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The Equivalence Theorem And Its Radiative Correction-Free Formulation For All R_ξ Gauges

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Abstract

The electroweak equivalence theorem quantitatively connects the physical amplitudes of longitudinal massive gauge bosons to those of the corresponding *unphysical* would-be Goldstone bosons. Its precise form depends on both the gauge fixing condition and the renormalization scheme. Our previous modification-free schemes have applied to a broad class of R_ξ gauges including 't Hooft-Feynman gauge but excluding Landau gauge. In this paper we construct a new renormalization scheme in which the radiative modification factor, $C^a_{\rm mod}$, is equal to unity for all R_ξ-gauges, including both 't Hooft-Feynman and Landau gauges. This scheme makes $C^a_{\rm mod}$ equal to unity by specifying a convenient subtraction condition for the would-be Goldstone boson wavefunction renormalization constant Z_{ϕ^a}. We build the new scheme for both the standard model and the effective Lagrangian formulated electroweak theories (with either linearly or non-linearly realized symmetry breaking sector).

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1. Introduction

The electroweak equivalence theorem (ET) [1]-[10], quantitatively connects the high energy scattering amplitudes of longitudinally polarized weak gauge bosons ($V_L^a = W_L^{\pm}, Z_L$) to the corresponding amplitudes of would-be Goldstone bosons ($\phi^a = \phi^{\pm}, \phi^0$). The ET has been widely used and has proven to be a powerful tool in studying the electroweak symmetry breaking (EWSB) mechanism, which remains a mystery and awaits experimental exploration at the CERN Large Hadron Collider (LHC) and the future linear colliders.

After some initial proposals [1], Chanowitz and Gaillard [2] gave the first general formulation of the ET for an arbitrary number of external longitudinal vector bosons and pointed out the non-trivial cancellation of terms growing like powers of the large energy which arise from external longitudinal polarization vectors. The existence of radiative modification factors to the ET was revealed by Yao and Yuan and further discussed by Bagger and Schmidt [3]. In recent systematic investigations, the precise formulation for the ET has been given for both the standard model (SM) [4, 5, 7] and chiral Lagrangian formulated electroweak theories (CLEWT) [6], in which convenient renormalization schemes for exactly simplifying these modification factors have been proposed for a class of R_ξ-gauges. A further general study of both multiplicative and additive modification factors [cf. eq. (1.1)] has been performed in Ref. [8, 9] for both the SM and CLEWT, by analyzing the longitudinal-transverse ambiguity and the physical content of the ET as a criterion for probing the EWSB sector. According to these studies, the ET can be precisely formulated as [2]-[9]

$$T[V_L^{a_1},\cdots,V_L^{a_n};\Phi_\alpha] = C\cdot T[i\phi^{a_1},\cdots,i\phi^{a_n};\Phi_\alpha] + B \ , \tag{1.1}$$

$$\begin{aligned}
C \ &\equiv C_{\rm mod}^{a_1}\cdots C_{\rm mod}^{a_n} = 1 + O(\text{loop}) \ , \\
B \ &\equiv \textstyle\sum_{l=1}^{n}(\ C_{\rm mod}^{a_{l+1}}\cdots C_{\rm mod}^{a_n} T[v^{a_1},\cdots,v^{a_l},i\phi^{a_{l+1}},\cdots,i\phi^{a_n};\Phi_\alpha] + \ \text{permutations of } v\text{'s and } \phi\text{'s}\ ) \\
&= O(M_W/E_j)\text{−suppressed} \\
&v^a \equiv v^\mu V_\mu^a \ , \quad v^\mu \equiv \epsilon_L^\mu - k^\mu/M_V = O(M_V/E) \ , \ \ (M_V = M_W, M_Z) \ ,
\end{aligned} \tag{1.1a, b, c}$$

with the conditions

$$\begin{aligned}
&E_j \sim k_j \gg M_W \ , \qquad (\ j = 1,2,\cdots,n\ ) \ , \\
&C\cdot T[i\phi^{a_1},\cdots,i\phi^{a_n};\Phi_\alpha] \gg B \ ,
\end{aligned} \tag{1.2a, b}$$

where ϕ^a's are the Goldstone boson fields and Φ_α denotes other possible physical in/out states. $C^a_{\rm mod} = 1 + O(\rm loop)$ is a renormalization-scheme and gauge dependent constant called the modification factor, and E_j is the energy of the j-th external line. For $E_j \gg M_W$, the B-term is only $O(M_W/E_j)$-suppressed relative to the leading term [8],

$$B = O\left(\frac{M_W^2}{E_j^2}\right) T[i\phi^{a_1}, \cdots, i\phi^{a_n}; \Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) T[V_{T_j}^{a_{r_1}}, i\phi^{a_{r_2}}, \cdots, i\phi^{a_{r_n}}; \Phi_\alpha] \quad . \qquad (1.3)$$

Therefore it can be either larger or smaller than $O(M_W/E_j)$, depending on the magnitudes of the ϕ^a-amplitudes on the RHS of (1.3) [8, 9]. For example, in the CLEWT, it was found that $B = O(g^2)$ [8, 9, 10], which is a constant depending only on the SM gauge coupling constant and does not vanish with increasing energy. Thus, the condition (1.2a) is necessary but not sufficient for ignoring the whole B-term. For sufficiency, the condition (1.2b) must also be imposed [8]. In section 3.3, we shall discuss minimizing the approximation from ignoring the B-term when going beyond lowest order calculations.

In the present work, we shall primarily study the simplification of the radiative modification factors, $C^a_{\rm mod}$, to unity. As shown in (1.1), the modification factors differ from unity at loop levels for all external would-be Goldstone bosons, and are not suppressed by the M_W/E_j-factor. Furthermore, these modification factors depend on both the gauge and scalar coupling constants [4, 5]. Although $C^a_{\rm mod} - 1 = O(\rm loop)$, this does *not* mean that $C^a_{\rm mod}$-factors cannot appear at the leading order of a perturbative expansion. An example is the $1/\mathcal{N}$-expansion [11] in which the leading order contributions include an infinite number of Goldstone boson loops so that the $C^a_{\rm mod}$'s will survive the large-$\mathcal{N}$ limit if the renormalization scheme is not properly chosen. In general, the appearance of $C^a_{\rm mod}$'s at loop levels alters the high energy equivalence between V_L and Goldstone boson amplitudes and potentially invalidates the naïve intuition gained from tree level calculations. For practical applications of the ET at loop levels, the modification factors complicate the calculations and reduce the utility of the equivalence theorem. Thus, the simplification of $C^a_{\rm mod}$ to unity is very useful.

The factor $C^a_{\rm mod}$ has been derived in the general R_ξ-gauges for both the SM [4, 5] and CLEWT [6], and been simplified to unity in a renormalization scheme, called *Scheme-II* in those references, for a broad class of R_ξ-gauges. *Scheme-II* is particularly convenient for 't Hooft-Feynman gauge, but cannot be applied to Landau gauge. In the present work, we make a natural generalization of our formalism and construct a new scheme, which we call *Scheme-IV*, for *all* R_ξ-gauges including both 't Hooft-Feynman and Landau gauges. In the Landau gauge, the exact simplification of $C^a_{\rm mod}$ is straightforward for the $U(1)$ Higgs theory [5, 7]; but, for the realistic non-Abelian theories (such as the SM and CLEWT) the situation is much more complicated. Earlier Landau gauge formulations of the non-Abelian case relied on explicit calculation of new loop level quantities, Δ_i^a, involving the Faddeev-Popov ghosts [5, 12].

This new *Scheme-IV* proves particularly convenient for Landau gauge. This is very useful since Landau gauge has been widely used in the literature and proves particularly convenient for studying dynamical EWSB. For instance, in the CLEWT, the complicated non-linear Goldstone boson-ghost interaction vertices from the Faddeev-Popov term (and the corresponding higher dimensional counter terms) vanish in Landau gauge, while the Goldstone boson and ghost fields remain exactly massless [13].

In the following analysis, we shall adopt the notation of references [4, 5] unless otherwise specified. This paper is organized as follows: In section 2 we derive the necessary Ward-Takahashi (WT) identities and construct our new renormalization scheme. In section 3, we derive the precise formulation of *Scheme-IV* such that the ET is free from radiative modifications (i.e., $C^a_{\mathrm{mod}} = 1$) in all R_ξ-gauges including both Landau and 't Hooft-Feynman gauges. This is done for a variety of models including the $SU(2)_L$ Higgs theory, the full SM, and both the linearly and non-linearly realized CLEWT. We further propose a convenient new prescription, called the " Divided Equivalence Theorem " (DET), for minimizing the error caused by ignoring the B-term. Finally, we discuss the relation of *Scheme-IV* to our previous schemes. In section 4, we perform explicit one-loop calculations to demonstrate our results. Conclusions are given in section 5.

2. The Radiative Modification Factor C^a_{mod} and Renormalization *Scheme-IV*

In the first part of this section, we shall define our model and briefly explain how the radiative modification factor to the ET (C^a_{mod}) originates from the quantization and renormalization procedures. Then, we analyze the properties of the C^a_{mod} in different gauges and at different loop levels. This will provide the necessary preliminaries for our main analyses and make this paper self-contained. In the second part of this section, using WT identities, we construct the new renormalization *Scheme-IV* for the exact simplification of the C^a_{mod}-factor in *all* R_ξ-gauges including both 't Hooft-Feynman and Landau gauges. Our prescription for obtaining $C^a_{\mathrm{mod}} = 1$ *does not require any explicit calculations beyond* *those needed for the usual on-shell renormalization program.*

2.1. The Radiative Modification Factor $C^a_{\rm mod}$

For simplicity, we shall first derive our results in the $SU(2)_L$ Higgs theory by taking $g'=0$ in the electroweak $SU(2)_L\otimes U(1)_Y$ standard model (SM). The generalizations to the full SM and to the effective Lagrangian formulations are straightforward (though there are some further complications) and will be given in later sections. The field content for the $SU(2)_L$ Higgs theory consists of the physical fields, H, W^a_μ, and $f(\bar f)$ representing the Higgs, the weak gauge bosons and the fermions, respectively, and the unphysical fields ϕ^a, c^a, and $\bar c^a$, representing the would-be Goldstone bosons, the Faddeev-Popov ghosts, and the anti-ghosts respectively. We quantize the theory using the following general R_ξ-gauge fixing condition

$$
\begin{aligned}
\mathcal{L}_{\rm GF} &= -\frac{1}{2}(F^a_0)^2 \;, \\
F^a_0 &= (\xi^a_0)^{-\frac{1}{2}}\partial_\mu W^{a\mu}_0-(\xi^a_0)^{\frac{1}{2}}\kappa^a_0\phi^a_0 \;=\; (\underline{\mathbf{K}}^a_0)^T\underline{\mathbf{W}}^a_0 \;, \\
\underline{\mathbf{K}}^a_0 &\equiv \left((\xi^a_0)^{-\frac{1}{2}}\partial_\mu,\,-(\xi^a_0)^{\frac{1}{2}}\kappa^a_0\right)^T \;,\quad \underline{\mathbf{W}}^a_0\equiv (W^{a\mu}_0,\phi^a_0)^T \;,
\end{aligned}
\tag{2.1}
$$

where the subscript "$_0$" denotes bare quantities. For the case of the $SU(2)_L$ theory, we can take $\xi^a_0=\xi_0$, $\kappa^a_0=\kappa_0$, for $a=1,2,3$. The quantized bare Lagrangian for the $SU(2)_L$ model is

$$
\mathcal{L}_{SU(2)_L}=-\frac{1}{4}W^{a\mu\nu}_0W^a_{0\mu\nu}+|D^\mu_0\Phi_0|^2-U_0\left(|\Phi_0|^2\right)-\frac{1}{2}(F^a_0)^2+(\xi^a_0)^{\frac{1}{2}}\bar c^a_0\hat s F^a_0+\;\mathcal{L}_{\rm fermion} \tag{2.2}
$$

where $\hat s$ is the Becchi-Rouet-Stora-Tyutin (BRST) [15] transformation operator. Since our analysis and formulation of the ET do not rely on any details of the Higgs potential or the fermionic part, we do not list their explicit forms here.

The Ward-Takahashi (WT) and Slavnov-Taylor (ST) identities of a non-Abelian gauge theory are most conveniently derived from the BRST symmetry of the quantized action. The transformations of the bare fields are

$$
\begin{aligned}
&\hat sW^{a\mu}_0=D^{a\mu}_0c^a_0=\partial^\mu c^a_0+g_0\varepsilon^{abc}\left[\!\left[W^{\mu b}_0c^c_0\right]\!\right] \;, && \hat sH_0=-\frac{g_0}{2}\left[\!\left[\phi^a_0c^a_0\right]\!\right] \;,\\
&\hat s\phi^a_0=D^\phi_0c^a_0=M_{W0}c^a_0+\frac{g_0}{2}\left[\!\left[H_0c^a_0\right]\!\right]+\frac{g_0}{2}\varepsilon^{abc}\left[\!\left[\phi^b_0c^c_0\right]\!\right] \;, && \hat sc^a_0=-\frac{g_0}{2}\varepsilon^{abc}\left[\!\left[c^b_0c^c_0\right]\!\right] \;,\\
&\hat sF^a_0=\xi_0^{-\frac{1}{2}}\cdot\partial_\mu\hat sW^{a\mu}_0-\xi_0^{\frac{1}{2}}\kappa_0\cdot\hat s\phi^a_0 \;, && \hat s\bar c^a_0=-\xi_0^{-\frac{1}{2}}F^a_0 \;,
\end{aligned}
\tag{2.3}
$$

where expressions such as $\left[\!\left[W_0^{\mu b} c_0^c\right]\!\right](x)$ indicate the local composite operator fields formed from $W_0^{\mu b}(x)$ and $c_0^c(x)$.

The appearance of the modification factor $C_{\rm mod}^a$ to the ET is due to the amputation and the renormalization of external massive gauge bosons and their corresponding Goldstone boson fields. For the amputation, we need a general ST identity for the propagators of the gauge boson, Goldstone boson and their mixing [3]-[5]. By introducing the external source term $\int dx^4 [J_i \chi_0^i + \bar{I}^a c_0^a + \bar{c}_0^a I^a]$ (where χ_0^i denotes any possible fields except the (anti-)ghost fields) to the generating functional, we get the following generating equation for connected Green functions:

$$0 = J_i(x) < 0|T\hat{s}\chi_i^a(x)|0> - \bar{I}^a(x) < 0|T\hat{s}c_0^a(x)|0> + < 0|T\hat{s}\bar{c}_0^a(x)|0> I^a(x) \qquad (2.4)$$

from which we can derive the ST identity for the matrix propagator of $\underline{\mathbf{W}}_0^a$,

$$\underline{\mathbf{K}}_0^T \underline{\mathbf{D}}_0^{ab}(k) \;=\; -\left[\underline{\mathbf{X}}^{ab}\right]^T(k) \qquad (2.5)$$

with

$$\underline{\mathbf{D}}_0^{ab}(k) \;=\; < 0|T\underline{\mathbf{W}}_0^a(\underline{\mathbf{W}}_0^b)^T|0>(k) \;\;, \qquad S_0(k)\delta^{ab} \;=\; < 0|Tc_0^b\bar{c}_0^a|0>(k) \;\;, \qquad (2.5a)$$

$$\underline{\mathbf{X}}^{ab}(k) \;\equiv\; \underline{\hat{\mathbf{X}}}^{ab}(k) S_0(k) \;\equiv\; \begin{pmatrix} \xi_0^{\frac{1}{2}} < 0|T\hat{s}W_0^{b\mu}|0> \\ \xi_0^{\frac{1}{2}} < 0|T\hat{s}\phi_0^b|0> \end{pmatrix}_{(k)} \cdot S_0(k) \;. \qquad (2.5b)$$

To explain how the modification factor $C_{\rm mod}^a$ to the ET arises, we start from the well-known ST identity [2]-[5] $< 0|F_0^{a_1}(k_1)\cdots F_0^{a_n}(k_n)\Phi_\alpha|0>= 0$ and set $n = 1$, i.e.,

$$0 = G[F_0^a(k);\Phi_\alpha] = \underline{\mathbf{K}}_0^T G[\underline{\mathbf{W}}_0^a(k);\Phi_\alpha] = -[\underline{\mathbf{X}}^{ab}]^T T[\underline{\mathbf{W}}_0^a(k);\Phi_\alpha] \;. \qquad (2.6)$$

Here $G[\cdots]$ and $T[\cdots]$ denote the Green function and the S-matrix element, respectively. The identity (2.6) leads directly to

$$\frac{k_\mu}{M_{W0}} T[W_0^{a\mu}(k);\Phi_\alpha] = \widehat{C}_0^a(k^2) T[i\phi_0^a;\Phi_\alpha] \qquad (2.7)$$

with $\widehat{C}_0^a(k^2)$ defined as

$$\widehat{C}_0^a(k^2) \equiv \frac{1+\Delta_1^a(k^2)+\Delta_2^a(k^2)}{1+\Delta_3^a(k^2)} \;\;, \qquad (2.8)$$

in which the quantities Δ_i^a are the proper vertices of the composite operators

$$\begin{aligned}
\Delta_1^a(k^2)\delta^{ab} &= \frac{g_0}{2M_{W0}} < 0|T\left[\!\left[H_0 c_0^b\right]\!\right]|\bar{c}_0^a > (k) \ , \\
\Delta_2^a(k^2)\delta^{ab} &= -\frac{g_0}{2M_{W0}}\varepsilon^{bcd} < 0|T\left[\!\left[\phi_0^c c_0^d\right]\!\right]|\bar{c}_0^a > (k) \ , \\
ik^\mu\Delta_3^a(k^2)\delta^{ab} &= -\frac{g_0}{2}\varepsilon^{bcd} < 0|T\left[\!\left[W_0^{\mu b} c_0^c\right]\!\right]|\bar{c}_0^a > (k) \ ,
\end{aligned} \tag{2.9}$$

which are shown diagrammatically in figure 1.

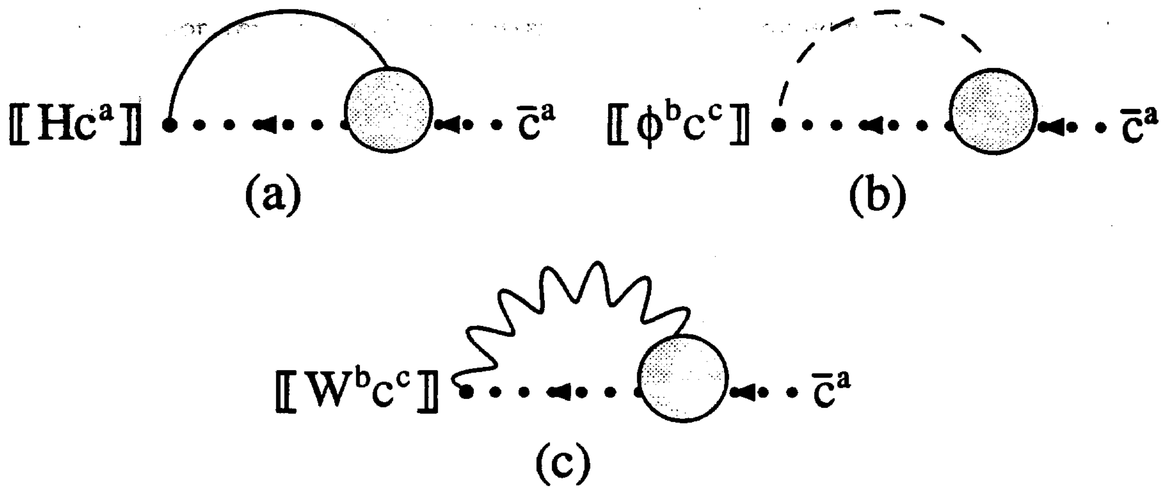

Figure 1.

Composite operator diagrams contributing to radiative modification factor of the equivalence theorem in non-Abelian Higgs theories. (a). Δ_1^a ; (b). Δ_2^a ; (c). Δ_3^a .

After renormalization, (2.8) becomes

$$\frac{k_\mu}{M_W}T[W^{a\mu}(k);\Phi_\alpha] = \widehat{C}^a(k^2)T[i\phi^a;\Phi_\alpha] \tag{2.10}$$

with the finite renormalized coefficient

$$\widehat{C}^a(k^2) = Z_{M_W}\left(\frac{Z_W}{Z_\phi}\right)^{\frac{1}{2}}\widehat{C}_0^a(k^2) \ . \tag{2.11}$$

The renormalization constants are defined as $W_0^{a\mu} = Z_W^{\frac{1}{2}}W^{a\mu}$, $\phi_0^a = Z_\phi^{\frac{1}{2}}\phi^a$, and $M_{W0} = Z_{M_W}M_W$. The modification factor to the ET is precisely the value of this finite renormalized coefficient $\widehat{C}^a(k^2)$ on the gauge boson mass-shell:

$$C_{\rm mod}^a = \widehat{C}^a(k^2)\Big|_{k^2=M_W^2} \ , \tag{2.12}$$

provided that the usual on-shell subtraction for M_W is adopted. In Sec. 3, we shall transform the identity (2.10) into the final form of the ET (which connects the W_L^a-amplitude to that of the corresponding ϕ^a-amplitude) for an arbitrary number of external longitudinal gauge bosons and obtain a *modification-free* formulation of the ET with $C_{\rm mod}^a = 1$ to all loop orders.

As shown above, the appearance of the $C_{\rm mod}^a$ factor to the ET is due to the amputation and renormalization of external $W^{a\mu}$ and ϕ^a lines by using the ST identity (2.5). Thus it is natural that the $C_{\rm mod}^a$ factor contains $W^{a\mu}$-ghost, ϕ^a-ghost and Higgs-ghost interactions expressed in terms of these Δ_i^a-quantities. Further simplification can be made by re-expressing $C_{\rm mod}^a$ in terms of known $W^{a\mu}$ and ϕ^a proper self-energies using WT identities as first proposed in Refs. [4]-[6]. This step is the basis of our simplification of $C_{\rm mod}^a = 1$ and will be also adopted for constructing our new *Scheme-IV* in Sec. 2.2. We must emphasize that, ***our simplification of*** $C_{\rm mod}^a = 1$ ***does not need any explicit calculation of the new loop-level*** Δ_i^a***-quantities*** which involve ghost interactions and are quite complicated. This is precisely why our simplification procedure is useful.

Finally, we analyze the properties of the Δ_i^a-quantities in different gauges and at different loop-levels. The loop-level Δ_i^a-quantities are non-vanishing in general and make $\widehat{C}_0(k^2) \neq 1$ and $C_{\rm mod}^a \neq 1$ order by order. In Landau gauge, these Δ_i^a-quantities can be partially simplified, especially at the one-loop order, because the tree-level Higgs-ghost and ϕ^a-ghost vertices vanish. This makes $\Delta_{1,2}^a = 0$ at one loop.[a] In general,

$$\Delta_1^a = \Delta_2^a = 0 + O(2\ \text{loop})\ , \quad \Delta_3^a = O(1\ \text{loop})\ , \qquad (\text{ in Landau gauge })\ . \qquad (2.13)$$

Beyond the one-loop order, $\Delta_{1,2}^a \neq 0$ since the Higgs and Goldstone boson fields can still indirectly couple to the ghosts via loop diagrams containing internal gauge fields, as shown in Figure 2.

[a] We note that, in the non-Abelian case, the statement that $\Delta_{1,2}^a = 0$ for Landau gauge in Refs. [3, 5] is only valid at the one-loop order.

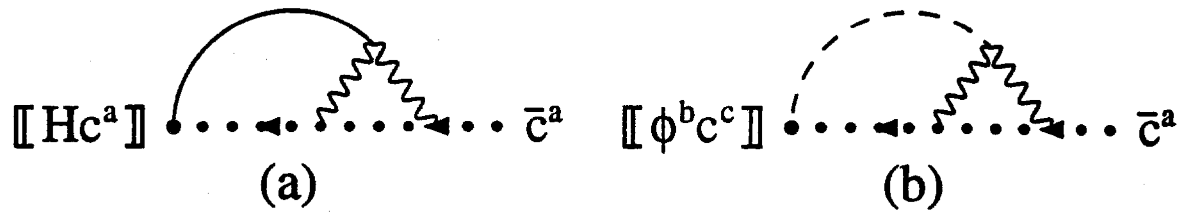

Figure 2.

The lowest order diagrams contributing to $\Delta_1^a(k^2)$ and $\Delta_2^a(k^2)$ in Landau gauge.

We note that the 2-loop diagram Figure 2b is non-vanishing in the full SM due to the tri-linear A_μ-$W_\nu^\pm$-$\phi^\mp$ and Z_μ-$W_\nu^\pm$-$\phi^\mp$ vertices, while it vanishes in the $SU(2)_L$ theory since the couplings of these tri-linear vertices are proportional to $\sin^2\theta_W$.

2.2. Construction of Renormalization Scheme-IV

From the generating equation for WT identities [5, 14], we obtain a set of identities for bare inverse propagators which contain the bare modification factor $\widehat{C}_0^a(k^2)$ [derived in (2.8), (2.9) and (2.12)] [4, 5]

$$\begin{aligned} ik^\mu[i\mathcal{D}_{0,\mu\nu,ab}^{-1}(k)+\xi_0^{-1}k_\mu k_\nu] &= -M_{W0}\widehat{C}_0^a(k^2)[i\mathcal{D}_{0,\phi\nu,ab}^{-1}(k)-i\kappa_0 k_\nu] \\ ik^\mu[-i\mathcal{D}_{0,\phi\mu,ab}^{-1}(k)+i\kappa_0 k_\mu] &= -M_{W0}\widehat{C}_0^a(k^2)[i\mathcal{D}_{0,\phi\phi,ab}^{-1}(k)+\xi_0\kappa_0^2] \\ i\mathcal{S}_{0,ab}^{-1}(k) &= [1+\Delta_3^a(k^2)][k^2-\xi_0\kappa_0 M_{W0}\widehat{C}_0^a(k^2)]\delta_{ab} \end{aligned} \tag{2.14}$$

where $\mathcal{D}_{0,\mu\nu}$, $\mathcal{D}_{0,\phi\nu}$, $\mathcal{D}_{0,\phi\phi}$, $\mathcal{S}_{0,ab}$ are the unrenormalized full propagators for gauge boson, gauge-Goldstone-boson mixing and ghost, respectively.

The renormalization program is chosen to match the on-shell scheme [16] for the physical degrees of freedoms, since this is very convenient and popular for computing the electroweak radiative corrections (especially for high energy processes). Among other things, this choice means that the proper self energies of physical particles are renormalized so as to vanish on their mass-shells, and that the vacuum expectation value of the Higgs field is renormalized such that the tadpole graphs are exactly cancelled. If the vacuum expectation value were not renormalized in this way, there would be tadpole contributions to figure 1a.

The renormalization constants of the unphysical degrees of freedoms are defined as

$$\phi_0^a = Z_\phi^{\frac{1}{2}}\phi^a\ ,\quad c_0^a = Z_c c^a\ ,\quad \bar{c}_0^a = \bar{c}^a\ ,\quad \xi_0^a = Z_\xi \xi^a\ ,\quad \kappa_0^a = Z_\kappa \kappa^a\ . \tag{2.15}$$

Some of these renormalization constants will be chosen such that the ET is free from radiative modifications, while the others are left to be determined as usual [16] so that our scheme is most convenient for the practical application of the ET. Using (2.15) and the relations $\mathcal{D}_{0,\mu\nu} = Z_W \mathcal{D}_{\mu\nu}$, $\mathcal{D}_{0,\phi\nu} = Z_\phi^{\frac{1}{2}} Z_W^{\frac{1}{2}} \mathcal{D}_{\phi\nu}$, and $\mathcal{S}_0 = Z_c \mathcal{S}$, we obtain the renormalized identities

$$\begin{aligned}
ik^\mu[i\mathcal{D}^{-1}_{\mu\nu,ab}(k) + \tfrac{Z_W}{Z_\xi}\xi^{-1}k_\mu k_\nu] &= -\widehat{C}^a(k^2)M_W[i\mathcal{D}^{-1}_{\phi\nu,ab}(k) - Z_\kappa Z_W^{\frac{1}{2}} Z_\phi^{\frac{1}{2}} ik_\nu\kappa] \\
ik^\mu[-i\mathcal{D}^{-1}_{\phi\mu,ab}(k) + Z_\kappa Z_W^{\frac{1}{2}} Z_\phi^{\frac{1}{2}} ik_\mu\kappa] &= -\widehat{C}^a(k^2)M_W[i\mathcal{D}^{-1}_{\phi\phi,ab}(k) + Z_\kappa^2 Z_\xi Z_\phi \xi\kappa^2] \qquad (2.16) \\
i\mathcal{S}^{-1}_{ab}(k) &= Z_c[1+\Delta_3(k^2)][k^2 - \xi\kappa M_W Z_\xi Z_\kappa (\tfrac{Z_\phi}{Z_W})^{\frac{1}{2}} \widehat{C}^a(k^2)]\delta_{ab}
\end{aligned}$$

Note that the renormalized coefficient $\widehat{C}^a(k^2)$ appearing in (2.16) is precisely the same as that in (2.12).

Constraints on Z_ξ, Z_κ, Z_ϕ and Z_c can be drawn from the fact that the coefficients in the renormalized identities of (2.16) are finite. This implies that

$$\begin{aligned}
&Z_\xi = \Omega_\xi Z_W \ , && Z_\kappa = \Omega_\kappa Z_W^{\frac{1}{2}} Z_\phi^{-\frac{1}{2}} Z_\xi^{-1} \ , \\
&Z_\phi = \Omega_\phi Z_W Z^2_{M_W} \hat{C}_0(sub.\ point) \ , && Z_c = \Omega_c[1+\Delta_3(sub.\ point)]^{-1} \ ,
\end{aligned} \qquad (2.17)$$

with

$$\Omega_{\xi,\kappa,\phi,c} \ = \ 1 + O(\text{loop}) \ = \ \text{finite} \ , \qquad (2.17a)$$

where Ω_ξ , Ω_κ , Ω_ϕ and Ω_c are unphysical and arbitrary finite constants to be determined by the subtraction conditions.

The propagators are expressed in terms of the proper self-energies as

$$\begin{aligned}
i\mathcal{D}^{-1}_{\mu\nu,ab}(k) &= \Big[\left(g_{\mu\nu} - \tfrac{k_\mu k_\nu}{k^2}\right)\left(-k^2 + M_W^2 - \Pi^a_{WW}(k^2)\right) \\
&\qquad + \tfrac{k_\mu k_\nu}{k^2}\left(-\xi^{-1}k^2 + M_W^2 - \widetilde{\Pi}^a_{WW}(k^2)\right)\Big]\delta_{ab} \ , \\
i\mathcal{D}^{-1}_{\phi\mu,ab}(k) &= -ik_\mu\left(M_W - \kappa + \widetilde{\Pi}^a_{W\phi}(k^2)\right)\delta_{ab} \ , \qquad (2.18) \\
i\mathcal{D}^{-1}_{\phi\phi,ab}(k) &= \left(k^2 - \xi\kappa^2 - \widetilde{\Pi}^a_{\phi\phi}(k^2)\right)\delta_{ab} \ , \\
i\mathcal{S}^{-1}_{ab}(k) &= \left(k^2 - \xi\kappa M_W - \widetilde{\Pi}^a_{c\bar{c}}(k^2)\right)\delta_{ab} \ ,
\end{aligned}$$

where Π^a_{WW} is the proper self-energy of the physical part of the gauge boson, and the $\widetilde{\Pi}^a_{ij}$'s are the unphysical proper self-energies.

Expanding the propagators in (2.16) in terms of proper self-energies yields the following identities containing $\widehat{C}(k^2)$:

$$\begin{aligned}
\widehat{C}^a(k^2) &= \frac{\xi^{-1}k^2\left(\Omega_\xi^{-1}-1\right)+M_W^2-\widetilde{\Pi}^a_{WW}(k^2)}{M_W\kappa\left(\Omega_\kappa\Omega_\xi^{-1}-1\right)+M_W^2+M_W\widetilde{\Pi}^a_{W\phi}(k^2)} \ , \\
\widehat{C}^a(k^2) &= \frac{k^2}{M_W}\frac{\kappa\left(\Omega_\kappa\Omega_\xi^{-1}-1\right)+M_W+\widetilde{\Pi}^a_{W\phi}(k^2)}{\xi\kappa^2\left(\Omega_\kappa^2\Omega_\xi^{-1}-1\right)+k^2-\widetilde{\Pi}^a_{\phi\phi}(k^2)} \ , \\
\widetilde{\Pi}^a_{c\bar{c}}(k^2) &= k^2-\xi\kappa M_W-Z_c\left[1+\Delta_3(k^2)\right]\left[M_W^2-\xi\kappa\Omega_\kappa M_W\widehat{C}^a(k^2)\right] \ .
\end{aligned} \tag{2.19}$$

We are now ready to construct of our new renormalization scheme, *Scheme-IV* , which will insure $\widehat{C}(M_W^2) = 1$ for all R_ξ-gauges, including Landau gauge. The R_ξ-gauges are a continuous one parameter family of gauge-fixing conditions [cf. (2.1)] in which the parameter ξ takes values from 0 to ∞ . In practice, however, there are only three important special cases: the Landau gauge ($\xi = 0$), the 't Hooft-Feynman gauge ($\xi = 1$) and unitary gauge ($\xi \to \infty$). In the unitary gauge, the unphysical degrees of freedom freeze out and one cannot discuss the amplitude for the would-be Goldstone bosons. In addition, the loop renormalization becomes inconvenient in this gauge due to the bad high energy behavior of massive gauge-boson propagators and the resulting complication of the divergence structure. The 't Hooft-Feynman gauge offers great calculational advantages, since the gauge boson propagator takes a very simple form and the tree-level mass poles of each weak gauge boson and its corresponding Goldstone boson and ghost are all the same. The Landau gauge proves very convenient in the electroweak chiral Lagrangian formalism [13] by fully removing the complicated tree-level non-linear Goldstone boson-ghost interactions [cf. Sec. 3.2] and in this gauge unphysical would-be Goldstones are exactly massless like true Goldstone bosons.

To construct the new *Scheme-IV* , we note that *a priori*, we have six free parameters to be specified: ξ , κ , Z_ϕ , Z_c , Ω_ξ , and Ω_κ in a general R_ξ-gauge. For Landau gauge ($\xi = 0$), the gauge-fixing term $\mathcal{L}_{\rm GF}$ [cf. (2.1)] gives vanishing Goldstone-boson masses without any κ-dependence, and the bi-linear gauge-boson vertex $-\frac{1}{2\xi_0}(\partial_\mu W_0^\mu)^2$ diverges, implying that the W-propagator is transverse and independent of Ω_ξ . The only finite term left in $\mathcal{L}_{\rm GF}$ for Landau gauge is the gauge-Goldstone mixing vertex $\kappa_0\phi_0\partial_\mu W_0^\mu = \Omega_\xi^{-1}\Omega_\kappa\kappa\phi\partial_\mu W^\mu$ [cf. (2.17)], which will cancel the tree-level W-ϕ mixing from the Higgs kinetic term $|D_0^\mu\Phi_0|^2$ in (2.2) provided that we choose $\kappa = M_W$. Hence, for the purpose of including Landau gauge into our *Scheme-IV* , we shall not make use of the degree of freedoms from Ω_ξ and Ω_κ , and in order to remove the tree-level W-ϕ mixing, we shall set $\kappa = M_W$. Thus, we fix the free parameters Ω_ξ , Ω_κ and κ as follows

$$\Omega_\xi \;=\; \Omega_\kappa \;=\; 1 \;, \qquad \kappa \;=\; M_W \;, \qquad (\text{ in } Scheme-IV\;)\;. \tag{2.20}$$

From (2.17), the choice $\Omega_\xi = \Omega_\kappa = 1$ implies

$$F_0^a = F^a \;, \tag{2.21}$$

i.e., the gauge-fixing function F_0^a is unchanged after the renormalization. For the remaining three unphysical parameters ξ , Z_ϕ and Z_c , we shall leave ξ free to cover all R_ξ-gauges and leave Z_c determined by the usual on-shell normalization condition

$$\left.\frac{d}{dk^2}\widetilde{\Pi}^a_{c\bar{c}}(k^2)\right|_{k^2=\xi M_W^2} \;=\; 0\;. \tag{2.22}$$

Therefore, in our *Scheme-IV* , *the only free parameter,* which we shall specify for insuring $\widehat{C}(M_W^2) = 1$, is the wavefunction renormalization constant Z_ϕ for the unphysical Goldstone boson.

Under the above choice (2.20), the first two equations of (2.19) become

$$\begin{aligned}\widehat{C}^a(M_W^2) \;&=\; \frac{M_W^2 - \widetilde{\Pi}^a_{WW}(M_W^2)}{M_W^2 + M_W\widetilde{\Pi}^a_{W\phi}(M_W^2)} \;=\; \frac{M_W^2 + M_W\widetilde{\Pi}^a_{W\phi}(M_W^2)}{M_W^2 - \widetilde{\Pi}^a_{\phi\phi}(M_W^2)} \\ &=\; \left[\frac{M_W^2 - \widetilde{\Pi}^a_{WW}(M_W^2)}{M_W^2 - \widetilde{\Pi}^a_{\phi\phi}(M_W^2)}\right]^{\frac{1}{2}}\;,\end{aligned} \tag{2.23}$$

at $k^2 = M_W^2$. Note that (2.23) re-expresses the factor $\widehat{C}^a(M_W^2)$ in terms of only two renormalized proper self-energies: $\widetilde{\Pi}_{\phi\phi}$ and $\widetilde{\Pi}_{WW}$ (or $\widetilde{\Pi}_{W\phi}$). We emphasize that, unlike the most general relations (2.19) adopted in Refs. [4, 5], the identity (2.23) compactly takes the *same symbolic form for any R_ξ-gauge including both 't Hooft-Feynman and Landau gauges* under the choice (2.20).[b]

From the new identity (2.23), we deduce that the modification factor $C^a(M_W^2)$ can be made equal to unity provided the condition

$$\widetilde{\Pi}^a_{\phi\phi}(M_W^2) = \widetilde{\Pi}^a_{WW}(M_W^2) \tag{2.24}$$

[b]In fact, (2.23) holds for arbitrary κ .

is imposed. This is readily done by adjusting Z_ϕ in correspondence to the unphysical arbitrary finite quantity $\Omega_\phi = 1+\delta\Omega_\phi$ in (2.17). The precise form of the needed adjustment can be determined by expressing the renormalized proper self-energies in terms of the bare proper self-energies plus the corresponding counter terms [5],

$$\begin{aligned}
\tilde{\Pi}^a_{WW}(k^2) &= \tilde{\Pi}_{WW,0}(k^2) + \delta\tilde{\Pi}_{WW} &&= Z_W\tilde{\Pi}^a_{WW,0}(k^2) + (1 - Z_W Z^2_{M_W})M^2_W \ , \\
\tilde{\Pi}^a_{W\phi}(k^2) &= \tilde{\Pi}_{W\phi,0}(k^2) + \delta\tilde{\Pi}_{W\phi} &&= (Z_W Z_\phi)^{\frac{1}{2}}\tilde{\Pi}^a_{W\phi,0}(k^2) + [(Z_W Z_\phi Z^2_{M_W})^{\frac{1}{2}} - 1]M_W \ , \\
\tilde{\Pi}^a_{\phi\phi}(k^2) &= \tilde{\Pi}_{\phi\phi,0}(k^2) + \delta\tilde{\Pi}_{\phi\phi} &&= Z_\phi\tilde{\Pi}^a_{\phi\phi,0}(k^2) + (1 - Z_\phi)k^2 \ ,
\end{aligned} \tag{2.25}$$

which, at the one-loop order, reduces to

$$\begin{aligned}
\tilde{\Pi}_{WW}(k^2) &= \tilde{\Pi}_{WW,0}(k^2) - [\delta Z_W + 2\delta Z_{M_W}]M^2_W \ , \\
\tilde{\Pi}_{W\phi}(k^2) &= \tilde{\Pi}_{W\phi,0}(k^2) - [\tfrac{1}{2}(\delta Z_W + \delta Z_\phi) + \delta Z_{M_W}]M_W \ , \\
\tilde{\Pi}_{\phi\phi}(k^2) &= \tilde{\Pi}_{\phi\phi,0}(k^2) - \delta Z_\phi k^2 \ .
\end{aligned} \tag{2.26}$$

Note that, in the above expressions for the R_ξ-gauge counter terms under the choice $\Omega_\xi = \Omega_\kappa = 1$ [cf. (2.20)], there is no explicit dependence on the gauge parameters ξ and κ so that (2.25) and (2.26) take the *same* forms for all R_ξ-gauges. From either (2.25) or (2.26), we see that in the counter terms to the self-energies there are three independent renormalization constants Z_W, Z_{M_W}, and Z_ϕ. Among them, Z_W and Z_{M_W} have been determined by the renormalization of the physical sector, such as in the on-shell scheme (which we shall adopt in this paper) [16],

$$\begin{aligned}
&\left.\frac{d}{dk^2}\Pi^a_{WW}(k^2)\right|_{k^2=M^2_W} = 0 \ , \qquad (\text{ for } Z_W \text{ }) \ ; \\
&\Pi^a_{WW}(k^2)|_{k^2=M^2_W} = 0 \ , \qquad (\text{ for } Z_{M_W} \text{ }) \ .
\end{aligned} \tag{2.27}$$

We are just left with Z_ϕ from the unphysical sector which can be adjusted, as shown in eq. (2.17). Since the ghost self-energy $\tilde{\Pi}^a_{c\bar{c}}$ is irrelevant to above identity (2.23), we do not list, in (2.25) and (2.26), the corresponding counter term $\delta\tilde{\Pi}^a_{c\bar{c}}$ which contains one more renormalization constant Z_c and will be determined as usual [cf. (2.22)]. Finally, note that we have already included the Higgs-tadpole counter term $-i\delta T$ in the bare Goldstone boson and Higgs boson self-energies, through the well-known tadpole $= 0$ condition [5, 16, 17].

Now, equating $\widetilde{\Pi}^a_{\phi\phi}(M^2_W)$ and $\widetilde{\Pi}^a_{WW}(M^2_W)$ according to (2.24), we solve for Z_ϕ:

$$Z_\phi \;=\; Z_W \frac{Z^2_{M_W} M^2_W - \widetilde{\Pi}^a_{WW,0}(M^2_W)}{M^2_W - \widetilde{\Pi}^a_{\phi\phi,0}(M^2_W)} \quad , \qquad (\text{ in } Scheme - IV\ ) \ . \tag{2.28}$$

Z_ϕ is thus expressed in terms of known quantities, that is, in terms of the renormalization constants of the physical sector and the bare unphysical proper self-energies of the gauge fields and the Goldstone boson fields, which must be computed in any practical renormalization program. We thus obtain $\widehat{C}^a(M^2_W) = 1$ without the extra work of explicitly evaluating the complicated Δ^a_i 's. At the one-loop level, the solution for $Z_\phi = 1 + \delta Z_\phi$ in (2.27) reduces to

$$\delta Z_\phi = 1 + \delta Z_W + 2\delta Z_{M_W} + M_W^{-2}\left[\widetilde{\Pi}^a_{\phi\phi,0}(M^2_W) - \widetilde{\Pi}^a_{WW,0}(M^2_W)\right] \quad . \tag{2.28a}$$

If we specialize to Landau gauge, (2.28a) can be alternatively expressed in terms of the bare ghost self-energy $\widetilde{\Pi}_{c\bar{c},0}$ plus the gauge boson renormalization constants:

$$\delta Z_\phi = 1 + \delta Z_W + 2\delta Z_{M_W} + 2M_W^{-2}\widetilde{\Pi}^a_{c\bar{c},0}(M^2_W) \quad , \qquad (\ \xi = 0\ ) \ , \tag{2.28a$'$}$$

due to the Landau gauge WT identity (valid up to one loop)

$$\widetilde{\Pi}_{\phi\phi,0}(M^2_W) - \widetilde{\Pi}_{WW,0}(M^2_W) \;=\; 2\widetilde{\Pi}_{c\bar{c},0}(M^2_W) + \ O(2\ \text{loop}) \ . \tag{2.29}$$

The validity of (2.29) can be proven directly. From the first two identities of our (2.14) we derive

$$\begin{aligned} \widehat{C}^a_0(M^2_{W0}) \;&=\; \left[\frac{M^2_{W0} - \widetilde{\Pi}^a_{WW,0}(M^2_{W0})}{M^2_{W0} - \widetilde{\Pi}^a_{\phi\phi,0}(M^2_{W0})}\right]^{\frac{1}{2}} \;=\; \widehat{C}^a_0(M^2_W) + \ O(2\ \text{loop}) \\ &=\; 1 + \frac{1}{2}M_W^{-2}\left[\widetilde{\Pi}_{\phi\phi,0}(M^2_W) - \widetilde{\Pi}_{WW,0}(M^2_W)\right] + \ O(2\ \text{loop}) \ , \end{aligned} \tag{2.30}$$

and from the third identity of (2.14) plus (2.8) and (2.13) we have

$$\begin{aligned} \widehat{C}^a_0(M^2_W) \;&=\; 1 - \Delta^a_3(M^2_W) + \ O(2\ \text{loop}) \ , \qquad (\ \xi = 0\ ) \\ &=\; 1 + M_W^{-2}\widetilde{\Pi}^a_{c\bar{c},0}(M^2_W) + \ O(2\ \text{loop}) \ , \qquad (\ \xi = 0\ ) \ . \end{aligned} \tag{3.31}$$

Thus, comparison of (2.30) with (2.31) gives our one-loop order Landau-gauge WT identity (2.29) so that (2.28a′) can be simply deduced from (2.28a). As a consistency check, we note that the same one-loop result (2.28a′) can also be directly derived from (2.8), (2.11) and (2.13) for Landau gauge by using (3.31) and requiring $C^a_{\text{mod}} = 1$, In summary, the complete definition of the *Scheme-IV* for the $SU(2)_L$ Higgs theory is as follows: The physical sector is renormalized in the conventional on-shell scheme [5, 16]. This means that the vacuum expectation value is renormalized so that tadpoles are exactly cancelled, the proper self-energies of physical states vanish on their mass-shells, and the residues of the propagator poles are normalized to unity. For the gauge sector, this means that Z_W and Z_{M_W} are determined by (2.27).

In the unphysical sector, the parameters κ, Ω_κ and Ω_ξ are chosen as in (2.20). The ghost wavefunction renormalization constant Z_c is determined as usual [cf. (2.22)]. The Goldstone wavefunction renormalization constant Z_ϕ is chosen as in (2.28) [or (2.28a)] so that $\widehat{C}(M_W^2) = 1$ is ensured. From (2.12), we see that this will automatically render the ET *modification-free.*

2.3. Scheme-IV in the Standard Model

For the full SM, the renormalization is greatly complicated due to the various mixings in the neutral sector [5, 16]. However, the first two WT identities in (2.19) take the *same* symbolic forms for both the charged and neutral sectors as shown in Ref. [5]. This makes the generalization of our *Scheme-IV* to the SM straightforward. Even so, we still have a further complication in our final result for determining the wavefunction renormalization constant Z_{ϕ^z} of the neutral Goldstone field ϕ^Z, due to the mixings in the counter term to the bare Z boson self-energy.

The SM gauge-fixing term can be compactly written as follows [5]

$$\mathcal{L}_{\rm GF} = -\frac{1}{2}(F_0^+F_0^- + F_0^-F_0^+) - \frac{1}{2}(\mathbf{F}_0^N)^T\mathbf{F}_0^N \ , \qquad (2.32)$$

where

$$\begin{aligned} F_0^\pm &= (\xi_0^\pm)^{-\frac{1}{2}}\partial_\mu W_0^{\pm\mu}(\xi_0^\pm)^{\frac{1}{2}}\kappa_0^\pm\phi_0^\pm \ , \\ \mathbf{F}_0^N &= (F_0^Z, F_0^A)^T = (\xi_0^N)^{-\frac{1}{2}}\partial_\mu \mathbf{N}_0^\mu - \bar{\kappa}_0\phi_0^Z \ , \end{aligned} \qquad (2.33)$$

and

$$\mathbf{N}_0^\mu = (Z_0^\mu, A_0^\mu)^T \ , \quad \mathbf{N}_0^\mu = \mathbf{Z}_N^{\frac{1}{2}}\mathbf{N} \ ; \quad (\xi_0^N)^{-\frac{1}{2}} = (\xi^N)^{-\frac{1}{2}}\mathbf{Z}_{\xi_N}^{-\frac{1}{2}} \ ,$$

$$(\xi_0^N)^{-\frac{1}{2}} = \begin{bmatrix} (\xi_0^Z)^{-\frac{1}{2}} & (\xi_0^{ZA})^{-\frac{1}{2}} \\ (\xi_0^{AZ})^{-\frac{1}{2}} & (\xi_0^A)^{-\frac{1}{2}} \end{bmatrix} \ , \qquad (\xi^N)^{-\frac{1}{2}} = \begin{bmatrix} (\xi^Z)^{-\frac{1}{2}} & 0 \\ 0 & (\xi^A)^{-\frac{1}{2}} \end{bmatrix} \ ; \qquad (2.34)$$

$$\bar{\kappa}_0 = \left((\xi_0^Z)^{-\frac{1}{2}}\kappa_0^Z, \ (\xi_0^A)^{-\frac{1}{2}}\kappa_0^A\right)^T \ , \quad \bar{\kappa} = \left((\xi^Z)^{-\frac{1}{2}}\kappa^Z, \ 0\right)^T \ , \quad \bar{\kappa}_0 = \mathbf{Z}_{\bar{\kappa}}\bar{\kappa} \ .$$

The construction of *Scheme-IV* for the charged sector is essentially the same as the $SU(2)_L$ theory and will be summarized below in (2.41). So, we only need to take care of the neutral sector. We can derive a set of WT identities parallel to (2.14) and (2.16) as in Ref. [5] and obtain the following constraints on the renormalization constants for ξ_N and $\bar{\kappa}$

$$\mathbf{Z}_{\xi_N}^{-\frac{1}{2}} = \Omega_{\xi_N}^{-\frac{1}{2}} \mathbf{Z}_N^{-\frac{1}{2}} \ , \quad \mathbf{Z}_{\bar{\kappa}} = \left(\xi_N^{\frac{1}{2}}\right)^T \left[\Omega_{\xi_N}^{-\frac{1}{2}}\right]^T \left(\xi_N^{-\frac{1}{2}}\right)^T \Omega_{\bar{\kappa}} Z_{\phi^z}^{-\frac{1}{2}} \ , \tag{2.35}$$

with

$$\begin{aligned} \Omega_{\xi_N}^{-\frac{1}{2}} &\equiv \begin{bmatrix} (\Omega_\xi^{ZZ})^{-\frac{1}{2}} & (\Omega_\xi^{ZA})^{-\frac{1}{2}} \\ (\Omega_\xi^{AZ})^{-\frac{1}{2}} & (\Omega_\xi^{AA})^{-\frac{1}{2}} \end{bmatrix} \equiv \begin{bmatrix} (1+\delta\Omega_\xi^{ZZ})^{-\frac{1}{2}} & -\frac{1}{2}\delta\Omega_\xi^{ZA} \\ -\frac{1}{2}\delta\Omega_\xi^{AZ} & (1+\delta\Omega_\xi^{AA})^{-\frac{1}{2}} \end{bmatrix} , \\ \Omega_{\bar{\kappa}} &\equiv \begin{bmatrix} \Omega_\kappa^{ZZ} & 0 \\ \Omega_\kappa^{AZ} & 0 \end{bmatrix} \equiv \begin{bmatrix} 1+\delta\Omega_\kappa^{ZZ} & 0 \\ \delta\Omega_\kappa^{AZ} & 0 \end{bmatrix} . \end{aligned} \tag{2.35a}$$

As in (2.20), we choose

$$\Omega_{\xi_N} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} , \quad \Omega_{\bar{\kappa}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} , \quad \kappa_Z = M_Z \ , \quad (\text{ in } Scheme-IV\ ) \ . \tag{2.36}$$

As mentioned above, in the full SM, the corresponding identities for $\widehat{C}^W(M_W^2)$ and $\widehat{C}^Z(M_Z^2)$ take the same symbolic forms as (2.23)

$$\widehat{C}^W(M_W^2) = \left[\frac{M_W^2 - \widetilde{\Pi}_{W^+W^-}(M_W^2)}{M_W^2 - \widetilde{\Pi}_{\phi^+\phi^-}(M_W^2)}\right]^{\frac{1}{2}} , \quad \widehat{C}^Z(M_Z^2) = \left[\frac{M_Z^2 - \widetilde{\Pi}_{ZZ}(M_Z^2)}{M_Z^2 - \widetilde{\Pi}_{\phi^z\phi^z}(M_Z^2)}\right]^{\frac{1}{2}} , \tag{2.37}$$

which can be simplified to unity provided that

$$\widetilde{\Pi}_{\phi^+\phi^-}(M_W^2) = \widetilde{\Pi}_{W^+W^-}(M_W^2) \ , \quad \widetilde{\Pi}_{\phi^z\phi^z}(M_Z^2) = \widetilde{\Pi}_{ZZ}(M_Z^2) \ . \tag{2.38}$$

The solution for $Z_{\phi^\pm}$ from the first condition of (2.38) is the same as in (2.28) or (2.28a), but the solution for Z_{ϕ^z} from the second condition of (2.38) is complicated due to the mixings in the $\widetilde{\Pi}_{ZZ,0}$ counter term:

$$\begin{aligned} \widetilde{\Pi}_{ZZ}(k^2) &= \widetilde{\Pi}_{ZZ,0}(k^2) + \delta\widetilde{\Pi}_{ZZ} = Z_{ZZ}\widehat{\widetilde{\Pi}}_{ZZ,0}(k^2) + (1 - Z_{ZZ}Z_{M_z}^2)M_Z^2 \ , \\ &\quad \widehat{\widetilde{\Pi}}_{ZZ,0}(k^2) \equiv \widetilde{\Pi}_{ZZ,0}(k^2) + Z_{ZZ}^{-\frac{1}{2}} Z_{AZ}^{\frac{1}{2}}[\widetilde{\Pi}_{ZA,0}(k^2) + \widetilde{\Pi}_{AZ,0}(k^2)] + Z_{ZZ}^{-1} Z_{AZ}\widetilde{\Pi}_{AA,0}(k^2) \ ; \\ \widetilde{\Pi}_{\phi^z\phi^z}(k^2) &= Z_{\phi^z}\widetilde{\Pi}_{\phi^z\phi^z,0}(k^2) + (1 - Z_{\phi^z})k^2 \ . \end{aligned} \tag{2.39}$$

Substituting (2.39) into the second condition of (2.38), we find

$$\begin{aligned} Z_{\phi^z} &= Z_{ZZ}\frac{Z_{M_z}^2 M_Z^2 - \widehat{\widetilde{\Pi}}_{ZZ,0}(M_Z^2)}{M_Z^2 - \widetilde{\Pi}_{\phi^z\phi^z,0}(M_Z^2)} \;, \quad (\text{ in } \mathit{Scheme}-IV\ ) \\ &= 1 + \delta Z_{ZZ} + 2\delta Z_{M_z} + M_Z^{-2}\left[\widetilde{\Pi}_{\phi^z\phi^z,0}(M_Z^2) - \widetilde{\Pi}_{ZZ,0}(M_Z^2)\right] \;, \quad (\text{ at 1 loop })\,, \end{aligned} \tag{2.40}$$

where the quantity $\widehat{\widetilde{\Pi}}_{ZZ,0}$ is defined in the second equation of (2.39). The added complication to the solution of Z_{ϕ^z} due to the mixing effects in the neutral sector vanishes at one loop.

Finally, we summarize *Scheme-IV* for the full SM. For both the physical and unphysical parts, the renormalization conditions will be imposed separately for the charged and neutral sectors. The conditions for the charged sector are identical to those for the $SU(2)_L$ theory. In the neutral sector, for the physical part, the photon and electric charge are renormalized as in *QED* [16], while for the unphysical part, we choose (2.36) and (2.40). The constraints on the whole unphysical sector in the *Scheme-IV*. are as follows:

$$\begin{aligned} &\kappa^{\pm} = M_W \;, \qquad \Omega_{\xi^{\pm}} = 1 \;, \qquad \Omega_{\kappa^{\pm}} = 1 \;, \\ &\widetilde{\Pi}_{\phi^+\phi^-}(M_W^2) = \widetilde{\Pi}_{W^+W^-}(M_W^2) \implies Z_{\phi^{\pm}} = Z_{W^{\pm}}\frac{Z_{M_W}^2 M_W^2 - \widetilde{\Pi}_{W^+W^-,0}(M_W^2)}{M_W^2 - \widetilde{\Pi}_{\phi^+\phi^-,0}(M_W^2)} \;, \\ &\delta Z_{\phi^{\pm}} = \delta Z_{W^{\pm}} + 2\delta Z_{M_W} + M_W^{-2}\left[\widetilde{\Pi}_{\phi^+\phi^-,0}(M_W^2) - \widetilde{\Pi}_{W^+W^-,0}(M_W^2)\right] \;, \quad (\text{ at 1 loop })\,; \end{aligned} \tag{2.41}$$

and

$$\begin{aligned} &\kappa_Z = M_Z \;, \quad \Omega_{\xi_N} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \;, \quad \Omega_{\bar{\kappa}} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \;, \\ &\widetilde{\Pi}_{\phi^z\phi^z}(M_Z^2) = \widetilde{\Pi}_{ZZ}(M_Z^2) \implies Z_{\phi^z} = Z_{ZZ}\frac{Z_{M_z}^2 M_Z^2 - \widehat{\widetilde{\Pi}}_{ZZ,0}(M_Z^2)}{M_Z^2 - \widetilde{\Pi}_{\phi^z\phi^z,0}(M_Z^2)} \;, \\ &\delta Z_{\phi^z} = \delta Z_{ZZ} + 2\delta Z_{M_z} + M_Z^{-2}\left[\widetilde{\Pi}_{\phi^z\phi^z,0}(M_Z^2) - \widetilde{\Pi}_{ZZ,0}(M_Z^2)\right] \;, \quad (\text{ at 1 loop })\,; \end{aligned} \tag{2.42}$$

which insure

$$\widehat{C}^W(M_W^2) = 1 \;, \qquad \widehat{C}^Z(M_Z^2) = 1 \;, \quad (\text{ in } \mathit{Scheme}-IV\ )\,. \tag{2.43}$$

Note that in (2.42) the quantity $\widehat{\widetilde{\Pi}}_{ZZ,0}$ is defined in terms of the bare self-energies of the neutral gauge bosons by the second equation of (2.39) and reduces to $\tilde{\Pi}_{ZZ,0}$ at one loop.

3. Precise Modification-Free Formulation of the ET for All R_ξ-Gauges

In this section, we first give the modification-free formulation of the ET within our new *Scheme-IV* for both $SU(2)_L$ Higgs theory and the full SM. In Sec. 3.2, we further generalize our result to the electroweak chiral Lagrangian (EWCL) formalism [13, 18] which provides the most economical description of the strongly coupled EWSB sector below the scale of new physics denoted by the effective cut-off $\Lambda(\leq 4\pi v \approx 3.1$ TeV). Numerous applications of the ET in this formalism have appeared in recent years [25]. The generalization to linearly realized effective Lagrangians [19] is much simpler and will be briefly discussed at the end of Sec. 3.2. Also, based upon our modification-free formulation of the ET, we propose a new prescription, called " Divided Equivalence Theorem " (DET), for minimizing the approximation due to ignoring the additive B-term in the ET. Finally, in Sec. 3.3, we analyze the relation of *Scheme-IV* to our previous schemes for the precise formulation of the ET.

3.1. The Precise Formulation in the $SU(2)_L$ theory and the SM

From our general formulation in Sec. 2.1, we see that the radiative modification factor $C^a_{\rm mod}$ to the ET is precisely equal to the factor $\widehat{C}^a(k^2)$ evaluated at the physical mass pole of the longitudinal gauge boson in the usual on-shell scheme. This is explicitly shown in (2.12) for $SU(2)_L$ theory and the generalization to the full SM is straightforward [4, 5]

$$C^W_{\rm mod} = \widehat{C}^W(M^2_W) \ , \ C^Z_{\rm mod} = \widehat{C}^Z(M^2_Z) \ , \tag{3.1}$$

for the on-shell subtraction of the gauge boson masses M_W and M_Z .

We then directly apply our renormalization *Scheme-IV* to give a new modification-free formulation of the ET *for all R_ξ-gauges*. For $SU(2)_L$ Higgs theory, we have

$$C^a_{\rm mod} = 1 \ , \quad (\ Scheme-IV \text{ for } SU(2)_L \text{ Higgs theory }) \tag{3.2}$$

where the *Scheme-IV* is defined in (2.20) and (2.28,28a). For the SM, we have

$$C^W_{\rm mod} = 1 \ , \quad C^Z_{\rm mod} = 1 \ , \quad (\ Scheme-IV \text{ for SM }) \tag{3.3}$$

where the *Scheme-IV* is summarized in (2.41) and (2.42). We emphasize that *the only special step to exactly ensure* $C^a_{\rm mod} = 1$ *and* $C^{W,Z}_{\rm mod} = 1$ *is to choose the unphysical Goldstone boson wavefunction renormalization constants* Z_ϕ as in (2.28) for the $SU(2)_L$ theory and $Z_{\phi^\pm}$ and Z_{ϕ^z} as in (2.41)-(2.42) for the SM.

Therefore, we can re-formulate the ET (1.1)-(1.2) in *Scheme-IV* with the radiative modifications removed:

$$T[V_L^{a_1},\cdots,V_L^{a_n};\Phi_\alpha] = T[i\phi^{a_1},\cdots,i\phi^{a_n};\Phi_\alpha] + B \ , \tag{3.4}$$

$$\begin{aligned} B \ &\equiv \textstyle\sum_{l=1}^n (\ T[v^{a_1},\cdots,v^{a_l},i\phi^{a_{l+1}},\cdots,i\phi^{a_n};\Phi_\alpha] + \ \text{permutations of } v\text{'s and } \phi\text{'s} \ ) \\ &= O(M_W/E_j)\text{--suppressed} \\ v^a &\equiv v^\mu V^a_\mu \ , \quad v^\mu \equiv \epsilon^\mu_L - k^\mu/M_V = O(M_V/E) \ , \ \ (M_V = M_W, M_Z) \ , \end{aligned} \tag{3.4a, b}$$

with the conditions

$$\begin{aligned} &E_j \sim k_j \gg M_W \ , \qquad (\ j = 1,2,\cdots,n\ ) \ , \\ &T[i\phi^{a_1},\cdots,i\phi^{a_n};\Phi_\alpha] \gg B \ . \end{aligned} \tag{3.5a, b}$$

Once *Scheme-IV* is chosen, we need not worry about the $C^a_{\rm mod}$-factors in (1.1)-(1.2) in *any* R_ξ-gauges and to any loop order. We remark that *Scheme-IV* is also valid for the $1/\mathcal{N}$-expansion [11] since the above formulation is based upon the WT identities (for two-point self-energies) which take the *same* form in any perturbative expansion. For the sake of many phenomenological applications, the explicit generalization to the important effective Lagrangian formalisms will be summarized in the following section.

3.2. Generalization to the Electroweak Chiral Lagrangian Formalism

The radiative modification-free formulation of the ET for the electroweak chiral Lagrangian (EWCL) formalism was given in Ref. [6] for *Scheme-II* which cannot be used in Landau gauge. However, since Landau gauge is widely used for the EWCL in the literature due to its special convenience for this non-linear formalism [13], it is important and useful to generalize our *Scheme-IV* to the EWCL. As to be shown below, this generalization is straightforward. We shall summarize our main results for the full $SU(2)\otimes U(1)_Y$ EWCL. For the convenience of practical applications of the ET within this formalism, some useful technical details will be provided in Appendices-A and -B. In the following analyses, we shall not distinguish the notations between bare and renormalized quantities unless it is necessary.

We start from the quantized $SU(2)_L \otimes U(1)_Y$ bare EWCL

$$
\begin{aligned}
\mathcal{L}^{[q]} &= \mathcal{L}_{\rm eff} + \mathcal{L}_{\rm GF} + \mathcal{L}_{\rm FP} \\
\mathcal{L}_{\rm eff} &= \left[\mathcal{L}_{\rm G} + \mathcal{L}^{(2)} + \mathcal{L}_{\rm F}\right] + \mathcal{L}'_{\rm eff} \\
\mathcal{L}_{\rm G} &= -\frac{1}{2}{\rm Tr}(\mathbf{W}_{\mu\nu}\mathbf{W}^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} \quad , \\
\mathcal{L}^{(2)} &= \frac{v^2}{4}{\rm Tr}[(D_\mu U)^\dagger (D^\mu U)] \quad , \\
\mathcal{L}_{\rm F} &= \overline{F_{Lj}} i\gamma^\mu D_\mu F_{Lj} + \overline{F_{Rj}} i\gamma^\mu D_\mu F_{Rj} - (\overline{F_{Lj}} U M_j F_{Rj} + \overline{F_{Rj}} M_j U^\dagger F_{Lj}) \quad ,
\end{aligned}
\tag{3.6}
$$

with

$$
\begin{aligned}
& \mathbf{W}_{\mu\nu} \equiv \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + ig[\mathbf{W}_\mu, \mathbf{W}_\nu] \; , \qquad B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu \; , \\
& U = \exp[i\tau^a \pi^a / v] \quad , \qquad D_\mu U = \partial_\mu U + ig\mathbf{W}_\mu U - ig' U \mathbf{B}_\mu \quad , \\
& \mathbf{W}_\mu \equiv W^a_\mu \frac{\tau^a}{2} \quad , \quad \mathbf{B}_\mu \equiv B_\mu \frac{\tau^3}{2} \quad , \\
& D_\mu F_{Lj} = \left[\partial_\mu - ig\frac{\tau^a}{2}W^a_\mu - ig'\frac{Y}{2}B_\mu\right] F_{Lj} \quad , \qquad D_\mu F_{Rj} = \left[\partial_\mu - ig'\left(\frac{\tau^3}{2} + \frac{Y}{2}\right) B_\mu\right] F_{Rj} \quad , \\
& F_{Lj} \equiv (f_{1j}, \; f_{2j})^T_L \quad , \quad F_{Rj} \equiv (f_{1j}, \; f_{2j})^T_R \quad ,
\end{aligned}
\tag{3.7}
$$

where π^a 's are the would-be Goldstone fields in the non-linear realization; f_{1j} and f_{2j} are the up- and down- type fermions of the j-th family (either quarks or leptons) respectively, and all right-handed fermions are $SU(2)_L$ singlet.

In (3.6), the leading order Lagrangian $\left[\mathcal{L}_{\rm G} + \mathcal{L}^{(2)} + \mathcal{L}_{\rm F}\right]$ denotes the model-independent contributions; the model-dependent next-to-leading order effective Lagrangian $\mathcal{L}'_{\rm eff}$ is given in Appendix-A. Many effective operators contained in $\mathcal{L}'_{\rm eff}$ (cf. Appendix-A), as reflections of the new physics, can be tested at the LHC and possible future electron (and photon) Linear Colliders (LC) through longitudinal gauge boson scattering processes [9, 25, 26]. Nonetheless, the analysis of the ET and the modification factors $C^a_{\rm mod}$ do not depend on the details of $\mathcal{L}'_{\rm eff}$.

The $SU(2)_L \otimes U(1)_Y$ gauge-fixing term, $\mathcal{L}_{\rm GF}$, in (3.6) is the same as that defined in (2.29) for the SM except that the linearly realized Goldstone boson fields ($\phi^{\pm,Z}$) are replaced by the non-linearly realized fields ($\pi^{\pm,Z}$). The BRST transformations for the bare gauge and Goldstone boson fields are

$$
\begin{aligned}
\hat{s}W^{\pm}_{\mu} &= -\partial_\mu c^{\pm} \mp i\left[e(A_\mu c^{\pm} - W^{\pm}_{\mu}c^A) + g\mathrm{c_w}(Z_\mu c^{\pm} - W^{\pm}c^Z)\right] \\
\hat{s}Z_\mu &= -\partial_\mu c^Z - ig\mathrm{c_w}\left[W^{+}_{\mu}c^- - W^{-}_{\mu}c^+\right] \\
\hat{s}A_\mu &= -\partial_\mu c^A - ie\left[W^{+}_{\mu}c^- - W^{-}_{\mu}c^+\right] \\
\hat{s}\pi^{\pm} &= M_W\left[\pm i(\underline{\pi}^Z c^{\pm} + \underline{\pi}^{\pm}\tilde{c}^3) - \eta\underline{\pi}^{\pm}(\underline{\pi}^+c^- + \underline{\pi}^-c^+) - \frac{\eta}{\mathrm{c_w}}\underline{\pi}^{\pm}\underline{\pi}^Z c^Z + \zeta\, c^{\pm}\right] , \\
\hat{s}\pi^Z &= M_Z\left[i(\underline{\pi}^-c^+ - \underline{\pi}^+c^-) - \mathrm{c_w}\eta\underline{\pi}^Z(\underline{\pi}^+c^- + \underline{\pi}^-c^+) - \eta\underline{\pi}^Z\underline{\pi}^Z c^Z + \zeta\, c^Z\right] ,
\end{aligned}
\tag{3.8}
$$

where $\mathrm{c_w} \equiv \cos\theta_{\mathrm{W}}$ and

$$
\begin{aligned}
\tilde{c}^3 &\equiv [\cos 2\theta_{\mathrm{W}}]c^Z + [\sin 2\theta_{\mathrm{W}}]c^A , \\
\eta &\equiv \frac{\underline{\pi}\cot\underline{\pi} - 1}{\underline{\pi}^2} = -\frac{1}{3} + O(\pi^2) , \\
\zeta &\equiv \underline{\pi}\cot\underline{\pi} = 1 - \frac{1}{3v^2}\vec{\pi}\cdot\vec{\pi} + O(\pi^4) , \\
\underline{\pi} &\equiv \frac{\pi}{v} , \quad \pi \equiv (\vec{\pi}\cdot\vec{\pi})^{\frac{1}{2}} = \left(2\pi^+\pi^- + \pi^Z\pi^Z\right)^{\frac{1}{2}} .
\end{aligned}
\tag{3.9}
$$

The derivations for $\hat{s}\pi^{\pm}$ and $\hat{s}\pi^Z$ are given in Appendix-B. Note that the non-linear realization greatly complicates the BRST transformations for the Goldstone boson fields. This makes the Δ_i^a-quantities which appear in the modification factors much more complex.

With the BRST transformations (3.8), we can write down the R_ξ-gauge Faddeev-Popov ghost Lagrangian in this non-linear formalism as:

$$
\mathcal{L}_{\mathrm{FP}} = \xi_W^{\frac{1}{2}}\left[\bar{c}^+\hat{s}F^+ + \bar{c}^+\hat{s}F^-\right] + \xi_Z^{\frac{1}{2}}\bar{c}^+\hat{s}F^Z + \xi_A^{\frac{1}{2}}\bar{c}^+\hat{s}F^A . \tag{3.10}
$$

The full expression is very lengthy due to the complicated non-linear BRST transformations for π^a's. In the Landau gauge, $\mathcal{L}_{\mathrm{FP}}$ is greatly simplified and has the *same* form as that in the linearly realized SM, due to the decoupling of ghosts from the Goldstone bosons at tree-level. This is clear from (3.10) after substituting (2.33) and setting $\xi_W = \xi_Z = \xi_A = 0$ [cf. ($B6$) in Appendix-B]. This is why the inclusion of Landau gauge into the modification-free formulation of the ET is particularly useful.

With these preliminaries, we can now generalize our precise formulation of the ET to the EWCL formalism. In Sec. 2 and 3, our derivation of the factor-C^a_{mod} and construction of the renormalization *Scheme-IV* for simplifying it to unity are based upon the general ST and WT identities. The validity of these general identities does *not* rely on any explicit expression of the Δ_i^a-quantities and the proper self-energies, and this makes our generalization straightforward. Our results are summarized as follows.

First we consider the derivation of the modification factor-$C_{\rm mod}^a$'s from the amputation and renormalization of external V_L and π lines. Symbolically, the expressions for $C_{\rm mod}^a$'s still have the *same* dependences on the renormalization constants and the Δ_i^a-quantities but their explicit expressions are changed in the EWCL formalism [6]. We consider the charged sector as an example of the changes.

$$C_{\rm mod}^W = \hat{C}^W(M_W^2) = \left(\frac{Z_W}{Z_{\pi^\pm}}\right)^{\frac{1}{2}} Z_{M_W} \left.\frac{1+\Delta_1^W(k^2)+\Delta_2^W(k^2)}{1+\Delta_3^W(k^2)}\right|_{k^2=M_W^2} , \quad (3.11)$$

which has the same symbolic form as the linear SM case [5] [see also (2.18), (2.11) and (2.12) for the $SU(2)_L$ Higgs theory in the present paper], but the expressions for these Δ_i 's are given by

$$\begin{aligned} 1+\Delta_1^W(k^2)+\Delta_2^W(k^2) &\equiv \frac{1}{M_W} < 0|T(\hat{s}\pi^\mp)|\bar{c}^\pm > (k) , \\ ik_\mu\left[1+\Delta_3^W(k^2)\right] &\equiv - < 0|T(\hat{s}W_\mu^\mp)|\bar{c}^\pm > (k) , \end{aligned} \quad (3.12)$$

where all fields and parameters are bare, and the BRST transformations for π^a and $W_\mu^\pm$ are given by (3.8). From (3.8)-(3.10) we see that the expression for $\Delta_1^W(k^2)+\Delta_2^W(k^2)$ has been greatly complicated due to the non-linear transformation of the Goldstone bosons, while the Δ_3^W takes the same symbolic form as in the linear SM. For Landau gauge, these Δ_i 's still satisfy the relation (2.13) and the two-loop graph of the type of Fig. 2b also appears in the $\Delta_1^W(k^2)+\Delta_2^W(k^2)$ of (3.10). We do not give any further detailed expressions for these Δ_i 's in either charged or neutral sector, because the following formulation of the ET within the *Scheme-IV* does *not* rely on any of these complicated quantities for $C_{\rm mod}^a$.

Second, consider the WT identities derived in Sec. 2, which enable us to re-express $C_{\rm mod}^a$ in terms of the proper self-energies of the gauge and Goldstone fields and make our construction of the *Scheme-IV* possible. For the non-linear EWCL, the main difference is that we now have higher order effective operators in $\mathcal{L}'_{\rm eff}$ (cf. Appendix-A) which parameterize the new physics effects below the effective cutoff scale Λ . However, they do not affect the WT identities for self-energies derived in Sec. 2 because their contributions, by definition, can always be included into the bare self-energies, as was done in Ref. [13]. Thus, our the construction for the *Scheme-IV* in Sec. 2 holds for the EWCL formalism.

Hence, our final conclusion on the modification-free formulation of the ET in this formalism is the same as that given in (3.1)-(3.5) of Sec. 3.1, after simply replacing the linearly realized Goldstone boson fields (ϕ^a 's) by the non-linearly realized fields (π^a 's).

Before concluding this section, we remark upon another popular effective Lagrangian formalism [19] for the weakly coupled EWSB sector (also called the decoupling scenario). In this formalism, the lowest order Lagrangian is just the linear SM with a relatively light Higgs boson and all higher order effective operators must have dimensionalities larger than four and are suppressed by the cutoff scale Λ :

$$\mathcal{L}_{\rm eff}^{\rm linear} = \mathcal{L}_{\rm SM} + \sum_n \frac{1}{\Lambda^{d_n-4}}\mathcal{L}_n \tag{3.13}$$

where d_n (≥ 5) is the dimension of the effective operator $\mathcal{L}_n$ and the effective cut-off Λ has, in principle, no upper bound. The generalization of our modification-free formulation of the ET to this formalism is extremely simple. All our discussions in Sec. 2 and 3.1 hold and the only new thing is to put the new physics contributions to the self-energies into the bare self-energies so that the general relations between the bare and renormalized self-energies [cf. (2.25) and (2.39)] remain the same. This is similar to the case of the non-linear EWCL (the non-decoupling scenario).

3.3. Divided Equivalence Theorem: a New Improvement

In this section, for the purpose of minimizing the approximation from ignoring the additive B-term in the ET (3.4) or (1.1), we propose a convenient new prescription, called " Divided Equivalence Theorem " (DET), based upon our modification-free formulation (3.4).

We first note that the rigorous *Scheme-IV* and the previous *Scheme-II* [4, 5] (cf. Sec. 3.4) ***do not rely on the size of the B-term.*** Furthermore, the result $C^a_{\rm mod} = 1$ greatly simplifies the expression for the B-term [cf. (1.1) and (3.4)]. This makes any further exploration and application of either the physical or technical content of the ET very convenient. In the following, we show how the error caused by ignoring B-term in the ET can be minimized through the new prescription DET.

For any given perturbative expansion up to a finite order N, the S-matrix T (involving V^a_L or ϕ^a) and the B-term can be generally written as $T = \sum_{\ell=0}^{N} T_\ell$ and $B = \sum_{\ell=0}^{N} B_\ell$. Within our modification-free formulation (3.4) of the ET, we have no complication due to the expansion of each $C^a_{\rm mod}$-factor on the RHS of (1.1) [i.e., $C^a_{\rm mod} = \sum_{\ell=0}^{N} (C^a_{\rm mod})_\ell$]. Therefore, at ℓ-th order and with $C^a_{\rm mod} = 1$ insured, the exact ET identity in (3.4) can be expanded as

$$T_\ell[V_L^{a_1}, \cdots, V_L^{a_n}; \Phi_\alpha] \;=\; T_\ell[i\phi^{a_1}, \cdots, i\phi^{a_n}; \Phi_\alpha] \;+\; B_\ell \;, \tag{3.14}$$

and the conditions (3.5ab) become, at the ℓ-th order,

$$\begin{array}{l} E_j \sim k_j \gg M_W \;, \qquad (j = 1, 2, \cdots, n) \;, \\ T_\ell[i\phi^{a_1}, \cdots, i\phi^{a_n}; \Phi_\alpha] \gg B_\ell \;, \qquad (\ell = 0, 1, 2, \cdots) \;. \end{array} \tag{3.15a, b}$$

We can estimate the ℓ-th order B-term as [cf. (1.3)]

$$B_\ell = O\left(\frac{M_W^2}{E_j^2}\right) T_\ell[i\phi^{a_1}, \cdots, i\phi^{a_n}; \Phi_\alpha] + O\left(\frac{M_W}{E_j}\right) T_\ell[V_{T_j}^{a_{r_1}}, i\phi^{a_{r_2}}, \cdots, i\phi^{a_{r_n}}; \Phi_\alpha] \;\;, \tag{3.16}$$

which is $O(M_W/E_j)$-suppressed for $E_j \gg M_W$. When the next-to-leading order (NLO: $\ell = 1$) contributions (containing possible new physics effects, cf. Appendix-A for instance) are included, the main limitation[c] on the predication of the ET for the V_L-amplitude via computing the Goldstone boson amplitude is due to ignoring the leading order B_0-term. This leading B_0-term is of $O(g^2)$ [8, 9] in the heavy Higgs SM and the CLEWT and cannot always be ignored in comparison with the NLO ϕ^a-amplitude T_1 though we usually have $T_0 \gg B_0$ and $T_1 \gg B_1$ respectively [8, 9] because of (3.16). It has been shown [9] that, except some special kinetic regions, $T_0 \gg B_0$ and $T_1 \gg B_1$ for all effective operators containing pure Goldstone boson interactions (cf. Appendix-A), as long as $E_j \gg M_W$. Based upon the above new equations (3.14)-(3.16), we can precisely formulate the ET at *each given order-ℓ* of the perturbative expansion where only B_ℓ , *but not the whole B-term*, will be ignored to build the longitudinal/Goldstone boson equivalence. Hence, *the equivalence is divided order by order* in the perturbative expansion, and the condition for this divided equivalence is $T_\ell \gg B_\ell$ (at the ℓ-th order) which is much weaker than $T_\ell \gg B_0$ [deduced from (3.5b)] for $\ell \geq 1$. For convenience, we call this formulation as " Divided Equivalence Theorem " (DET). Therefore, to improve the prediction of V_L-amplitude for the most interesting NLO contributions (in T_1) by using the ET, we propose the following simple prescription:

[c]We must clarify that, for the discussion of the ***physical content*** of the ET as a criterion for probing the EWSB, as done in Refs. [8, 9], the issue of including/ignoring the B-term is essentially ***irrelevant*** because both the Goldstone boson amplitude and the B-term are explicitly estimated order by order and are compared to each other [8, 9].

(i). Perform a direct and precise unitary gauge calculation for the tree-level V_L-amplitude $T_0[V_L]$ which is quite simple.

(ii). Make use of the DET (3.14) and deduce $T_1[V_L]$ from the Goldstone boson amplitude $T_1[GB]$, by ignoring B_1 only.

To see how simple the direct unitary gauge calculation of the tree-level V_L-amplitude is, we calculate the $W_L^+W_L^- \to W_L^+W_L^-$ scattering amplitude in the EWCL formalism as a typical example. The exact tree-level amplitude $T_0[W_L]$ only takes three lines:

$$\begin{aligned} T_0[W_L] = ig^2 \Bigg[& -(1+x)^2\sin^2\theta + 2x(1+x)(3\cos\theta - 1) - \mathrm{c}_{\mathrm{w}}^2 \frac{4x(2x+3)^2\cos\theta}{4x+3-\mathrm{s}_{\mathrm{w}}^2\mathrm{c}_{\mathrm{w}}^{-2}} \\ & + \mathrm{c}_{\mathrm{w}}^2 \frac{8x(1+x)(1-\cos\theta)(1+3\cos\theta) + 2[(3+\cos\theta)x+2][(1-\cos\theta)x - \cos\theta]^2}{2x(1-\cos\theta)+\mathrm{c}_{\mathrm{w}}^{-2}} \Bigg] \\ & + ie^2 \left[-\frac{x(2x+3)^2\cos\theta}{x+1} + 4(1+x)(1+3\cos\theta) + \frac{[(3+\cos\theta)x+2][(1-\cos\theta)x-\cos\theta]^2}{x(1-\cos\theta)} \right], \end{aligned} \tag{3.17}$$

where θ is the scattering angle, $x \equiv p^2/M_W^2$ with p denoting the C.M. momentum, and $\mathrm{s}_{\mathrm{w}} \equiv \sin\theta_W$, $\mathrm{c}_{\mathrm{w}} \equiv \cos\theta_W$. (3.17) contains five diagrams: one contact diagram, two s-channel diagrams by Z and photon exchange, and two similar t-channel diagrams. The corresponding Goldstone boson amplitude also contain five similar diagrams except all external lines being scalars. However, even for just including the leading B_0-term which contains only one external v^a-line [cf. (3.4a) or (1.1b)], one has to compute extra $5 \times 4 = 20$ tree-level graphs due to all possible permutations of the external v^a-line. It is easy to figure out how the number of these extra graphs will be greatly increased if one explicitly calculates the whole B-term. Therefore, we point out that, as the lowest order tree-level V_L-amplitude is concerned, it is *much simpler* to directly calculate the precise tree-level V_L-amplitude in the unitary gauge than to indirectly calculate the R_ξ-gauge Goldstone boson amplitude *plus* the complicated B_0 or the whole B term [which contains much more diagrams due to the permutations of v_μ-factors in (1.1b) or (3.4a)] as proposed in some literature [24]. To minimize the numerical error related to the B-term, our new prescription DET is the best and the most convenient.

Then, let us further exemplify, up to the NLO of the EWCL formalism, how the precision of the ET is improved by the above new prescription DET. Consider the lowest order contributions from $\mathcal{L}_G + \mathcal{L}^{(2)} + \mathcal{L}_F$ [cf. (3.6)] and the NLO contributions from the important operators $\mathcal{L}_{4,5}$ (cf. Appendix A). For the typical process $W_LW_L \to W_LW_L$

up to the NLO, both explicit calculations and the power counting analysis [8, 9] give

$$\begin{aligned} T_0 &= O\left(\frac{E^2}{v^2}\right) , & B_0 &= O(g^2) ; \\ T_1 &= O\left(\frac{E^2}{v^2}\frac{E^2}{\Lambda^2}\right) , & B_1 &= O\left(g^2\frac{E^2}{\Lambda^2}\right) ; \end{aligned} \tag{3.18}$$

where $v = 246$ GeV and $\Lambda \simeq 4\pi v \simeq 3.1$ TeV. From the condition (3.5b) and (3.15b) and up to the NLO, we have

$$\begin{aligned} (3.5b): \quad & T_1 \gg B_0 \quad \Longrightarrow \quad 1 \gg 24.6\% , \quad (\text{for } E = 1 \text{ TeV}) ; \\ (3.15b): \quad & T_0 \gg B_0 \quad \Longrightarrow \quad 1 \gg 2.56\% , \quad (\text{for } E = 1 \text{ TeV}) ; \\ & T_1 \gg B_1 \quad \Longrightarrow \quad 1 \gg 2.56\% , \quad (\text{for } E = 1 \text{ TeV}) . \end{aligned} \tag{3.19}$$

Here we see that, up to the NLO and for $E = 1$ TeV, the precision of the DET (3.14)-(3.16) is increased by about a factor of 10 in comparison with the usual prescription of the ET [cf. (3.5a,b)] as ignoring the B-term is concerned. It is clear that the DET (3.14)-(3.16) can be applied to a much wider high energy region than the usual ET due to the much weaker condition (3.15b).

In general, to do a calculation up to any order $\ell \geq 1$, we can apply the DET to minimize the approximation due to the B-term by following way: computing the full V_L-amplitude up to the $(\ell - 1)$-th order and applying the DET (3.14) at ℓ-th order with B_ℓ ignored. The practical applications of this DET up to NLO ($\ell = 1$) turns out most convenient. It is obvious that the above formulation for DET generally holds for both the SM and the effective Lagrangian formalisms.

3.4. Comparison of Scheme-IV with Other Schemes

The fact that we call the new renormalization scheme, *Scheme-IV*, implies that there are three other previous renormalization schemes for the ET. *Schemes-I* and *-II* were defined in references [4, 5], while *Scheme-III* was defined in reference [8].

Scheme-I [4, 5] is a generalization of the usual one-loop level on-shell scheme [16] to all orders. In this scheme, the unphysical sector is renormalized such that, for example, in the pure $SU(2)_L$ Higgs theory

$$\widetilde{\Pi}^a_{WW}(\xi\kappa M_W) = \widetilde{\Pi}^a_{W\phi}(\xi\kappa M_W) = \widetilde{\Pi}^a_{\phi\phi}(\xi\kappa M_W) = \widetilde{\Pi}^a_{c\bar{c}}(\xi\kappa M_W) = 0 , \tag{3.20a}$$

$$\frac{d}{dk^2}\widetilde{\Pi}^a_{\phi\phi}(k^2)\bigg|_{k^2=\xi\kappa M_W} = 0 \ , \qquad \frac{d}{dk^2}\widetilde{\Pi}^a_{c\bar{c}}(k^2)\bigg|_{k^2=\xi\kappa M_W} = 0 \ , \qquad (3.20b)$$

where $k^2 = \xi\kappa M_W$ is the tree level mass pole of the unphysical sector. In this scheme, the modification factor is not unity, but does take a very simple form in terms of a single parameter determined by the renormalization scheme [4, 5],

$$C^a_{\rm mod} = \Omega_\kappa^{-1} \ , \qquad (\ Scheme-I \text{ with } \kappa = M_W \text{ and } \xi = 1\ ) \ . \qquad (3.21)$$

Scheme-II [4, 5] is a variation of the usual on-shell scheme, in which the unphysical sector is renormalized such that C^a_{mod} is set equal to unity. The choice here is to impose all of the conditions in (3.20) except that the Goldstone boson wavefunction renormalization constant Z_ϕ is not determined by (3.20b) but specially chosen. To accomplish this, Ω_ξ is adjusted so that $\widetilde{\Pi}^a_{WW}(\xi\kappa M_W) = 0$, and Z_ϕ is adjusted so that $\widetilde{\Pi}^a_{\phi\phi}(\xi\kappa M_W) = 0$. Ω_κ is set to unity, which ensures that $\widetilde{\Pi}^a_{W\phi}(\xi\kappa M_W) = \widetilde{\Pi}^a_{c\bar{c}}(\xi\kappa M_W) = 0$, and Z_c is adjusted to ensure that the residue of the ghost propagator is unity. The above conditions guarantee that $\widehat{C}^a(\xi\kappa M_W) = 1$. The final choice is to set $\kappa = \xi^{-1}M_W$ so that $C^a_{\rm mod} = 1$. This scheme is particularly convenient for the 't Hooft-Feynman gauge, where $\kappa = M_W$. For $\xi \neq 1$, there is a complication due to the tree level gauge-Goldstone-boson mixing term proportional to $\kappa - M_W = (\xi^{-1} - 1)M_W$. But this is not a big problem since the mixing term corresponds to a tree level gauge-Goldstone-boson propagator similar to that found in the Lorentz gauge ($\kappa = 0$) [20]. The main shortcoming of this scheme is that it does not include Landau gauge since, for $\xi = 0$, the choice $\kappa = \xi^{-1}M_W$ is singular and the quantities $\Omega_{\xi,\kappa}$ have no meaning. In contrast to *Scheme-II* , *Scheme-IV* is valid for all R_ξ-gauges including both Landau and 't Hooft-Feynman gauges. The primary inconvenience of *Scheme-IV* is that for non-Landau gauges all unphysical mass poles deviate from their tree-level values [21, 16, 5], thereby invalidating condition (3.20a).[d] This is not really a problem since these poles have no physical effect.

Scheme-III [8] is specially designed for the pure V_L-scatterings in the strongly coupled EWSB sector. For a $2 \to n-2$ ($n \geq 4$) strong pure V_L-scattering process, the B-term defined in (1.1) is of order $O(g^2)v^{n-4}$, where $v = 246$ GeV. By the electroweak power counting analysis [9, 8], it has been shown [8] that all g-dependent contributions from either vertices or the mass poles of gauge-boson, Goldstone boson and ghost fields are

[d]The violation of (3.20a) in non-Landau gauges is not special to *Scheme-IV* , but is a general feature of all schemes [21]-[23] which choose the renormalization prescription (2.21) for the gauge-fixing condition [21, 16, 5].

at most of $O(g^2)$ and the contributions of fermion Yukawa couplings (y_f) coming from fermion-loops are of $O(\frac{y_f^2}{16\pi^2}) \leq O(\frac{g^2}{16\pi^2})$ since $y_f \leq y_t \simeq O(g)$. Also, in the factor $C^a_{\rm mod}$ all loop-level Δ^a_i-quantities [cf. eq. (2.9) and Fig.1] are of $O(\frac{g^2}{16\pi^2})$ since they contain at least two ghost-gauge-boson or ghost-scalar vertices. Hence, if the B-term (of $O(g^2)f_\pi^{n-4}$) is ignored in the strong pure V_L-scattering amplitude, all other g- and y_f-dependent terms should also be ignored. Consequently we can simplify the modification factor such that $C^a_{\rm mod} \simeq 1 + O(g^2)$ by choosing [8]

$$Z_{\phi^a} = \left[\left(\frac{M_V}{M_V^{\rm phys}}\right)^2 Z_{V^a} Z^2_{M_V}\right]\Bigg|_{g,e,y_f=0} \quad , \quad (\ Scheme - III\ )\ . \tag{3.22}$$

All other renormalization conditions can be freely chosen as in any standard renormalization scheme. (Here $M_V^{\rm phys}$ is the physical mass pole of the gauge boson V^a. Note that we have set $M_V^{\rm phys} = M_V$ in *Scheme-IV* for simplicity.) In this scheme, because of the neglect of all gauge and Yukawa couplings, all gauge-boson, Goldstone-boson and ghost mass poles are approximately zero. Thus, all R_ξ-gauges (including both 't Hooft-Feynman and Landau gauges) become *equivalent*, for the case of strong pure V_L-scatterings in both *the heavy Higgs SM or the EWCL formalism. But, for processes involving fields other* than longitudinal gauge bosons, only *Scheme-II* and *Scheme-IV* are suitable.[e] Even in the case of pure V_L-scattering, we note that in the kinematic regions around the t and u channel singularities, photon exchange becomes important and must be retained [9]. In this case, *Scheme-IV* or *Scheme-II* is required to remove the $C^a_{\rm mod}$-factors.

In summary, renormalization *Scheme-IV* ensures the modification-free formulation of the ET [cf. (3.1)-(3.5)]. It is valid for all R_ξ-gauges, but is particularly convenient for the Landau gauge where all unphysical Goldstone boson and ghost fields are exactly massless [5, 17]. *Scheme-II* [4, 5], on the other hand, is particularly convenient for 't Hooft-Feynman gauge. For all other R_ξ-gauges, both schemes are valid, but the *Scheme-IV* may be more

[e]Some interesting examples are $W_LW_L, Z_LZ_L \to t\bar{t}$, $V_LH \to V_LH$, and $AA \to W_LW_L, WWV_LV_L$, etc. (A =photon).

convenient due to the absence of the tree-level gauge-Goldstone boson mixing.

4. Explicit One-Loop Calculations

4.1. One-loop Calculations for the Heavy Higgs Standard Model

To demonstrate the effectiveness of *Scheme-IV*, we first consider the heavy Higgs limit of the standard model. The complete one-loop calculations for proper self-energies and renormalization constants in the heavy Higgs limit have been given for general R_ξ-gauges in reference [5] for renormalization *Scheme-I*. Since, in this scheme, $\Omega_{\xi,\kappa}^{W,ZZ} = 1 + \delta\Omega_{\xi,\kappa}^{W,ZZ} \approx 1$ at one-loop in the heavy Higgs limit, *Scheme-I* coincides with *Scheme-IV* to this order. Hence, we can directly use those results to demonstrate that $C_{mod}^{W,Z}$ is equal to unity in

Scheme-IV. The results for the charged and neutral sectors are [5]:

$$\begin{aligned}
\widetilde{\Pi}_{WW,0}(k^2) &= -\frac{g^2}{16\pi^2}\left[\frac{1}{8}m_H^2+\frac{3}{4}M_W^2\ln\frac{m_H^2}{M_W^2}\right] \quad , \\
\widetilde{\Pi}_{ZZ,0}(k^2) &= -\frac{g^2}{16\pi^2c_w^2}\left[\frac{1}{8}m_H^2+\frac{3}{4}M_Z^2\ln\frac{m_H^2}{M_Z^2}\right] \quad , \\
\widetilde{\Pi}_{\phi^\pm\phi^\pm,0}(k^2) &= -\frac{g^2}{16\pi^2}k^2\left[\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(\frac{3}{4}-\frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right] \quad , \\
\widetilde{\Pi}_{\phi^z\phi^z,0}(k^2) &= -\frac{g^2}{16\pi^2c_w^2}k^2\left[\frac{1}{8}\frac{m_H^2}{M_Z^2}+\left(\frac{3}{4}-\frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right] \quad , \\
\delta Z_{M_W} &= -\frac{g^2}{16\pi^2}\left[\frac{1}{16}\frac{m_H^2}{M_W^2}+\frac{5}{12}\ln\frac{m_H^2}{M_W^2}\right] \quad , \\
\delta Z_{M_z} &= -\frac{g^2}{16\pi^2c_w^2}\left[\frac{1}{16}\frac{m_H^2}{M_Z^2}+\frac{5}{12}\ln\frac{m_H^2}{M_Z^2}\right] \quad , \\
\delta Z_W &= -\frac{g^2}{16\pi^2}\frac{1}{12}\ln\frac{m_H^2}{M_W^2} \quad , \\
\delta Z_{ZZ} &= -\frac{g^2}{16\pi^2}\frac{1}{12c_w^2}\ln\frac{m_H^2}{M_Z^2} \quad , \\
\delta Z_{\phi^\pm} &= -\frac{g^2}{16\pi^2}\left[\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(\frac{3}{4}-\frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right] \quad , \\
\delta Z_{\phi^z} &= -\frac{g^2}{16\pi^2c_w^2}\left[\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(\frac{3}{4}-\frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right] \quad , \\
\Delta_1^W(k^2) &= -\frac{g^2}{16\pi^2}\frac{\xi_W}{4}\ln\frac{m_H^2}{M_W^2} \quad , \\
\Delta_1^{ZZ}(k^2) &= -\frac{g^2}{16\pi^2}\frac{\xi_Z}{4c_w^2}\ln\frac{m_H^2}{M_Z^2} \quad .
\end{aligned} \tag{4.1}$$

Note that $\Delta_{2,3}^W$ and the corresponding neutral sector terms (cf. Fig. 1) are not enhanced by powers or logarithms of the large Higgs mass, and thus are ignored in this approximation. Substituting δZ_W, δZ_{M_W}, $\widetilde{\Pi}_{\phi^\pm\phi^\pm,0}$, $\widetilde{\Pi}_{WW,0}$ and δZ_{ZZ}, δZ_{M_z}, $\widetilde{\Pi}_{\phi^z\phi^z,0}$, $\widetilde{\Pi}_{ZZ,0}$ into the right hand sides of eqs. (2.41) and (2.42) respectively, we obtain

$$\begin{aligned}
\delta Z_{\phi^\pm} &= \frac{g^2}{16\pi^2}\left[-\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(-\frac{3}{4}+\frac{\xi_W}{2}\right)\ln\frac{m_H^2}{M_W^2}\right] \quad , \\
\delta Z_{\phi^z} &= \frac{g^2}{16\pi^2c_w^2}\left[-\frac{1}{8}\frac{m_H^2}{M_W^2}+\left(-\frac{3}{4}+\frac{\xi_Z}{2}\right)\ln\frac{m_H^2}{M_Z^2}\right] \quad ,
\end{aligned} \tag{4.2}$$

verifying the equivalence of *Schemes-I* and *-IV* in this limit. This means that the one-loop value of $C_{\rm mod}^{W,Z}$ should be equal to unity. Using (2.8), (2.11) and the renormalization constants given in (4.1), we directly compute the $C_{\rm mod}^{W,Z}$ up to one-loop in the R_ξ-gauges for the heavy Higgs case as

$$\begin{aligned}
C_{\rm mod}^{W} &= 1+\frac{1}{2}(\delta Z_W-\delta Z_{\phi^\pm}+2\delta Z_{M_W})+\Delta_1^W(M_W^2)\\
&= 1+\frac{g^2}{16\pi^2}\left\{\left(\frac{1}{16}-\frac{1}{16}\right)\frac{m_H^2}{M_W^2}+\left(\frac{1}{24}+\frac{3}{8}-\frac{5}{12}-\frac{\xi_W}{4}+\frac{\xi_W}{4}\right)\ln\frac{m_H^2}{M_W^2}\right\}\\
&= 1+O(2\text{ loop})\ ,\\
C_{\rm mod}^{Z} &= 1+\frac{1}{2}\left(\delta Z_{ZZ}-\delta Z_{\phi^z}+2\delta Z_{M_z}\right)+\Delta_1^Z(M_Z^2)\\
&= 1+\frac{g^2}{16\pi^2c_w^2}\left\{\left(\frac{1}{16}-\frac{1}{16}\right)\frac{m_H^2}{M_W^2}+\left(\frac{1}{24}+\frac{3}{8}-\frac{5}{12}-\frac{\xi_Z}{4}+\frac{\xi_Z}{4}\right)\ln\frac{m_H^2}{M_Z^2}\right\}\\
&= 1+O(2\text{ loop})\ .
\end{aligned}\tag{4.3}$$

Equation (4.3) is an explicit one-loop proof that $C_{mod}^{W,Z}=1$ in *Scheme-IV*. The agreement of *Schemes-I* and *-IV* only occurs in the heavy Higgs limit up to one-loop order. When the Higgs is not very heavy, the full one-loop corrections from all scalar and gauge couplings must be included, so that *Scheme-IV* and *Scheme-I* are no longer equivalent.

4.2. Complete One-Loop Calculations for the $U(1)$ Higgs theory

The simplest case to explicitly demonstrate *Scheme-IV* for arbitrary Higgs mass is the $U(1)$ Higgs theory. In this section, we use complete one-loop calculations in the $U(1)$ Higgs theory (for any value of m_H) to explicitly verify that $C_{mod}=1$ in *Scheme-IV* for both Landau and 't Hooft-Feynman gauges.

The $U(1)$ Higgs theory contains minimal field content: the physical Abelian gauge field A_μ (with mass M), the Higgs field H (with mass m_H), as well as the unphysical Goldstone boson field ϕ and the Faddeev-Popov ghost fields $c,\bar{c}$ (with mass poles at $\xi\kappa M$). Because the symmetry group is Abelian, Δ_2 and Δ_3 do not occur and the modification factor is given by

$$C_{\rm mod}=\left(\frac{Z_A}{Z_\phi}\right)^{\frac{1}{2}}Z_M[1+\Delta_1(M^2)]\ ,\tag{4.4}$$

with

$$\Delta_1(k^2) = \frac{Z_g Z_H^{\frac{1}{2}}}{Z_M} \frac{g\mu^\epsilon}{M} \int_q < 0|H(-k-q)c(q)|\bar{c}(k) > \tag{4.5}$$

where $\int_q \equiv \int \frac{dq^D}{(2\pi)^D}$ and $D = 4 - 2\epsilon$. Δ_1 vanishes identically in Landau gauge ($\xi = 0$), because in the $U(1)$ theory the ghosts couple only to the Higgs boson and that coupling is proportional to ξ . In *Scheme-IV* , the wavefunction renormalization constant Z_ϕ of the Goldstone boson field is defined to be

$$Z_\phi = Z_A \frac{Z_M^2 M^2 - \tilde{\Pi}_{AA,0}(M^2)}{M^2 - \tilde{\Pi}_{\phi\phi,0}(M^2)} \quad . \tag{4.6}$$

Substituting (4.6) into (4.5), we obtain the following one-loop expression for C_{mod}

$$C_{\mathrm{mod}} = 1 + \frac{1}{2} M^{-2} \left[\tilde{\Pi}_{AA,0}(M^2) - \tilde{\Pi}_{\phi\phi,0}(M^2)\right] + \Delta_1(M^2) \quad . \tag{4.7}$$

We shall now explicitly verify that C_{mod} is equal to unity in both Landau and 't Hooft-Feynman gauges.

In Landau gauge:

$$\begin{aligned}
\tilde{\Pi}_{AA,0}(k^2)\Big|_{\xi=0} = ig^2 \quad & \Big\{-I_1(m_H^2) - 4M^2 I_2(k^2; M^2, m_H^2) + k^2 I_2(k^2; 0, m_H^2) \\
& +4k^2 I_3(k^2; 0, m_H^2) + 4\left(I_{41}(k^2; M^2, m_H^2) + k^2 I_{42}(k^2; a^2, b^2)\right)\Big\} \quad , \\
\tilde{\Pi}_{\phi\phi,0}(k^2)\Big|_{\xi=0} = ig^2 \quad & \Bigg\{-\frac{m_H^2}{M^2} I_1(m_H^2) + \frac{m_H^4}{M^2} I_2(k^2; 0, m_H^2) - 4k^2 I_2(k^2; M^2, m_H^2) \\
& +4\frac{k^2}{M^2}\left(I_{41}(k^2; M^2, m_H^2) - I_{41}(k^2; 0, m_H^2)\right) \\
& +4\frac{m_H^4}{M^2}\left(I_{42}(k^2; M^2, m_H^2) - I_{42}(k^2, 0, m_H^2)\right)\Bigg\} \quad , \\
\Delta_1(k^2)\Big|_{\xi=0} = 0 \quad , &
\end{aligned} \tag{4.8}$$

where the quantities I_j's denote the one-loop integrals:

$$
\begin{aligned}
I_1(a^2) &= \mu^{2\epsilon}\int \frac{d^{4-2\epsilon}p}{(2\pi)^{4-2\epsilon}}\frac{1}{p^2-a} \equiv \mu^{2\epsilon}\int_p \frac{1}{p^2-a} \ , \\
I_2(k^2;a^2,b^2) &= \mu^{2\epsilon}\int_p \frac{1}{(p^2-a)[(p+k)^2-b]} \ , \\
I_3^\mu(k;a^2,b^2) &= \mu^{2\epsilon}\int_p \frac{p^\mu}{(p^2-a)[(p+k)^2-b]} = k^\mu I_3(k^2;a^2,b^2) \ , \\
I_4^{\mu\nu}(k;a^2,b^2) &= \mu^{2\epsilon}\int_p \frac{p^\mu p^\nu}{(p^2-a)[(p+k)^2-b]} = g^{\mu\nu}I_{41}(k^2;a^2,b^2) + k^\mu k^\nu I_{42}(k^2;a^2,b^2) \ ,
\end{aligned}
\tag{4.9}
$$

which are evaluated in Appendix-C. Substituting (4.9) into the right hand side of (4.7), we obtain

$$
C_{\rm mod} = 1 + \ O(2\ \text{loop}) \ , \qquad (\ \text{in Landau gauge}\ ) \ , \tag{4.10}
$$

as expected.

We next consider the 't Hooft-Feynman gauge in which $\Delta_1(M^2)$ is non-vanishing:

$$
\begin{aligned}
\tilde{\Pi}_{AA,0}(k^2)\Big|_{\xi=1} &= ig^2\left\{-I_1(m_H^2) - I_1(M^2) + (k^2-4M^2)I_2 + 4k^2I_3 + 4(I_{41}+k^2I_{42})\right\} \ , \\
\tilde{\Pi}_{\phi\phi,0}(k^2)\Big|_{\xi=1} &= ig^2\left\{\left(1+\frac{m_H^2}{M^2}\right)\left(I_1(M^2)-I_1(m_H^2)\right) + \left(\frac{m_H^4}{M^2} - M^2 - 4k^2\right)I_2 - 4k^2I_3\right\} \ , \\
\Delta_1(k^2)\Big|_{\xi=1} &= ig^2 I_2(M^2;M^2,m_H^2) \ ,
\end{aligned}
\tag{4.11}
$$

where $I_j = I_j(k^2;M^2,m_H^2)$ for $j \geq 2$. Again, we substitute (4.11) into equation (4.7) and find that

$$
C_{\rm mod} = 1 + \ O(2\ \text{loop}) \ , \qquad (\ \text{in 't Hooft} - \text{Feynman gauge}\ ) \ . \tag{4.12}
$$

5. Conclusions

In this paper, we have constructed a convenient new renormalization scheme, called *Scheme-IV*, which rigorously reduces all radiative modification factors to the equivalence theorem ($C^a_{\rm mod}$'s) to unity in all R_ξ-gauges including both 't Hooft-Feynman and Landau gauges. This new *Scheme-IV* proves particularly convenient for Landau gauge which cannot be included in the previously described *Scheme-II* [4, 5]. Our formulation is explicitly constructed for both the $SU(2)_L$ and $SU(2)_L \otimes U(1)_Y$ theories [cf. sections 2 and 3].

Furthermore, we have generalized our formulation to the important effective Lagrangian formalisms for both the non-linear [13] and linear [19] realizations of the electroweak symmetry breaking (EWSB) sector, where the new physics (due to either strongly or weakly coupled EWSB mechanisms) has been parameterized by effective operators (cf. section 3.2 and Appendix-A). In the construction of the *Scheme-IV* (cf. section 2.2), we first re-express the C^a_{mod}-factors in terms of proper self-energies of the unphysical sector by means of the R_ξ-gauge WT identities. Then, we simplify the C^a_{mod}-factors to unity by specifying the subtraction condition for the Goldstone boson wavefunction renormalization constant Z_{ϕ^a} [cf. (2.28) and (2.41-42)]. This choice for Z_{ϕ^a} is determined by the known gauge and Goldstone boson self-energies (plus the gauge boson wavefunction and mass renormalization constants) which must be computed in any practical renormalization scheme. We emphasize that the implementation of the *Scheme-IV* requires no additional calculation (of Δ^a_i-quantities, for instance) beyond what is required for the *standard* radiative correction computations [16]. Based upon this radiative modification-free formulation for the equivalence theorem [cf. (3.4)], we have further proposed a new prescription, which we call the " Divided Equivalence Theorem " (DET) [cf. (3.14)-(3.15) and discussions followed], for minimizing the approximation due to ignoring the additive B-term in the equivalence theorem (3.4) or (1.1). Finally, we have explicitly verified that, in *Scheme-IV*, the C^a_{mod}-factor is reduced to unity in the heavy Higgs limit of the standard model (cf. section 4.1) and for arbitrary Higgs mass in the $U(1)$ Higgs theory (cf. section 4.2).

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Appendix A. Next-to-leading Order Effective Operators in the EWCL

Within the EWCL formalism, the next-to-leading order effective operators arising from new physics can be parameterized as [13, 9]

$$\mathcal{L}'_{\mathrm{eff}} \equiv \mathcal{L}'_{\mathrm{GB}} + \mathcal{L}'_{\mathrm{F}} \qquad (A1)$$

The bosonic part $\mathcal{L}'_{\rm GB}$ is given by

$$
\begin{aligned}
\mathcal{L}'_{\rm GB} &= \mathcal{L}^{(2)\prime} + \sum_{n=1}^{14} \mathcal{L}_n \ , \\
\mathcal{L}^{(2)\prime} &= \ell_0 (\tfrac{v}{\Lambda})^2 \ \tfrac{v^2}{4} [{\rm Tr}(\mathcal{T}\mathcal{V}_\mu)]^2 \ , \\
\mathcal{L}_1 &= \ell_1 (\tfrac{v}{\Lambda})^2 \ \tfrac{gg'}{2} B_{\mu\nu} {\rm Tr}(\mathcal{T}{\bf W}^{\mu\nu}) \ , \\
\mathcal{L}_2 &= \ell_2 (\tfrac{v}{\Lambda})^2 \ \tfrac{ig'}{2} B_{\mu\nu} {\rm Tr}(\mathcal{T}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_3 &= \ell_3 (\tfrac{v}{\Lambda})^2 \ ig {\rm Tr}({\bf W}_{\mu\nu}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_4 &= \ell_4 (\tfrac{v}{\Lambda})^2 [{\rm Tr}(\mathcal{V}_\mu \mathcal{V}_\nu)]^2 \ , \\
\mathcal{L}_5 &= \ell_5 (\tfrac{v}{\Lambda})^2 [{\rm Tr}(\mathcal{V}_\mu \mathcal{V}^\mu)]^2 \ , \\
\mathcal{L}_6 &= \ell_6 (\tfrac{v}{\Lambda})^2 [{\rm Tr}(\mathcal{V}_\mu \mathcal{V}_\nu)] {\rm Tr}(\mathcal{T}\mathcal{V}^\mu) {\rm Tr}(\mathcal{T}\mathcal{V}^\nu) \ , \\
\mathcal{L}_7 &= \ell_7 (\tfrac{v}{\Lambda})^2 [{\rm Tr}(\mathcal{V}_\mu \mathcal{V}^\mu)] {\rm Tr}(\mathcal{T}\mathcal{V}_\nu) {\rm Tr}(\mathcal{T}\mathcal{V}^\nu) \ , \\
\mathcal{L}_8 &= \ell_8 (\tfrac{v}{\Lambda})^2 \ \tfrac{g^2}{4} [{\rm Tr}(\mathcal{T}{\bf W}_{\mu\nu})]^2 \ , \\
\mathcal{L}_9 &= \ell_9 (\tfrac{v}{\Lambda})^2 \ \tfrac{ig}{2} {\rm Tr}(\mathcal{T}{\bf W}_{\mu\nu}) {\rm Tr}(\mathcal{T}[\mathcal{V}^\mu, \mathcal{V}^\nu]) \ , \\
\mathcal{L}_{10} &= \ell_{10} (\tfrac{v}{\Lambda})^2 \tfrac{1}{2} [{\rm Tr}(\mathcal{T}\mathcal{V}^\mu) {\rm Tr}(\mathcal{T}\mathcal{V}^\nu)]^2 \ , \\
\mathcal{L}_{11} &= \ell_{11} (\tfrac{v}{\Lambda})^2 \ g \epsilon^{\mu\nu\rho\lambda} {\rm Tr}(\mathcal{T}\mathcal{V}_\mu) {\rm Tr}(\mathcal{V}_\nu {\bf W}_{\rho\lambda}) \ , \\
\mathcal{L}_{12} &= \ell_{12} (\tfrac{v}{\Lambda})^2 \ 2g {\rm Tr}(\mathcal{T}\mathcal{V}_\mu) {\rm Tr}(\mathcal{V}_\nu {\bf W}^{\mu\nu}) \ , \\
\mathcal{L}_{13} &= \ell_{13} (\tfrac{v}{\Lambda})^2 \ \tfrac{gg'}{4} \epsilon^{\mu\nu\rho\lambda} B_{\mu\nu} {\rm Tr}(\mathcal{T}{\bf W}_{\rho\lambda}) \ , \\
\mathcal{L}_{14} &= \ell_{14} (\tfrac{v}{\Lambda})^2 \ \tfrac{g^2}{8} \epsilon^{\mu\nu\rho\lambda} {\rm Tr}(\mathcal{T}{\bf W}_{\mu\nu}) {\rm Tr}(\mathcal{T}{\bf W}_{\rho\lambda}) \ ,
\end{aligned}
\tag{A2}
$$

with

$$
\mathcal{V}_\mu \equiv (D_\mu U) U^\dagger \ , \qquad \mathcal{T} \equiv U \tau_3 U^\dagger \ , \tag{A3}
$$

where $\mathcal{T}$ is the custodial $SU(2)_C$-violating operator. In ($A2$), $\mathcal{L}^{(2)\prime}$ and $\mathcal{L}_1 \sim \mathcal{L}_{11}$ are CP-conserving while $\mathcal{L}_{12} \sim \mathcal{L}_{14}$ are CP-violating. Many of these effective operators can be probed at the LHC and LC via longitudinal gauge boson scattering processes [9, 25, 26]. For the fermionic part $\mathcal{L}'_{\rm F}$, we refer the reader to the reference [27] for details.

Appendix B. Non-linear BRST Transformations of Goldstone Boson Fields and the Faddeev-Popov Term in the EWCL

In this Appendix, we first summarize the derivation for the non-linear $SU(2)_L \otimes U(1)_Y$ BRST transformations of the Goldstone boson fields [cf. (3.8)] and then give the complete Landau gauge Faddeev-Popov term.

For simplicity of notation, we need only derive the usual $SU(2)_L \otimes U(1)_Y$ gauge transformations for the π^a 's, since their BRST transformations are obtained by simply replacing the usual gauge parameters ($\theta^a(x)$) by the corresponding ghost fields $c^a(x)$. Consider the infinitesimal $SU(2)_L \otimes U(1)_Y$ gauge transformation for the U-matrix [cf. (3.7)]:

$$
\begin{aligned}
U \Longrightarrow U' &= \mathcal{G}_L(\theta_L) U \mathcal{G}_Y^\dagger(\theta_Y) \equiv U + \delta U \\
&= U + \frac{ig}{2}\theta_L^a \tau^a U - \frac{ig'}{2}\theta_Y U \tau^3 + O(\theta_L^2,\ \theta_Y^2)
\end{aligned}
\tag{B1}
$$

with

$$
\begin{aligned}
\mathcal{G}_L &= \exp\left[ig\theta_L^a(x)\tau^a/2\right] = 1 + ig\theta_L^a\tau^a/2 + O(\theta_L^2) \in SU(2)_L \ , \\
\mathcal{G}_Y &= \exp\left[ig\theta_Y(x)\tau^3/2\right] = 1 + ig'\theta_Y\tau^3/2 + O(\theta_Y^2) \in U(1)_Y \ .
\end{aligned}
\tag{B2}
$$

Expanding the U-matrix in ($B1$), we obtain

$$
\begin{aligned}
\delta U = & \\
& \frac{g}{2}\cos\underline{\pi}\begin{bmatrix} \theta_L^3(i - \omega\underline{\pi}_3) + (\theta_L^1 - i\theta_L^2)(-\underline{\pi}_1 - i\underline{\pi}_2)\omega & \theta_L^3(-\underline{\pi}_1 + i\underline{\pi}_2)\omega + \left(\theta_L^1 - i\theta_L^2\right)(i + \omega\underline{\pi}_3) \\ \theta_L^3(\underline{\pi}_1 + i\underline{\pi}_2)\omega + \left(\theta_L^1 + i\theta_L^2\right)(i - \omega\underline{\pi}_3) & \theta_L^3(-i - \omega\underline{\pi}_3) + \left(\theta_L^1 + i\theta_L^2\right)(-\underline{\pi}_1 + i\underline{\pi}_2)\omega \end{bmatrix} \\
& + \frac{g'}{2}\cos\underline{\pi}\begin{bmatrix} (-i + \omega\underline{\pi}_3)\theta_Y & \omega(-\underline{\pi}_1 + i\underline{\pi}_2)\theta_Y \\ \omega(\underline{\pi}_1 + i\underline{\pi}_2)\theta_Y & (i + \omega\underline{\pi}_3)\theta_Y \end{bmatrix}
\end{aligned}
\tag{B3}
$$

where $\omega \equiv \zeta^{-1}$ and all notations (including ζ) are defined in (3.9).

By differential variation of U with respect to the π^a-field, we obtain

$$
\delta U = \frac{\sin\underline{\pi}}{\underline{\pi}}\begin{bmatrix} -\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} + i\left(\eta\underline{\pi}_3\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} + \delta\underline{\pi}_3\right) & \eta\left(i\underline{\pi}_1 + \underline{\pi}_2\right)\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} + \left(i\delta\underline{\pi}_1 + \delta\underline{\pi}_2\right) \\ \eta\left(i\underline{\pi}_1 - \underline{\pi}_2\right)\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} + \left(i\delta\underline{\pi}_1 - \delta\underline{\pi}_2\right) & -\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} - i\left(\eta\underline{\pi}_3\vec{\underline{\pi}}\cdot\delta\vec{\underline{\pi}} + \delta\underline{\pi}_3\right) \end{bmatrix}
\tag{B4}
$$

where

$$
\pi^\pm = \frac{1}{\sqrt{2}}(\pi_1 \mp i\pi_2) \ , \quad \pi^Z = \pi_3 \ .
\tag{B5}
$$

Equating the two expressions for δU in ($B3$) and ($B4$), we derive the BRST transformations for $\pi^\pm$ and π^Z given in (3.8). The BRST transformations for the gauge fields [cf. (3.8)] are the same as in the SM.

After setting up the BRST transformations (3.8), we can write down the complete R_ξ-gauge Faddeev-Popov term for the EWCL from (3.10). Since *Scheme-IV* is particularly useful for Landau gauge, we give only the Landau gauge Faddeev-Popov term. From (3.10) and (3.8), we obtain, in Landau gauge,

$$
\begin{aligned}
\mathcal{L}_{\rm FP} \;=\;& -\bar{c}^+\partial^2 c^+ + \bar{c}^+ i\partial^\mu\left[e\left(W_\mu^+ c^A - A_\mu c^+\right) + g c_{\rm w}\left(W_\mu^+ c^Z - Z_\mu c^+\right)\right]\\
& -\bar{c}^-\partial^2 c^- - \bar{c}^- i\partial^\mu\left[e\left(W_\mu^- c^A - A_\mu c^-\right) + g c_{\rm w}\left(W_\mu^- c^Z - Z_\mu c^-\right)\right]\\
& -\bar{c}^Z\partial^2 c^Z + i g c_{\rm w}\bar{c}^Z\partial^\mu\left[W_\mu^- c^+ - W_\mu^+ c^-\right]\\
& -\bar{c}^A\partial^2 c^A + i e\bar{c}^A\partial^\mu\left[W_\mu^- c^+ - W_\mu^+ c^-\right] \quad .
\end{aligned}
\tag{B6}
$$

Finally, we remark that, although the Faddeev-Popov term in 't Hooft-Feynman gauge is much more complicated than in Landau gauge, it is still useful due to the simplicity of the gauge boson propagators. The main advantage of the new *Scheme-IV* is its applicability to *all* R_ξ-gauges including both Landau and 't Hooft-Feynman gauges.

Appendix C. Analytic Expressions for the One-Loop Integrals

Finally, we give the complete analytic expressions for the one-loop integrals (I_i 's) used in section 4.2 [cf. (4.9)] for explicit calculations. For simplicity of notation, we define

$$
\frac{1}{\hat{\epsilon}} \equiv \frac{1}{\epsilon} - \gamma + \ln(4\pi\mu^2) \ . \tag{C1}
$$

Then, we have

$$
I_1(a^2) \;=\; \mu^{2\epsilon}\int_p \frac{1}{p^2-a} \;=\; \frac{i}{16\pi^2}a^2\left[\frac{1}{\hat{\epsilon}} - \ln a^2 + 1\right] \ . \tag{C2}
$$

$$
\begin{aligned}
I_2(k^2;a^2,b^2) \;&=\; \mu^{2\epsilon}\int_p \frac{1}{(p^2-a)[(p+k)^2-b]}\\
&=\; \frac{i}{16\pi^2}\left[\frac{1}{\hat{\epsilon}} - \ln(ab) + 2 + \frac{a^2-b^2}{k^2}\ln\frac{b}{a} - \bar{I}_{20}(k^2;a^2,b^2)\right] \ ,
\end{aligned}
$$

$$
\bar{I}_{20}(k^2;a^2,b^2) \;=\; \begin{cases}
-\dfrac{\sqrt{AB}}{k^2}\ln\dfrac{\sqrt{-A}+\sqrt{-B}}{\sqrt{-A}-\sqrt{-B}} \ , & (\ k^2 \le (a-b)^2\ ) \ ,\\[2ex]
\dfrac{2\sqrt{-AB}}{k^2}\arctan\sqrt{\dfrac{B}{-A}} \ , & (\ (a-b)^2 < k^2 < (a+b)^2\ ) \ ,\\[2ex]
+\dfrac{\sqrt{AB}}{k^2}\left[\ln\dfrac{\sqrt{B}+\sqrt{A}}{\sqrt{B}-\sqrt{A}} - i\pi\right] \ , & (\ k^2 \ge (a+b)^2\ ) \ ,
\end{cases}
\tag{C3}
$$

where $A = k^2 - (a+b)^2$ and $B = k^2 - (a-b)^2$.

$$\begin{aligned}
I_3^{\mu}(k;a^2,b^2) &= \mu^{2\epsilon}\int_p \frac{p^{\mu}}{(p^2-a)[(p+k)^2-b]} = k^{\mu} I_3(k^2;a^2,b^2) \ , \\
I_3(k^2;a^2,b^2) &= -\frac{i}{16\pi^2}\frac{a^2}{2}\left[\frac{1}{\hat{\epsilon}} - \bar{I}_{30}(k^2;a^2,b^2)\right] \ , \\
\bar{I}_{30}(k^2;a^2,b^2) &= \ln a^2 - 2 + \frac{b^2-a^2}{k^2} + \frac{1}{2}\left[1 + \frac{2b^2}{k^2} - \frac{(a^2-b^2)^2}{k^4}\right]\ln\frac{b^2}{a^2} \\
&\quad + \left[1 + \frac{a^2-b^2}{k^2}\right]\bar{I}_{20}(k^2;a^2,b^2) \ .
\end{aligned} \tag{C4}$$

$$\begin{aligned}
I_4^{\mu\nu}(k;a^2,b^2) &= \mu^{2\epsilon}\int_p \frac{p^{\mu}p^{\nu}}{(p^2-a)[(p+k)^2-b]} = g^{\mu\nu} I_{41}(k^2;a^2,b^2) + k^{\mu}k^{\nu} I_{42}(k^2;a^2,b^2) \ , \\
I_{41}(k^2;a^2,b^2) &= \frac{i}{16\pi^2}\frac{1}{4}\Bigg[\left(\frac{1}{\hat{\epsilon}} + \frac{1}{2}\right)\left(a^2+b^2-\frac{k^2}{3}\right) - \frac{13}{18}k^2 + \frac{11}{6}(a^2+b^2) - \frac{(a^2-b^2)^2}{3k^2}k^2 \\
&+ \left(\frac{k^2}{3} - (a^2+b^2) + \frac{a^4-b^4}{k^2} - \frac{(a^2-b^2)^3}{3k^4}\right)\ln\frac{b}{a} - \left(a^2+b^2-\frac{k^2}{3}\right)\ln a^2 + \frac{AB}{3k^2}\bar{I}_{20}(k^2;a^2,b^2)\Bigg] \ , \\
I_{42}(k^2;a^2,b^2) &= \frac{i}{16\pi^2}\frac{1}{3}\Bigg[\frac{1}{\hat{\epsilon}} + \frac{13}{6} + \frac{a^2-5b^2}{2k^2} + \frac{(a^2-b^2)^2}{k^4} - \Bigg(1 + 3\frac{b^2}{k^2} + 3\frac{b^2(a^2-b^2)}{k^4} \\
&- \frac{(a^2-b^2)^3}{k^6}\Bigg)\ln\frac{b}{a} - \ln a^2 - \left(1 + \frac{a^2-2b^2}{k^2} + \frac{(a^2-b^2)^2}{k^4}\right)\bar{I}_{20}(k^2;a^2,b^2)\Bigg] \ .
\end{aligned} \tag{C5}$$

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