frac{Z \pi}{L_\omega} \right) \right] \left[ \theta(z) - \theta(z - L_\omega) \right] + v(L_\omega) \theta(z - L_\omega) \\
\beta(z) = \Delta \beta \left[ 1 - \cos \left( \frac{Z \pi}{L_\omega} \right) \right] \left[ \theta(z) - \theta(z - L_\omega) \right] + \beta(z = 0) + \Delta \beta \theta(z - L_\omega) \quad (25)
\]

which leads to a current vanishing smoothly at \( z = 0 \) and \( z = L_\omega \).

For \( z < 0 \) one obtains,

\[
\mathcal{H}(z) = A e^{2\omega \sqrt{D} t},
\]

and for \( z > 0 \),

\[
\mathcal{H}(z) = \left( \mathcal{B}_+ - \frac{1}{D(\lambda_+ - \lambda_-)} \int_0^z du \tilde{\gamma}(u) e^{-\lambda_+ u} \right) e^{\lambda_+ z} \\
+ \left( \mathcal{B}_- - \frac{1}{D(\lambda_- - \lambda_+)} \int_0^z du \tilde{\gamma}(u) e^{-\lambda_- u} \right) e^{\lambda_- z}.
\]

where

\[
\lambda_\pm = \frac{v_\omega \pm \sqrt{v_\omega^2 + 4 \Delta \bar{D}}}{2 \bar{D}},
\]

and \( \tilde{\gamma}(z) = v_\omega \partial_z J^0(z) f(k_i) \), \( J_0 \) being the total CP-violating current resulting from the sum of the right-handed stop and Higgsino contributions and \( f(k_i) \) a coefficient depending on the number of degrees of freedom present in the thermal bath and related to the definition of the effective source \( \tilde{B} \). Imposing the continuity of \( \mathcal{H} \) and \( \mathcal{H}' \) at the boundaries, we find

\[
A = \mathcal{B}_+ \left( 1 - \frac{\lambda_-}{\lambda_+} \right) = \mathcal{B}_- \left( \frac{\lambda_+}{\lambda_-} - 1 \right) = \frac{1}{D(\lambda_+ - \lambda_-)} \int_0^\infty du \tilde{\gamma}(u) e^{-\lambda_+ u}.
\]

From the form of the above equations one can see that CP-violating densities are non-zero for a time \( t \sim \bar{D}/v_\omega^2 \), and the assumptions leading to the analytical form of \( \mathcal{H}(z) \) are valid provided \( \Gamma_i, \Gamma_{\text{w}} \gg v_\omega^2 / \bar{D} \).

The equation governing the baryon asymmetry \( n_B \) is given by \( \tilde{B} \)

\[
D_q n_B'' - v_\omega n_B' - \theta(-z) n_f \Gamma_{\text{w}} n_L = 0,
\]

where \( \Gamma_{\text{w}} = 6 \kappa_\omega^{-4} T \) is the weak sphaleron rate \( \kappa \approx 1 \) \( \tilde{B} \), and \( n_L \) is the total number density of left-handed weak doublet fermions; \( n_f = 3 \) is the number of families and we have assumed that the baryon asymmetry gets produced only in the symmetric phase. Expressing \( n_L(z) \) in terms of the Higgs number density

\[
n_L = \frac{9k_1 k_T - 8k_1 k_T - 5k_1 k_1}{k_1(k_1 + k_2 + 9k_T - 9k_T) \mathcal{H}}
\]

\footnote{We have checked that numerical results using the ansatz \( \tilde{B} \) differ by less than 10% from those obtained by using functional kinks. This leads us to the belief that our results are robust even in the absence of a numerical calculation of bubble solutions in the MSSM.}

\footnote{The correct value of \( \kappa \) is at present the subject of debate; see, for instance, Ref. \( \tilde{B} \).}
and making use of Eqs. (26)-(30), we find that

\[
\frac{n_B}{s} = -g(k_i) \frac{v_{\text{ms}}}{v_\omega^2}.
\]  

(32)

where \( s = 2\pi^2 g_\nu T^3/45 \) is the entropy density (\( g_\nu \) being the effective number of relativistic degrees of freedom) and \( g(k_i) \) is a numerical coefficient depending upon the light degrees of freedom present in the thermal bath.

5. The expression for the baryon asymmetry derived in the last section assumes that the order parameter \( v(T_c)/T_c \geq 1 \), in order to assure that anomalous processes are sufficiently suppressed in the broken phase. As we already discussed, this demands low values of the right-handed stop effective plasma mass at the critical temperature and low values of the mixing mass parameter \( A_t \). We will now present numerical results based on the previous calculations. Since \( \Delta \beta \), Eq. (24), denotes the variation of the ratio of vacuum expectation values of the Higgs fields along the bubble wall, it will be negligible unless the CP-odd Higgs mass takes values of the order of the critical temperature. Indeed, for large values of \( m_A \), \( \Delta \beta \sim 1/m_A^2 \).

Values of the CP-odd Higgs mass \( m_A \leq 200 \text{ GeV} \) are associated with a weaker first order phase transition. Fig. 2 shows the behaviour of the order parameter \( v/T \) in the \( m_A - \tan \beta \) plane, for \( A_t = 0 \), \( m_Q = 500 \text{ GeV} \) and values of \( \tilde{m}_U \) close to its upper bound, Eq. (25). In order to correctly interpret the results of Fig. 2 one should remember that the Higgs mass bounds are somewhat weaker for values of \( m_A < 150 \text{ GeV} \). However, even for values of \( m_A \) of order 80 GeV, in the low \( \tan \beta \) regime the lower bound on the Higgs mass is of order 60 GeV. Hence, it follows from Fig. 2 that, to obtain a sufficiently strong first order phase transition, \( v/T \geq 1 \), the CP-odd Higgs mass \( m_A \geq 150 \text{ GeV} \). In our analysis, we shall fix \( m_A = 200 \text{ GeV} \), which is well inside the acceptable range for this parameter. As anticipated above, this choice leads to values of \( \Delta \beta \sim 0.015 \).

Since for large values of \( m_Q \) the stop contribution to the baryon asymmetry is strongly suppressed compared to the chargino and neutralino ones, the numerical values of the baryon asymmetry depend linearly on the phase of the Higgsino mass parameter. In general, the numerical values obtained following the above procedure are too low to explain the observed baryon asymmetry unless this phase is of order one. This conclusion may only be avoided for certain specific values of the gaugino and Higgsino mass parameters. In order to make this result explicit, in Fig. 3 we show the value of this phase necessary to obtain a value \( n_B/s \sim 4 \times 10^{-11} \) in the \( \mu - M_2 \) plane. The wall velocity is taken to be \( v_\omega = 0.1 \), while the bubble wall width is taken to be \( L_\omega = 25/T \), and, for simplicity, we consider \( M_1 = M_2 \). Our results, are, however, quite insensitive to the specific choice of \( v_\omega \) and \( L_\omega \). Due to the fact that, as already stated, stop contributions to the baryon asymmetry are negligible as compared to the chargino/neutralino ones, we have taken \( |\sin(\phi_A + \phi_\mu)| = 0 \) in Fig. 3.

\(^7\)Recent analyses \( ^{[23]} \) of the effective three-dimensional theory of the MSSM essentially confirm the early results of Ref. \( ^{[20]} \), while a similar analysis for the light stop scenario is still lacking.

\(^8\)We have checked that, as expected, the final results do not depend much on the value of \( M_1 \). In particular \( M_1 \) can be chosen such that the lightest neutralino is lighter than the light stop.
Values of the phases lower than 0.1 are only consistent with the observed baryon asymmetry for values of $|\mu|$ of order of the gaugino mass parameters. This is due to a large enhancement of the computed baryon asymmetry for these values of the parameters. Since the Higgs background in the Feynman diagrams of Fig. 1b is carrying a very low momentum (of order of the inverse of the bubble wall width $L_\omega$), this resonant behaviour is associated with the possibility of absorption (or emission) of Higgs quanta by the propagating particles. For momenta of order of the critical temperature, this can only take place when the Higgsinos and gauginos are nearly degenerate in mass, $\mu \sim M_2$. By using the Uncertainty Principle, it is easy to understand that the width of this resonance is expected to be proportional to the decay width of the -inos, which is what comes out from our explicit computation. In the case of the stops, instead, since $m_Q^2 \gg \tilde{m}_\chi^2$, the same phenomenon can only happen for momenta larger than $m_Q$. Such configurations are exponentially suppressed and do not give any relevant contribution to the one loop diagram of Fig. 1a. The resonant behaviour is certainly enhanced by our approximations. Indeed, since we are working at first order in the Higgs insertions, the relevant masses propagating in the loops are given by their expressions in the symmetric phase. Hence, strong degeneracies are possible when $\mu$ is close to $M_2$. This degeneracy, however, will be broken by further Higgs insertions.

Therefore, taking the results from Fig. 3 at face value, one would find an absolute lower bound on $|\sin(\phi_\mu)| \gtrsim 10^{-2}$. However, given all uncertainties in the calculation, including those associated to the different damping rates for squarks, Higgsinos and gauginos, and the sphaleron parameters, in particular $\kappa = \Gamma_\omega/[6\alpha_w^4 T]$ we will prefer to consider, as a conservative estimate, the previous bound on $|\sin(\phi_\mu)|$ accurate only up to about one order of magnitude. If the phases of the parameters $A_t$ and $\mu$ are $\mathcal{O}(1)$, CP-violation will be induced in the Higgs sector and the Higgs VEVs will become complex. However, it is easy to prove that, whenever the tree level CP-odd mass is larger than the vacuum expectation values and the trilinear mass parameters $|A_t|$ and $|\mu|$ take values lower than $m_Q$, the induced phase of the product of vacuum expectation values will be much lower than 1, hence our results will be only weakly modified by this effect.

6. In conclusion, we made a self-consistent computation of the baryon asymmetry generated at the electroweak phase transition in the Minimal Supersymmetric Standard Model. We proved that baryon asymmetry is mainly generated by CP-violating Higgsino currents, provided that Higgsinos and gauginos are not much heavier than the electroweak critical temperature ($T_c \sim 100$ GeV), and the phase $\phi_\mu$ is not much smaller than 0.1. Observe that these relatively large values of the phases are only consistent with the constraints from the electric dipole moment of the neutron if the squarks of the first and second generation have masses of the order of a few TeV $^{[10]}$. On the other hand, the generated baryon asymmetry is not washed out inside the bubbles, after the phase transition, provided that the lightest stop is lighter than the top quark (the so-called light stop scenario), the pseudoscalar Higgs boson is heavier than $\sim 150$ GeV and the lightest Higgs boson is lighter than $\sim 80$ GeV.

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9 We have adopted in the calculation the value $\kappa = 1$. The final result scales with $\kappa$. 

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The most direct experimental way of testing this scenario is through the search for the lightest Higgs at LEP2. If the Higgs is found, the second test will come from the search for the lightest stop at the Tevatron collider (the stop mass is typically too large for the stop to be seen at LEP). If both particles are found, the last crucial test will come from $B$ physics, more specifically, in relation to the CP-violating effects. Depending on their gaugino-Higgsino composition, light neutralinos or charginos, at the reach of LEP2, although not essential, would further restrict the allowed parameter space consistent with electroweak baryogenesis.

Finally, let us expand on the Higgs searches. If the next run of LEP at $\sqrt{s} = 186$ GeV collects the planned luminosity, it will be able to search for Higgs masses up to about 75 GeV [14]. If no Higgs signal is found, this will pose additional constraints on the present scenario. In particular, in order to get still a sufficiently strong first order phase transition it will demand $A_t/m_Q \simeq 0$. Moreover, the selected parameter space leads to values of the branching ratio $\text{BR}(b \to s \gamma)$ different from the Standard Model case. Although the exact value of this branching ratio depends strongly on the value of the $\mu$ and $A_t$ parameters, the typical difference with respect to the Standard Model prediction is of the order of the present experimental sensitivity and hence in principle testable in the near future. Indeed, for the typical spectrum considered here, due to the light charged Higgs, the branching ratio $\text{BR}(b \to s \gamma)$ is somewhat higher than in the SM case, unless negative values of $A_t \mu$ are present.

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References


Figure 1: a) One-loop Feynman diagrams contributing to $\langle J_R^\mu(z) \rangle$. Here $m_{LR}^2$ indicates the combination $h_i (A_i H_2 - \mu^* H_1)$. b) One-loop diagrams contributing to $\langle J^0_H(z) \rangle$. Here $\mu_a$, $a = 1, \ldots, 4$ describes the interactions of Higgsinos with gauginos $\tilde{W}_a$, $a = 1, \ldots, 3$ and $\tilde{B}$, $a = 4$. More precisely, $\mu_a = g_a \left[ H_1 P_L + \mu^* H_2 P_R \right]$ where $P_{L(R)} = \frac{1}{2} (1 \mp \gamma_5)$, $g_a = g_2$, $a = 1, \ldots, 3$, and $g_a = g_1$ for $a = 4$. 
Figure 2: Contour plots of constant values of $v(T_c)/T_c$ (solid lines) and $m_H$ in GeV (dashed lines) in the plane $(m_A, \tan \beta)$. We have fixed $m_t = 175$ GeV and the values of supersymmetric parameters: $m_Q = 500$ GeV, $m_U = m_U^{\text{crit}}$ fixed by the charge and color breaking constraint, and $A_t = \mu / \tan \beta$. 
Figure 3: Contour plot of $|\sin \phi_\mu|$ in the plane $(\mu, M_2)$ for fixed $n_B/s = 4 \times 10^{-11}$ and $v_\omega = 0.1$, $L_\omega = 25/T$, $m_Q = 500$ GeV, $m_U = m_U^{\text{crit}}$, $\tan \beta = 2$ and $A_t = \mu/\tan \beta$. 