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GRAVITON SPECTRA IN STRING COSMOLOGY

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We propose to uncover the signature of a stringy era in the primordial Universe by searching for a prominent peak in the relic graviton spectrum. This feature, which in our specific model terminates an ω^3 increase and initiates an ω^{-7} decrease, is induced during the so far overlooked bounce of the scale factor between the collapsing deflationary era (or pre-Big Bang) and the expanding inflationary era (or post-Big Bang). We evaluate both analytically and numerically the frequency and the intensity of the peak and we show that they may likely fall in the realm of the new generation of interferometric detectors. The existence of a peak is at variance with ordinarily monotonic (either increasing or decreasing) graviton spectra of canonical cosmologies; its detection would therefore offer strong support to string cosmology.

I. INTRODUCTION

A stochastic background of gravitational waves (GW) is produced during inflationary eras (whether exponential, power-law or pole-like). The resulting spectra differ from each other, depending upon the underlying inflationary model. Thus one could in principle obtain information about the physics at the Planck epoch from the observation of the GW background.

The most appealing candidate for a description of physics at the Planck scale is superstring theory: hence comes therefore the most promising cosmological model.

The crucial problem is to give a detailed and consistent description of the Universe approaching the singularity. By using the $O(D-1, D-1)$ symmetry of the tree-level superstring action [1]

$$S = -\frac{1}{2l_{st}^{D-2}} \int d^D x \sqrt{-g} e^{-\phi} [R + \nabla_\mu \phi \nabla^\mu \phi + V(\phi)] , \quad (1.1)$$

where $D \geq 4$, one can construct non-singular cosmologies [2]. The fundamental requirement for such a model is the existence of the so called “branch-changing” solutions [3] which

smoothly interpolate between a contracting Universe (*pre-big-bang*, PBB) and an expanding one (*post-big-bang*), related to one another by a duality transformation. This feature is quite general. In fact, the full string loop expansion, that includes the case where the spatial hypersurfaces have non-zero curvature, shows “bouncing” solutions resembling the “branch-changing” ones [4].

In PBB a pole-like inflation (also referred to as superinflation) is always present at $t < 0$ due to the theory symmetries (foremost among these a particular $O(D - 1, D - 1)$ symmetry known as scale factor duality, SFD). In fact, under SFD the ordinary Friedmann-Robertson-Walker radiation dominated solution (FRW) $a(t) \propto t^{1/2}, t > 0$ is mapped into the superinflationary one $a(t) \propto (-t)^{-1/2}, t < 0$. The two eras can be smoothly connected (*graceful exit*) if a potential $V(\Phi)$ for the dilaton Φ is provided.¹ Around $t = 0$ the curvature of space-time approaches the string scale, the dilaton settles down to a minimum of its potential and the coupling evolves towards the present ratio of the Planck to the string scale [3]

$$\frac{l_P}{l_{st}} \simeq g_s (\sim g_{\text{gauge}}) = e^{\phi/2} \rightarrow 10^{-1} \div 10^{-2}.$$

After this high curvature regime (stringy era) the Universe reexpands again, finally joining the $a(t) \propto t^{1/2}$ radiation dominated era at $t \rightarrow \infty$.

This picture holds in the string frame (SF), where the dilaton is non-minimally coupled to the scalar curvature. However the SF is not the most convenient for our objectives: for instance, the exact solution for the dilaton during the joining era is not known. On the contrary in the Einstein frame (EF, minimal coupling) the dependence from the dilaton of the GW equation disappears, which is a very good reason for staying henceforth in the EF. Here the PBB picture partially changes, in that superinflation is turned into accelerated contraction, $H := \dot{a}/a < 0$, $\dot{H} < 0$, *i.e.* into *deflation*. Now, it has been shown [7] that deflation is, under almost all the effects, equivalent to inflation. In particular this is true for the background gravitational spectrum [8]. We will exploit this equivalence whenever deflation appears in cosmological solutions. After the conformal transformation to the EF

$$g_{\mu\nu}^{(EF)} = g_{\mu\nu}^{(SF)} e^{-\phi^{(SF)}}, \quad dt^{(EF)} = e^{-\phi^{(SF)}/2} dt^{(SF)},$$

one gets [15] for $t \ll 0$ and $D = 4$:

$$a_{EF}(t_{EF}) \propto (-t_{EF})^{1/3}, \quad \phi_{EF}(t_{EF}) \propto -\left(\frac{2}{\sqrt{3}}\right) \ln(-t_{EF}). \quad (1.2)$$

Deflation appears as the EF expression of superinflation in the SF, PBB scenario. This means that as a general feature the universe contracts, reaches a minimum size and after

¹Alternatively the two eras can be connected by a quantum cosmological scattering [5].

that reexpands. The detailed behaviour of the scale factor $a(t)$ depends on the model. It is possible, however, to extract the qualitative effects from general considerations, independently from the knowledge of the exact solutions for $a(t)$ and $\phi(t)$. In fact the classical GW equation [see (2.2) below] does not depend explicitly on the dilaton dynamics: in fact, it is sufficient to fix the evolution of the metric, regardless of that of the dilaton. Obviously, the era around the minimum between collapse and expansion, that is commonly referred to as *stringy*, the solution can be approximated with:

$$a = a_{min} + \left(\frac{t}{t_0}\right)^2, \quad t \simeq 0. \quad (1.3)$$

A clear example of complete solution of string cosmology in the SF is reported in [6]. It shows the expected three phases: 1) the $t \rightarrow -\infty$ inflationary solution; 2) the $t \simeq 0$ smooth connection in the full stringy era; 3) the post-Big-Bang radiation dominated era. With respect to the original PBB model a new feature arises: in the full stringy era the scale factor $a(t)$ decreases at first, reaches a minimum and reexpands, finally joining the $t^{1/2}$ run. It turns out that the contraction is accelerated, i.e. deflation appears again.

We will exploit the property of string cosmological models of possessing a deflationary era. In particular, this paper is devoted to studying the modifications in the spectrum of primordial GW's produced by the stringy era joining the deflationary stage to the radiation dominated one.

This paper is organized as follows. Sec.II is dedicated to the graviton spectra. We first introduce our formula for the energy density per octave Ω_g . After that, in Sec. II.A we show the known results for the spectrum from the dilaton era; in Sec. II.B we present our new results on the contributions from the stringy era. Our conclusions are reported in Sec. III.

II. PRIMORDIAL GRAVITON SPECTRA

Spectra of primordial gravitational waves are well known. Differences in the form of the spectra depend on the evolution of the cosmological background in the two relevant phases: the *goodbye* phase in which the wavelength becomes larger than the effective horizon, H^{-1} , and the *hallo again* phase in which the waves reenter the horizon. Suppose $a \propto t^b$ (for $b \rightarrow \infty$ exponential inflation is recovered) in the former and $a \propto t^c$ in the latter. For exponential inflation, the fraction of critical density contributed per octave $\Omega(k)$ does not depend on k , i.e. the spectrum is flat; in the power-law case the spectrum is decreasing; in the pole-like one the spectrum increases. The different behaviors can be collected in one formula:

$$\Omega(k) \propto k^\beta, \quad \beta \equiv 2 \frac{3c + b - 2bc - 2}{(1 - b)(c - 1)}, \quad (2.1)$$

which can be easily derived with both quantum and classical arguments and is given in a slightly different (but fully equivalent) form by Sahni [9]. The quantum derivation consists

in considering the asymptotic wave-equation solutions that are related by a Bogoliubov transformation [9]. (The complete solutions are reported in [10] and in [11].) The positive frequency modes (the asymptotically vacuum state at $t \rightarrow -\infty$) will be in general a linear superposition of modes of positive and negative frequencies with respect to the vacuum to the right ($t \rightarrow +\infty$). The coefficient of the negative mode of the vacuum to the right, determines the number of gravitons created by the interaction with the background curvature [9]:

$$\Omega_g^q(\nu) := \frac{1}{\rho_{cr}} \frac{d\rho_g}{d \ln \nu} \propto \frac{\nu^4}{\rho_{cr}} |c_-(\nu)|^2 .$$

At the classical level the following expression holds:

$$\Omega_g^c(\omega) := \frac{1}{\rho_{cr}} \frac{d\rho_g}{d \ln \omega} = \frac{\pi c^2}{4G\rho_{cr}} \omega^2 |\Delta h_k|^2 ;$$

here $|\Delta h_k| := k^3 h_k^2$ is the spectral amplitude, $\omega(t) = k/a(t)$ and $h_k(t)$ is the amplitude of the linear wave of wave vector k . One gets² formula (2.1) in the EF, by considering the classical solution of the wave equation

$$\ddot{h}_k + 3H\dot{h}_k + \omega^2 h_k = 0 , \tag{2.2}$$

where a dot means derivation wrt standard time.

Note that in the model discussed here new problems arise. In fact, as the the scale factor undergoes a bounce, the modes that exit the horizon during the dilaton-driven era (PBB), necessarily cross it again twice during the pure stringy era (see Sec. II.B and Fig. 3 below). Furthermore their amplitudes grow, instead of being frozen once outside the horizon. As a consequence the quantum derivation cannot be straightforwardly extended. On the contrary, one can still apply the classical derivation: with the appropriate correction for the bounce in the form of $a(t) \propto (-t)^g$, $t < 0$, $g > 1$, we find the new analytic result [12]:

$$\Omega_g \propto k^{\beta'}, \quad \beta' = \beta + 2(3g - 1)/(1 - g) , \tag{2.3}$$

which is fully supported by numerical computations.

A. Spectrum from the dilaton era

From (2.1) it is easily seen that for deflation+radiation the spectrum is growing in k FIG.1, like in the superinflation case.

²A rigorous proof of the equivalence of the equation in the different frames is reported in [12]

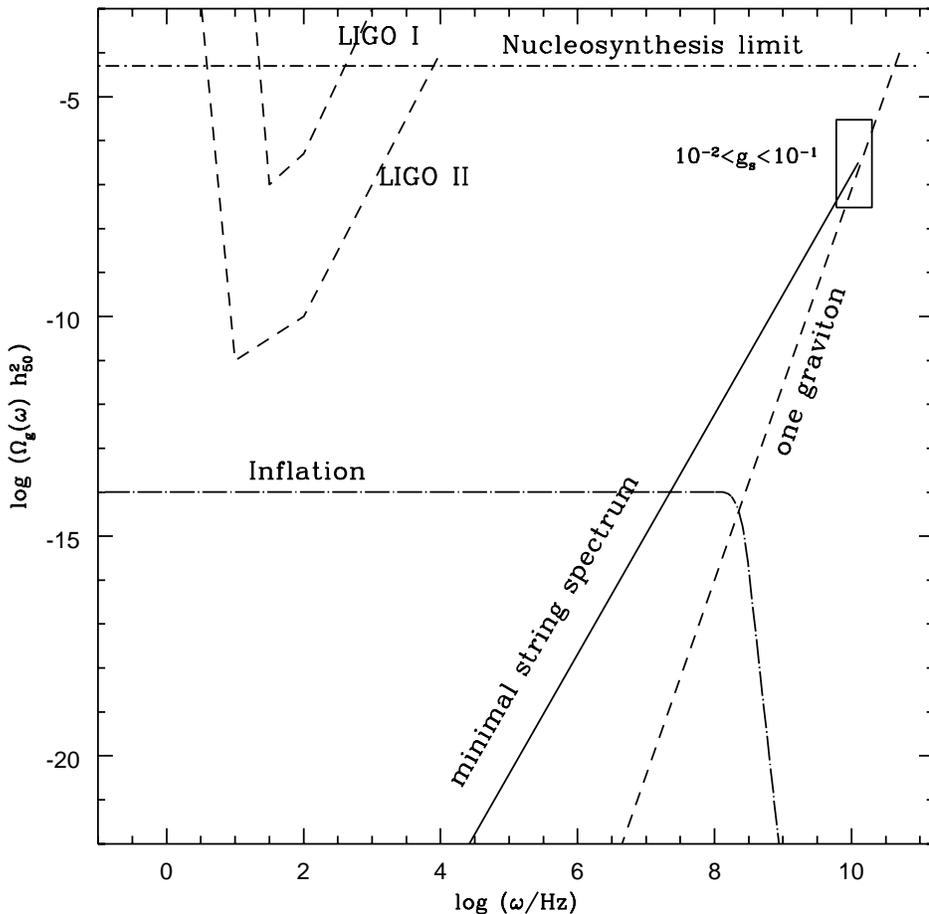


FIG. 1. The spectra in the two cases above compared to the LIGO sensitivity, and to the nucleosynthesis bound. Also shown is the limit for graviton creation

The largest value of k depends on the value of H at the end of (deflation) inflation. Typical values in the range $(10^{-8} \div 1)m_P$, i.e. temperatures $(10^{15} \div 10^{19})\text{Gev}$, imply frequencies in the range $(10^7 \div 10^{11})\text{Hz}$, Fig.2. Thus it makes sense to consider the possibility of a detection of the gravitational background by the LIGO detectors that work in the range $(1 \div 10^3)\text{ Hz}$ [13]. Such a detection could fix spectral parameters, thus defining the cosmological background. Quadrupole anisotropies in the Cosmic Microwave Background (CMB) and Pulsar Timings (PT) already constrain the ranges of the spectral parameters, but the region they allow in the plane of the parameters is still large [14].

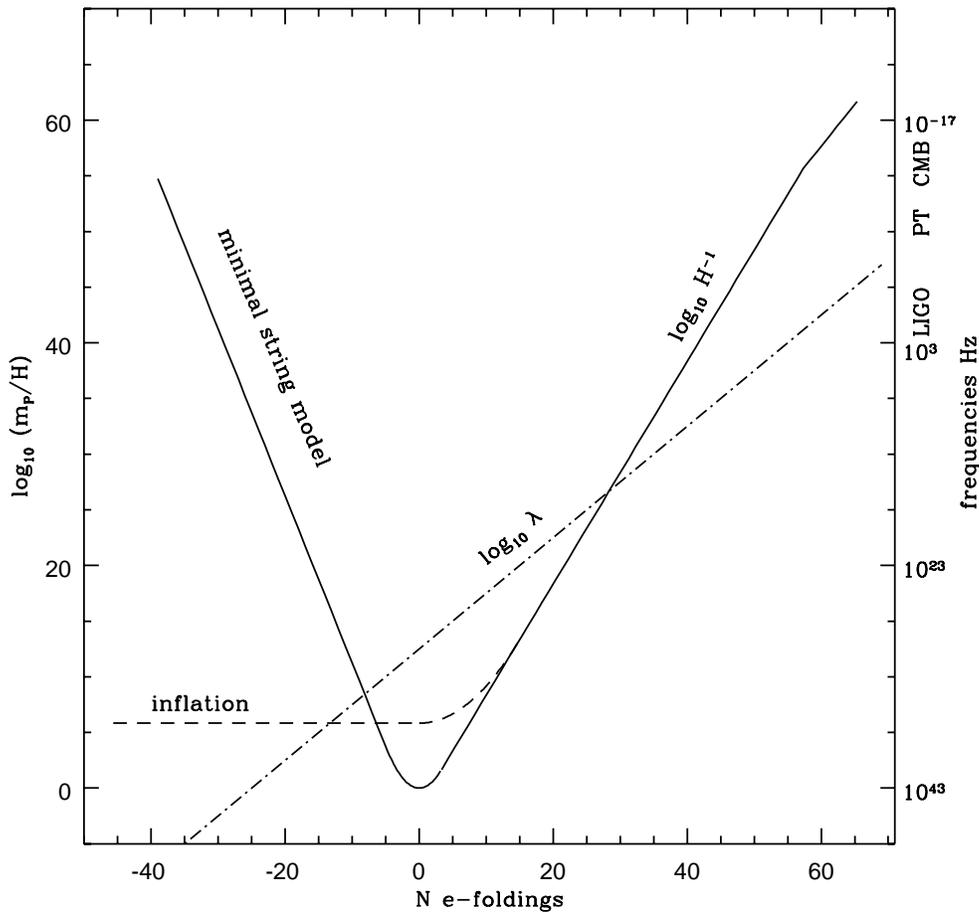


FIG. 2. The behavior of the Hubble horizon and of the physical wavelength in the cases of standard inflation and deflation followed by the canonical radiation dominated era.

B. Spectrum from the stringy era

As discussed in the Introduction, the stringy era is better studied in the EF. Two relevant consequences follow from *ansatz* (1.3).

First, H^{-1} goes to zero, as $t \rightarrow 0$, slower than the physical wavelength $\lambda = a(2\pi/k)$:

$$\frac{\lambda}{H^{-1}} \sim \frac{4\pi}{t_0 k} \sqrt{a - a_{min}} . \quad (2.4)$$

This means, Fig. 3, that any wave, that crossed out the horizon in the deflationary stage at (1), reenters it at (2), since for $t < 0$ $a \rightarrow a_{min}$ at (3).

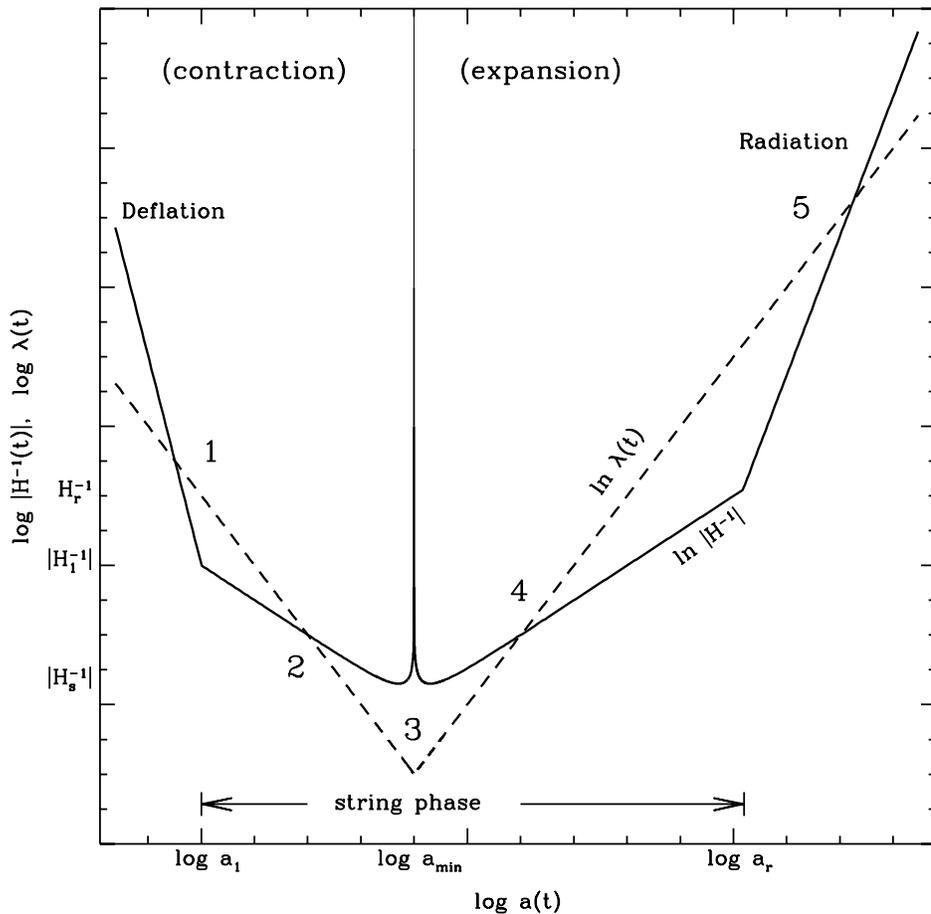


FIG. 3. A typical gravitational wave generated during the PBB deflationary presents the features described in the test.

On the contrary, for $t > 0$, a grows and alongwith the perturbation reexits at (4). The subsequent evolution matches the ordinary picture, with a final *hallo again* in the radiation (or matter, depending on k) dominated era at (5). Something similar happens in the case of double inflation [16] where the subhorizon crossing regards a certain range of wavenumbers, and produces a typical modulation in the spectrum. Here, due to deflation, all the waves cross the subhorizon region.

Secondly, according to (1.3), there is always an era of power law inflation after a deflationary period, when $t > 0$, since $a \sim t^2$. Thus the correct spectrum has the form of a deflationary one at small k and of a power-law inflationary one at large k . We underline that, in order to obtain the correct description of the physics involved, we must take into

account both the deflationary and the subsequent power-law inflationary era, whereas by neglecting the latter we would miss the signature in the spectrum described in this paper. This signature consists of a peak in the spectrum at a frequency ω_M .

For very small frequencies ($\omega \ll 10^{-16}$, waves reentering during the matter dominated era) the slope of the spectrum is easily obtained from (2.1) with $b = 1/3$, and $c = 2/3$ $\Omega_g(\omega) \propto \omega$, while for frequencies in the range $\omega \in \{10^{-16}, \omega_M\}$ we have $b = 1/3$, $c = 1/2$ and then $\Omega_g(\omega) \propto \omega^3$. Similar results were found in a different context [14]. There, the maximum frequency (one graviton created per Hubble volume) is given, Fig. 1, by

$$\omega_r = 10^{11} \sqrt{\frac{H_r}{m_P}} \quad \text{Hz} ,$$

where the subscript r refers to the beginning of the radiation dominated era, and m_P is the Planck mass: furthermore ω_r coincides with the frequency of the peak. In our model, the frequency of the peak is related to the free parameters by

$$\omega_M = \frac{10^{11}}{(z_{st} z_{infl})^{1/2}} \sqrt{\frac{H_r}{m_P}} \quad \text{Hz} ,$$

where

$$1 + z_{st} = \left(\frac{a_{stend}}{a_{min}} \right) , \quad 1 + z_{infl} = \left(\frac{a_r}{a_{stend}} \right)$$

are the two parameters of the model and a_{stend} and a_{min} are respectively the value of the scale factor at the end of pure stringy era (beginning of the power-law string era), and its minimum value at $t = 0$, and a_r is the value of the scale factor at the beginning of the radiation era. The value of spectral density at the peak is:

$$\Omega_M = \Omega_\gamma \left(\frac{H_r}{m_P} \right)^2 z_{st}^{7/2} z_{infl}$$

where $\Omega_\gamma h_{50}^2 \simeq 10^{-4}$ is the present ratio of the photon density to critical density.

For the waves crossing the horizon during the string era we must consider the growing of amplitudes due to contracting metric [$a(t) \sim (-t)^2$] for wavelength larger than the Hubble horizon. The slope of the spectrum can be derived from (2.3) with $g = 2$, $\beta = 3$ and results to be $\Omega_g(\omega) \propto \omega^{-7}$.

Finally for very large frequencies the power-law inflation determines the slope $\Omega_g(\omega) \propto \omega^{-2}$, [from (2.1) with $b = 2$, $c = 1/2$]. Summarizing:

$$\begin{aligned} \omega_o < \omega < \omega_e & \quad \Omega_g \simeq \Omega_\gamma \left(\frac{H_r}{m_P} \right)^2 \left(\frac{\omega_e}{\omega_M} \right)^3 \left(\frac{\omega}{\omega_e} \right) z_{st}^{7/2} z_{infl}, \\ \omega_e < \omega < \omega_M & \quad \Omega_g \simeq \Omega_\gamma \left(\frac{H_r}{m_P} \right)^2 \left(\frac{\omega}{\omega_M} \right)^3 z_{st}^{7/2} z_{infl}, \\ \omega_M < \omega < \omega_1 & \quad \Omega_g \simeq \Omega_\gamma \left(\frac{H_r}{m_P} \right)^2 \left(\frac{\omega}{\omega_M} \right)^{-7} z_{st}^{7/2} z_{infl}, \\ \omega_1 < \omega < \omega_r & \quad \Omega_g \simeq \Omega_\gamma \left(\frac{H_r}{m_P} \right)^2 \left(\frac{\omega}{\omega_M} \right)^{-2} (z_{st} z_{infl})^{1/2}, \end{aligned} \quad (2.5)$$

where $\omega_e = 10^{-16}\text{Hz}$.

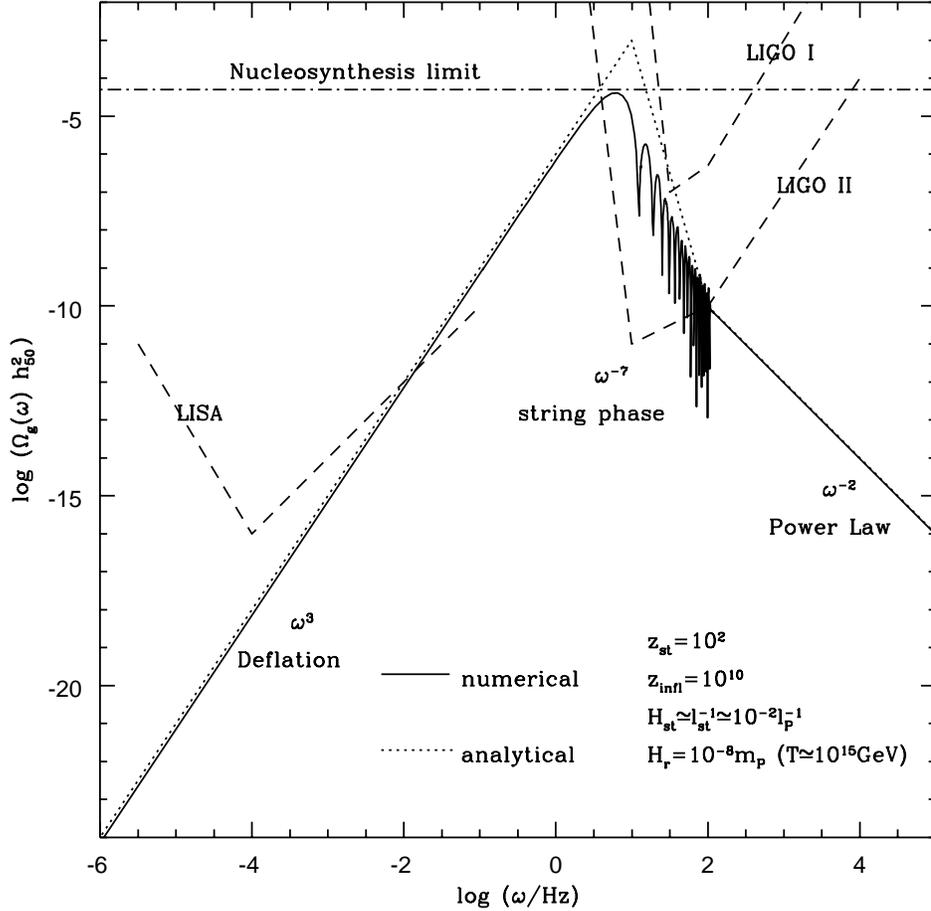


FIG. 4. The graviton spectrum of the stochastic background, with the structured peak due to the stringy era, superimposed to the known observational bounds and to the expected sensitivities of the LIGO's.

The full spectrum is shown in Fig.4. Three features emerge: 1) the growth due to deflation at small ω ; 2) the existence of a sharp peak at ω_M ; 3) the descent due to power-law inflation at large ω . Concerning 2), we underline that the maximum of the spectrum $\Omega(\omega_M)$ is higher than expected. This is due to the fact that the perturbations that exit the horizon in the stringy era are amplified more than those coming from the deflationary era. Furthermore, a comment must be made about the oscillations showing up in Fig. 4: they arise because waves of different ω and different phases remain subhorizon for different time

lengths. Finally, the total, deflation+power-law, duration N_T must be at least of 60 e-folds.

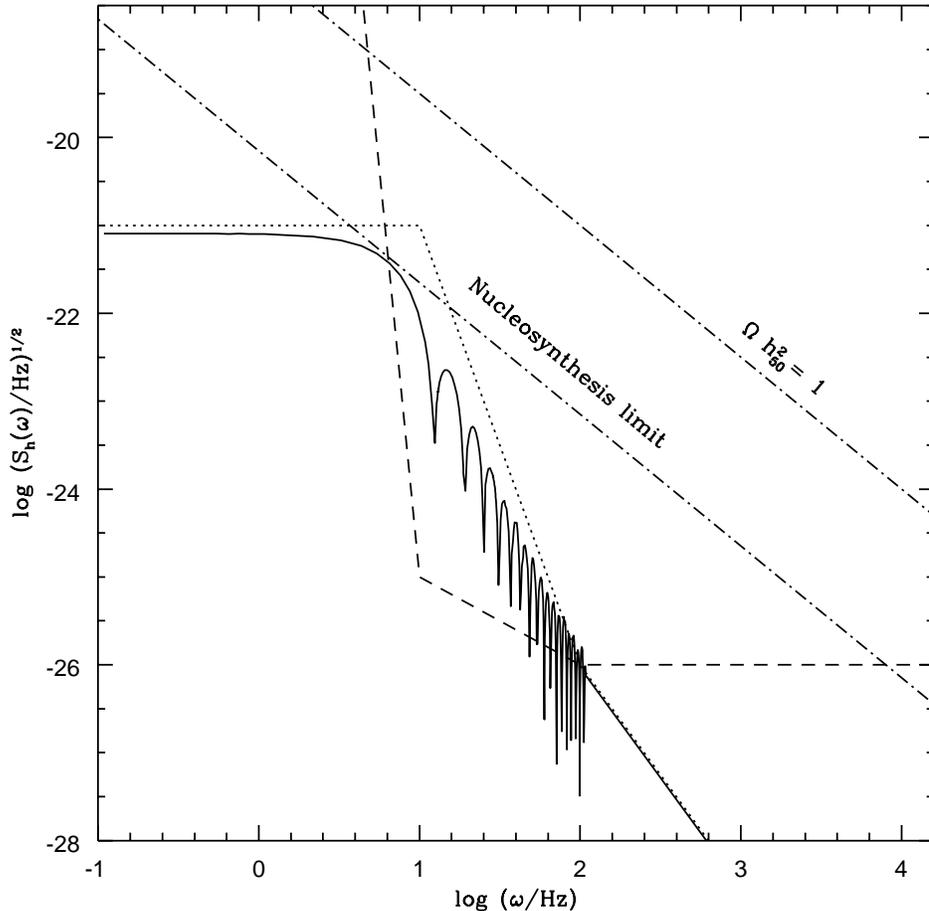


FIG. 5. The spectral amplitude of the stochastic background of Fig. 4

This is a three parameter model: $N_{st} = \ln z_{st}$, the number of e-folds of the stringy era, $N_{infl} = \ln z_{infl}$, the number of e-folds of the power-law era; H_r , the value of the Hubble parameter at the end of inflation and the starting of the radiation dominated era. The shaded area in the Fig. 6 shows the region of the parameter space (z_{infl}, H_r) for which the peak of the spectrum falls in the range of detectability of LIGO/VIRGO interferometers [19], for all z_{st} values.

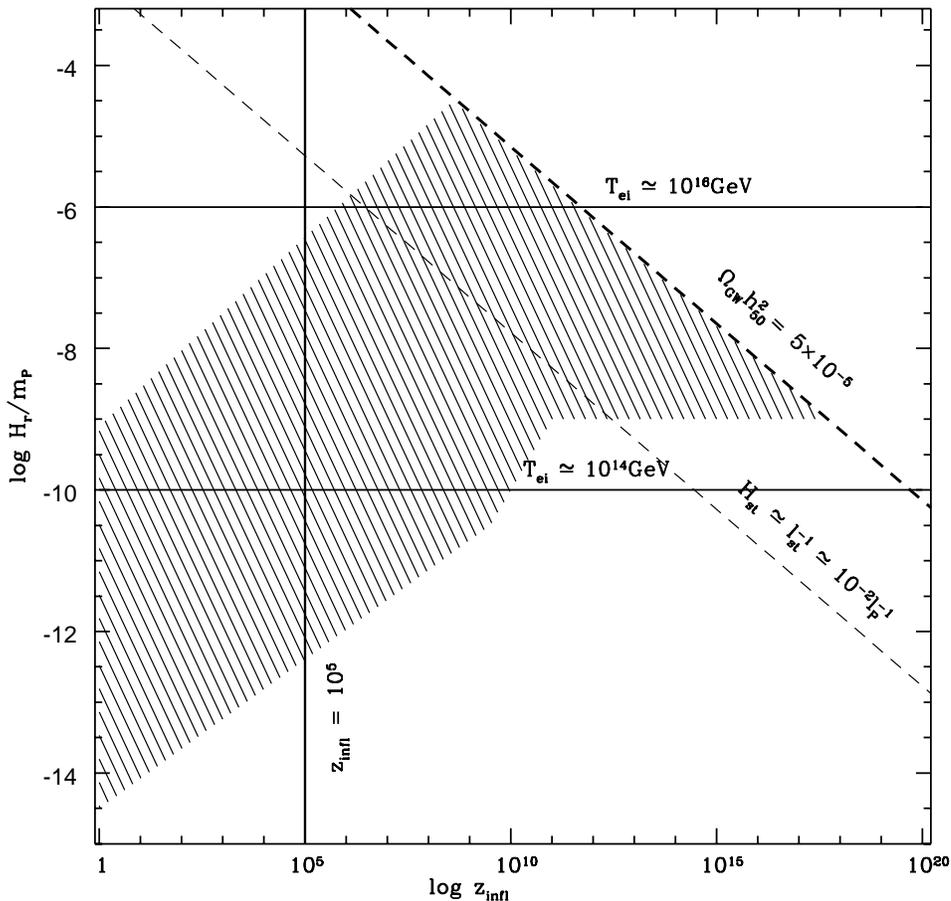


FIG. 6. The shading shows the parameter region for which the structured peak could be detected by LIGO. A sensible model falls in the area $z_{infl} > 10^5$ (monopoles), $10^{-10} < H_r/m_P < 10^{-6}$ (GUT), and $\Omega_g h_{50}^2 < 5 \cdot 10^{-5}$ (nucleosynthesis). The final constraint $10^{-2} < g_s < 10^{-1}$ gives the region between the dashed lines.

In addition, we must require that the graviton energy density does not exceed the nucleosynthesis bound [17], ($\Omega_g h_{50}^2 < 5 \times 10^{-5}$ and that the inflationary era solves the monopole problem (limit from the nucleosynthesis, $N_{infl} \geq 10$, i.e. $z_{infl} \geq 5$). Furthermore the low energy limit of string theory and GUT models constrain H_r to be in the range $10^{-10} < H_r/m_P < 10^{-6}$, that means $10^{-16} < T/GeV < 10^{-14}$ [18]. Finally, the value of z_{st} is such that the maximum of $H = H_{st} \simeq l_{st}^{-1}$ is comparable to the string value $10^{-2} < H_{st}/m_P < 10^{-1}$. The region defined by the above constraints has non zero intersec-

tion with the shaded region.

III. CONCLUSIONS

We have shown that the full stringy phase ($t \simeq 0$) is fundamental for detecting a primordial signature of string cosmology, while it is commonly assumed that only the PBB, pure dilaton era is relevant.

To obtain this result we have studied both numerically and analytically the spectra of primordial gravitons and we have proved that the latter has a distinct peak. Our model has three free parameters: we find the region of parameter space which satisfies all the known constraints. In this region both the frequency and the amplitude of the peak may fall within the realm of the new interferometric detectors. A typical value for the peak is $h_{50}^2 \Omega_M \simeq 5 \times 10^{-5}$ at the frequency $\omega_M \simeq 10$ Hz.

Finally we remark that, for simplicity's sake, we have framed our formulation in a four dimensional space-time. In reality, our results hold true for any $D \geq 4$ dimensional space-time which allows the compactification of the internal dimensions [22]. This is so because, when the proper conformal frame is chosen [20], [21], the gravitational wave equation still takes the form (2.2), that is independent of D .

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