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A Non-Minimal SO(10) × U(1)$_F$ SUSY - GUT Model Obtained from a Bottom-Up Approach

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Abstract

A non-minimal supersymmetric $SO(10) \times U(1)_F$ grand unified model is developed based on the Yukawa structure of a model previously constructed in collaboration with S. Nandi to explain the quark and lepton masses and mixings in a particular neutrino scenario. The $U(1)_F$ family symmetry can be made anomaly-free with the introduction of one conjugate pair of $SO(10)$-singlet neutrinos with the same $U(1)_F$ charge. Due to a plethora of conjugate pairs of supermultiplets, the model develops a Landau singularity within a factor of 1.5 above the GUT scale. With the imposition of a Z_2 discrete symmetry, all higgsino triplets can be made superheavy while just one pair of higgsino doublets remains light and results in mass matrix textures previously obtained from the bottom-up approach. Diametrically opposite splitting of the first and third family scalar quark and lepton masses away from the second family ones results from the nonuniversal D-term contributions.

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I. INTRODUCTION

In a recent series of papers [1, 2], the author in collaboration with S. Nandi began a program to construct a viable model for the fermion quark and lepton masses and mixings at the supersymmetric grand unification scale. The program envisaged by us has evolved in three stages, beginning with a bottom-up approach which ensures accurate results for the known low-energy data without introducing an undue amount of theoretical bias at the outset. This is to be contrasted with most theoretical model construction which has been carried out by various authors [3] using a top-down approach. In that case, some well-defined theoretical principles are selected at the outset with the model parameters then picked to fit the known low-energy data as well as possible.

The general framework chosen by us was that of supersymmetric $SO(10)$ grand unification (SUSY-GUTS), since this appeared to give the most satisfactory explanation for the unification of the standard model gauge couplings [4] at a high energy scale of the order of 10^{16} GeV, as well as accommodating the 16 fermions of each family in a simple representation of the gauge group. The low energy data for most quark and charged lepton masses as well as the quark Cabibbo-Kobayashi-Maskawa quark mixing matrix [5] are reasonably well-known [6], while various scenarios must be entertained at this time for the neutrino masses and mixings according to which experimental results one is willing to accept at face value.

The first bottom-up stage [1] of our program for a given scenario then consisted of evolving [7] the masses and mixing matrices to the SUSY-GUT scale, where the up, down, charged lepton and neutrino mass matrices can be constructed by making use of Sylvester's theorem [8]. Two free parameters, one for the quark sector and one for the lepton sector, which control the choice of bases for the mass matrices were tuned and different neutrino scenarios selected to search for mass matrices exhibiting simple $SO(10)$ structure. For this purpose, complete unification of all third family quark and lepton Yukawa couplings was assumed [9] corresponding to a pure **10** Higgs contribution to the 33 mass matrix elements, while simplicity in the sense of pure **10** or pure $\overline{\mathbf{126}}$ Higgs contributions was sought for as many of the other mass matrix elements as possible. This choice of procedure was influenced by earlier work such as that of Georgi and Jarlskog [10], where the 33 mass matrix elements transformed as pure **10**'s and the 22 elements as pure **126**'s. We are aware that level-5 $\overline{\mathbf{126}}$ $SO(10)$ multiplets do not arise naturally in superstring models [11] and must be treated as effective operators; hence such a model should be treated as an effective theory at best. We shall return to this point at the end of Sect. II.

The simplest $SO(10)$ structure at the SUSY-GUT scale was obtained in the neutrino scenario incorporating the Mikheyev - Smirnov - Wolfenstein (MSW) [12] nonadiabatic resonant conversion interpretation of the depletion of solar electron-neutrinos [13] together with the observed depletion of atmospheric muon-neutrinos [14]. In this scenario, no eV-scale neutrino masses exist to contribute a hot dark matter component to mixed dark matter [15]; moreover, since no sterile neutrinos were incorporated into the model at that time, in the version under consideration we are unable to explain the $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ mixing results obtained by the LSND collaboration [16]. The mass matrices constructed at the SUSY-GUT scale have the following textures

$$M^U \sim M^{N_{Dirac}} \sim diag(\overline{126};\ \overline{126};\ 10) \tag{1.1a}$$

$$M^D \sim M^E \sim \begin{pmatrix} 10', \overline{126} & 10', \overline{126}' & 10' \\ 10', \overline{126}' & \overline{126} & 10' \\ 10' & 10' & 10 \end{pmatrix} \tag{1.1b}$$

with M^D_{11}, M^E_{12} and M^E_{21} anomalously small and only the 13 and 31 elements complex. Entries in the matrices stand for the Higgs representations contributing to those elements. We assumed that vacuum expectation values (VEV's) develop only for the symmetric representations **10** and **126**. The **10**'s contribute equally to (M^U, M^D) and ($M^{N_{Dirac}}$, M^E), while the **126**'s weight (M^U, M^D) and ($M^{N_{Dirac}}$, M^E) in the ratio of 1 : -3. The Majorana neutrino mass matrix M^R, determined from the seesaw formula [17] with use of $M^{N_{Dirac}}$ and the reconstructed light neutrino mass matrix, exhibits a nearly geometrical structure [18] given by

$$M^R \sim \begin{pmatrix} F & -\sqrt{FE} & \sqrt{FC} \\ -\sqrt{FE} & E & -\sqrt{EC} \\ \sqrt{FC} & -\sqrt{EC} & C \end{pmatrix} \tag{1.1c}$$

where $E \simeq \frac{5}{6}\sqrt{FC}$ with all elements relatively real. It can not be purely geometrical, however, since the singular rank-1 matrix above can not be inverted as required by the seesaw formula, $M^{N_{eff}} \simeq -M^{N_{Dirac}}(M^R)^{-1}M^{N^T_{Dirac}}$.

In the second stage [2] of our program, we attempted to find a model incorporating a family symmetry which yields the above matrices determined phenomenologically from our bottom-up approach. Success was obtained by introducing a global $U(1)_F$ family symmetry [19] which uniquely labels each one of the three light families, as well as conjugate pairs of heavy families and various Higgs representations. In addition to controlling the evolution of the Yukawa couplings from the SUSY-GUT scale to the supersymmetry-breaking weak scale, the supersymmetric nature of the SUSY-GUT model played a key role in that the nonrenormalization theorems [20] of supersymmetry allow one to focus solely on Dimopoulos-type tree diagrams [21], in order to calculate the contributions to the mass matrix elements. With twelve input parameters in the form of Yukawa couplings times VEV's, the numerical results obtained for the 3 heavy Majorana masses and 20 low energy parameters for the quark and lepton masses and two mixing matrices were found to be in exceptionally good agreement with the low energy data in the neutrino scenario in question as shown in [2].

In this paper the author has pursued the third stage of the program which is to construct a consistent supersymmetric grand unified model of all the interactions in an $SO(10) \times U(1)_F$ framework. A number of important issues [22] must be addressed such as the anomaly-free nature of the superpotential, the requirement that supersymmetry remain unbroken at the SUSY-GUT scale and only effectively broken at the electroweak scale, the requirement that all colored Higgs triplets become superheavy, so that proton decay remains sufficiently suppressed while only one pair of Higgs doublets remains light to break the electroweak symmetry and to preserve the good prediction for the electroweak angle in $\sin^2\theta_W$. The evolution equations for the gauge and Yukawa couplings should also not be greatly altered by the presence of any extra light fields in the model. In this paper we shall address these issues and show that some degree of success is achieved. In addition, we find that a Landau singularity develops slightly above the SUSY-GUT scale as a result of the plethora of conjugate pairs of supermultiplets present in the model. Some indication of the type of splitting of the squark and slepton masses is also gained from the nonuniversal D-term contributions.

II. SUPERMULTIPLETS AND HIGGS SUPERPOTENTIAL

In order to build a satisfactory supersymmetric model [22], we require that the superpotential be analytic and anomaly-free. For this purpose we replace the fermion and Higgs $SO(10)$ multiplets by chiral supermultiplets and double the number of supermultiplets with non-zero $U(1)_F$ charges by adding supermultiplets with equal and opposite $U(1)_F$ charges.

We begin by listing the fermion and boson fields introduced in the $SO(10)\times U(1)_F$ model in the second stage of our program.

- **Left-Handed Fermion Fields:**

$$\mathbf{16}_i: \quad \psi_3^{(9)}, \quad \psi_2^{(-1)}, \quad \psi_1^{(-8)} \tag{2.1a}$$

$$\begin{array}{lllllllllll} \mathbf{16}: & f: & -0.5, & 1.0, & 2.0, & 4.0, & 4.5, & -4.5, & -7.5, & 11.0, & 12.5, \\ & & & & & 1.5, & -6.0, & -6.5 & & & \\ \overline{\mathbf{16}}: & f^c: & 0.5, & -1.0, & -2.0, & -4.0, & -4.5, & 4.5, & 7.5, & -11.0, & -12.5, \\ & & & & & -1.5, & 6.0, & 6.5 & & & \end{array} \tag{2.1b}$$

We have identified with a subscript the three light fermion family fields belonging to the **16** representations of $SO(10)$ and indicated their assigned $U(1)_F$ charges with a superscript, while for the conjugate superheavy fermion fields we have just listed their $U(1)_F$ charges. The corresponding Higgs boson fields comprise the following:

- **Higgs Fields**:

$$\begin{aligned} \mathbf{10}:&\ \ H_1^{(-18)},\ H_2^{(-8)} \\ \mathbf{45}:&\ \ A_1^{(3.5)},\ A_2^{(0.5)} \\ \overline{\mathbf{126}}:&\ \ \bar{\Delta}^{(2)},\ \bar{\Delta}'^{(-22)} \\ \mathbf{1}:&\ \ S_1^{(2)},\ S_2^{(6.5)} \end{aligned} \tag{2.1c}$$

As customary, for each of the above fields we introduce a chiral superfield with the same $U(1)_F$ charge and components as indicated:

$$\begin{aligned} \mathbf{\Psi}_i &= (\tilde{\psi}_i,\ \psi_i,\ \chi_{\psi_i}), & i &= 1,2,3 \\ \mathbf{F}_i &= (\tilde{f}_i,\ f_i,\ \chi_{f_i}), & i &= 1-12 \\ \bar{\mathbf{F}}_i &= (\tilde{f}_i^c,\ f_i^c,\ \chi_{f_i^c}) & i &= 1-12 \\ \mathbf{H}_i &= (H_i,\ \tilde{H}_i,\ \chi_{H_i}), & i &= 1,2 \\ \mathbf{A}_i &= (A_i,\ \tilde{A}_i,\ \chi_{A_i}), & i &= 1,2 \\ \bar{\mathbf{\Delta}} &= (\bar{\Delta},\ \tilde{\bar{\Delta}},\ \chi_{\bar{\Delta}}), & \bar{\mathbf{\Delta}}' &= (\bar{\Delta}',\ \tilde{\bar{\Delta}}',\ \chi_{\bar{\Delta}'}) \\ \mathbf{S}_i &= (S_i,\ \tilde{S}_i,\ \chi_{S_i}), & i &= 1,2 \end{aligned} \tag{2.2}$$

All chiral superfields are taken to be left-handed $SO(10)$ supermultiplets; the tildes indicate superpartners of the ordinary fermions or bosons with odd R-parity; and the $\chi's$ refer to the corresponding auxilary fields.

In order that the superpotential to be constructed will be analytic and anomaly-free, we double the superfields containing the ordinary Higgs scalars by introducing superfields with the opposite $U(1)_F$ charges and conjugate $SO(10)$ representations:

$$\begin{aligned}
\bar{\mathbf{H}}_i &= (\bar{H}_i,\ \tilde{\bar{H}}_i,\ \chi_{\bar{H}_i}), & i &= 1,2 \\
\bar{\mathbf{A}}_i &= (\bar{A}_i,\ \tilde{\bar{A}}_i,\ \chi_{\bar{A}_i}), & i &= 1,2 \\
\mathbf{\Delta} &= (\Delta,\ \tilde{\Delta},\ \chi_{\Delta}), & \mathbf{\Delta}' &= (\Delta',\ \tilde{\Delta}',\ \chi_{\Delta'}) \\
\bar{\mathbf{S}}_i &= (\bar{S}_i,\ \tilde{\bar{S}}_i,\ \chi_{\bar{S}_i}), & i &= 1,2
\end{aligned} \tag{2.3}$$

Since the sum of the $U(1)_F$ charges for the three light fermion families is zero, the $[SO(10)]^2 \times U(1)_F$ triangle anomaly vanishes. The remaining $[U(1)_F]^3$ triangle anomaly can be canceled with the introduction of two singlet (sterile) neutrinos, n and n^c, both with $U(1)_F$ charge of -12 which prevents them from pairing off and becoming superheavy [23]. The corresponding superfields are

$$\begin{aligned}
\mathbf{N} &= (\tilde{n},\ n,\ \chi_n) \\
\bar{\mathbf{N}} &= (\tilde{n}^c,\ n^c,\ \chi_{n^c})
\end{aligned} \tag{2.4}$$

In addition to the analyticity and anomaly-free requirements for the superpotential, we must ensure that many fields become superheavy at the SUSY-GUT scale Λ_{SGUT}, while three fermion families of **16**'s remain light. Moreover, just one pair of Higgs doublets should remain light [24] to ensure a good value for $\sin^2\theta_W$, while all colored Higgs triplets must get superheavy to avoid rapid proton decay via dimension 5 and 6 operators [25]. This can be accomplished by introducing some additional chiral Higgs superfields transforming as $SO(10)$ representations which do not participate in the Yukawa interactions for which the original $SO(10) \times U(1)_F$ model was constructed.

To help identify a suitable choice of additional superfields, we elaborate the maximal

$SU(2)_L \times SU(2)_R \times SU(4)$ subgroup content of various $SO(10)$ representations [26].

$$
\begin{array}{lrcl}
\mathbf{H}: & \mathbf{10} & = & (2,2,1)+(1,1,6) \\
\mathbf{A}: & \mathbf{45} & = & (1,1,15)+(1,3,1)+(3,1,1)+(2,2,6) \\
\mathbf{\Sigma}: & \mathbf{54} & = & (1,1,1)+(3,3,1)+(1,1,20')+(2,2,6) \\
\mathbf{\Delta}: & \mathbf{126} & = & (1,3,\overline{10})+(3,1,10)+(1,1,6)+(2,2,15) \\
\bar{\mathbf{\Delta}}: & \overline{\mathbf{126}} & = & (1,3,10)+(3,1,\overline{10})+(1,1,6)+(2,2,15) \\
\mathbf{\Phi}: & \mathbf{210} & = & (1,1,1)+(1,1,15)+(1,3,15)+(3,1,15) \\
 & & & +(2,2,6)+(2,2,10)+(2,2,\overline{10})
\end{array}
\tag{2.5}
$$

This suggests that Λ_{SGUT} scale VEV's can be generated for each $SO(10)$ representation while preserving the standard model gauge group according to

$$
\begin{array}{rrcl}
\mathbf{1}: & <S> & = & ts_{1,1,1} \\
\mathbf{45}: & <A> & = & pa_{1,1,15}+qa_{1,3,1} \\
\mathbf{54}: & <\Sigma> & = & r\sigma_{1,1,1} \\
\mathbf{126}: & <\Delta> & = & v_R\delta_{1,3,\overline{10}} \\
\overline{\mathbf{126}}: & <\bar{\Delta}> & = & \bar{v}_R\delta_{1,3,10} \\
\mathbf{210}: & <\Phi> & = & a\phi_{1,1,1}+b\phi_{1,1,15}+c\phi_{1,3,15}
\end{array}
\tag{2.6}
$$

where the VEV directions in the $SU(2)_L \times SU(2)_R \times SU(4)$ subspace follow from (2.5) and the coefficients are in general complex.

Higgsino doublets containing a neutral field are generated when the $SO(10)$ representations break down to the standard model (SM) $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group according to

$$
\begin{aligned}
\mathbf{10} &\supset (2,2,1) &&\supset \tilde{H}_u = \begin{pmatrix} h^+ \\ h^o \end{pmatrix}, \quad \tilde{H}_d = \begin{pmatrix} \bar{h}^o \\ h^- \end{pmatrix} \\
\overline{\mathbf{126}} &\supset (2,2,15) &&\supset \tilde{\bar{\Delta}}_u = \begin{pmatrix} \delta^+ \\ \delta^o \end{pmatrix}, \quad \tilde{\bar{\Delta}}_d = \begin{pmatrix} \bar{\delta}^o \\ \delta^- \end{pmatrix} \\
\mathbf{210} &\supset (2,2,\overline{10}) &&\supset \tilde{\Phi}_u = \begin{pmatrix} \phi^+ \\ \phi^o \end{pmatrix} \\
\mathbf{210} &\supset (2,2,10) &&\supset \tilde{\Phi}_d = \begin{pmatrix} \bar{\phi}^o \\ \phi^- \end{pmatrix}
\end{aligned}
\tag{2.7}
$$

Electroweak scale VEV's are generated by Higgs scalars in the $\mathbf{10}$ and $\overline{\mathbf{126}}$ superfields when the standard model breaks to $U(1)_{em}$ and can give masses to the three families of fermions in the ψ_i of (2.1a) according to

$$
\begin{aligned}
\mathbf{10}: &\quad < H > = v_u h_{2,1,1} + v_d h_{2,-1,1} \\
\overline{\mathbf{126}}: &\quad < \bar{\Delta} > = w_u \delta_{2,1,1} + w_d \delta_{2,-1,1}
\end{aligned}
\tag{2.8}
$$

Here the subscripts refer to the VEV directions in the $SU(2)_L \times U(1)_Y \times SU(3)_c$ basis. As noted earlier, just one pair of Higgs doublets should remain light at the electroweak scale, so a good value for $\sin^2\theta_W$ is obtained. How this can come about is discussed in detail in Sect. III.

Higgsino colored triplets of charges $\pm 1/3$ which can couple to a pair of quarks and a quark and lepton and hence be exchanged in a diagram leading to proton decay appear in

$$
\begin{aligned}
\mathbf{10} &\supset (1,1,6) &&\supset \tilde{H}_t = h^{-1/3}, \quad \tilde{H}_{\bar{t}} = h^{1/3} \\
\mathbf{126} &\supset (1,1,6) &&\supset \tilde{\Delta}_t^{(1,1,6)} = \delta^{-1/3}, \quad \tilde{\Delta}_{\bar{t}}^{(1,1,6)} = \delta^{1/3} \\
\mathbf{126} &\supset (1,3,\overline{10}) &&\supset \tilde{\Delta}_{\bar{t}}^{(1,3,\overline{10})} = \delta'^{1/3} \\
\overline{\mathbf{126}} &\supset (1,1,6) &&\supset \tilde{\bar{\Delta}}_t^{(1,1,6)} = \bar{\delta}^{-1/3}, \quad \tilde{\bar{\Delta}}_{\bar{t}}^{(1,1,6)} = \bar{\delta}^{1/3} \\
\overline{\mathbf{126}} &\supset (1,3,10) &&\supset \tilde{\bar{\Delta}}_t^{(1,3,10)} = \bar{\delta}'^{-1/3} \\
\mathbf{210} &\supset (1,3,15) &&\supset \tilde{\Phi}_t = \phi^{-1/3}, \quad \tilde{\Phi}_{\bar{t}} = \phi^{1/3}
\end{aligned}
\tag{2.9}
$$

We shall discuss the issue of surviving light Higgs triplets in Sect. IV.

In order to generate a satisfactory higgsino doublet mass matrix, we find it necessary to add the following Higgs superfields:

$$\mathbf{54}: \quad \mathbf{\Sigma}_0^{(0)},\ \mathbf{\Sigma}_1^{(-16)},\ \bar{\mathbf{\Sigma}}_1^{(16)},\ \mathbf{\Sigma}_2^{(-10)},\ \bar{\mathbf{\Sigma}}_2^{(10)} \tag{2.10a}$$

$$\mathbf{210}: \quad \mathbf{\Phi}_0^{(0)},\ \mathbf{\Phi}_1^{(-20)},\ \bar{\mathbf{\Phi}}_1^{(20)},\ \mathbf{\Phi}_2^{(-10)},\ \bar{\mathbf{\Phi}}_2^{(10)} \tag{2.10b}$$

To make all the higgsino triplets of type (2.9) superheavy, we introduce the additional Higgs superfield:

$$\mathbf{45}: \quad \mathbf{A}_0^{(0)} \tag{2.10c}$$

Finally, we must introduce the following Higgs superfields to guarantee F-flat directions, so supersymmetry is only softly broken at Λ_{SGUT} as discussed in Sect. V.

$$\begin{aligned} \mathbf{45}: &\quad \mathbf{A}_3^{(8)},\ \bar{\mathbf{A}}_3^{(-8)} \\ \mathbf{1}: &\quad \mathbf{S}_3^{(8.5)},\ \bar{\mathbf{S}}_3^{(-8.5)} \end{aligned} \tag{2.10d}$$

We are now in a position to write down all the terms in the superpotential which conserve the $U(1)_F$ charge. The Higgs superpotential for the quadratic and cubic terms is given by

$$\begin{aligned} W_H^{(2)} = \ & \mu_0\mathbf{\Phi_0\Phi_0} + \mu_1\mathbf{\Phi_1\bar{\Phi}_1} + \mu_2\mathbf{\Phi_2\bar{\Phi}_2} + \mu_3\mathbf{\Delta'\bar{\Delta}'} + \mu_0'\mathbf{\Sigma_0\Sigma_0} + \mu_1'\mathbf{\Sigma_1\bar{\Sigma}_1} + \mu_2'\mathbf{\Sigma_2\bar{\Sigma}_2} \\ & + \mu_0''\mathbf{A_0A_0} + \mu_1''\mathbf{A_1\bar{A}_1} + \mu_2''\mathbf{A_2\bar{A}_2} + \mu_3''\mathbf{A_3\bar{A}_3} + \mu_1'''\mathbf{S_1\bar{S}_1} + \mu_2'''\mathbf{S_2\bar{S}_2} + \mu_3'''\mathbf{S_3\bar{S}_3} \end{aligned} \tag{2.11a}$$

$$\begin{aligned} W_H^{(3)} = \ & \lambda_0\mathbf{\Phi_0\Phi_0\Phi_0} + \lambda_1\mathbf{\Phi_1\bar{\Phi}_1\Phi_0} + \lambda_2\mathbf{\Phi_2\bar{\Phi}_2\Phi_0} + \lambda_3\mathbf{\Phi_1\bar{\Phi}_2\bar{\Phi}_2} + \lambda_4\mathbf{\Phi_2\Phi_2\bar{\Phi}_1} \\ & + \rho_0\mathbf{\Phi_0\Phi_0\Sigma_0} + \rho_1\mathbf{\Phi_1\bar{\Phi}_1\Sigma_0} + \rho_2\mathbf{\Phi_2\bar{\Phi}_2\Sigma_0} + \rho_3\mathbf{\Phi_1\bar{\Phi}_2\bar{\Sigma}_2} + \rho_4\mathbf{\Phi_2\bar{\Phi}_1\Sigma_2} \\ & + \rho_0'\mathbf{\Phi_0\Phi_0A_0} + \rho_1'\mathbf{\Phi_1\bar{\Phi}_1A_0} + \rho_2'\mathbf{\Phi_2\bar{\Phi}_2A_0} + \sigma_0\mathbf{\Sigma_0\Sigma_0\Sigma_0} + \sigma_1\mathbf{\Sigma_1\bar{\Sigma}_1\Sigma_0} \\ & + \sigma_2\mathbf{\Sigma_2\bar{\Sigma}_2\Sigma_0} + \sigma_0'\mathbf{\Sigma_0\Sigma_0A_0} + \sigma_1'\mathbf{\Sigma_1\bar{\Sigma}_1A_0} + \sigma_2'\mathbf{\Sigma_2\bar{\Sigma}_2A_0} + \kappa_0\mathbf{A_0A_0\Phi_0} \end{aligned}$$

$$
\begin{aligned}
&+\kappa_1\mathbf{A_1\bar{A}_1\Phi_0}+\kappa_2\mathbf{A_2\bar{A}_2\Phi_0}+\kappa_3\mathbf{A_3\bar{A}_3\Phi_0}+\kappa_0'\mathbf{A_0A_0\Sigma_0}+\kappa_1'\mathbf{A_1\bar{A}_1\Sigma_0}\\
&+\kappa_2'\mathbf{A_2\bar{A}_2\Sigma_0}+\kappa_3'\mathbf{A_3\bar{A}_3\Sigma_0}+\kappa_4'\mathbf{A_3A_3\Sigma_1}+\kappa_5'\mathbf{\bar{A}_3\bar{A}_3\bar{\Sigma}_1}+\kappa_0''\mathbf{A_0A_0A_0}\\
&+\kappa_1''\mathbf{A_1\bar{A}_1A_0}+\kappa_2''\mathbf{A_2\bar{A}_2A_0}+\kappa_3''\mathbf{A_3\bar{A}_3A_0}+\eta_0\mathbf{\Delta\bar{\Delta}\Phi_0}+\eta_1\mathbf{\Delta\bar{\Delta}A_0}\\
&+\eta_0'\mathbf{\Delta'\bar{\Delta}\Phi_0}+\eta_1'\mathbf{\Delta'\bar{\Delta}A_0}+\tau_1\mathbf{\Delta H_1\bar{\Phi}_1}+\tau_2\mathbf{\bar{\Delta}\bar{H}_1\Phi_1}+\tau_3\mathbf{\Delta H_2\bar{\Phi}_2} \qquad (2.11b)\\
&+\tau_4\mathbf{\bar{\Delta}\bar{H}_2\Phi_2}+\delta_1\mathbf{H_1\bar{H}_1\Sigma_0}+\delta_2\mathbf{H_2\bar{H}_2\Sigma_0}+\delta_3\mathbf{H_2H_2\bar{\Sigma}_1}+\delta_4\mathbf{\bar{H}_2\bar{H}_2\Sigma_1}\\
&+\delta_5\mathbf{\bar{H}_1H_2\Sigma_2}+\delta_6\mathbf{H_1\bar{H}_2\bar{\Sigma}_2}+\delta_1'\mathbf{H_1\bar{H}_1A_0}+\delta_2'\mathbf{H_2\bar{H}_2A_0}+\gamma_0\mathbf{A_0A_0A_0}\\
&+\gamma_1\mathbf{A_0A_1\bar{A}_1}+\gamma_2\mathbf{A_0A_2\bar{A}_2}+\gamma_3\mathbf{A_0A_3\bar{A}_3}+\varepsilon_1\mathbf{S_1S_2\bar{S}_3}+\varepsilon_2\mathbf{\bar{S}_1\bar{S}_2S_3}
\end{aligned}
$$

All the μ parameters in (2.11a) are taken to be of order of the SUSY-GUT scale. We have not introduced corresponding superheavy mass terms for the $\mathbf{H_1}, \mathbf{\bar{H}_1}, \mathbf{H_2}, \mathbf{\bar{H}_2}$ and $\mathbf{\Delta}$, $\mathbf{\bar{\Delta}}$ superfields in order to keep components of them light. As shown in Sect. III, we shall introduce a Z_2 discrete symmetery in order to place further restrictions on the terms which can appear in the superpotential.

The result of having so many Higgs and matter superfields in the model is to introduce a Landau singularity between the SUSY-GUT scale and the Planck scale. But this should be the case, for the model is to be considered only an effective theory at best. As pointed out earlier, the level-5 $\overline{126}$ $SO(10)$ multiplets do not arise naturally in superstring models [11] and must be treated as effective operators. In order to see the origin of the singularity more quantitatively, we note the one-loop approximation to the renormalization group equation for the running $SO(10)$ gauge coupling is given by

$$
\frac{dg_{10}}{dt}=\frac{1}{16\pi^2}\left[N_{10}+8N_{45}+12N_{54}+35(N_{126}+N_{\overline{126}})+56N_{210}+2(N_{16}+N_{\overline{16}})-24\right]g_{10}^3 \tag{2.12}
$$

With $N_{16}=15,\ N_{\overline{16}}=12,\ N_{10}=4,\ N_{45}=7,\ N_{54}=N_{210}=5$, and $N_{126}=N_{\overline{126}}=2$, we find a Landau singularity arises at the energy scale

$$
\mu=\mu_{10}\exp\left[\frac{4\pi^2}{285g_{10}^2(\mu_{10})}\right] \tag{2.13}
$$

where $\mu_{10}=\Lambda_{SGUT}\simeq 2\times 10^{16}$ GeV. With a gauge coupling of $g_{10}(\mu_{10})=0.67$, the singularity occurs within a factor of 1.5 of the SUSY-GUT scale. This value is close to the mass scale assumed in [2] for the mass of conjugate fermions which pair off and get superheavy and enter the Dimopoulos tree diagrams for the fermion mass matrix contributions. The suggestion then is that the model, representing an effective theory, perhaps arises from a superstring theory which becomes confining within two orders of magnitude of the string scale. The higher-dimensional Higgs representations that appear phenomenologically in the model can then be regarded as composite states of the simpler confining theory holding above the singularity. The possible existence of an infrared fixed point structure at an energy scale beyond 10^{16} GeV has been suggested and explored in models without grand unification by Lanzagorta and Ross [27].

III. ONE PAIR OF LIGHT HIGGS DOUBLETS

We now address the issue of how one can obtain just one pair of light Higgs doublets, in order to preserve a satisfactory electroweak scale value for $\sin^2\theta_W$ in evolution from the grand unification scale [24]. For this purpose we use the technique of Lee and Mohapatra [28] by constructing the doublet Higgsino mass matrix.

As indicated in (2.7), Higgsino doublets arise from the $\mathbf{10}, \mathbf{126}, \overline{\mathbf{126}}$ and $\mathbf{210}$ representations of $SO(10)$. If we drop the tildes and order the bases for the columns and rows, respectively, according to

$$\begin{aligned} B_u &= \left\{\Phi_{1u},\ \bar{\Phi}_{1u},\ \Phi_{2u},\ \bar{\Phi}_{2u},\ \Phi_{0u},\ \Delta'_u,\ \bar{\Delta}'_u,\ \Delta_u,\ \bar{\Delta}_u,\ H_{1u},\ \bar{H}_{1u},\ H_{2u},\ \bar{H}_{2u}\right\} \\ B_d &= \left\{\bar{\Phi}_{1d},\ \Phi_{1d},\ \bar{\Phi}_{2d},\ \Phi_{2d},\ \Phi_{0d},\ \bar{\Delta}'_d,\ \Delta'_d,\ \bar{\Delta}_d,\ \Delta_d,\ \bar{H}_{1d},\ H_{1d},\ \bar{H}_{2d},\ H_{2d}\right\} \end{aligned} \tag{3.1}$$

we find the 13×13 matrix separates into two block diagonal pieces, the first 7×7 and the second 6×6. Since the first submatrix is full rank 7, the first 7 Higgsino doublets all become superheavy. In order for just one pair of Higgs doublets to remain light at the Λ_{SGUT} scale, the second block diagonal matrix must be rank 5.

To achieve that goal, we first introduce a Z_2 discrete symmetry [29] whereby the following superfields are assigned the quantum number -1:

$$\bar{\mathbf{\Phi}}_1,\ \mathbf{\Phi}_2,\ \bar{\mathbf{\Phi}}_2,\ \bar{\mathbf{\Sigma}}_2,\ \mathbf{\Delta},\ \mathbf{A}_0,\ \mathbf{A}_3,\ \bar{\mathbf{A}}_3 \tag{3.2}$$

while all other superfields are assigned the quantum number +1. If we then demand that the allowed $W_H^{(3)}$ cubic superpotential terms respect the Z_2 symmetry, while the $W_H^{(2)}$ quadratic superpotential terms are allowed to violate it softly, [30] (2.11a) remains unchanged and is repeated here for convenience

$$\begin{aligned} W_H^{(2)} \;=\;& \mu_0\mathbf{\Phi}_0\mathbf{\Phi}_0 + \mu_1\mathbf{\Phi}_1\bar{\mathbf{\Phi}}_1 + \mu_2\mathbf{\Phi}_2\bar{\mathbf{\Phi}}_2 + \mu_3\mathbf{\Delta}'\bar{\mathbf{\Delta}}' + \mu_0'\mathbf{\Sigma}_0\mathbf{\Sigma}_0 + \mu_1'\mathbf{\Sigma}_1\bar{\mathbf{\Sigma}}_1 + \mu_2'\mathbf{\Sigma}_2\bar{\mathbf{\Sigma}}_2 \\ &+\mu_0''\mathbf{A}_0\mathbf{A}_0 + \mu_1''\mathbf{A}_1\bar{\mathbf{A}}_1 + \mu_2''\mathbf{A}_2\bar{\mathbf{A}}_2 + \mu_3''\mathbf{A}_3\bar{\mathbf{A}}_3 + \mu_1'''\mathbf{S}_1\bar{\mathbf{S}}_1 + \mu_2'''\mathbf{S}_2\bar{\mathbf{S}}_2 + \mu_3'''\mathbf{S}_3\bar{\mathbf{S}}_3 \end{aligned} \tag{3.3a}$$

while (2.11b) reduces to

$$\begin{aligned} W_H^{(3)} \;=\;& \lambda_0\mathbf{\Phi}_0\mathbf{\Phi}_0\mathbf{\Phi}_0 + \lambda_2\mathbf{\Phi}_2\bar{\mathbf{\Phi}}_2\mathbf{\Phi}_0 + \lambda_3\mathbf{\Phi}_1\bar{\mathbf{\Phi}}_2\bar{\mathbf{\Phi}}_2 + \rho_0\mathbf{\Phi}_0\mathbf{\Phi}_0\mathbf{\Sigma}_0 \\ &+\rho_2\mathbf{\Phi}_2\bar{\mathbf{\Phi}}_2\mathbf{\Sigma}_0 + \rho_3\mathbf{\Phi}_1\bar{\mathbf{\Phi}}_2\bar{\mathbf{\Sigma}}_2 + \rho_4\mathbf{\Phi}_2\bar{\mathbf{\Phi}}_1\mathbf{\Sigma}_2 + \rho_1'\mathbf{\Phi}_1\bar{\mathbf{\Phi}}_1\mathbf{A}_0 \\ &+\sigma_0\mathbf{\Sigma}_0\mathbf{\Sigma}_0\mathbf{\Sigma}_0 + \sigma_1\mathbf{\Sigma}_1\bar{\mathbf{\Sigma}}_1\mathbf{\Sigma}_0 + \sigma_2'\mathbf{\Sigma}_2\bar{\mathbf{\Sigma}}_2\mathbf{A}_0 + \kappa_0\mathbf{A}_0\mathbf{A}_0\mathbf{\Phi}_0 \\ &+\kappa_1\mathbf{A}_1\bar{\mathbf{A}}_1\mathbf{\Phi}_0 + \kappa_2\mathbf{A}_2\bar{\mathbf{A}}_2\mathbf{\Phi}_0 + \kappa_3\mathbf{A}_3\bar{\mathbf{A}}_3\mathbf{\Phi}_0 + \kappa_0'\mathbf{A}_0\mathbf{A}_0\mathbf{\Sigma}_0 \\ &+\kappa_1'\mathbf{A}_1\bar{\mathbf{A}}_1\mathbf{\Sigma}_0 + \kappa_2'\mathbf{A}_2\bar{\mathbf{A}}_2\mathbf{\Sigma}_0 + \kappa_3'\mathbf{A}_3\bar{\mathbf{A}}_3\mathbf{\Sigma}_0 + \kappa_4'\mathbf{A}_3\mathbf{A}_3\mathbf{\Sigma}_1 \\ &+\kappa_5'\bar{\mathbf{A}}_3\bar{\mathbf{A}}_3\bar{\mathbf{\Sigma}}_1 + \eta_1\mathbf{\Delta}\bar{\mathbf{\Delta}}\mathbf{A}_0 + \eta_0'\mathbf{\Delta}'\bar{\mathbf{\Delta}}'\mathbf{\Phi}_0 + \tau_1\mathbf{\Delta}\mathbf{H}_1\bar{\mathbf{\Phi}}_1 \\ &+\tau_2\bar{\mathbf{\Delta}}\bar{\mathbf{H}}_1\mathbf{\Phi}_1 + \tau_3\mathbf{\Delta}\mathbf{H}_2\bar{\mathbf{\Phi}}_2 + \delta_1\mathbf{H}_1\bar{\mathbf{H}}_1\mathbf{\Sigma}_0 + \delta_2\mathbf{H}_2\bar{\mathbf{H}}_2\mathbf{\Sigma}_0 \\ &+\delta_3\mathbf{H}_2\mathbf{H}_2\bar{\mathbf{\Sigma}}_1 + \delta_4\bar{\mathbf{H}}_2\bar{\mathbf{H}}_2\mathbf{\Sigma}_1 + \delta_5\bar{\mathbf{H}}_1\mathbf{H}_2\mathbf{\Sigma}_2 + \varepsilon_1\mathbf{S}_1\mathbf{S}_2\bar{\mathbf{S}}_3 + \varepsilon_2\bar{\mathbf{S}}_1\bar{\mathbf{S}}_2\mathbf{S}_3 \end{aligned} \tag{3.3b}$$

The two terms $\mu_1\mathbf{\Phi}_1\bar{\mathbf{\Phi}}_1$ and $\mu_2'\mathbf{\Sigma}_2\bar{\mathbf{\Sigma}}_2$ of (3.3a) violate Z_2 invariance and break the Z_2 symmetry softly.

We shall assume that the VEV for $\mathbf{A}_0$, $< A_0 >$, which helps to make all colored Higgsino triplets superheavy does not contribute to the doublet Higgsino mass matrix as explained in

Sect. IV. The 6×6 doublet Higgsino submatrix then becomes

$$\mathcal{M}_H = \begin{pmatrix} 0 & 0 & 0 & \frac{1}{\sqrt{2}}\tau_2(b_1+c_1) & 0 & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}}\tau_1(\bar{b}_1+\bar{c}_1) & 0 & \frac{1}{\sqrt{2}}\tau_3(\bar{b}_2+\bar{c}_2) & 0 \\ 0 & \frac{1}{\sqrt{2}}\tau_2(b_1-c_1) & \delta_1 r_0 & 0 & \delta_5 r_2 & 0 \\ \frac{1}{\sqrt{2}}\tau_1(\bar{b}_1-\bar{c}_1) & 0 & 0 & \delta_1 r_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \delta_2 r_0 & \delta_4 r_1 \\ \frac{1}{\sqrt{2}}\tau_3(\bar{b}_2-\bar{c}_2) & 0 & 0 & \delta_5 r_2 & \delta_3 \bar{r}_1 & \delta_2 r_0 \end{pmatrix} \tag{3.4}$$

We have used the notation of (2.6) for the VEV's involved. If we assume the chiral symmetry is broken so some $c_i \neq 0$, and in particular that $\bar{c}_1 = -\bar{b}_1$ while $c_1 \neq \pm b_1$, the 23 element of the above matrix vanishes, and we obtain a rank 5 matrix. The massless Higgsino doublet at the Λ_{SGUT} scale is then given by

$$\tilde{H}_u = \alpha_{12}\tilde{\bar{\Delta}}_u + \alpha_{13}\tilde{H}_{1u} \tag{3.5a}$$

while the other massless Higgsino doublet is obtained from the transpose of $\mathcal{M}_H$ and is found to be

$$\tilde{H}_d = \alpha'_{11}\tilde{\bar{\Delta}}_d + \alpha'_{12}\tilde{\Delta}_d + \alpha'_{14}\tilde{H}_{1d} + \alpha'_{15}\tilde{\bar{H}}_{2d} + \alpha'_{16}\tilde{H}_{2d} \tag{3.5b}$$

The coefficients in the two expansions are related by

$$\alpha_{12} = -\sqrt{2}(\delta_1 r_0)/(\tau_2(b_1-c_1))\alpha_{13} \tag{3.6a}$$

and by

$$\begin{aligned} \alpha'_{11} &= -\sqrt{2}/(\tau_2(b_1+c_1))\left[\delta_5 r_2 - \delta_1 r_0 \tau_3(\bar{b}_2-\bar{c}_2)/(\tau_1(\bar{b}_1-\bar{c}_1))\right]\alpha'_{16} \\ \alpha'_{12} &= -\sqrt{2}/(\tau_3(\bar{b}_2+\bar{c}_2))\left[\delta_3\bar{r}_1 - \delta_2^2 r_0^2/(\delta_4 r_1)\right]\alpha'_{16} \\ \alpha'_{14} &= -\tau_3(\bar{b}_2-\bar{c}_2)/(\tau_1(\bar{b}_1-\bar{c}_1))\alpha'_{16} \\ \alpha'_{15} &= -\delta_2 r_0/(\delta_4 r_1)\alpha'_{16} \end{aligned} \tag{3.6b}$$

Note that by our choice of chiral symmetry breaking, $\bar{c}_1 = -\bar{b}_1$, for the VEV's of $\bar{\Phi}_1$, the corresponding Higgs doublet H_u has components only in the $\bar{\Delta}_u$ and H_{1u} directions, and can contribute only to the diagonal 33 and 22 elements of the up quark and Dirac neutrino mass matrices in lowest-order tree level as a result of the $U(1)_F$ charges. On the other hand, the Higgs doublet H_d has components in the $\bar{\Delta}_d$, Δ_d, H_{1d}, $\bar{H}_{2d}$ and H_{2d} directions, with lowest-order tree-level contributions to all four (33, 23, 32 and 22) elements of the down quark and charged lepton mass matrices. This helps to explain how it is possible that the basis with up quark and Dirac neutrino mass matrices diagonal can be selected as the preferred basis leading to simple $SO(10)$ mass matrices. For details see Ref. [1, 2].

The other Higgsino doublets are superheavy and are general linear combinations of all six basis vectors in the subspace.

$$\tilde{\mathcal{H}}_{iu} = \alpha_{i1}\tilde{\Delta}_u + \alpha_{i2}\tilde{\bar{\Delta}}_u + \alpha_{i3}\tilde{H}_{1u} + \alpha_{i4}\tilde{\bar{H}}_{1u} + \alpha_{i5}\tilde{H}_{2u} + \alpha_{i6}\tilde{\bar{H}}_{2u}, \qquad i = 2,3...6 \quad (3.7a)$$

$$\tilde{\mathcal{H}}_{id} = \alpha'_{i1}\tilde{\bar{\Delta}}_d + \alpha'_{i2}\tilde{\Delta}_d + \alpha'_{i3}\tilde{\bar{H}}_{1d} + \alpha'_{i4}\tilde{H}_{1d} + \alpha'_{i5}\tilde{\bar{H}}_{2d} + \alpha'_{i6}\tilde{H}_{2d}, \qquad i = 2,3...6 \quad (3.7b)$$

By inverting Eqs. (3.5) and (3.7), we obtain with suitable normalization

$$\tilde{\bar{\Delta}}_u = \alpha^*_{12}\tilde{H}_u + \sum_{i=2}^{6}\alpha^*_{i2}\tilde{\mathcal{H}}_{iu} \quad (3.8a)$$

$$\tilde{H}_{1u} = \alpha^*_{13}\tilde{H}_u + \sum_{i=2}^{6}\alpha^*_{i3}\tilde{\mathcal{H}}_{iu} \quad (3.8b)$$

$$\tilde{\Delta}_u,\ \tilde{\bar{H}}_{1u},\ \tilde{H}_{2u},\ \tilde{\bar{H}}_{2u} = \sum_{i=2}^{6}\alpha'^*_{ik}\tilde{\mathcal{H}}_{iu}, \qquad k = 1,4,5,6 \quad (3.8c)$$

and

$$\tilde{\bar{\Delta}}_d = \alpha'^*_{11}\tilde{H}_d + \sum_{i=2}^{6}\alpha'^*_{i1}\tilde{\mathcal{H}}_{id} \quad (3.9a)$$

$$\tilde{\Delta}_d = \alpha'^*_{12}\tilde{H}_d + \sum_{i=2}^{6}\alpha'^*_{i2}\tilde{\mathcal{H}}_{id} \quad (3.9b)$$

$$\tilde{\bar{H}}_{1d} = \sum_{i=2}^{6}\alpha'^*_{i3}\tilde{\mathcal{H}}_{id} \quad (3.9c)$$

$$\tilde{H}_{1d} = \alpha'^*_{14}\tilde{H}_d + \sum_{i=2}^{6}\alpha'^*_{i4}\tilde{\mathcal{H}}_{id} \quad (3.9d)$$

$$\tilde{\bar{H}}_{2d} = \alpha'^*_{15}\tilde{H}_d + \sum_{i=2}^{6}\alpha'^*_{i5}\tilde{\mathcal{H}}_{id} \quad (3.9e)$$

$$\tilde{H}_{2d} = \alpha'^{*}_{16}\tilde{H}_d + \sum_{i=2}^{6} \alpha'^{*}_{i6}\tilde{\mathcal{H}}_{id} \tag{3.9f}$$

The superheavy fields decouple at the Λ_{SGUT} scale, and electroweak VEV's are generated only by the light Higgs doublets as follows:

$$\begin{aligned}
< \bar{\Delta}_u > &= \alpha^{*}_{12} < H_u >, & < H_{1u} > &= \alpha^{*}_{13} < H_u > \\
< \bar{\Delta}_d > &= \alpha'^{*}_{11} < H_d >, & < \Delta_d > &= \alpha'^{*}_{12} < H_d > \\
& & < H_{1d} > &= \alpha'^{*}_{14} < H_d > \\
< \bar{H}_{2d} > &= \alpha'^{*}_{15} < H_d >, & < H_{2d} > &= \alpha'^{*}_{16} < H_d >
\end{aligned} \tag{3.10}$$

We observe from the above that one pair of light Higgs doublets makes several electroweak tree-level VEV contributions as found earlier in our $SO(10) \times U(1)_F$ model summarized in Sect. I. Since the **10** VEV's, $< H_{1u} >$ and $< H_{1d} >$, contributing to the 33 mass matrix elements are considerably larger than the $\mathbf{10'}$ and $^{(}\mathbf{1\bar{2}6}^{)}$ VEV's, it is clear from the above that $\tilde{H}_u$ and $\tilde{H}_d$ point mainly in the **10** direction.

IV. SUPERHEAVY HIGGS TRIPLETS

We now turn our attention to the Higgs doublet-triplet splitting problem. The point is that unless all Higgs triplets get superheavy, too rapid proton decay can take place by the exchange of a Higgsino color triplet leading to a dimension-5 contribution or by the exchange of a Higgs color triplet leading to a dimension-6 contribution to proton decay [25]. This problem can be alleviated through the Dimopoulos - Wilczek type mechanism [31].

The Higgsino triplets appear in the representations singled out in (2.9). We thus choose to order the bases for the triplet Higgsino mass matrix as follows where we again have dropped tildes:

$$\begin{aligned}
B_u = \Big\{ & \Phi_{1t},\ \bar{\Phi}_{1t},\ \Phi_{2t},\ \bar{\Phi}_{2t},\ \Phi_{0t},\ \Delta_t'^{(1,1,6)},\ \bar{\Delta}_t'^{(1,1,6)},\ \bar{\Delta}_t'^{(1,3,10)}, \\
& \Delta_t^{(1,1,6)},\ \bar{\Delta}_t^{(1,1,6)},\ \bar{\Delta}^{(1,3,10)},\ H_{1t},\ \bar{H}_{1t},\ H_{2t},\ \bar{H}_{2t} \Big\}
\end{aligned} \tag{4.1a}$$

and

$$B_d = \Big\{ \bar{\Phi}_{1\bar{t}},\ \Phi_{1\bar{t}},\ \bar{\Phi}_{2\bar{t}},\ \Phi_{2\bar{t}},\ \Phi_{0\bar{t}},\ \bar{\Delta}'^{(1,1,6)}_{\bar{t}},\ \Delta'^{(1,1,6)}_{\bar{t}},\ \Delta'^{(1,3,\overline{10})}_{\bar{t}}, \\ \bar{\Delta}^{(1,1,6)}_{\bar{t}},\ \Delta^{(1,1,6)}_{\bar{t}},\ \Delta^{(1,3,\overline{10})}_{\bar{t}},\ \bar{H}_{1\bar{t}},\ H_{1\bar{t}},\ \bar{H}_{2\bar{t}},\ H_{2\bar{t}} \Big\} \tag{4.1b}$$

We now assume that the Dimopoulos - Wilczek type mechanism operates, as the VEV for $\mathbf{A_0}$ takes the form

$$< A_0 >= diag(0,\ 0,\ a,\ a,\ a) \otimes \epsilon = p_0 a_{1,1,15} \tag{4.2}$$

where ϵ is the 2×2 antisymmetric matrix. This contributes to the colored triplet Higgsino mass matrix but not to the doublet Higgsino mass matrix given in (3.4). Again we find that the colored triplet Higgsino mass matrix splits into two block diagonal submatrices of dimensions 8×8 and 7×7 in terms of the bases given above. The first is trivially full rank, while the second assumes the following form:

$$\mathcal{M}_{H'} = \begin{pmatrix} \eta_1 p_0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}}\tau_2(a_1+b_1) & 0 & 0 \\ 0 & \eta_1 p_0 & 0 & \frac{1}{\sqrt{2}}\tau_1(\bar{a}_1+\bar{b}_1) & 0 & \frac{1}{\sqrt{2}}\tau_3(\bar{a}_2+\bar{b}_2) & 0 \\ 0 & 0 & \eta_1 p_0 & \tau_1\bar{c}_1 & 0 & \tau_3\bar{c}_2 & 0 \\ 0 & \frac{1}{\sqrt{2}}\tau_2(a_1-b_1) & \tau_2 c_1 & \delta_1 r_0 & 0 & \delta_5 r_2 & 0 \\ \frac{1}{\sqrt{2}}\tau_1(\bar{a}_1-\bar{b}_1) & 0 & 0 & 0 & \delta_1 r_0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \delta_2 r_0 & \delta_4 r_1 \\ \frac{1}{\sqrt{2}}\tau_3(\bar{a}_2-\bar{b}_2) & 0 & 0 & 0 & \delta_5 r_2 & \delta_3 \bar{r}_1 & \delta_2 r_0 \end{pmatrix} \tag{4.3}$$

By inspection the above matrix is also full rank, so all color triplet Higgsinos become superheavy. Thus splitting of one pair of doublet and triplet Higgsinos is achieved through a Dimopoulos - Wilczek type mechanism. The important point is that $< A_0 >= p_0 a_{1,1,15}$ does not contribute a mass term to the $^{(}\overline{\mathbf{126}}^{)}(2,2,15)$ Higgsino doublets, since the $SU(4)$ Clebsch-Gordan coefficient yielding an antisymmetric $\mathbf{45}$ representation vanishes [32].

V. GUT SCALE CONDITIONS FOR WEAK SCALE SUPERSYMMETRY

We now turn our attention to the subject of weak scale supersymmetry and the conditions which must obtain for the supersymmetry to remain unbroken at the Λ_{SGUT} scale. We are referring to the conditions which preserve some F-flat and D-flat directions for which the minimum $V = 0$ of the scalar potential is maintained [22]. This requires that

$$V(\{\phi_i\}) = \sum_i |F_i|^2 + \frac{1}{2}\sum_a |D^a|^2 \tag{5.1a}$$

vanishes for the directions singled out by the VEV's of the scalar fields. The sum goes over all fields present in the Higgs superpotential, where

$$F_i = \frac{\partial W}{\partial \phi_i}, \qquad D^a = -g\phi_i^* T_{ij}^a \phi_j \tag{5.1b}$$

For the purposes of this Section, we have ignored any explicit soft supersymmetry-breaking terms.

The F-terms appearing in (5.1) then involve the following derivatives as indicated by an obvious shorthand notation:

$$\begin{array}{ll} F_{\Phi_0},\ F_{\Phi_i},\ F_{\bar{\Phi}_i}, & i = 1,2 \\ F_{\Delta'},\ F_{\bar{\Delta}'}, & \\ F_{\Sigma_0},\ F_{\Sigma_i},\ F_{\bar{\Sigma}_i}, & i = 1,2 \\ F_{A_0},\ F_{A_i},\ F_{\bar{A}_i}, & i = 1,2,3 \\ F_{S_0},\ F_{S_i},\ F_{\bar{S}_i}, & i = 1,2,3 \end{array} \tag{5.2}$$

In Appendix A, we write down the F-flat conditions in terms of the VEV's appearing in (2.6) and keep only those terms whose Λ_{SGUT} VEV's are non-vanishing [33]. For $\{F_{\Phi_i},\ F_{\bar{\Phi}_i}\}$ and $\{F_{A_i},\ F_{\bar{A}_i}\}$ for each i there are three and two conditions, respectively, since the coefficient for each possible VEV direction must vanish. For F_{Σ_i} and $F_{\bar{\Sigma}_i}$, two conditions also arise, for the contributions point not only in the $\sigma_{1,1,1}$ direction, but also in the $s_{1,1,1}$ direction. Note that the conditions allow all the masses present in (3.3a) to be superheavy, while the VEV's in (2.6) are also near the Λ_{SGUT} scale. No F-flat directions are lifted in so doing. Nor are any Goldstone bosons introduced by the $SO(10)$ symmetry breaking.

In order for $p_0 \neq 0,\ q_0 = 0$ to be satisfied so all colored Higgs triplets are superheavy while one pair of Higgs doublets can remain light, the second condition in (A13) requires that we set $c_0 = 0$. Consistency of the remaining conditions is easily maintained by setting $\lambda_3 = 0$. Some additional simple relations that follow are

$$\frac{a_1}{a_2} = -\frac{3}{2}\frac{b_1}{b_2} = 6\frac{c_1}{c_2} \tag{5.3a}$$

$$\frac{\bar{a}_1}{\bar{a}_2} = -\frac{3}{2}\frac{\bar{b}_1}{\bar{b}_2} = 6\frac{\bar{c}_1}{\bar{c}_2} \tag{5.3b}$$

$$\rho_3\bar{r}_2(\bar{a}_2/\bar{a}_1) = \rho_4 r_2(a_2/a_1) \tag{5.3c}$$

$$p_3/\bar{p}_3 = q_3/\bar{q}_3 \tag{5.3d}$$

$$\kappa_4' r_1 p_3^2 = \kappa_5' \bar{r}_1 \bar{p}_3^2 \tag{5.3e}$$

$$\mu_3 = -\frac{1}{10}\eta_0'\left[\frac{1}{\sqrt{6}}a_0 + \frac{1}{\sqrt{2}}b_0\right] \tag{5.3f}$$

$$\mu_1' = -\frac{1}{2\sqrt{15}}\kappa_4'(3p_3^2 - 2q_3^2)/\bar{r}_1 \tag{5.3g}$$

$$\mu_2' = \frac{1}{\sqrt{15}}\rho_4(a_1\bar{a}_1 + b_1\bar{b}_1 + c_1\bar{c}_1)(c_2/c_1\bar{r}_2) \tag{5.3h}$$

$$\kappa_1'/\kappa_1 = \kappa_2'/\kappa_2 = \kappa_3'/\kappa_3 + 2(\kappa_4' r_1 p_3)/(\kappa_3 r_0 \bar{p}_3) = \left[\sqrt{\frac{2}{5}}a_0 - \frac{2\sqrt{2}}{\sqrt{15}}b_0\right]\frac{1}{r_0} \tag{5.3i}$$

$$\mu_0'' = -\frac{2}{3\sqrt{2}}\kappa_0 b_0 - \frac{1}{\sqrt{15}}\kappa_0' r_0 \tag{5.3j}$$

$$\mu_1''/\kappa_1 = \mu_2''/\kappa_2 = \mu_3''/\kappa_3 = -\frac{2}{5}\left[\frac{1}{\sqrt{2}}b_0 + \frac{1}{\sqrt{6}}a_0\right] \tag{5.3k}$$

$$\mu_1''' t_1\bar{t}_1 = \mu_2''' t_2\bar{t}_2 = \mu_3''' t_3\bar{t}_3 \tag{5.3l}$$

The additional restriction that $\bar{b}_1 = -\bar{c}_1$, needed to ensure that only one pair of Higgs doublets remains light, further implies that $4\bar{b}_2 = \bar{c}_2$. No restrictions are found on $p_i,\ \bar{p}_i,\ q_i,\ \bar{q}_i$ for $i = 1, 2$ which appear in the VEV's of the **45**'s needed to break the $SO(10)$ symmetry down to the SM at the Λ_{SGUT} scale. Several special cases of interest for the **45** VEV's in addition to that employed for A_0 in (4.2) are the following:

$$\begin{aligned}
< A_{45_d} >= &\ diag(q,\ q,\ 0,\ 0,\ 0)\otimes\epsilon && \sim q a_{1,3,1} \\
< A_{45_X} >= &\ diag(u,\ u,\ u,\ u,\ u)\otimes\epsilon && \sim \left(\sqrt{\tfrac{3}{5}}a_{1,1,15}+\sqrt{\tfrac{2}{5}}a_{1,3,1}\right)u \\
< A_{45_Y} >= &\ diag(3u,\ 3u,\ -2u,\ -2u,\ -2u)\otimes\epsilon && \sim \left(\sqrt{\tfrac{2}{5}}a_{1,1,15}-\sqrt{\tfrac{3}{5}}a_{1,3,1}\right)u \\
< A_{45_Z} >= &\ diag(3u,\ 3u,\ 2u,\ 2u,\ 2u)\otimes\epsilon && \sim \left(\sqrt{\tfrac{2}{5}}a_{1,1,15}+\sqrt{\tfrac{3}{5}}a_{1,3,1}\right)u
\end{aligned} \tag{5.4}$$

In [2] we have chosen the VEV's in the A_{45_X} and A_{45_Z} directions to be non-vanishing, so the $SO(10)$ symmetry is broken directly to the SM: $SO(10)\to SU(3)_c\times SU(2)_L\times U(1)_Y$. While such VEV's appear to be allowed by our analysis, unfortunately they are not uniquely singled out.

If we gauge the $U(1)_F$ family symmetry, D-terms can arise from the spontaneous breaking of the $U(1)_F$ and $SO(10)$ at the SUSY-GUT scale and involve only D_F and D_X, if $SO(10)\times U(1)_F$ breaks directly to the SM as we have assumed in [2]. These terms will vanish in the limit that the soft supersymmetry breaking terms are neglected, as the VEV's for the conjugate fields ϕ_i and $\bar{\phi}_i$ which break the $U(1)$ symmetries become equal. We shall address the soft supersymmetry-breaking in the next Section.

VI. SOFT SUSY-BREAKING CONTRIBUTIONS

Here we present the supersymmetric part of the scalar potential which applies when the supersymmetry is softly broken:

$$V(\{\phi_i\})=\sum_i|F_i|^2+\frac{1}{2}\sum_a|D^a|^2+V_{soft} \tag{6.1a}$$

where

$$F_i=\frac{\partial W}{\partial\phi_i},\qquad D^a=-g\phi_i^*T^a_{ij}\phi_j, \tag{6.1b}$$

The soft SUSY-breaking part of the scalar potential, so far as the Higgs mass terms are concerned, is given by

$$\begin{aligned}
V_{soft} &= m_0^2|\Phi_0|^2 + m_1^2|\Phi_1|^2 + \bar{m}_1^2|\bar{\Phi}_1|^2 + m_2^2|\Phi_2|^2 + \bar{m}_2^2|\bar{\Phi}_2|^2 + m_3^2|\Delta'|^2 + \bar{m}_3^2|\bar{\Delta}'|^2 \\
&+m_0'^2|\Sigma_0|^2 + m_1'^2|\Sigma_1|^2 + \bar{m}_1'^2|\bar{\Sigma}_1|^2 + m_2'^2|\Sigma_2|^2 + \bar{m}_2'^2|\bar{\Sigma}_2|^2 + m_0''^2|A_0|^2 \\
&+m_1''^2|A_1|^2 + \bar{m}_1''^2|\bar{A}_1|^2 + m_2''^2|A_2|^2 + \bar{m}_2''^2|\bar{A}_2|^2 + m_3''^2|A_3|^2 + \bar{m}_3''^2|\bar{A}_3|^2 \\
&+m_1'''^2|S_1|^2 + \bar{m}_1'''^2|\bar{S}_1|^2 + m_2'''^2|S_2|^2 + \bar{m}_2'''^2|\bar{S}_2|^2 + m_3'''^2|S_3|^2 + \bar{m}_3'''^2|\bar{S}_3|^2 \\
&+m_u^2|H_u|^2 + m_d^2|H_d|^2 + m_{ud}^2(\epsilon_{ij}H_u^i H_d^j + h.c.)
\end{aligned} \tag{6.2}$$

The D-terms include contributions from the broken $U(1)_F$ and $U(1)_X$, as well as the $SU(2)_L$ and $U(1)_Y$, which are given by

$$\begin{aligned}
V_D &= \tfrac{1}{2}g_F^2 \Big[2(|S_1|^2 - |\bar{S}_1|^2) + 6.5(|S_2|^2 - |\bar{S}_2|^2) + 8.5(|S_3|^2 - |\bar{S}_3|^2) \\
&+3.5(|A_1|^2 - |\bar{A}_1|^2) + 0.5(|A_2|^2 - |\bar{A}_2|^2) - 16(|\Sigma_1|^2 - |\bar{\Sigma}_1|^2) \\
&-10(|\Sigma_2|^2 - |\bar{\Sigma}_2|^2) - 20(|\Phi_1|^2 - |\bar{\Phi}_1|^2) - 10(|\Phi_2|^2 - |\bar{\Phi}_2|^2) \\
&+22(|\Delta'|^2 - |\bar{\Delta}'|^2) - 2(|\Delta|^2 - |\bar{\Delta}|^2) - 18(|H_1|^2 - |\bar{H}_1|^2) \\
&-8(|H_2|^2 - |\bar{H}_2|^2) - 8|\tilde{\psi}_1|^2 - |\tilde{\psi}_2|^2 + 9|\tilde{\psi}_3|^2\Big]^2 \\
&+\tfrac{1}{2}g_X^2 \Big[-10(|\Delta'|^2 - |\bar{\Delta}'|^2) - 2(|\Delta|^2 - |\bar{\Delta}|^2) + 2(|H_1|^2 - |\bar{H}_1|^2) \\
&+2(|H_2|^2 - |\bar{H}_2|^2) + \ldots\Big]^2 \\
&+\tfrac{1}{8}g^2 \Big[|H_u|^4 + |H_d|^4 - 2|H_u|^2|H_d|^2 + 4|H_u^\dagger H_d|^2\Big] \\
&+\tfrac{1}{8}g'^2 \left[|H_u|^4 + |H_d|^4 - 2|H_u|^2|H_d|^2\right]
\end{aligned} \tag{6.3}$$

Once the soft SUSY-breaking masses are allowed to become nonuniversal, sizable D-term contributions to the scalar potential can result. The F-terms can be found by differentiating the last few terms in (3.3b) which are linear in one superheavy field with respect to that field. We find

$$\begin{aligned}
V_F &= |\tau_1 \Delta H_1|^2 + |\tau_2 \bar{\Delta}\bar{H}_1|^2 + |\delta_1 H_1 \bar{H}_1 + \delta_2 H_2 \bar{H}_2|^2 \\
&\quad +|\delta_3 H_2 H_2|^2 + |\delta_4 \bar{H}_2 \bar{H}_2|^2 + |\delta_5 \bar{H}_1 H_2|^2 + |\eta_1 \Delta\bar{\Delta}|^2
\end{aligned} \tag{6.4}$$

Upon minimizing the full scalar potential, one finds the VEV's generated for the scalar fields and their conjugates become unequal provided some m^2's are driven negative as shown in [34]. Supersymmetry is broken along a nearly D-flat direction with $|m| = O(1\ \text{TeV})$.

By making use of (3.8) and (3.9) to replace the original Higgs doublets by the pair H_u and H_d which remains light down to the electroweak scale and integrating out the fields which become superheavy, we find the scalar potential for the Higgs sector can be written as

$$\begin{aligned} V(Higgs) &= m_u^2(H_u^\dagger H_u) + m_d^2(H_d^\dagger H_d) + m_{12}^2(\varepsilon_{ij} H_u^i H_d^j + h.c.) \\ &\quad + \tfrac{1}{8}(g^2 + g'^2)\left[(H_u^\dagger H_u)^2 + (H_d^\dagger H_d)^2 - 2(H_u^\dagger H_u)(H_d^\dagger H_d)\right] \\ &\quad + (\tfrac{1}{2}g^2 + g'^2)|H_u^\dagger H_d|^2 \end{aligned} \tag{6.5}$$

Despite the apparent non-minimal nature of our model at the SUSY-GUT scale due to the presence of many Higgs contributions, since only one pair of Higgs doublets survives at the electroweak scale, the scalar potential at that scale is similar to that of the minimal supersymmetric standard model. Thus the good result for $\sin^2\theta_W$ achieved in the MSSM is maintained, and the evolution of all the gauge and Yukawa couplings from Λ_{SGUT} is unaltered.

In integrating out the superheavy fields, one also finds nonuniversal corrections to the squark and slepton fields given by

$$\Delta m_a^2 = Q_{Fa} D_F + Q_{Xa} D_X \tag{6.6}$$

in the notation of Kolda and Martin [34], where the Q's are the $U(1)_F$ and $U(1)_X$ charges and the D's are parameters which summarize the symmetry-breaking process at the SUSY-GUT scale. The main point we wish to make here is that the first and third family squark and slepton masses will be split further away from their universal values than the second family, due to their larger $U(1)_F$ charges. Recall $Q_F = -8,\ -1,\ 9$ for the first, second and third family, respectively. Which family emerges with the smallest mass depends on the sign of D_F. In any case, the splitting will be limited by the present experimental constraints on flavor-changing neutral currents.

VII. YUKAWA SUPERPOTENTIAL

The superpotential for the Yukawa interactions can be simply constructed from the superfields introduced earlier, where every term remains invariant under the $U(1)_F$ and Z_2 symmetries. For this purpose we assign a Z_2 charge of $+1$ to each of the matter superfields $\mathbf{\Psi}_i$, $\mathbf{F}^{(k)}$ and $\bar{\mathbf{F}}^{(k)}$. We then find for the Yukawa superpotential

$$
\begin{aligned}
W_Y = \; & g_{10}\mathbf{\Psi_3\Psi_3 H_1} + g_{10'}\Big\{\Big[\mathbf{\Psi_2\Psi_3} + \mathbf{F}^{(-4.5)}\mathbf{F}^{(12.5)} + \mathbf{F}^{(4)}\mathbf{F}^{(4)} + \bar{\mathbf{F}}^{(0.5)}\bar{\mathbf{F}}^{(7.5)}\Big]\mathbf{H_2} \\
& \qquad + \Big[\mathbf{F}^{(-0.5)}\mathbf{F}^{(-7.5)} + \bar{\mathbf{F}}^{(4.5)}\bar{\mathbf{F}}^{(-12.5)} + \bar{\mathbf{F}}^{(-4)}\bar{\mathbf{F}}^{(-4)}\Big]\bar{\mathbf{H}}_2\Big\} \\
+ \; & g_{126}\Big[\mathbf{\Psi_2\Psi_2} + \mathbf{F}^{(-6)}\mathbf{F}^{(4)} + \mathbf{F}^{(4.5)}\mathbf{F}^{(-6.5)}\Big]\bar{\mathbf{\Delta}} + g_{126'}\Big[\mathbf{F}^{(11)}\mathbf{F}^{(11)}\bar{\mathbf{\Delta}}' + \bar{\mathbf{F}}^{(-11)}\bar{\mathbf{F}}^{(-11)}\mathbf{\Delta}'\Big] \\
+ \; & g'_{45}\Big\{\Big[\mathbf{\Psi}_1\bar{\mathbf{F}}^{(4.5)} + \mathbf{\Psi}_3\bar{\mathbf{F}}^{(-12.5)} + \mathbf{F}^{(1)}\bar{\mathbf{F}}^{(-4.5)}\Big]\mathbf{A}_1 + \Big[\mathbf{\Psi}_2\bar{\mathbf{F}}^{(4.5)} + \mathbf{F}^{(4.5)}\bar{\mathbf{F}}^{(-1)}\Big]\bar{\mathbf{A}}_1\Big\} \\
+ \; & g''_{45}\Big\{\Big[\mathbf{\Psi}_2\bar{\mathbf{F}}^{(0.5)} + \mathbf{\Psi}_1\bar{\mathbf{F}}^{(7.5)} + \mathbf{F}^{(1)}\bar{\mathbf{F}}^{(-1.5)} + \mathbf{F}^{(4)}\bar{\mathbf{F}}^{(-4.5)} + \mathbf{F}^{(1.5)}\bar{\mathbf{F}}^{(-2)} + \mathbf{F}^{(-6.5)}\bar{\mathbf{F}}^{(6)}\Big]\mathbf{A}_2 \\
& \qquad + \Big[\mathbf{F}^{(2)}\bar{\mathbf{F}}^{(-1.5)} + \mathbf{F}^{(4.5)}\bar{\mathbf{F}}^{(-4)} + \mathbf{F}^{(1.5)}\bar{\mathbf{F}}^{(-1)} + \mathbf{F}^{(-6)}\bar{\mathbf{F}}^{(6.5)}\Big]\bar{\mathbf{A}}_2\Big\} \\
+ \; & g'_1\Big\{\Big[\mathbf{\Psi}_3\bar{\mathbf{F}}^{(-11)} + \mathbf{\Psi}_2\bar{\mathbf{F}}^{(-1)} + \mathbf{\Psi}_1\bar{\mathbf{F}}^{(6)} + \mathbf{F}^{(-0.5)}\bar{\mathbf{F}}^{(-1.5)} + \mathbf{F}^{(2)}\bar{\mathbf{F}}^{(-4)} + \mathbf{F}^{(-6.5)}\bar{\mathbf{F}}^{(4.5)}\Big]\mathbf{S}_1 \\
& \qquad + \Big[\mathbf{F}^{(4)}\bar{\mathbf{F}}^{(-2)} + \mathbf{F}^{(-4.5)}\bar{\mathbf{F}}^{(6.5)} + \mathbf{F}^{(1.5)}\bar{\mathbf{F}}^{(0.5)}\Big]\bar{\mathbf{S}}_1\Big\} \\
+ \; & g''_1\Big\{\Big[\mathbf{F}^{(4.5)}\bar{\mathbf{F}}^{(-11)} + \mathbf{F}^{(-4.5)}\bar{\mathbf{F}}^{(-2)}\Big]\mathbf{S}_2 + \Big[\mathbf{\Psi}_2\bar{\mathbf{F}}^{(7.5)} + \mathbf{F}^{(2)}\bar{\mathbf{F}}^{(4.5)} + \mathbf{F}^{(11)}\bar{\mathbf{F}}^{(-4.5)}\Big]\bar{\mathbf{S}}_2\Big\} \\
+ \; & g'''_1\Big\{\Big[\mathbf{F}^{(4)}\bar{\mathbf{F}}^{(-12.5)} + \mathbf{F}^{(-4.5)}\bar{\mathbf{F}}^{(-4)} + \mathbf{F}^{(-7.5)}\bar{\mathbf{F}}^{(-1)} + \mathbf{F}^{(-6.5)}\bar{\mathbf{F}}^{(-2)}\Big]\mathbf{S}_3 \\
& \qquad + \Big[\mathbf{F}^{(1)}\bar{\mathbf{F}}^{(7.5)} + \mathbf{F}^{(2)}\bar{\mathbf{F}}^{(6.5)} + \mathbf{F}^{(4)}\bar{\mathbf{F}}^{(4.5)} + \mathbf{F}^{(12.5)}\bar{\mathbf{F}}^{(-4)}\Big]\bar{\mathbf{S}}_3\Big\} \\
+ \; & g_{210}\Big[\mathbf{F}^{(-0.5)}\bar{\mathbf{F}}^{(0.5)} + \mathbf{F}^{(1)}\bar{\mathbf{F}}^{(-1)} + \mathbf{F}^{(2)}\bar{\mathbf{F}}^{(-2)} + \mathbf{F}^{(4)}\bar{\mathbf{F}}^{(-4)} \\
& \qquad + \mathbf{F}^{(4.5)}\bar{\mathbf{F}}^{(-4.5)} + \mathbf{F}^{(-4.5)}\bar{\mathbf{F}}^{(4.5)} + \mathbf{F}^{(-7.5)}\bar{\mathbf{F}}^{(7.5)} + \mathbf{F}^{(11)}\bar{\mathbf{F}}^{(-11)} \\
& \qquad + \mathbf{F}^{(12.5)}\bar{\mathbf{F}}^{(-12.5)} + \mathbf{F}^{(1.5)}\bar{\mathbf{F}}^{(-1.5)} + \mathbf{F}^{(-6)}\bar{\mathbf{F}}^{(6)} + \mathbf{F}^{(-6.5)}\bar{\mathbf{F}}^{(6.5)}\Big]\mathbf{\Phi}_0 \\
+ \; & g'_{210}\Big[\mathbf{F}^{(12.5)}\bar{\mathbf{F}}^{(7.5)}\Big]\mathbf{\Phi}_1
\end{aligned}
\tag{7.1}
$$

where we have assumed the Yukawa couplings are real. All but the last three terms involving $(\bar{\mathbf{S}}_3)$, $\mathbf{\Phi}_0$ and $\mathbf{\Phi}_1$ have previously appeared in the $SO(10)\times U(1)_F$ model constructed earlier in [2]. These new terms can alter the numerical results previously obtained in that reference if their corresponding Yukawa couplings do not vanish; their effects will be discussed elsewhere.

VIII. SUMMARY

In this paper, as the third stage of an extended program, the author has attempted to construct a consistent supersymmetric grand unified model in the $SO(10) \times U(1)_F$ framework, based on the results obtained earlier with a bottom-up approach carried out in collaboration with S. Nandi. In that earlier work, supersymmetry simply controlled the running of the Yukawa couplings and enabled us to restrict our attention to Dimopoulos tree diagrams to evaluate various mass matrix elements. Here we introduce complete supermultiplets, a superpotential and soft-breaking terms in order to study more thoroughly the consequences of such a SUSY-GUT model.

For this purpose, we started with the **16** and $\overline{\mathbf{16}}$ fermion and **1**, **10**, **45** and $\overline{\mathbf{126}}$ Higgs multiplets and their associated $U(1)_F$ family charges required in [2] for the $SO(10) \times U(1)_F$ model construction of the quark and lepton mass matrices. We extend these same assignments to $SO(10)$ supermultiplets and add $U(1)_F$-conjugate Higgs supermultiplets to make the $[SO(10)]^2 \times U(1)_F$ triangle anomaly vanish. The $[U(1)_F]^3$ triangle anomaly will also vanish, so the model is anomaly-free with the addition of a pair of $SO(10)$ singlet supermultiplets, both with $U(1)_F$ charge -12. Since these supermultiplets correspond to a sterile neutrino, a conjugate sterile neutrino and their scalar neutrino partners, but with the same $U(1)_F$ charges, they do not pair off and get superheavy.

To this set of supermultiplets derived from the Yukawa sector of the model must be added additional pairs of $U(1)_F$-conjugate Higgs supermultiplets belonging to **54** and **210** representations for the Higgs sector of the superpotential. These are needed in order to generate appropriate higgsino mass matrices and to ensure that some F-flat direction exists after the breaking of the GUT symmetry, so that the supersymmetry remains unbroken at [redacted]

The large multiplicity of superfields introduced results in the development of a Landau singularity within a factor of 1.5 of Λ_{SGUT} when the $SO(10)$ gauge coupling is evolved beyond the SUSY-GUT scale toward the Planck scale. We have argued that this should occur, for the model is an effective theory at best since the higher level $SO(10)$ supermultiplets do not arise naturally in superstring models, for example. The appearance of the Landau singularity suggests that the true theory near the Planck scale becomes confining when evolved downward through two orders of magnitude with the higher-dimensional Higgs representations emerging as composite states of that theory.

By the introduction of a Z_2 discrete symmetry and the judicious choice of chiral symmetry breaking, we find that the it can be arranged that only one pair of higgsino (Higgs) doublets remains light at the electroweak scale; on the other hand, all higgsino triplets become superheavy. Moreover, the electroweak VEV's generated by the light pair of Higgs doublets make lowest-order tree-level contributions only to the diagonal 22 and 33 elements of the up quark and Dirac neutrino mass matrices, while all four elements in the 2-3 sector of the down quark and charged lepton mass matrices receive such tree-level contributions. This is in agreement with the phenomenological results obtained earlier in Ref. [2].

By the addition of soft SUSY-breaking terms to the scalar part of the Higgs potential, nonuniversal corrections to the masses of the squark and slepton fields can be generated which involve the $U(1)_F$ family charges when the VEV's for the scalar fields and their conjugates become unequal. The first or third family squark and slepton masses will be split further away from the universal values than the second family, with the family receiving the smallest mass depending upon the sign of the splitting parameter present in (6.6). Although the model discussed is far from the usual minimal model, since only one pair of Higgs doublets survives at the electroweak scale, the scalar potential for the Higgs doublets at that scale is similar to that of the minimal supersymmetric standard model, MSSM. As such, the good result for $\sin^2\theta_W$ is maintained.

As a result of the additional Higgs supermultiplets introduced in the model for the Higgs sector, several new terms appear in the Yukawa superpotential involving an extra conjugate pair of Higgs singlets and two **210** representations. If their corresponding Yukawa couplings are not taken to vanish, they can alter the numerical results obtained earlier in Ref. [2]. We shall defer for future study this point and the possible role the added neutrino singlets may play as sterile neutrinos in neutrino oscillations.

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APPENDIX

In this Appendix we present the F-flat conditions, $|F_{\phi_i}|^2 = 0$, which arise from the requirement that supersymmetry remain unbroken at the SUSY-GUT scale, Λ_{SGUT}. The F-term derivatives were already listed in (5.2) and are spelled out explicitly here. We keep only those terms involving non-vanishing Λ_{SGUT} VEV's and require that, for each F-term derivative, the coefficient vanish for each possible VEV direction as given in (2.6). With the help of Ref. [33], we obtain the following results.

$$
\begin{aligned}
F_{\Phi_0} &= 2\mu_0\Phi_0 + 3\lambda_0\Phi_0\Phi_0 + \lambda_2\Phi_2\bar{\Phi}_2 + 2\rho_0\Phi_0\Sigma_0 + \kappa_0 A_0 A_0 + \kappa_1 A_1\bar{A}_1 + \kappa_2 A_2\bar{A}_2 \\
&\qquad +\kappa_3 A_3\bar{A}_3 + \eta_0'\Delta'\bar{\Delta}' \\
&\bullet\quad \mu_0 a_0 + \tfrac{1}{4\sqrt{6}}\lambda_0 c_0^2 + \tfrac{1}{12\sqrt{6}}\lambda_2 c_2\bar{c}_2 - \tfrac{3}{2\sqrt{15}}\rho_0 r_0 a_0 + \tfrac{1}{2\sqrt{6}}(\kappa_0 q_0^2 + \kappa_1 q_1\bar{q}_1 \\
&\qquad +\kappa_2 q_2\bar{q}_2 + \kappa_3 q_3\bar{q}_3) + \tfrac{1}{20\sqrt{6}}\eta_0' v_R\bar{v}_R = 0 \\
&\bullet\quad \mu_0 b_0 + \tfrac{1}{6\sqrt{2}}\lambda_0(b_0^2 + c_0^2) + \tfrac{1}{18\sqrt{2}}\lambda_2(b_2\bar{b}_2 + c_2\bar{c}_2) + \tfrac{1}{\sqrt{15}}\rho_0 r_0 b_0 \\
&\qquad +\tfrac{1}{3\sqrt{2}}(\kappa_0 p_0^2 + \kappa_1 p_1\bar{p}_1 + \kappa_2 p_2\bar{p}_2 + \kappa_3 p_3\bar{p}_3) + \tfrac{1}{20\sqrt{2}}\eta_0' v_R\bar{v}_R = 0 \qquad (A1) \\
&\bullet\quad \mu_0 c_0 + \lambda_0(\tfrac{1}{2\sqrt{6}}a_0 + \tfrac{1}{3\sqrt{2}}b_0)c_0 + \lambda_2\left[\tfrac{1}{12\sqrt{6}}(a_2\bar{c}_2 + c_2\bar{a}_2) + \tfrac{1}{18\sqrt{2}}(b_2\bar{c}_2 + c_2\bar{b}_2)\right] \\
&\qquad -\tfrac{1}{4\sqrt{15}}\rho_0 r_0 c_0 + \tfrac{1}{2\sqrt{6}}\left[2\kappa_0 p_o q_0 + \kappa_1(p_1\bar{q}_1 + q_1\bar{p}_1) + \kappa_2(p_2\bar{q}_2 + q_2\bar{p}_2)\right. \\
&\qquad \left.+\kappa_3(p_3\bar{q}_3 + q_3\bar{p}_3)\right] + \tfrac{1}{20}\eta_0' v_R\bar{v}_R = 0 \\
F_{\Phi_1} &= \mu_1\bar{\Phi}_1 + \lambda_3\bar{\Phi}_2\bar{\Phi}_2 + \rho_3\bar{\Phi}_2\bar{\Sigma}_2 + \rho_1'\bar{\Phi}_1 A_0 \\
&\bullet\quad \mu_1\bar{a}_1 + \tfrac{1}{6\sqrt{6}}\lambda_3\bar{c}_2^2 - \tfrac{3}{2\sqrt{15}}\rho_3\bar{a}_2\bar{r}_2 = 0 \\
&\bullet\quad \mu_1\bar{b}_1 + \tfrac{1}{9\sqrt{2}}\lambda_3(\bar{b}_2^2 + \bar{c}_2^2) + \tfrac{1}{\sqrt{15}}\rho_3\bar{b}_2\bar{r}_2 = 0 \qquad (A2) \\
&\bullet\quad \mu_1\bar{c}_1 + \lambda_3\left[\tfrac{1}{3\sqrt{6}}\bar{a}_2 + \tfrac{2}{9\sqrt{2}}\bar{b}_2\right]\bar{c}_2 - \tfrac{1}{4\sqrt{15}}\rho_3\bar{c}_2\bar{r}_2 = 0 \\
F_{\bar{\Phi}_1} &= \mu_1\Phi_1 + \rho_4\Phi_2\Sigma_2 + \rho_1'\Phi_1 A_0 \\
&\bullet\quad \mu_1 a_1 - \tfrac{3}{2\sqrt{15}}\rho_4 a_2 r_2 = 0 \\
&\bullet\quad \mu_1 b_1 + \tfrac{1}{\sqrt{15}}\rho_4 b_2 r_2 = 0 \qquad (A3) \\
&\bullet\quad \mu_1 c_1 - \tfrac{1}{4\sqrt{15}}\rho_4 c_2 r_2 = 0 \\
F_{\Phi_2} &= \mu_2\bar{\Phi}_2 + \lambda_2\bar{\Phi}_2\Phi_0 + \rho_2\bar{\Phi}_2\Sigma_0 + \rho_4\bar{\Phi}_1\Sigma_2 \\
&\bullet\quad \mu_2\bar{a}_2 + \tfrac{1}{6\sqrt{6}}\lambda_2 c_0\bar{c}_2 - \tfrac{3}{2\sqrt{15}}(\rho_2\bar{a}_2 r_0 + \rho_4\bar{a}_1 r_2) = 0 \\
&\bullet\quad \mu_2\bar{b}_2 + \tfrac{1}{9\sqrt{2}}\lambda_2(b_0\bar{b}_2 + c_0\bar{c}_2) + \tfrac{1}{\sqrt{15}}(\rho_2\bar{b}_2 r_0 + \rho_4\bar{b}_1 r_2) = 0 \qquad (A4) \\
&\bullet\quad \mu_2\bar{c}_2 + \lambda_2\left[\tfrac{1}{6\sqrt{6}}(a_0\bar{c}_2 + c_0\bar{a}_2) + \tfrac{1}{9\sqrt{2}}(b_0\bar{c}_2 + c_0\bar{b}_2)\right] \\
&\qquad -\tfrac{1}{4\sqrt{15}}(\rho_2\bar{c}_2 r_0 + \rho_4\bar{c}_1 r_2) = 0
\end{aligned}
$$

$$F_{\bar{\Phi}_2} = \mu_2\Phi_2 + \lambda_2\Phi_2\Phi_0 + 2\lambda_3\Phi_1\bar{\Phi}_2 + \rho_2\Phi_2\Sigma_0 + \rho_3\Phi_1\bar{\Sigma}_2$$

- $\mu_2 a_2 + \frac{1}{6\sqrt{6}}\lambda_2 c_0 c_2 + \frac{1}{3\sqrt{6}}\lambda_3 c_1\bar{c}_2 - \frac{3}{2\sqrt{15}}(\rho_2 a_2 r_0 + \rho_3 a_1\bar{r}_2) = 0$
- $\mu_2 b_2 + \frac{1}{9\sqrt{2}}\lambda_2(b_0 b_2 + c_0 c_2) + \frac{2}{9\sqrt{2}}\lambda_3(b_1\bar{b}_2 + c_1\bar{c}_2)$
  $+\frac{1}{\sqrt{15}}(\rho_2 b_2 r_0 + \rho_3 b_1\bar{r}_2) = 0 \qquad (A5)$
- $\mu_2 c_2 + \lambda_2\left[\frac{1}{6\sqrt{6}}(a_0 c_2 + c_0 a_2) + \frac{1}{9\sqrt{2}}(b_0 c_2 + c_0 b_2)\right]$
  $+\lambda_3\left[\frac{1}{3\sqrt{6}}(c_1\bar{a}_2 + a_1\bar{c}_2) + \frac{2}{9\sqrt{2}}(b_1\bar{c}_2 + c_1\bar{b}_2)\right]$
  $-\frac{1}{4\sqrt{15}}(\rho_2 c_2 r_0 + \rho_3 c_1\bar{r}_2) = 0$

$$F_{\Delta'} = \mu_3\bar{\Delta}' + \eta'_0\bar{\Delta}'\Phi_0$$

- $\mu_3\bar{v}_R + \eta'_0\left[\frac{1}{10\sqrt{6}}a_0 + \frac{1}{10\sqrt{2}}b_0 + \frac{1}{10}c_0\right]\bar{v}_R = 0 \qquad (A6)$

$$F_{\bar{\Delta}'} = \mu_3\Delta' + \eta'_0\Delta'\Phi_0$$

- $\mu_3 v_R + \eta'_0\left[\frac{1}{10\sqrt{6}}a_0 + \frac{1}{10\sqrt{2}}b_0 + \frac{1}{10}c_0\right]v_R = 0 \qquad (A7)$

$$F_{\Sigma_0} = 2\mu'_0\Sigma_0 + \rho_0\Phi_0\Phi_0 + \rho_2\Phi_2\bar{\Phi}_2 + 3\sigma_0\Sigma_0\Sigma_0 + \sigma_1\Sigma_1\bar{\Sigma}_1$$
$$+\kappa'_0 A_0 A_0 + \kappa'_1 A_1\bar{A}_1 + \kappa'_2 A_2\bar{A}_2 + \kappa'_3 A_3\bar{A}_3$$

- $\mu'_0 r_0 - \frac{1}{8\sqrt{15}}\left[\rho_0(6a_0^2 - 4b_0^2 + c_0^2) + \rho_2(6a_2\bar{a}_2 - 4b_2\bar{b}_2 + c_2\bar{c}_2)\right] + \frac{3}{4\sqrt{15}}\sigma_0 r_0^2$
  $+\frac{1}{4\sqrt{15}}\sigma_1 r_1\bar{r}_1 + \frac{1}{2\sqrt{15}}(\kappa'_0 p_0^2 + \kappa'_1 p_1\bar{p}_1 + \kappa'_2 p_2\bar{p}_2 + \kappa'_3 p_3\bar{p}_3) \qquad (A8)$
  $-\frac{3}{4\sqrt{15}}(\kappa'_0 q_0^2 + \kappa'_1 q_1\bar{q}_1 + \kappa'_2 q_2\bar{q}_2 + \kappa'_3 q_3\bar{q}_3) = 0$
- $\rho_0(a_0^2 + b_0^2 + c_0^2) + \rho_2(a_2\bar{a}_2 + b_2\bar{b}_2 + c_2\bar{c}_2) - 3\sigma_0 r_0^2 - \sigma_1 r_1\bar{r}_1 + \kappa'_0(p_0^2 + q_0^2)$
  $+\kappa'_1(p_1\bar{p}_1 + q_1\bar{q}_1) + \kappa'_2(p_2\bar{p}_2 + q_2\bar{q}_2) + \kappa'_3(p_3\bar{p}_3 + q_3\bar{q}_3) = 0$

$$F_{\Sigma_1} = \mu'_1\bar{\Sigma}_1 + \sigma_1\bar{\Sigma}_1\Sigma_0 + \kappa'_4 A_3 A_3$$

- $\mu'_1\bar{r}_1 + \frac{1}{2\sqrt{15}}\sigma_1 r_0\bar{r}_1 + \frac{1}{2\sqrt{15}}\kappa'_4(2p_3^2 - 3q_3^2) = 0 \qquad (A9)$
- $\sigma_1 r_0\bar{r}_1 - \kappa'_4(p_3^2 + q_3^2) = 0$

$$F_{\bar{\Sigma}_1} = \mu'_1\Sigma_1 + \sigma_1\Sigma_1\Sigma_0 + \kappa'_5\bar{A}_3\bar{A}_3$$

- $\mu'_1 r_1 + \frac{1}{2\sqrt{15}}\sigma_1 r_0 r_1 + \frac{1}{2\sqrt{15}}\kappa'_5(2\bar{p}_3^2 - 3\bar{q}_3^2) = 0 \qquad (A10)$
- $\sigma_1 r_0 r_1 - \kappa'_5(\bar{p}_3^2 + \bar{q}_3^2) = 0$

$$
\begin{aligned}
F_{\Sigma_2} &= \mu_2'\bar{\Sigma}_2 + \rho_4\Phi_2\bar{\Phi}_1 + \sigma_2'\bar{\Sigma}_2 A_0 \\
&\bullet \quad \mu_2'\bar{r}_2 + \tfrac{1}{4\sqrt{15}}\rho_4\left[-6a_2\bar{a}_1 + 4b_2\bar{b}_1 - c_2\bar{c}_1\right] = 0 \qquad (A11) \\
&\bullet \quad \rho_4(a_2\bar{a}_1 + b_2\bar{b}_1 + c_2\bar{c}_1) = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{\bar{\Sigma}_2} &= \mu_2'\Sigma_2 + \rho_3\Phi_1\bar{\Phi}_2 + \sigma_2'\Sigma_2 A_0 \\
&\bullet \quad \mu_2' r_2 + \tfrac{1}{4\sqrt{15}}\rho_3\left[-6a_1\bar{a}_2 + 4b_1\bar{b}_2 - c_1\bar{c}_2\right] = 0 \qquad (A12) \\
&\bullet \quad \rho_3(a_1\bar{a}_2 + b_1\bar{b}_2 + c_1\bar{c}_2) = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{A_0} &= 2\mu_0''A_0 + \rho_1'\Phi_1\bar{\Phi}_1 + \sigma_2'\Sigma_2\bar{\Sigma}_2 + 2\kappa_0 A_0\Phi_0 + 2\kappa_0' A_0\Sigma_0 \\
&\bullet \quad \mu_0'' p_0 + \kappa_0\left[\tfrac{2}{3\sqrt{2}}p_0 b_0 + \tfrac{1}{\sqrt{6}}q_0 c_0\right] + \tfrac{1}{\sqrt{15}}\kappa_0' p_0 r_0 = 0 \qquad (A13) \\
&\bullet \quad \mu_0'' q_0 + \tfrac{1}{\sqrt{6}}\kappa_0(p_0 c_0 + q_0 a_0) - \tfrac{3}{2\sqrt{15}}\kappa_0' q_0 r_0 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{A_1} &= \mu_1''\bar{A}_1 + \kappa_1\bar{A}_1\Phi_0 + \kappa_1'\bar{A}_1\Sigma_0 \\
&\bullet \quad \mu_1''\bar{p}_1 + \kappa_1\left[\tfrac{2}{3\sqrt{2}}b_0\bar{p}_1 + \tfrac{1}{\sqrt{6}}c_0\bar{q}_1\right] + \tfrac{1}{\sqrt{15}}\kappa_1' r_0\bar{p}_1 = 0 \qquad (A14) \\
&\bullet \quad \mu_1''\bar{q}_1 + \tfrac{1}{\sqrt{6}}\kappa_1(c_0\bar{p}_1 + a_0\bar{q}_1) - \tfrac{3}{2\sqrt{15}}\kappa_1' r_0\bar{q}_1 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{\bar{A}_1} &= \mu_1'' A_1 + \kappa_1 A_1\Phi_0 + \kappa_1' A_1\Sigma_0 \\
&\bullet \quad \mu_1'' p_1 + \kappa_1\left[\tfrac{2}{3\sqrt{2}}b_0 p_1 + \tfrac{1}{\sqrt{6}}c_0 q_1\right] + \tfrac{1}{\sqrt{15}}\kappa_1' r_0 p_1 = 0 \qquad (A15) \\
&\bullet \quad \mu_1'' q_1 + \tfrac{1}{\sqrt{6}}\kappa_1(c_0 p_1 + a_0 q_1) - \tfrac{3}{2\sqrt{15}}\kappa_1' r_0 q_1 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{A_2} &= \mu_2''\bar{A}_2 + \kappa_2\bar{A}_2\Phi_0 + \kappa_2'\bar{A}_2\Sigma_0 \\
&\bullet \quad \mu_2''\bar{p}_2 + \kappa_2\left[\tfrac{2}{3\sqrt{2}}b_0\bar{p}_2 + \tfrac{1}{\sqrt{6}}c_0\bar{q}_2\right] + \tfrac{1}{\sqrt{15}}\kappa_2' r_0\bar{p}_2 = 0 \qquad (A16) \\
&\bullet \quad \mu_2''\bar{q}_2 + \tfrac{1}{\sqrt{6}}\kappa_2(c_0\bar{p}_2 + a_0\bar{q}_2) - \tfrac{3}{2\sqrt{15}}\kappa_2' r_0\bar{q}_2 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{\bar{A}_2} &= \mu_2'' A_2 + \kappa_2 A_2\Phi_0 + \kappa_2' A_2\Sigma_0 \\
&\bullet \quad \mu_2'' p_2 + \kappa_2\left[\tfrac{2}{3\sqrt{2}}b_0 p_2 + \tfrac{1}{\sqrt{6}}c_0 q_2\right] + \tfrac{1}{\sqrt{15}}\kappa_2' r_0 p_2 = 0 \qquad (A17) \\
&\bullet \quad \mu_2'' q_2 + \tfrac{1}{\sqrt{6}}\kappa_2(c_0 p_2 + a_0 q_2) - \tfrac{3}{2\sqrt{15}}\kappa_2' r_0 q_2 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{A_3} &= \mu_3''\bar{A}_3 + \kappa_3\bar{A}_3\Phi_0 + \kappa_3'\bar{A}_3\Sigma_0 + 2\kappa_4' A_3\Sigma_1 \\
&\bullet \quad \mu_3''\bar{p}_3 + \kappa_3\left[\tfrac{2}{3\sqrt{2}}b_0\bar{p}_3 + \tfrac{1}{\sqrt{6}}c_0\bar{q}_3\right] + \tfrac{1}{\sqrt{15}}\kappa_3' r_0\bar{p}_3 + \tfrac{2}{\sqrt{15}}\kappa_4' r_1 p_3 = 0 \qquad (A18) \\
&\bullet \quad \mu_3''\bar{q}_3 + \tfrac{1}{\sqrt{6}}\kappa_3(c_0\bar{p}_3 + a_0\bar{q}_3) - \tfrac{3}{2\sqrt{15}}\kappa_3' r_0\bar{q}_3 - \tfrac{3}{\sqrt{15}}\kappa_4' r_1 q_3 = 0
\end{aligned}
$$

$$
\begin{aligned}
F_{\bar{A}_3} &= \mu_3'' A_3 + \kappa_3 A_3 \Phi_0 + \kappa_3' A_3 \Sigma_0 + 2\kappa_5' \bar{A}_3 \bar{\Sigma}_1 \\
& \bullet \quad \mu_3'' p_3 + \kappa_3 \left[\tfrac{2}{3\sqrt{2}} b_0 p_3 + \tfrac{1}{\sqrt{6}} c_0 q_3\right] + \tfrac{1}{\sqrt{15}} \kappa_3' r_0 p_3 + \tfrac{2}{\sqrt{15}} \kappa_5' \bar{r}_1 \bar{p}_3 = 0 \quad (A19) \\
& \bullet \quad \mu_3'' q_3 + \tfrac{1}{\sqrt{6}} \kappa_3 (c_0 p_3 + a_0 q_3) - \tfrac{3}{2\sqrt{15}} \kappa_3' r_0 q_3 - \tfrac{3}{\sqrt{15}} \kappa_5' \bar{r}_1 \bar{q}_3 = 0 \\
F_{S_1} &= \mu_1''' \bar{S}_1 + \varepsilon_1 S_2 \bar{S}_3 \\
& \bullet \quad \mu_1''' \bar{t}_1 + \varepsilon_1 t_2 \bar{t}_3 = 0 \qquad (A20) \\
F_{\bar{S}_1} &= \mu_1''' S_1 + \varepsilon_2 S_3 \bar{S}_2 \\
& \bullet \quad \mu_1''' t_1 + \varepsilon_2 t_3 \bar{t}_2 = 0 \qquad (A21) \\
F_{S_2} &= \mu_2''' \bar{S}_2 + \varepsilon_1 S_1 \bar{S}_3 \\
& \bullet \quad \mu_2''' \bar{t}_2 + \varepsilon_1 t_1 \bar{t}_3 = 0 \qquad (A22) \\
F_{\bar{S}_2} &= \mu_2''' S_2 + \varepsilon_2 S_3 \bar{S}_1 \\
& \bullet \quad \mu_2''' t_2 + \varepsilon_2 t_3 \bar{t}_1 = 0 \qquad (A23) \\
F_{S_3} &= \mu_3''' \bar{S}_3 + \varepsilon_2 \bar{S}_1 \bar{S}_2 \\
& \bullet \quad \mu_3''' \bar{t}_3 + \varepsilon_2 \bar{t}_1 \bar{t}_2 = 0 \qquad (A24) \\
F_{\bar{S}_3} &= \mu_3''' S_3 + \varepsilon_1 S_1 S_2 \\
& \bullet \quad \mu_3''' t_3 + \varepsilon_1 t_1 t_2 = 0 \qquad (A25)
\end{aligned}
$$

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