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Spin Correlations in Top-Quark Pair Production at e^+e^- Colliders

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Abstract

We show that top-quark pairs are produced in an essentially unique spin configuration in polarized e^+e^- colliders at all energies above the threshold region. Since the directions of the electroweak decay products of polarized top-quarks are strongly correlated to the top-quark spin axis, this unique spin configuration leads to a distinctive topology for top-quark pair events which can be used to constrain anomalous couplings to the top-quark. A significant interference effect between the *longitudinal* and *transverse* W-bosons in the decay of polarized top-quarks is also discussed. These results are obtained at leading order in perturbation theory but radiative corrections are expected to be small.

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Since the top-quark, with mass in the range of 175 GeV [1,2], decays electroweakly before hadronizing [3], there are significant angular correlations between the decay products of the top-quark and the spin of the top-quark. Therefore, if top-quark pairs are produced with large polarizations, there will be sizable angular correlations between the decay products and the spin orientation of each top-quark, as well as between the decay products of the top-quark and the decay products of the top anti-quark. These angular correlations depend sensitively on the top-quark couplings to the Z and the photon, and to the W and b-quark. Many authors have proposed to use the angular information in top-quark events produced at e^+e^- colliders to constrain deviations from Standard-Model couplings [4-8]. In most of these studies, the top-quark spin is decomposed in the helicity basis, i.e. along the direction of motion of the quark. Recently, Mahlon and Parke [9] have shown that this decomposition in the helicity basis is far from the optimal decomposition for top-quarks produced at the Tevatron. In this paper we address the issue of what is the optimal decomposition of the top-quark spins for e^+e^- colliders. Here, we only consider top-quark production at energies that are above the threshold region [10]. We give the differential cross-section for top-quark pair production for a generic spin basis. We have found that there is a decomposition for which the top-quark pairs are produced in an essentially unique spin configuration at polarized e^+e^- colliders. We call this spin basis the "Off-diagonal" basis because the contribution from like-spin pairs vanishes to leading order in perturbation theory. Finally, we discuss the angular correlations between the decay products of the top-quarks, and point out a significant interference effect between the *longitudinal* and *transverse* W-bosons in the decays of the polarized top-quarks.

The generic spin basis we will consider in this paper is shown in Figure 1. This choice is motivated by two features of the Standard Model prediction for top-quark pair production at e^+e^- colliders at leading-order in perturbation theory; the transverse top-quark polarization is zero, and CP is conserved. Therefore, we have defined our general basis so that the spins of the top quark and anti-quark are in the production plane, so there is no transverse polarization, and the spin four-vectors are back-to-back in the zero momentum frame, so that states with opposite spins are CP eigenstates.

The top-quark spin states are defined in the top-quark rest-frame, where we decompose the top spin along the direction $\hat{s}_t$, which makes an angle ξ with the anti-top momentum in the clockwise direction. Similarly, the top anti-quark spin states are defined in the anti-top rest-frame, along the direction $\hat{s}_{\bar{t}}$, which makes the *same* angle ξ with the top momentum also in the clockwise direction. Thus, the state $t_\uparrow \bar{t}_\uparrow$ ($t_\downarrow \bar{t}_\downarrow$) refers to a top with spin in the $+\hat{s}_t$ ($-\hat{s}_t$) direction in the top rest-frame, and an anti-top with spin $+\hat{s}_{\bar{t}}$ ($-\hat{s}_{\bar{t}}$) in the anti-top rest-frame.

Using this generic spin basis, the Standard Model leading-order polarized cross-sections for top-quark pair production at center-of-mass energy $\sqrt{s}$, top-quark speed β and top-quark scattering angle θ^*, are given by

$$\begin{aligned} \frac{d\sigma}{d\cos\theta^*}\,(e^-_L\, e^+_R \to t_\uparrow\, \bar{t}_\uparrow) &= \frac{d\sigma}{d\cos\theta^*}\,(e^-_L\, e^+_R \to t_\downarrow\, \bar{t}_\downarrow) \\ &= \Big(\frac{3\pi\alpha^2}{2s}\beta\Big)\ |A_{LR}\cos\xi \ - \ B_{LR}\sin\xi\ |^2 , \end{aligned}$$

$$\frac{d\sigma}{d\cos\theta^*}\,(e^-_L\, e^+_R \to t_\uparrow\, \bar{t}_\downarrow \text{ or } t_\downarrow\, \bar{t}_\uparrow) = \Big(\frac{3\pi\alpha^2}{2s}\beta\Big)\ |A_{LR}\sin\xi \ + \ B_{LR}\cos\xi \ \pm \ D_{LR}|^2. \tag{1}$$

Here α is the QED fine structure constant and the quantities A, B and D depend on the kinematic variables β and $\cos\theta^*$, and on the fermion couplings to the photon and Z-boson, in the following way

$$\begin{aligned} A_{LR} &= [(f_{LL} + f_{LR})\ \sqrt{1-\beta^2}\ \sin\theta^*]/2 , \\ B_{LR} &= [f_{LL}\ (\cos\theta^* + \beta) + f_{LR}\ (\cos\theta^* - \beta)]/2 , \\ D_{LR} &= [f_{LL}\ (1 + \beta\cos\theta^*) + f_{LR}\ (1 - \beta\cos\theta^*)]/2 , \end{aligned} \tag{2}$$

with

$$f_{IJ} = Q_\gamma(e)Q_\gamma(t) + Q^I_Z(e)Q^J_Z(t)\Big(\frac{1}{\sin^2\theta_W}\Big)\Big(\frac{s}{(s-M_Z^2)+iM_Z\Gamma_Z}\Big) , \tag{3}$$

where M_Z is the Z mass, Γ_Z is the Z width, and $I, J \in (L, R)$. The electron couplings are given by

$$Q_\gamma(e) = -1, \quad Q_Z^L(e) = \frac{2\sin^2\theta_W - 1}{2\cos\theta_W}, \quad Q_Z^R(e) = \frac{\sin^2\theta_W}{\cos\theta_W}\,, \tag{4}$$

and the top-quark couplings are given by

$$Q_\gamma(t) = \frac{2}{3}, \quad Q_Z^L(t) = \frac{3 - 4\sin^2\theta_W}{6\cos\theta_W}, \quad Q_Z^R(t) = \frac{-2\sin^2\theta_W}{3\cos\theta_W}\,. \tag{5}$$

The angle θ_W is the Weinberg angle. In the limit $s \gg M_Z^2$ the products of fermion couplings for top-quark production are

$$f_{LL} = -1.19, \quad f_{LR} = -0.434, \quad f_{RR} = -0.868, \quad f_{RL} = -0.217\,, \tag{6}$$

where, as throughout this paper, we use $\sin^2\theta_W = 0.232$.

The cross-sections for $e_R^- \, e_L^+$ may be obtained from eqns. (1-3) by interchanging L and R as well as $\uparrow$ and $\downarrow$. These cross-sections can be conveniently derived using the spinor helicity method for massive fermions described in [9].

Our generic basis reproduces the familiar helicity basis for the special case

$$\cos\xi = \pm 1\,, \tag{7}$$

for which the top-quark spin is defined along its direction of motion. Substituting eqn. (7) in our general polarized cross-section expressions (1), we recover the well known helicity cross-sections

$$\begin{aligned}\frac{d\sigma}{d\cos\theta^*}(e_L^- \, e_R^+ \to t_L \, \bar{t}_L) &= \frac{d\sigma}{d\cos\theta^*}(e_L^- \, e_R^+ \to t_R \, \bar{t}_R) \\ &= \Big(\frac{3\pi\alpha^2}{8s}\beta\Big)\, |f_{LL} + f_{LR}|^2\, (1-\beta^2)\sin^2\theta^*\,,\end{aligned}$$

$$\frac{d\sigma}{d\cos\theta^*}(e_L^- \, e_R^+ \to t_R \, \bar{t}_L \text{ or } t_L \, \bar{t}_R) = \Big(\frac{3\pi\alpha^2}{8s}\beta\Big)\, |f_{LL}(1\mp\beta) + f_{LR}(1\pm\beta)|^2\, (1\mp\cos\theta^*)^2\,. \tag{8}$$

Another useful basis is the "Beamline basis" [9], in which the top-quark spin axis is the *positron direction in the top rest-frame*, and the top anti-quark spin axis is the electron direction in the anti-top rest-frame. In terms of the spin angle ξ, this corresponds to

$$\cos\xi = \frac{\cos\theta^* + \beta}{1 + \beta\cos\theta^*}\,. \tag{9}$$

The polarized cross-sections in this basis are

$$\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\uparrow\,\bar{t}_\uparrow)=\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\downarrow\,\bar{t}_\downarrow)$$
$$=\left(\frac{3\pi\alpha^2}{2s}\beta\right)|f_{LR}|^2\,\frac{\beta^2(1-\beta^2)\sin^2\theta^*}{(1+\beta\cos\theta^*)^2}\,,$$

$$\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\downarrow\,\bar{t}_\uparrow)=\left(\frac{3\pi\alpha^2}{2s}\beta\right)|f_{LR}|^2\,\frac{\beta^4\sin^4\theta^*}{(1+\beta\cos\theta^*)^2}\,,$$

$$\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\uparrow\,\bar{t}_\downarrow)=\left(\frac{3\pi\alpha^2}{2s}\beta\right)\left|f_{LL}(1+\beta\cos\theta^*)+f_{LR}\frac{(1-\beta^2)}{(1+\beta\cos\theta^*)}\right|^2. \tag{10}$$

Note that in this basis, three out of the four polarized cross-sections are proportional to $|f_{LR}|^2$ which is much smaller $|f_{LL}|^2$. These three components are further suppressed by at least two powers of β. The remaining component, $t_\uparrow\bar{t}_\downarrow$, will therefore account for most of the total cross-section. Hence the top-quark spin is strongly correlated with the positron spin direction determined in the top-quark rest-frame.

The production threshold for top-quarks of mass 175 GeV is far above the Z-boson pole. In this region, the Z width is negligible and we can take the factors f_{IJ} as real. With real f_{IJ}'s one can choose a basis in which the $t_\uparrow\bar{t}_\uparrow$ and $t_\downarrow\bar{t}_\downarrow$ components vanish identically, see eqn. (1). In this basis, which we refer to as the "Off-diagonal basis", the spin angle ξ is given by

$$\tan\xi=\frac{(f_{LL}+f_{LR})\,\sqrt{1-\beta^2}\,\sin\theta^*}{f_{LL}\,(\cos\theta^*+\beta)+f_{LR}\,(\cos\theta^*-\beta)} \tag{11}$$

and the polarized cross-sections are

$$\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\uparrow\,\bar{t}_\uparrow)=\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\downarrow\,\bar{t}_\downarrow)=0\,,$$

$$\frac{d\sigma}{d\cos\theta^*}\,(e_L^-\,e_R^+\to t_\uparrow\,\bar{t}_\downarrow\ or\ t_\downarrow\,\bar{t}_\uparrow)=\left(\frac{3\pi\alpha^2}{8s}\beta\right)\Bigg[f_{LL}(1+\beta\cos\theta^*)+f_{LR}(1-\beta\cos\theta^*)$$
$$\pm\sqrt{(f_{LL}(1+\beta\cos\theta^*)+f_{LR}(1-\beta\cos\theta^*))^2-4f_{LL}f_{LR}\beta^2\sin^2\theta^*}\,\Bigg]^2. \tag{12}$$

For $|f_{LL}|\gg|f_{LR}|$ only the $t_\uparrow\bar{t}_\downarrow$ component is substantially different from zero, and to leading order in $\frac{f_{LR}}{f_{LL}}$, the $t_\downarrow \bar{t}_\uparrow$ component is given by the same expression as for the Beamline basis[1]. In general there are two "Off-diagonal" bases for fermion pair production, one for $e^-_L e^+_R$ scattering and the other for $e^-_R e^+_L$, since in general $\frac{f_{LR}}{f_{LL}} \neq \frac{f_{RL}}{f_{RR}}$. However for top-quark production the two ratios are approximately equal in both sign and magnitude, so that the two "Off-diagonal" bases are almost identical. In the rest of this paper we will only use the Off-diagonal basis for $e^-_L e^+_R$ defined by eqn. (11) even when discussing $e^-_R e^+_L$ scattering.

To illustrate the different spin bases we now consider top-quark pair production, at a 400 GeV e^+e^- collider. We take the top-quark mass to be 175 GeV ($\beta \sim 0.5$). Fig. 2 shows the dependence of the spin direction angle, ξ, on the scattering angle, θ^*, for the helicity, Beamline and Off-diagonal bases. The Beamline basis lies close to the Off-diagonal basis for all scattering angles. For $\cos\theta^*$ near zero there is a marked difference between the helicity basis and the other two bases. Note that as $\beta \to 1$, both the Beamline and Off-diagonal bases approach the helicity basis. Therefore, for lepton colliders with center-of-mass energies greater than 1 TeV, all three bases are equivalent.

The cross-sections for producing $t\bar{t}$ pairs of definite helicities are shown in Fig. 3 for both $e^-_L\ e^+_R$ and $e^-_R\ e^+_L$ scattering. While the dominant components of the signal are $t_L \bar{t}_R$ for a left-handed electron and $t_R \bar{t}_L$ for a right-handed electron, other spin components make up more than 40% of the total cross-section. In the Off-diagonal basis, in contrast, only one spin component is appreciably non-zero for all values of the scattering angle; $t_\uparrow\ \bar{t}_\downarrow$ for

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[1]The polarized cross-section formulae (1) are also valid for top-quark pair production in $q\bar{q}$-scattering, with an appropriate change of couplings. The Off-diagonal basis for $q\bar{q}$-scattering is given by eqn. (11) with $f_{LL} = f_{LR} = f_{RL} = f_{RR}$, for which $\tan\xi = \sqrt{1-\beta^2}\tan\theta^*$ with

$$\frac{d\sigma}{d\cos\theta^*}\ (q^-_L\ \bar{q}^+_R \to t_\uparrow\ \bar{t}_\uparrow\ or\ t_\downarrow\ \bar{t}_\downarrow) = 0$$

and

$$\frac{d\sigma}{d\cos\theta^*}\ (q^-_L\ \bar{q}^+_R \to t_\uparrow\ \bar{t}_\downarrow\ or\ t_\downarrow\ \bar{t}_\uparrow) = \left(\ \frac{\pi\alpha_s^2}{9s}\beta\ \right)\left[1 \pm \sqrt{1-\beta^2\sin^2\theta^*}\ \right]^2 .$$

$e_L^- \, e_R^+$ and $t_\downarrow \, \bar{t}_\uparrow$ for $e_R^- \, e_L^+$, see Fig. 4. All other components are more than two and a half orders of magnitude smaller than these dominant contributions. Thus, when defined in the Off-diagonal basis, the spins of the top quark and anti-quark produced in $e_L^- \, e_R^+$ and $e_R^- \, e_L^+$ scattering are essentially determined. This continues to hold for top-quark pair production at higher energies. In Fig. 5 we show, for $e_L^- \, e_R^+$ collisions, the fraction of top-quark events in the $t_\uparrow \bar{t}_\downarrow$ configuration, defined in the Off-diagonal basis, as a function of the top-quark speed β. Also shown is the fraction of top-quark pairs in the $t_L \bar{t}_R$ helicity configuration. The Off-diagonal basis gives a very clean $t_\uparrow \bar{t}_\downarrow$ spin state for all values of β in these collisions. Similarly, the spin state $t_\downarrow \bar{t}_\uparrow$ dominates $e_R^- \, e_L^+$ collisions at all energies.

We do not show here cross-section plots for the Beamline basis, since they are almost identical to those of Fig. 4, except that the non-dominant contributions are now at the 1% level for a 400 GeV collider. For both the Beamline basis and the Off-diagonal basis, the contribution of higher order corrections to the non-dominant components is expected to increase their total contribution to the few percent level.

Since the top-quark pairs are produced in an unique spin configuration, and the electroweak decay products of polarized top-quarks are strongly correlated to the spin axis, the top-quark events at e^+e^- collider have a very distinctive topology. Deviations from this topology would signal anomalous couplings. In the Standard Model, the predominant decay mode of the top-quark is $t \to bW^+$, with the W^+ decaying either hadronically or leptonically. For definiteness we consider here the decay $t \to bW^+ \to be^+\nu$. The differential decay width of a polarized top-quark depends non-trivially on three angles. The first is the angle, χ_w^t, between the top-quark spin and the direction of motion of the W-boson in the top-quark rest-frame. Next is the angle between the direction of motion of the b-quark and the positron in the W-boson rest-frame. We call this angle $\pi - \chi_e^w$. Finally, in the top-quark rest-frame, we have the azimuthal angle, Φ, between the positron direction of motion and the top-quark spin around the direction of motion of the W-boson.

The differential polarized top-quark decay distribution in terms of these three angles is given by

$$\frac{1}{\Gamma_T}\,\frac{d^3\,\Gamma}{d\cos\chi_w^t\;d\cos\chi_e^w\;d\Phi} = \frac{3}{16\pi\;(m_t^2+2m_W^2)}\left[m_t^2(1+\cos\chi_w^t)\sin^2\chi_e^w\right.$$

$$\left.+m_W^2(1-\cos\chi_w^t)(1-\cos\chi_e^w)^2 - 2m_t m_W(1-\cos\chi_e^w)\sin\chi_e^w\sin\chi_w^t\cos\Phi\right]\,, \qquad (13)$$

where m_t is the top-quark mass, m_W is the W mass, and Γ_T is the total decay width (we neglect the b-quark mass). The first and second terms in (13) give the contributions of longitudinal and transverse W-bosons respectively. The interference term, given by the third term in (13), does not contribute to the total width, but its effects on the angular distribution of the top-quark decay products are sizable. Fig. 6 shows contour plots of the differential angular decay distribution in the $\chi_e^w - \chi_w^t$ plane[2], after integrating over the azimuthal angle Φ. Fig. 7 shows analogous contours integrated over Φ for negative (solid lines) and for positive (dashed lines) values of $\cos\Phi$ separately. The pronounced difference between these is related to the size of the interference term, which can be seen from the Φ-distribution

$$\frac{1}{\Gamma_T}\,\frac{d\,\Gamma}{d\Phi} = \frac{1}{2\pi}\left[1 - \frac{3\pi^2 m_t m_W}{16(m_t^2+2m_W^2)}\cos\Phi\right]. \qquad (14)$$

For a 175 GeV top-quark the coefficient in front of the cosine term has a value equal to 0.59, therefore the maximum and minimum values of this distribution are approximately 4 to 1.

There are also significant correlations of the angle between the top-quark spin and the momentum of the i-th decay product, χ_i^t, measured in the top-quark rest-frame. The differential decay rate of the top-quark is given by

$$\frac{1}{\Gamma_T}\,\frac{d\,\Gamma}{d\cos\chi_i^t} = \frac{1}{2}\left[1+\alpha_i\cos\chi_i^t\right], \qquad (15)$$

where $\alpha_b = -0.41$, $\alpha_\nu = -0.31$ and $\alpha_{e^+} = 1$, for $m_t = 175$ GeV, see ref. [11].

In summary, we have presented simple analytic expressions for the polarized cross-section for top-quark pair production in polarized e^+e^- colliders. For a particular choice of axes,

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[2]We take $M_W = 80$ GeV.

the Off-diagonal basis, not only do the like-spin contributions vanish, but one spin configuration dominates the total cross-section. In this configuration, the top-quark spin is strongly correlated with the positron spin direction determined in the top-quark rest-frame. The subsequent electroweak decays of the top-quark pair give decay products whose angular distributions are highly correlated with the parent top-quark spin. Top-quark pair events thus have a distinctive topology. This topology is sensitive to the top-quark couplings to the Z-boson and to the photon, which determine the orientation and the size of the top-quark and top anti-quark polarizations, as well as to the top-quark couplings to the W and the b-quark, which determine its decay distributions. Angular correlations in top-quark events may therefore be used to constrain deviations from the Standard Model. We have also shown that the interference between the *longitudinal* and *transverse* W-bosons has a significant impact on the angular distribution of the top-quark decay products, and thus will provide additional means for testing the Standard Model predictions for top-quark decays.

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FIGURES

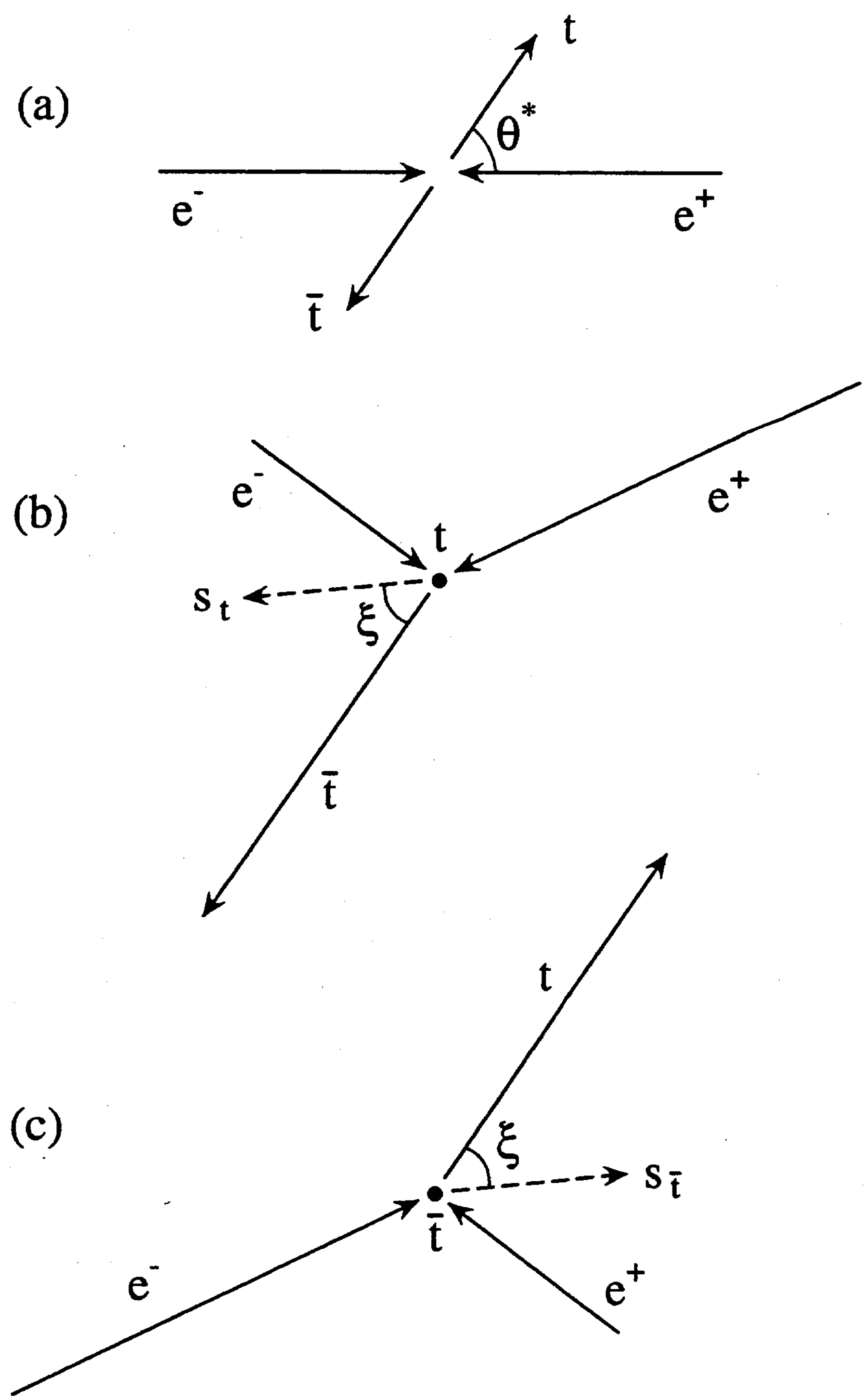

FIG. 1. The scattering process in the center-of-mass frame (a), in the top-quark rest-frame (b) and in the top anti-quark rest-frame (c). s_t ($s_{\bar{t}}$) is the top (anti-top) spin axis.

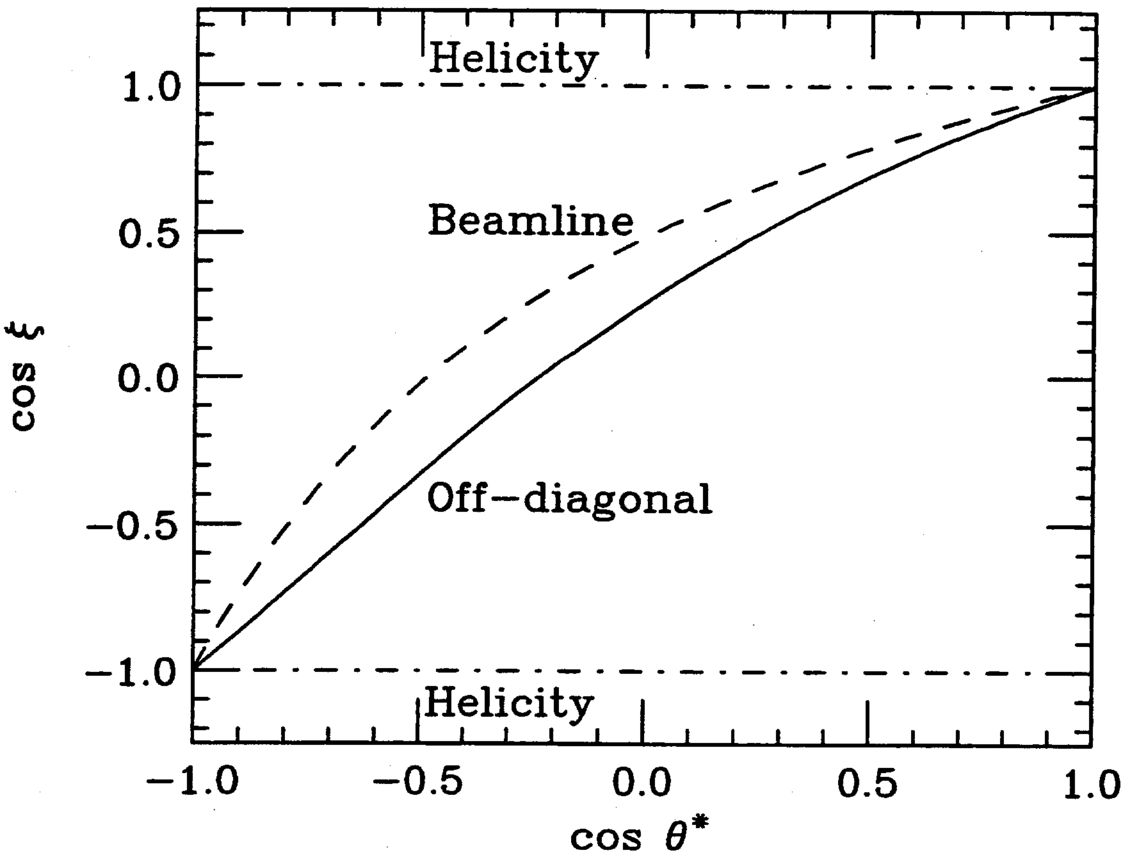

FIG. 2. The dependence of the spin angle, ξ, on the scattering angle, θ^*, for the helicity, Beamline and Off-diagonal (defined for $e_L^- \, e_R^+$ scattering) bases for a 175 GeV top-quark produced by a 400 GeV e^+e^- collider.

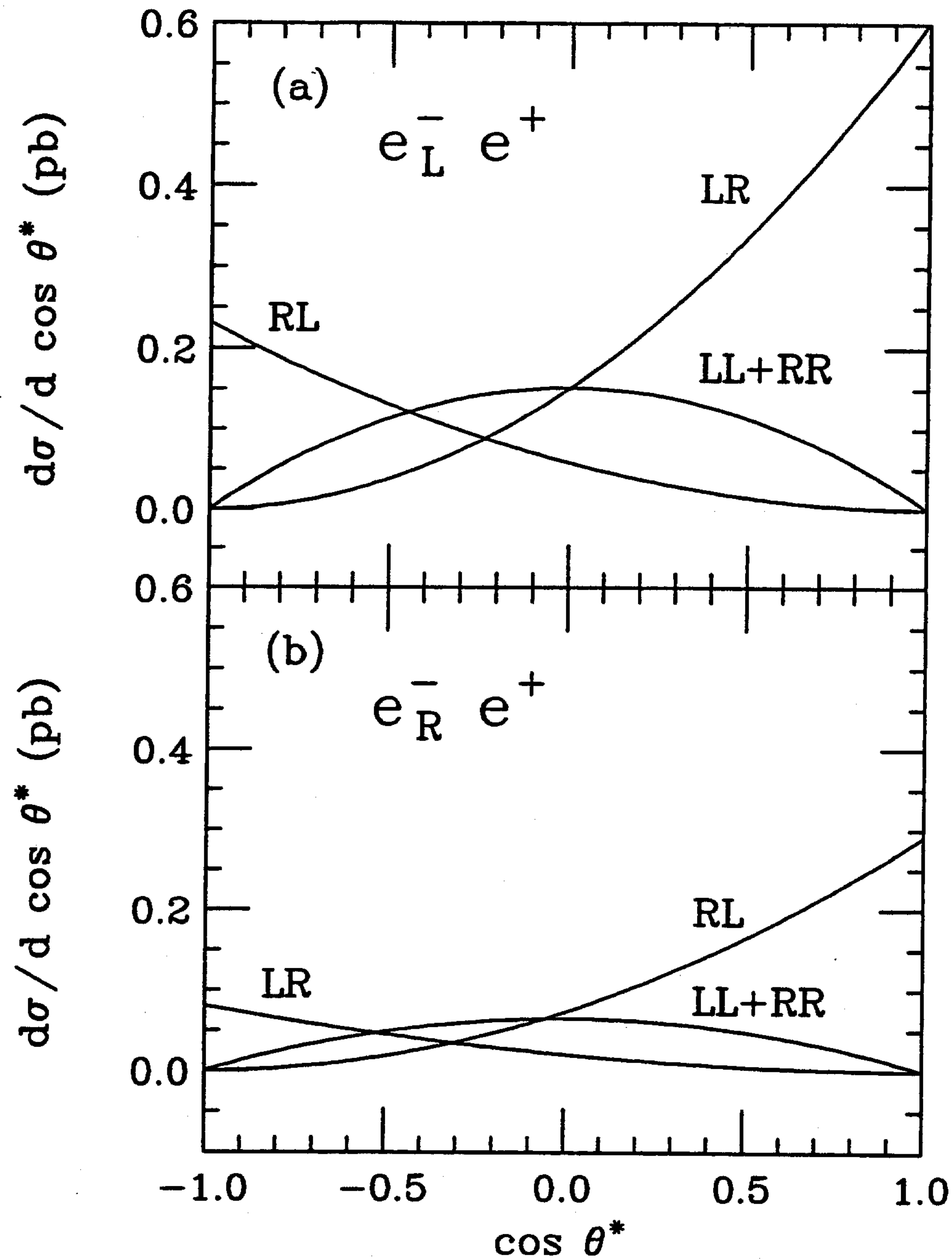

FIG. 3. The differential cross-sections for producing top-quark pairs at a 400 GeV e^+e^- collider in the following helicity configurations: $t_L\bar{t}_R$ (LR), $t_R\bar{t}_L$ (RL), and the sum of $t_L\bar{t}_L$ and $t_R\bar{t}_R$ (LL+RR), for left-handed and right-handed electron beams.

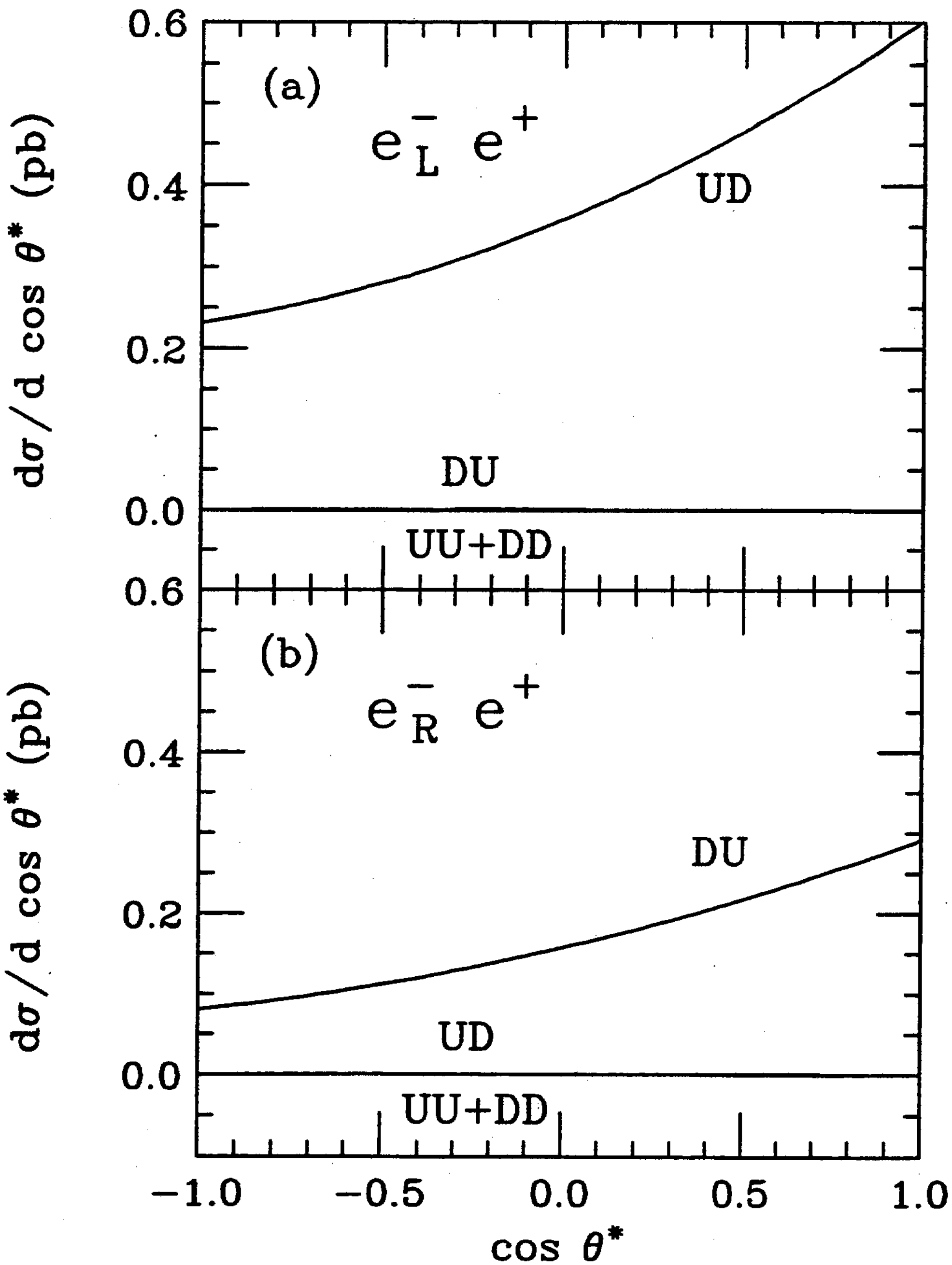

FIG. 4. The differential cross-sections for producing top-quark pairs at a 400 GeV e^+e^- collider in the following spin configurations in the Off-diagonal basis (defined for $e_L^-\ e_R^+$ scattering): $t_\uparrow\bar{t}_\downarrow$ (UD), $t_\downarrow\bar{t}_\uparrow$ (DU), and the sum of $t_\uparrow\bar{t}_\uparrow$ and $t_\downarrow\bar{t}_\downarrow$ (UU+DD), for left-handed and right-handed electron beams.

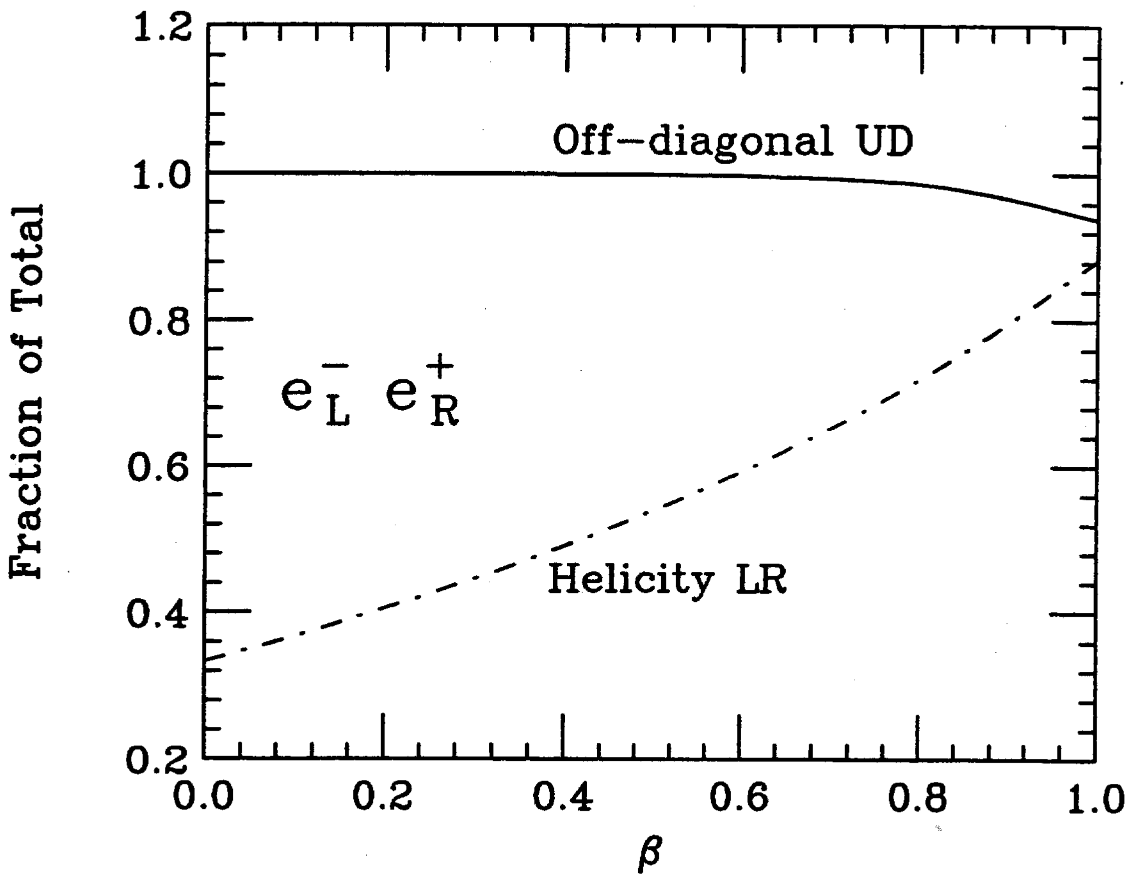

FIG. 5. The fraction of top-quark pairs in the dominant spin configuration in the Off-diagonal basis and in the helicity basis, as a function of the top-quark speed, β, in $e_L^- e_R^+$ scattering. The solid line gives $\sigma(e_L^- e_R^+ \to t_\uparrow \bar{t}_\downarrow)/\sigma_T$, defined in the Off-diagonal basis. The dot-dashed line gives $\sigma(e_L^- e_R^+ \to t_L \bar{t}_R)/\sigma_T$ in the helicity basis. Here σ_T is the total cross-section for $e_L^- e_R^+ \to t\bar{t}$.

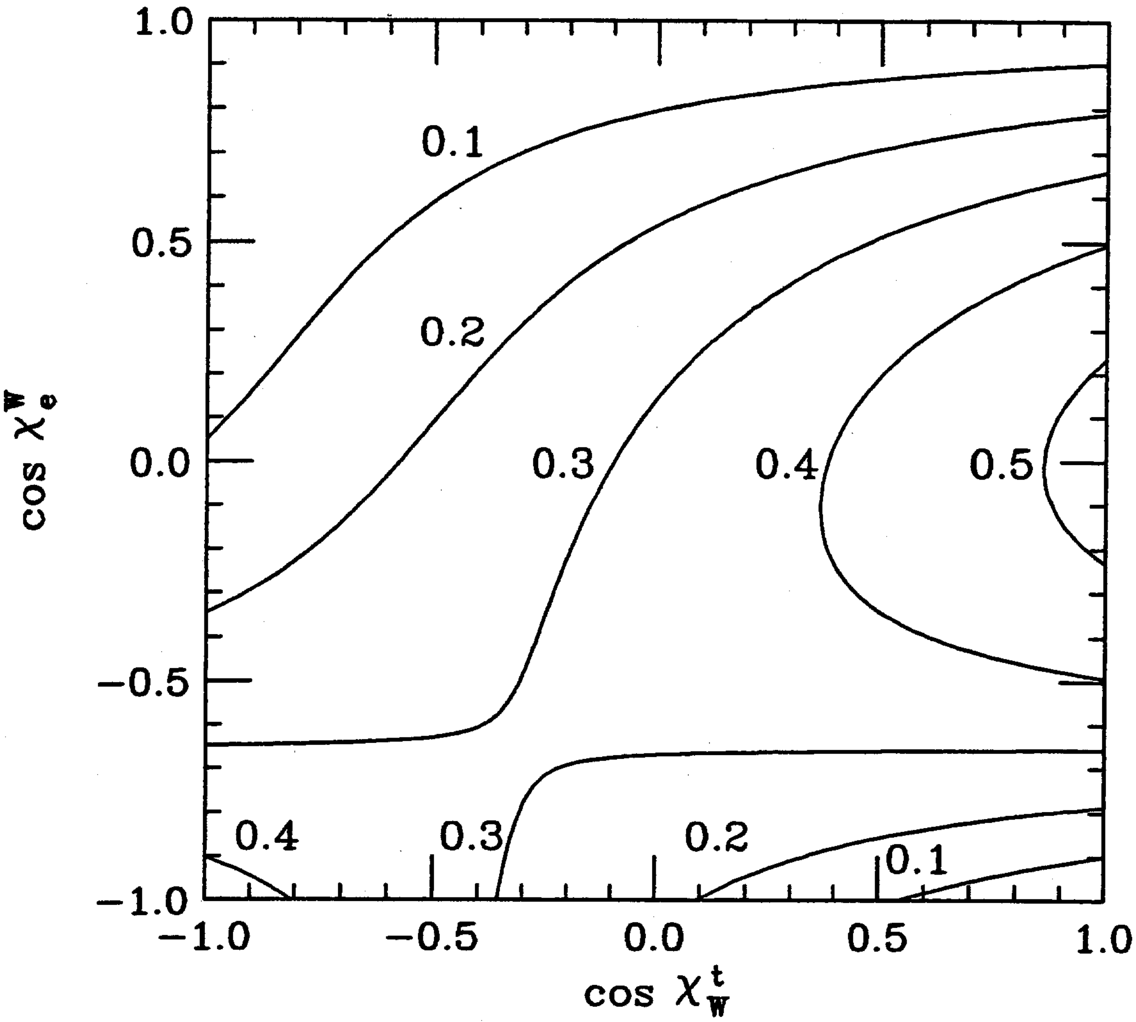

FIG. 6. Contours of the top-quark decay distribution, eqn. (13), integrated over all Φ, in the $\chi_e^w - \chi_w^t$ plane.

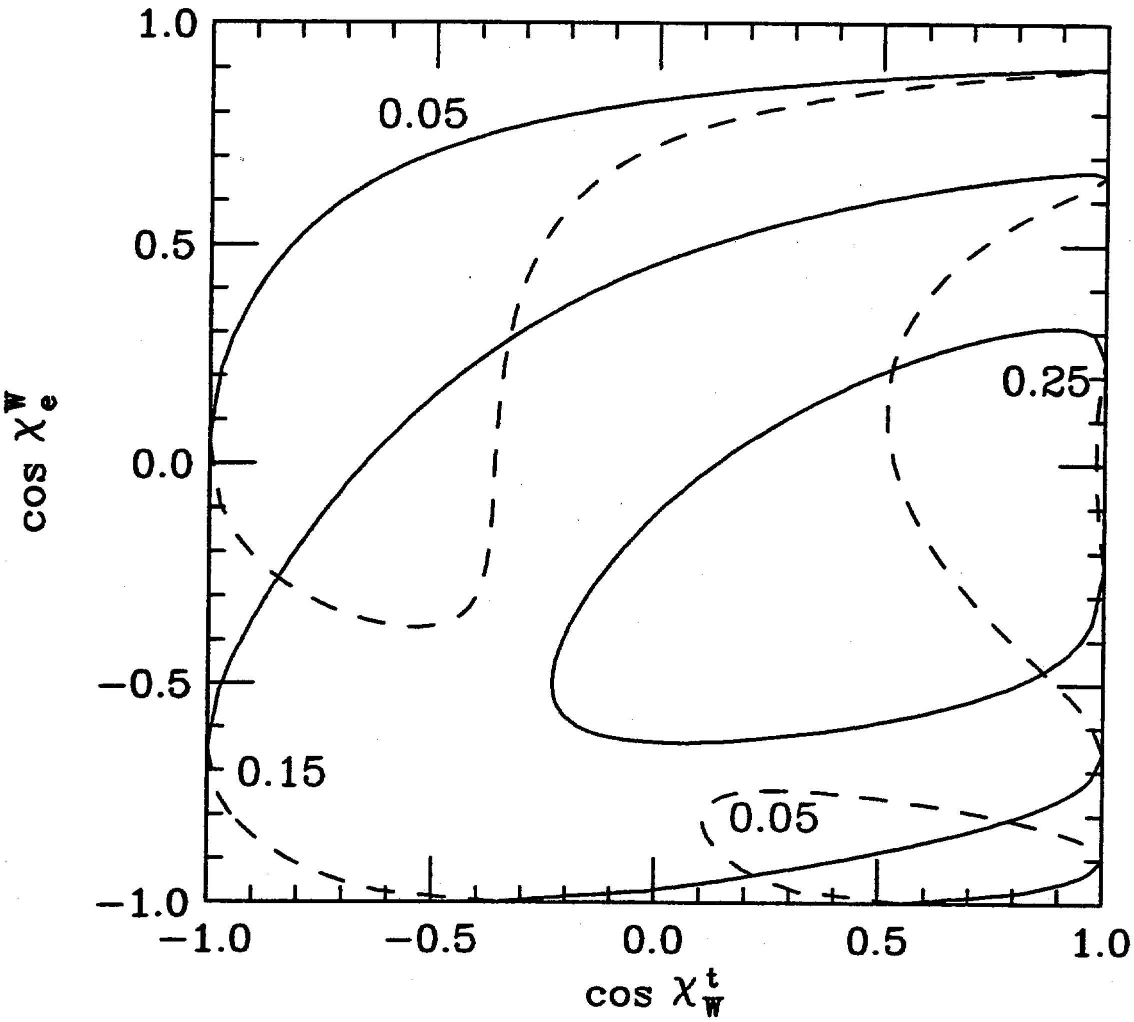

FIG. 7. Contours of the top-quark decay distribution, eqn. (13), integrated over Φ for $\cos\Phi < 0$ (solid), and for $\cos\Phi > 0$ (dashed) in the $\chi_e^w - \chi_w^t$ plane. The solid and dashed curves join continuously at the edges of the plot, where the interference term is zero.