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CDF

Determination of Top Quark Mass at CDF

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Current progress at CDF on the determination of top quark mass in different decay channels is reviewed. Results are based on the final statistics 110 pb^{-1} of combined run Ia and run Ib.

1 Introduction

In 1995 CDF and D0 collaborations discovered top quark in $p\bar{p}$ collisions at TEVATRON [1]. Almost two decades of intensive effort of high energy physics community was finally awarded by a success. The next task is to measure top quark parameters as precisely as possible. In this article we will concentrate on the CDF analysis of top quark mass measurements in a different decay channels.
For the determination of top quark mass following categories of $t\bar{t}$ candidates are used:

- lepton + $\geq$ 4 jets

$$\begin{array}{lllll} p\bar{p} \rightarrow & t & \bar{t} & + & X \\ & & \hookrightarrow & W^-\bar{b} & \\ & \hookrightarrow & W^+b & \hookrightarrow & l^-\bar{\nu} \\ & & \hookrightarrow & q_i\bar{q}_j & \end{array}$$

  This is a standard channel used for the reconstruction of top mass. Using constraints following from the cascade decay chain kinematics of the $t\bar{t}$ one does have two more constraints than number of independent variables. Therefore full reconstruction of the kinematic parameters of particles involved in the decay chain is possible. Method was described for the first time in [2].

- all hadronic channel

$$\begin{array}{lllll} p\bar{p} \rightarrow & t & \bar{t} & + & X \\ & & \hookrightarrow & W^-\bar{b} & \\ & \hookrightarrow & W^-b & \hookrightarrow & q_l\bar{q}_k \\ & & \hookrightarrow & q_i\bar{q}_j & \end{array}$$

  here both W decay into quarks. Therefore 6 jets are required in the final state. It means even more constraint system (3 more constrains than number of independent variables) but this advantage is more than balanced by disadvantage in a larger number of ambiguities in the correct assignments of jets to the correct items in decay chain.

- 2 leptons + $\geq$ 2 jet

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$$
\begin{array}{llllll}
p\bar{p} \rightarrow & t & \bar{t} & + & X & \\
 & & \hookrightarrow & W^-\bar{b} & & \\
 & \hookrightarrow & W^+b & \hookrightarrow & l^-\bar{\nu} & \\
 & & \hookrightarrow & l^+\nu & &
\end{array}
$$

In this case there is one more independent variable to describe decay chain than number of constrains. Therefore full reconstruction of the decay chain is impossible without additional information [3]. In our treatment top mass is not reconstructed on event by event basis anymore. We use two different methods to reconstruct top mass from the distribution of variables sensitive to the top mass.

2 lepton + $\geq$ 4 jets

Selection criteria consisted of requirement at least 3 jets with transverse energy $E_T \geq 15$ GeV each. All jets should be inside pseudorapidity window $|\eta| \leq 2$. In case when only three jets have been found, relaxed criteria for forth jet have been applied ($E_T \geq 8$ GeV and $|\eta| \leq 2.4$). In addition to 4 jets also one isolated lepton with $P_T \geq 20$ GeV/c inside $|\eta| \leq 1$ and missing transverse energy $\not{E}_T \geq 20$ GeV have been required. Above kinematical selection passed 163 events. This sample is still background dominated. To suppress significantly background it is necessary to employ b-tagging procedure. We use two concepts of the b-tagging. One is based on the presence of soft lepton (of nontrivial origin) associated with jet (so called SLT tagging). Other one takes an advantage of excellent resolution of the silicon vertex detector (SVX) and ability to reconstruct vertices of the short lived particles (hadrons with b or c quark component in theirs structure).
Direct top quark mass reconstruction in this channel is possible because decay chain kinematics provides here two more equations than is the number of the independent variables involved. System is overconstraint and this feature is used for selection of the best solution for top mass based on χ^2 criteria.

There are up to 24 ambiguities coming from a different possible associations of jets and leptons to the different vertices in the decay chain and a fact that there are two solutions for the longitudinal component of neutrino.
To the kinematically selected sample top mass reconstruction procedure is applied and only events which have best $\chi^2 \leq 10$ are reserved for further analysis. 153 events passed above selection. After application of SVX and SLT tagging procedure our final sample consists of 34 events and the background is expected to be on the level of 6.4 events.

The templates of the reconstructed top mass distributions are created for Monte Carlo (MC) $t\bar{t}$ events (generated by HERWIG [5]. passed through detector simulation and then through reconstruction chain as for real data) for the fixed top mass in range from 140 GeV/c^2 to 220 GeV/c^2. Reconstructed "top mass" distribution is created for the background W +jets events generated by VECBOS [6]. From the signal ($t\bar{t}$) and the background templates likelihood function is formed. Only free parameters in the likelihood function are the top mass and the signal to background ratio. Minimum of the negative likelihood in the top mass parameter space determines best estimate of top mass.
The reconstructed top mass distribution of 34 $t\bar{t}$ candidates is presented on Fig.1. On the

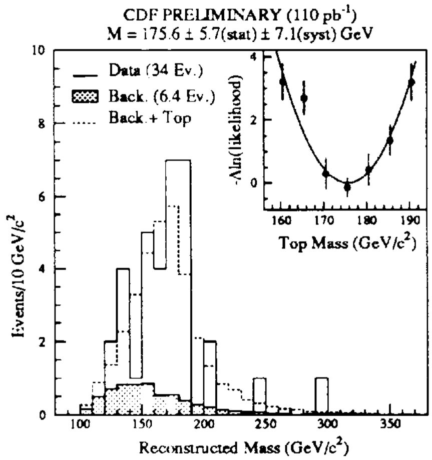

Figure 1: Reconstructed top mass distribution of 34 $t\bar{t}$ candidates.

same plot also likelihood fit is displayed. Result of the fit is:

$$M_t = 175.6 \pm 5.7(\mathbf{stat})\ \mathbf{GeV/c^2}.$$

Reconstructed mass distribution obtained from the likelihood fit of the thousand 34 MC events samples (generated top mass was 175 GeV/c^2) is presented on Fig.2.

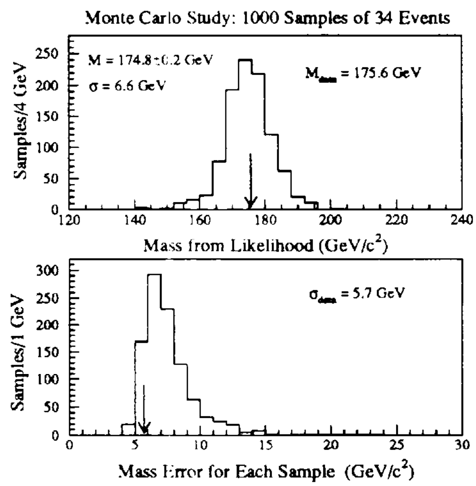

Figure 2: Distribution of the result of 1000 MC experiments of the top mass reconstruction from 34 event samples. Each sample consists of combination of $t\bar{t}$ events generated by HERWIG for M_t = 175 GeV/$c^2$and background generated by VECBOS. Background is kept on average 6.4 events in the samples. Lower plot represents distribution of uncertainty in top mass estimates. Arrows point to re sult for the real data.

Main effort over the last year was concentrated on better understanding of the systematic errors. Different contributions to systematic errors are summarized in the Tab.1. Individual contributions to the systematic errors are explained below.

- Jet E_T Scale

Table 1: Systematic uncertainties in top mass measurement

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Systematics	Value	
	GeV	(%)
Jet E_T Scale	3.1	1.8
Soft Gluon Effects	1.9	1.1
Different Generators	0.9	0.6
Hard Gluon Effects	3.6	2.1
Fit Configuration	2.5	1.4
b-tagging Bias	2.3	1.3
Background Spectrum	1.6	0.9
Likelihood method	2.0	1.1
Monte Carlo statistics	2.3	1.3
Total	7.1	4.0

Corrected transverse energy in calorimeter is calculated according following formula:

$$E_t^{cor} = E_t^{raw}(R) \times f_{rel}(R) \times f_{abs}(R) - UA(R) + OC(R) \qquad (1)$$

where R means radius of cone (in the (η, ϕ) space) that was used for jet reconstruction, f_{rel} relative correction (which makes response of calorimeter uniform over pseudorapidity range covered by calorimeter). UA underlying event correction, OC out of given cone (R) correction (correction that compensates for fact that in jet reconstruction fixed cone size was taken). Each item is moved by 1 σ up and down separately and changes in reconstructed top mass from nominal value of simulated sample give a measure of the systematic error on reconstructed top mass. Estimated systematic errors for specific items are added in quadrature to get final estimate of the systematic error.

- Soft Gluon Effects
  Estimate of the systematic error from the uncertainty in simulation of soft gluons was guessed from a comparison of ratio of corrected jet E_T obtained with cone radius 1.0 to the same quantity obtained with cone size 0.4 for data and MC. As an example above distribution for W + 1 jet events sample is plotted on Fig.3.

  Assuming that all difference between MC and data is due to soft gluons we apply a corresponding correction and estimate an effect on the reconstructed mass.

- Hard Gluon Effects
  Systematic errors caused by hard gluon radiation have been estimated following way: In the simulated data there was determined fraction of events when distance of closest

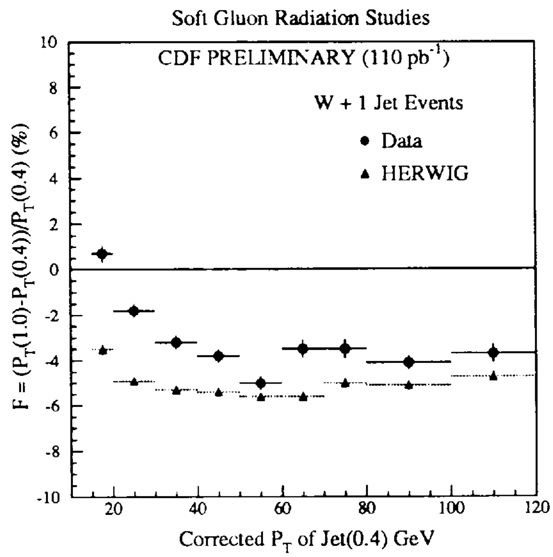

Figure 3: Fractional difference of corrected jet energy for radius of jet cone 1.0 and 0.4 . A comparison of real data with HERWIG simulation.

reconstructed jet (in the (η, ϕ) space) to original quark direction was greater than 0.4. These events we call events with gluon jets. Fraction of these kind of events is model dependent (e.g. in HERWIG it represents 55 % of events). In the study of systematic, fraction of gluon jet events was changed from 38 % to 72 % and change in the reconstructed mass was taken as a measure of a given kind of systematic.

- Different Generators
  The templates have been produced using HERWIG generator. In a study of the systematic also the generator ISAJET was used. Difference in the fitted mass was taken as a systematic error.

- Fit Configuration
  There are possible also other plausible strategies for the determination of the top quark mass than one based on minimum χ^2 criterium.

- b-tagging bias
  In principle the b-tagging procedure affects selected kinematic configuration of $t\bar{t}$ events and background in a different way.

- Background spectrum
  As a standard for background calculation was taken spectrum of W + n jets generated by VECBOS at $Q^2 = P_T^2$ scale. As a measure of the systematic error was used background spectrum by VECBOS calculated at $Q^2 = M_W^2$ scale.

- Likelihood method
  The systematic uncertainties from a choice of mass range for the likelihood fit, parabola or cubic fit of the likelihood fit etc.

- Monte Carlo statistics
  The templates have finite statistic which can be source of systematic error

Final result on the top mass estimate is:

$$M_t = 175.6 \pm 5.7(\mathbf{stat})\ \pm 7.1(\mathbf{syst})\ \mathbf{GeV/c^2}$$

There was also performed analysis on the subsample of events that have two b-tagged jets (relaxed criteria for the second tag have been applied). Ten events have been double tagged. Expected background is 0.4 events (additional suppression of background by factor 3 was obtained by requirement that effective mass of two untagged jets should be inside region (60., 100.) GeV/c^2). Apart from much lower background than in the standard sample advantage is also higher probability of the correct assignment of jets to the proper items in top decay chain and as consequence better resolution. Disadvantage is more than factor 3 lower statistics.
Estimate of systematic is according to schema described above. Final result of this analysis is:

$$M_t = 174.8 \pm 7.6(\mathbf{stat}) \pm 5.6(\mathbf{syst})\ \mathbf{GeV/c^2}$$

3 all hadronic channel

Our kinematic selection for these events requires $6 \geq n_j \leq 8$ jets with $E_T \geq 15$ GeV inside $|\eta| \leq 2$ separated by $R_{min} \geq 0.5$, $\Sigma E_T \geq 200$GeV, $\Sigma E_T/\sqrt{\hat{s}} \geq 0.75$ and aplanarity + $0.0025 \times \Sigma E_T \geq 0.54$. Only SVX tagging procedure is used for b-tagging.
There are three classes of kinematically selected $t\bar{t}$ candidates:

- no events b-tagged
  In this class signal to background ratio is 1/30. Because of the low content of signal, shape of distributions from this source is used to describe background.

- single tagged events

- double tagged events

Only single and double tagged events are used for top mass estimate. There are 90 possibilities to associate 6 most energetic jets to different items in $t\bar{t}$ decay chain. Requirement of association of at least one b tagged jet with one of the 2 b-jets in $t\bar{t}$ decay chain reduces number of ambiguities to 30. A combination with the smallest χ^2 for reconstructed top mass is selected. On Fig.4 reconstructed top mass distribution is displayed. Shaded area represents shape of expected background. white histogram is background + signal (simulated by HERWIG $t\bar{t}$ events for $M_t = 175$ GeV/c^2, bullet represents data. Estimate of systematic errors follows procedure described in previous section. Final result is:

$$M_t = 186 \pm 10(\mathbf{stat}) \pm 12(\mathbf{syst})\ \mathbf{GeV/c^2}$$

4 2 leptons + $\geq$ 2 jets

The selection criteria consist of selection of the two opposite sign isolated leptons $P_T \geq$ 20 GeV/c, at least two jets with raw $E_T \geq 10$ GeV and $\not{E}_T \geq 25$ GeV. Also events which

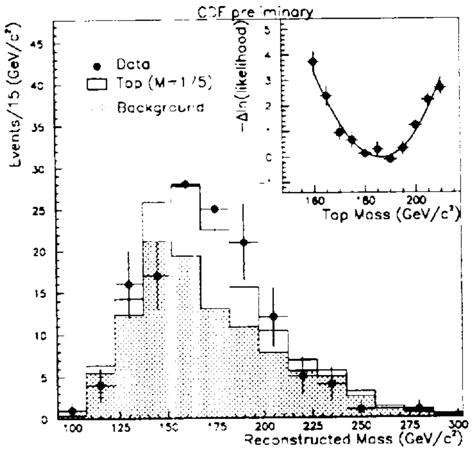

Figure 4: Reconstructed top mass distribution for b-tagged all hadronic $t\bar{t}$ candidates. Bullet = data, filled area = background normalized to its estimated size, white = background + top contribution (HERWIG, M_t = 175 GeV/c^2)

have $\not{E}_T \leq 50$ GeV and absolute value of difference in azimuthal angle between the direction of $\not{E}_T$ and transverse momentum of any isolated lepton or jet (above threshold) bellow 20^o are rejected. To suppress background even more scalar sum of E_T of jets $\not{E}_T$ and P_T of selected leptons is required to be larger than 170 GeV. These criteria passed 8 events and estimated background is 1.1 ± 0.3 events.
Because of the production of two neutrinos in this channel, system is underconstraint from the point of view of kinematic reconstruction of decay chain. For 17 independent variables decay chain kinematics offers only 16 equations.
We applied two independent methods to determine top mass in this channel. The first one is based on likelihood fit [7] to energy distribution of two most energetic jets in an event ($t\bar{t}$ candidates). Likelihood function is a combination of the top mass dependent templates for energy distribution of two most energetic jets and the same distribution for background. Number of background events is represented by Poisson distribution with average 1.1 smeared according to normal distribution with $\sigma = 0.3$. Method was tested on a Monte Carlo samples composed of 7 $t\bar{t}$ and 1 background event. Reconstructed top mass distribution for these samples and also a distribution of statistical errors are plotted on Fig.5. As one can see expected statistical error is close to 22 GeV/c^{2}virtually independent of top mass. When the same method is applied to the data we obtain top mass estimate:

$$M_t = 159 \pm 22(\text{stat}) \pm 17(\text{syst})$$

Another method is based on a combination of kinematics and specific result from the standard method calculation. The method was for the first time described in [8] (see also [9]). Assuming $t \to Wb \to l\nu b$ decay chain by simple exercise in cascade kinematics one easily arrives at formula for top mass in W rest frame:

$$M_t = \sqrt{2M_w E_b + m_b^2 + M_w^2} \qquad (2)$$

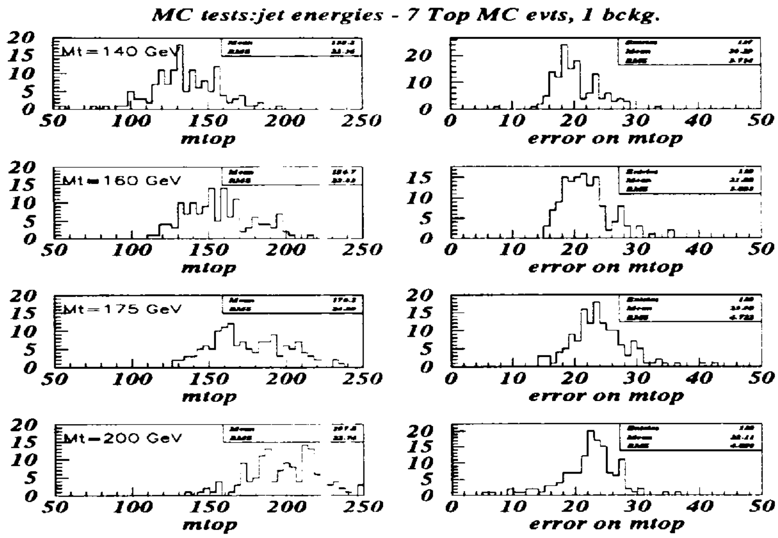

Figure 5: Expected resolution for different top mass hypothesises in case of likelihood fit to energy distribution of two most energetic jets

Energy of b-jet in W boson rest frame can be determined as follows:

$$E_b = \frac{A + |c|\sqrt{m_b^2(c^2-1)+A^2}}{1-c^2} \tag{3}$$

where $A = \frac{\langle m_{lb}^2 \rangle}{M_w} - \frac{m_b^2}{M_w}$, $c = \langle \cos(\theta_{lb}) \rangle$; m_b, M_w, m_{lb} are mass of b quark, W boson and effective mass of lepton and b-jet respectively, θ_{lb} is an angle between lepton from W decay and b-jet in W rest frame.

For the top mass greater than 100 GeV/c^{2}approximate formula can be derived from equation (2) and (3):

$$M_t^2 \simeq M_w^2 + 2\frac{< m_{lb}^2 >}{1-c} \tag{4}$$

Precision of this formula is better then 0.2 % for the top mass greater than 140 GeV/c^2 and is even improving at higher top mass.

In the tree level standard model calculation in the W rest frame [10]

$$\langle \cos(\theta_{lb}) \rangle = \frac{M_w^2}{M_t^2 + 2M_w^2} \tag{5}$$

Combining previous two formulas one obtains:

$$M_t \simeq \sqrt{< m_{lb}^2 > + \sqrt{< m_{lb}^2 >^2 + M_w^2(M_w^2 + 4 < m_{lb}^2 >)}} \tag{6}$$

As one can see average effective mass of lepton and b-jet from top decay is directly and unambiguously related to the top mass.

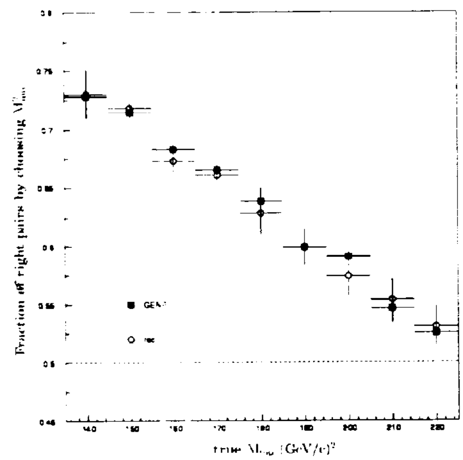

Figure 6: Probability to select correct combination of lepton and b-jet (from the same top) based on minimal sum of effective mass for different combinations of lepton and b-jet

In $t\bar{t}$ events there are two possibilities to combine two leptons with two b-jets. Only one combination correctly combines b-jets and leptons from the same $t(\bar{t})$ decay. We choose as a correct combination case that gives smallest sum of effective masses of lepton and b-jet. One can see on Fig.6 distribution of probability that this is the right choice as a function of the top mass. Average of the effective mass squared of lepton and b-jet (we consider two most energetic jets in an event as a b-jets) selected according to above prescription we denote as $< m_{lb}^2 >_{sm}$. In reality there arises also problem that sometimes at least one lepton does not come directly from the W decay or that one of the two most energetic jets is not a b-jet. Probability for different categories of $t\bar{t}$ events to pass dilepton $t\bar{t}$ selection criteria are displayed on Fig.7. To obtain estimate of $< m_{lb}^2 >$ from top quark decay from $< m_{lb}^2 >_{sm}$ correspondence function was derived based on simulation of $t\bar{t}$ production by HERWIG and detector simulation by CDF simulation package called QFL. This function is designed to correct for both selection bias and detector effects.

In the calculation of top mass from our data sample (8 events) background (1.1 events) was ignored in this case. Possible bias from the background is included in the systematic errors. Distribution of the statistical error of top mass estimated from 8 events samples of $t\bar{t}$ events generated by HERWIG (for M_t = 160 GeV/c^2) which passed through detector simulation, reconstruction and dilepton selection chain is displayed on fig.8. Final result of the top mass estimate is:

$$M_t = 162 \pm 21(\textbf{stat}) \pm 7(\textbf{syst})\ \textbf{GeV/c}^2$$

Systematic errors in both methods are dominated by jet energy scale uncertainty and an uncertainty in the background. Current estimates of the systematic error are very conservative.

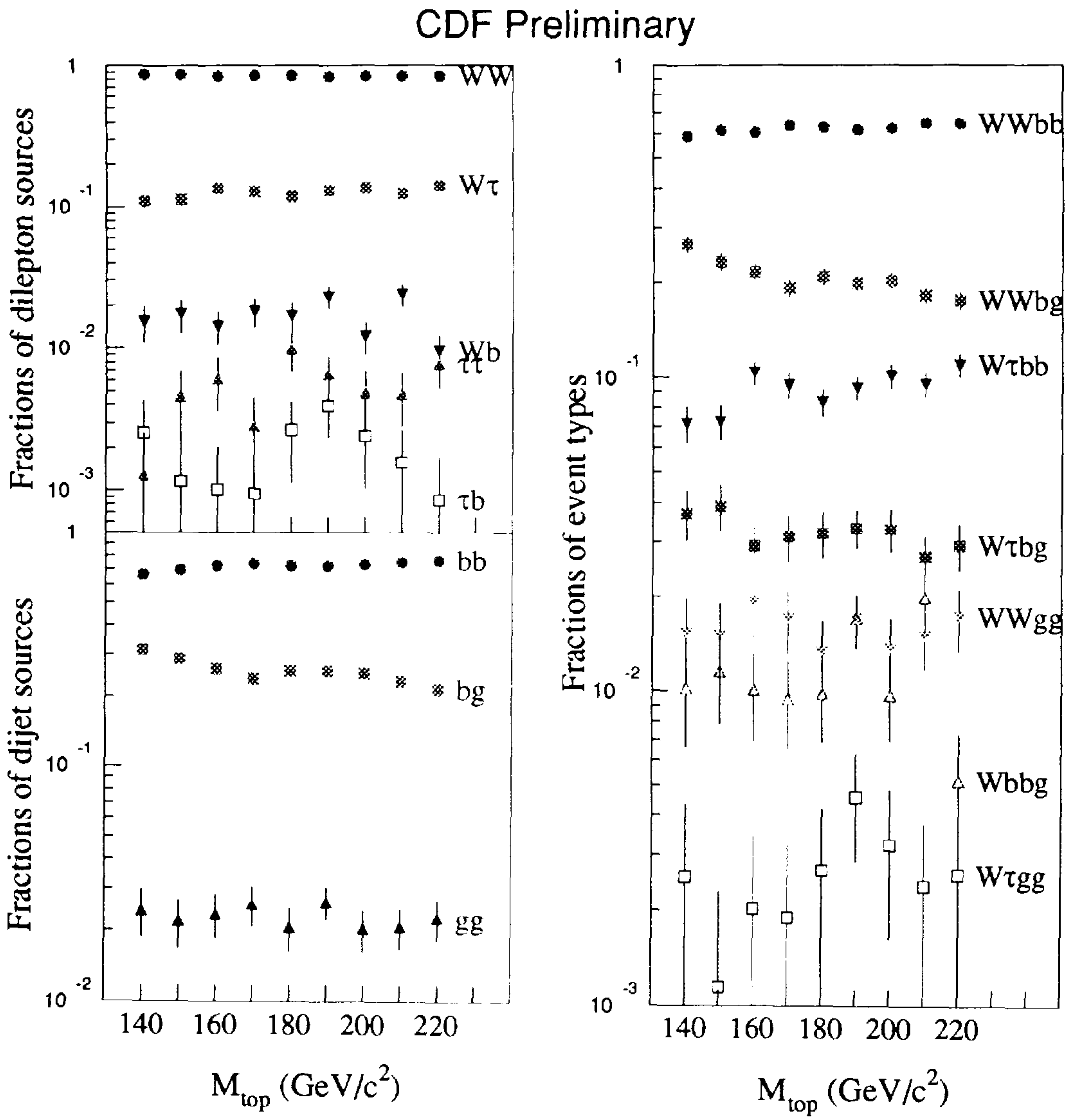

Figure 7: Fraction of different types of $t\bar{t}$ events passing selection criteria for dilepton $t\bar{t}$ events.

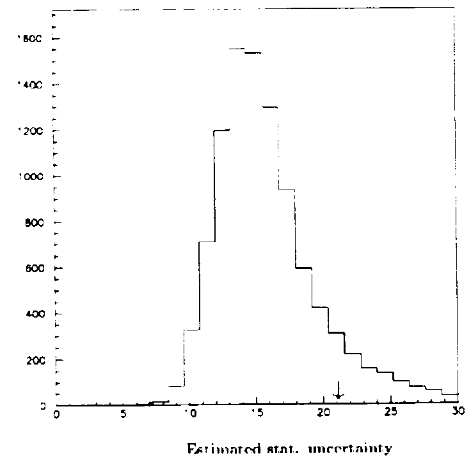

Figure 8: Distribution of expected statistical error for top mass estimated for 8 events sample ($t\bar{t}$ generated by HERWIG; M_t = 160 GeV/c^2)

5 Optimized Top Mass measurement

Collected statistic is final one. Next run is scheduled in 1999. Only possibility to get better measurement of top mass until next run is to combine existing measurements and to use as much of existing information as possible.
In original treatment of lepton + ≥ 4 jets channel only b-tagged events are used for top mass reconstruction. About one third of events which are not tagged is expected to be of $t\bar{t}$ origin.
SVX and SLT taggers have different efficiency and background suppression features. To achieve better resolution it is natural to treat separately SLT and SVX tagged $t\bar{t}$ candidates. These are possible improvements which can be incorporated into top mass determination [11].
Following samples form so called orthogonal subsets of lepton +jets channel.

- SVX single tagged (no SLT tag)
- SVX double tagged (no SLT tag)
- SLT tagged (no SVX tag)
- No tag

For orthogonal samples combined likelihood function is a product of the individual likelihood functions.
For tagged samples selection criteria are the same as described in section 2. Selection criteria for no tag sample requested all four jets to pass the same selection criteria (no relaxed criteria for forth jet). Different likelihood functions have been defined for each set. Likelihood fit was performed using combined likelihood function.

Final result of the top mass estimate is:

$$M_t = 176.8 \pm 4.4(\textbf{stat}) \pm 4.8(\textbf{syst})\ \textbf{GeV/c}^2$$

Samples used for top mass determination in section 3 and 4 are also orthogonal to the sets used in this section. Work is in progress to include them in the combined top mass estimate. Main problem is the correct treatment of systematic errors in the combined top quark mass determination.

6 Conclusion

Top quark mass was determined independently in the three different channels. Results are mutually consistent. Most precise determination comes from combined optimized measurement in the lepton + jets channel – $M_t = 176.8 \pm 4.4(\text{stat})\pm 4.8(\text{syst})$ GeV/c^2.

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