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FERMILAB-Conf-96/378

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October 1996

Submitted to the Proceedings of the *XII Advanced ICFA Workshop on Nonlinear and Collective Phenomena in Beam Physics: Theory and Experiments*, Arcidosso, Italy, September 2-9, 1996.

Operated by Universities Research Association Inc. under Contract No. DE-AC02-76CH03000 with the United States Department of Energy

Microwave Instability in α-like Quasi-Isochronous Buckets

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Abstract. The problem of microwave instability inside an α-like quasi-isochronous bucket is addressed. The coupling impedance at wavelengths shorter than the length of the short bunches is found to be not small. The Keil-Schnell criterion is modified for such a bucket using the concept of self-bunching. The mechanism of particle loss during a microwave growth is examined.

I INTRODUCTION

In order to obtain very short bunches and to avoid unreasonably large rf voltage, sometimes it is necessary to design a storage ring that operates with a very small slippage factor η_0. In that case, η_1, the next order in fractional momentum-offset δ of the slippage factor, will become important. If $\Delta\phi$, the rf phase offset, is used as a variable conjugate to the fractional momentum-offset δ, the Hamiltonian governing the motion of a beam particle can be written as

$$H=\left(\frac{\eta_0\delta^2}{2}+\frac{\eta_1\delta^3}{3}\right)h\omega_0+\frac{eV\omega_0}{2\pi\beta^2E}\left[\cos(\phi_s+\Delta\phi)+\Delta\phi\sin\phi_s\right]\ , \tag{1.1}$$

where ϕ_s is the synchronous phase, V the rf voltage, E the energy of the particle with revolution frequency $\omega_0/(2\pi)$ and velocity β relative to that of light. At large $|\eta_0/\eta_1|$, the Hamiltonian represents two series of distorted pendulum-like buckets as shown in Fig. 1(a). As $|\eta_0/\eta_1|$ decreases to a point when the values of the Hamiltonian through all unstable fixed points are equal, the two series merge as depicted in Fig. 1(b) [1]. When

$$\left|\frac{\eta_0}{\eta_1}\right|<\left\{\frac{6eV}{\pi\beta^2h\eta_0E}\left[\left(\frac{\pi}{2}-\phi_s\right)\sin\phi_s-\cos\phi_s\right]\right\}^{1/2}\ , \tag{1.2}$$

the buckets becomes α-like as in Fig. 1(c). By the way, the right-hand side of Eq. (1.2) is just $\sqrt{3}$ times the half bucket height when the η_1 term in the Hamiltonian is absent. This α-shaped bucket is asymmetric in fractional

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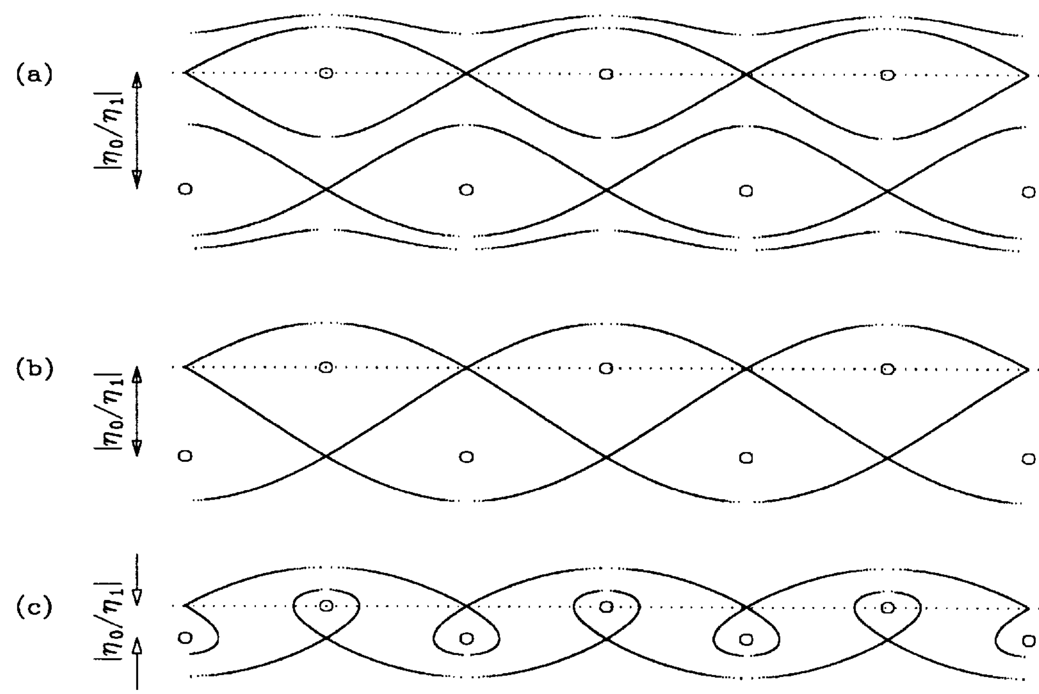

FIGURE 1. (a) When $|\eta_0/\eta_1|$ is not too small, the longitudinal phase space shows 2 series of distorted pendulum-like buckets. (b) As $|\eta_0/\eta_1|$ decreases to the critical value in Eq. (1.2), the 2 series merge. (c) Further reduction of $|\eta_0/\eta_1|$ leads to new pairing of stable and unstable fixed points and the buckets become α-like. In each case, the dotted line is the phase axis at zero momentum spread, and the small circles are the stable fixed points.

momentum spread, with fixed $|\eta_0/2\eta_1|$ in the positive direction and $|\eta_0/\eta_1|$ in the negative direction.

By the deployment of sextupoles, the contribution of η_1 can be eliminated, and the next order η_2 will restore the bucket to pendulum-like, even if the zeroth order η_0 vanishes. However, the α-like bucket has its own merit of being much narrower than the pendulum-like bucket. Therefore, if one needs an extremely short bunch, such a bucket may become indispensable.

Since the total slippage factor, $\eta = \eta_0 + \eta_1\delta + \eta_2\delta^2 + \cdots$, is vanishingly small, the spread in revolution frequency inside the bunch will be small. As a result, there will be very little Landau damping to counteract the growth in microwave instability. Since the microwave wavelength must be less than the size of the bunch, we would like to investigate in Sec. II whether there is any significant coupling impedance at such short wavelengths to drive the collective instability. In Sec. III, we study the concept of self-bunching and rewrite the Boussard-modified Keil-Schnell stability criterion [2–4] so that it is applicable for the α-like bucket. We also investigate the way particles are lost during microwave instability. Finally, conclusions are given in Sec. IV.

In our study, the recently proposed 2 TeV-2 TeV muon-muon collider [5] will be used as an example. Although the α-like bucket will be avoided by two families of sextupoles [6,8], an analysis of microwave instability inside such a bucket is still of much interest [7].

II AMOUNT OF IMPEDANCE

The usual Boussard-modified Keil-Schnell criterion for longitudinal microwave stability is

$$\left|\frac{Z_\parallel}{n}\right| \lesssim \left(\frac{2\pi|\eta_0|\beta^2 E}{eI_p}\right)\sigma_\delta^2 \ , \tag{2.1}$$

where $Z_\parallel/n$ is the longitudinal coupling impedance of the vacuum chamber per revolution harmonic,

$$I_p = \frac{eN}{\sqrt{2\pi}\sigma_\tau} \tag{2.2}$$

is the local peak current, E is the particle energy and β its velocity with respect to the velocity of light. Each bunch in the muon collider contains $N = 2\times 10^{12}$ muons with a rms length $\sigma_\tau = 10$ ps and a rms momentum spread of $\sigma_\delta = 0.15\%$. The short bunch length as well as a reasonable rf voltage limit the slippage factor of the collider to $\eta \lesssim 1\times 10^{-6}$ [7,8]. The circumference of the collider ring is $C_0 = 2\pi R \sim 8000$ m. The peak local current is therefore $I_p = 12.78$ kA, and the limit for microwave stability turns out to be $|Z_\parallel|/n \lesssim 0.0022$ Ohm [9], which is definitely exceeded by coupling impedance of any realistic vacuum chamber.

The Boussard-modified Keil-Schnell criterion can be rewritten as

$$n\omega_0\sqrt{\frac{eI_p|Z_\parallel/n||\eta_0|}{2\pi\beta^2 E}} \lesssim n\omega_0|\eta_0|\sigma_\delta \ , \tag{2.3}$$

where the left side is the growth rate and the right side the rate of Landau damping. Since the storage of the muons lasts for only about 1000 turns, if the growth rate is slow, the microwave growth may not be important.

Microwave instability develops inside a bunch with wavelengths less than the length of the bunch. For a bunch of length $\sigma_\tau = 10$ ps, the frequency of the microwave disturbance must therefore be

$$f_{\rm mw} \gtrsim \frac{1}{2\pi\sigma_\tau} = 15.6 \text{ GHz} \ , \tag{2.4}$$

or in revolution harmonics, $n \gtrsim 4.16\times 10^5$. The Landau damping time is therefore about 250 turns and the growth time is much less for a reasonable $Z_\parallel/n$.

Since these times are much less than the storage time of $\sim$ 1000 turns, the microwave growth will be important and the amount of longitudinal coupling impedance in the vacuum chamber becomes an important issue.

One may argue that the usual broad-band model has the coupling impedance centered around the cutoff frequency. Therefore, the $Z_{\|}/n$ at the high frequencies that is responsible for microwave instability must drop down tremendously. However, the designed pipe radius of the collider is $b = 1.5$ cm. With c denoting the velocity of light, the cutoff frequency is therefore already $f_c = 2.405\,c/(2\pi b) = 7.65$ GHz, which is not much lower than the $\sim$ 15 GHz that we talked about. Remember that the broad-band model is only a model and the coupling impedance may not actually be centered right at cutoff. It is also possible that the coupling impedance consists of more than one broad band.

Let us investigate the possibility of having a reasonable amount of longitudinal coupling impedance above 15 GHz. Henke [10] had computed analytically the longitudinal coupling impedance of a small pill-box cavity. With a depth to pipe radius ratio of $\Delta/b = 0.1$ and width to pipe radius $2g/b = 0.01$, the real part of the longitudinal coupling impedance is shown in Fig. 2 [11]. We see roughly a broad band with a peak value of $\sim$ 10 Ω. It is centered at $\omega b/c \approx 15$ or 48 GHz, which corresponds to about a half wavelength into the cavity depth Δ. For a collider ring circumference of 8000 m, this amounts to $\mathcal{R}e\, Z_{\|}/n \approx 7.9 \times 10^{-6}$ Ω. Note that this cavity depth is, in fact, $\Delta = 1.5$ mm and width $2g = 0.15$ mm; so it is like a scratch in the beam pipe. If we add up a lot of such "scratches" or small dings and bugglings for the whole vacuum chamber, the total $\mathcal{R}e\, Z_{\|}/n$ can be appreciable.

In Fig. 3, we plot the real part of the longitudinal coupling impedance for the same pill-box cavity with depth increased to $\Delta/b = 0.2$ and the total width increased to $2g/b = 0.2$. We see two broad peaks. The first one peaks at roughly 25 Ω around $\omega b/c \approx 6$ or $\sim$ 19 GHz, corresponding to $\mathcal{R}e\, Z_{\|}/n \approx 5 \times 10^{-5}$ Ω. This is a cavity of depth $\Delta = 3$ mm and width $2g = 3$mm. The small pill-box like cavity left behind by a shield bellows system can be of such a size. Assuming 1000 bellows systems for the more than a thousand elements, the longitudinal coupling impedance per revolution harmonic is therefore $\mathcal{R}e\, Z_{\|}/n \approx 0.05$ Ω. In an earlier design of the sliding shielded bellows of the Superconducting Super Collider, the pill-box like cavity left behind has a depth of $\Delta = 4$ mm and a width of as much as $2g = 10$ cm when the vacuum chamber is at superconducting temperature. Such a system gives a broad-band impedance [12] of $\sim$ 40 Ω at 15 GHz. For 1000 such bellows in the muon collider, the coupling impedance adds up to $\mathcal{R}e\, Z_{\|}/n \approx 0.1$ Ω.

There must also be some other small discontinuities in the vacuum chamber. As a result, it is not unreasonable to assume that a $Z_{\|}/n$ of magnitude from 0.1 to 1.0 Ω will be driving the microwave instability in the muon collider ring. According to Eq. (2.3), the growth time at 15 GHz can therefore be as short as 37.9 to 12.0 turns.

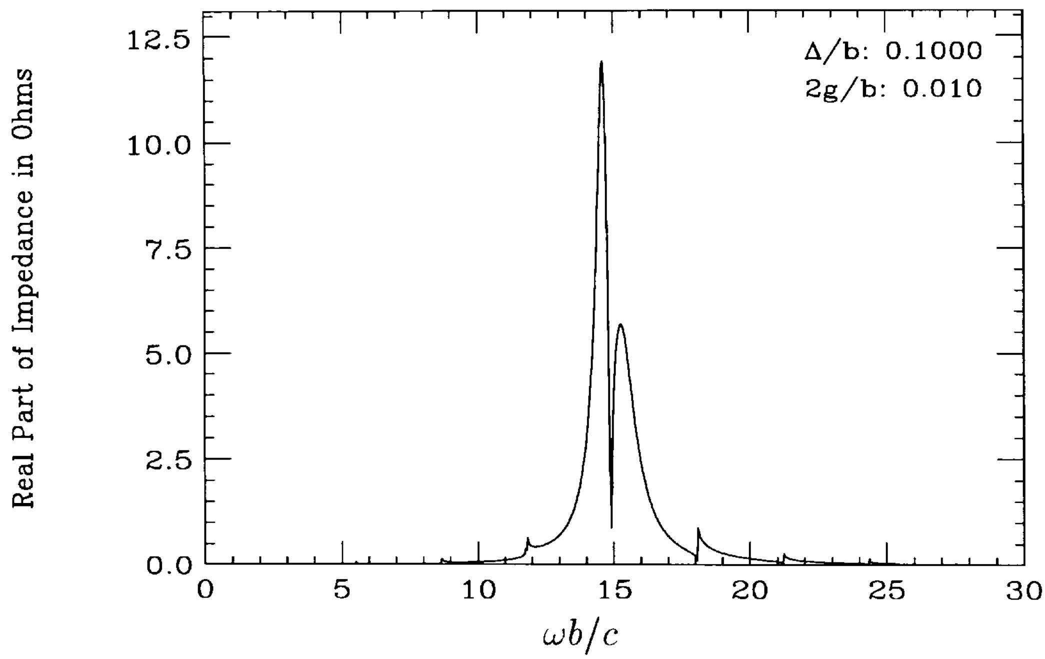

FIGURE 2. Real part of longitudinal impedance as a function of $\omega b/c$ for a pill-box cavity of depth $\Delta = 0.1\,b$ and length $2g = 0.01\,b$, where b is the radius of beam pipe and $\omega/2\pi$ is the frequency.

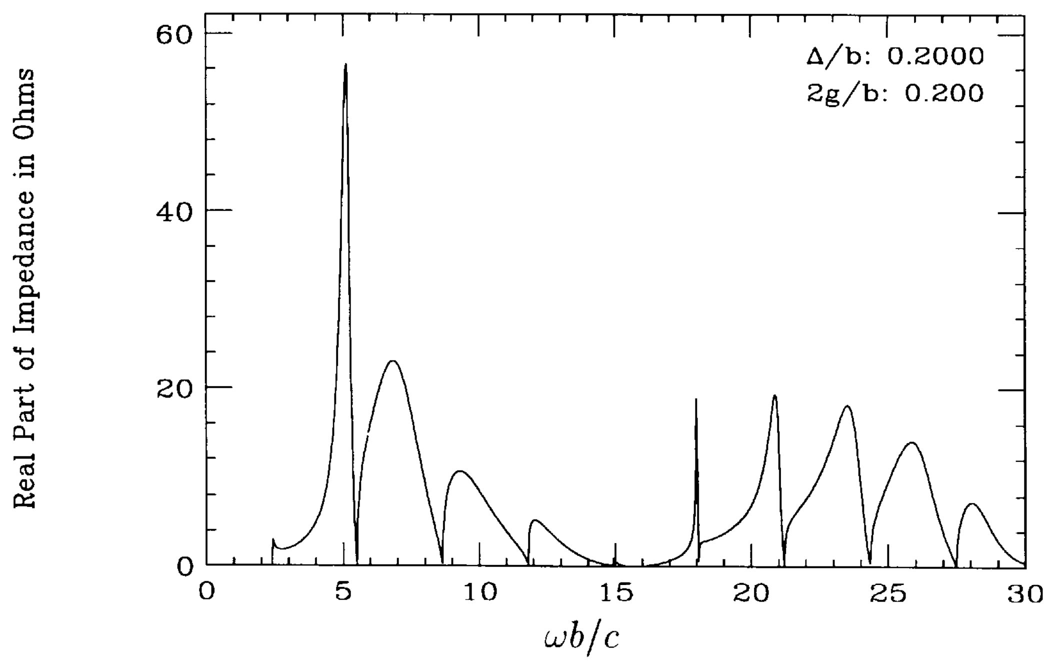

FIGURE 3. Real part of longitudinal impedance as a function of $\omega b/c$ for a pill-box cavity of depth $\Delta = 0.20\,b$ and length $2g = 0.20\,b$, where b is the radius of beam pipe and $\omega/2\pi$ is the frequency.

III SELF BUNCHING

The Boussard-modified Keil-Schnell criterion can also be rewritten as

$$\left(\frac{2eI_p|Z_\parallel/n|}{\pi\beta^2|\eta_0|E}\right)^{1/2} \lesssim 2\sigma_\delta \ . \tag{3.1}$$

The left side is the height of a bucket driven by a voltage $I_p|Z_\parallel|$ at harmonic n. Stability implies that this bucket height must be less than $2\sigma_\delta$ or roughly the half momentum spread $\delta_{\max}$. This is called self-bunching.

Now we can generalize this idea of self-bunching to the α-like bucket. When the driving impedance is small, the particles will self-bunch into pendulum-like buckets as shown in Fig. 4a. Again, when the height of the pendulum-like buckets is larger than the momentum spread of the bunch, we have microwave growth. Therefore, for no growth we require, according to Eq. (3.1),

$$\left(\frac{2eI_p|Z_\parallel/n|}{\pi\beta^2|\eta_0|E}\right)^{1/2} \lesssim \sigma_\delta \ . \tag{3.2}$$

This is because the bunch has a momentum spread in the negative direction that is twice the spread in the positive direction. The impedance limit becomes

$$\left|\frac{Z_\parallel}{n}\right| \lesssim \left(\frac{2\pi|\eta_0|\beta^2E}{eI_p}\right)\left(\frac{\sigma_\delta}{2}\right)^2 \ . \tag{3.3}$$

This amounts to only $|Z_\parallel/n| = 0.00055$ Ohm, which is just $\frac{1}{4}$ of the limit in Eq. (2.1). However, if the coupling impedance is large enough, the self-bunching buckets will be α-like instead as depicted in Fig. 4b. This occurs when

$$\left(\frac{6eI_p|Z_\parallel/n|}{\pi\beta^2|\eta_0|E}\right)^{1/2} > \left|\frac{\eta_0}{\eta_1}\right| \ , \tag{3.4}$$

according to Eq. (1.2). If we take $|\eta_0/\eta_1| = 3\delta_{\max} = 0.009$, the limit for stability becomes $|Z_\parallel/n| = 0.0066$ Ohm, which is most likely less than the impedance of the vacuum chamber. Note that there are two sets of self-bunching buckets. One set has bucket height from $-|\eta_0/\eta_1|$ to $|\eta_0/2\eta_1|$ and the other from $-|3\eta_0/2\eta_1|$ to 0. Unlike the microwave instability in the usual pendulum-like bucket which may lead only to a growth of longitudinal emittance, here particles will be lost by leaking out from the original α-like bucket of the bunch, while, for example, making synchrotron oscillation inside the self-bunching α-like buckets centered at $\delta = -|\eta_0/\eta_1|$. The rate is just the synchrotron frequency inside the self-bunching bucket or, in terms of number of turns,

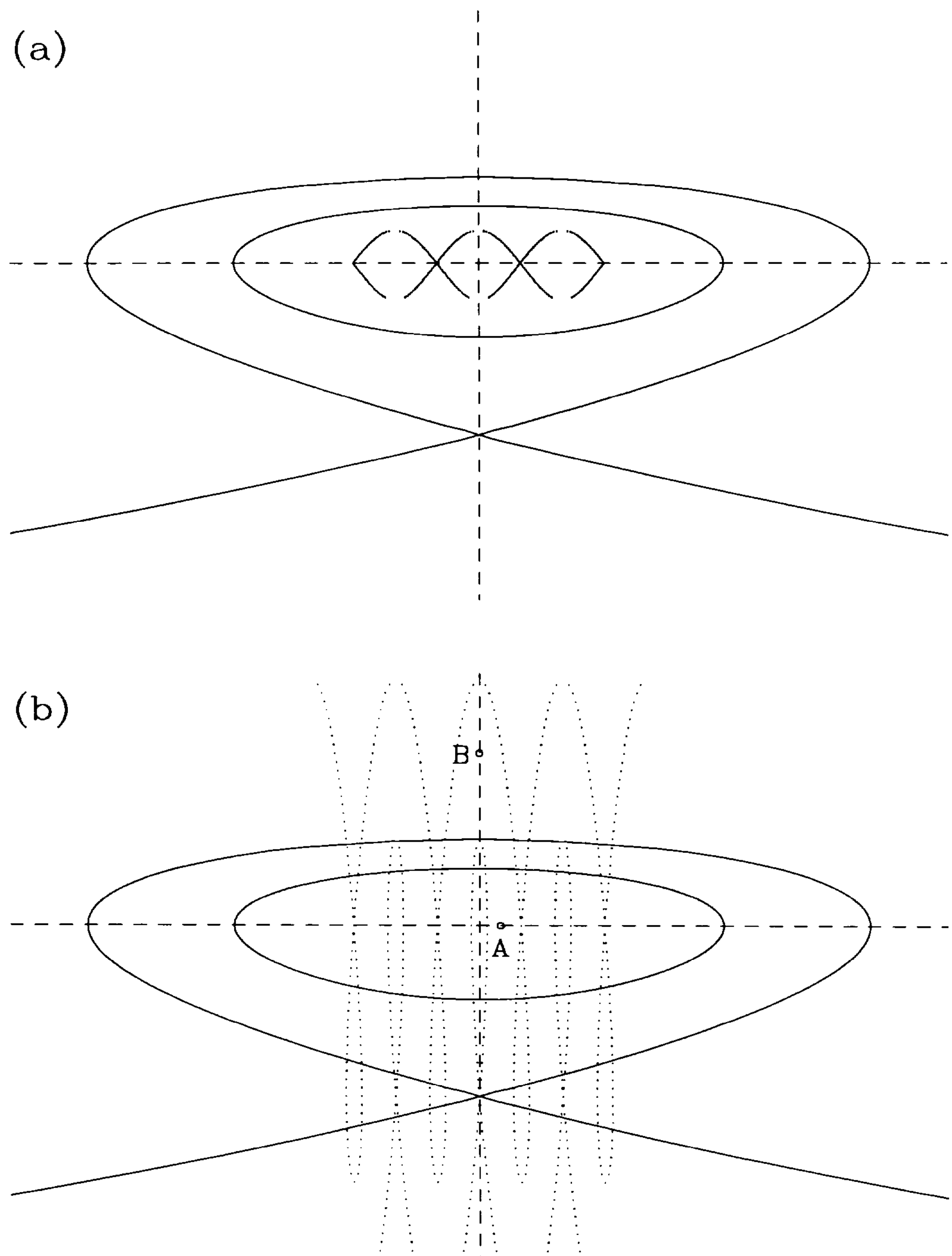

FIGURE 4. A bunch in a α-like bucket is subjected to self-bunching by a disturbance having a wavelength less than the size of the bunch. The self-bunching buckets are (a) pendulum-like when the coupling impedance is small, and change to (b) α-like when the coupling impedance is large.

$$\Delta N = \frac{1}{2\pi n}\left(\frac{2\pi\beta^2 E}{eI_p|Z_\parallel/n||\eta_0|}\right)^{1/2} . \tag{3.5}$$

For a broad-band disturbance of $|Z_\parallel/n| = 1$ Ohm, centered at 20 GHz or harmonic $n = 5.33 \times 10^5$, this amounts to $\Delta N = 9.36$ turns.

It is worth pointing out that the self-bunching buckets in Fig. 4 have been sketched over-simplified. The strength of the self-bunching force depends on $I_\ell|Z_\parallel|$ where I_ℓ is the local linear current. As we are moving from the center to the edge of the bunch, the self-bunching force will decrease to zero. Therefore, the two series of self-bunching α-like buckets will become fatter away from the bunch center and will eventually merge to form pendulum-like buckets instead near the edge of the bunch.

Bunch particles that are not inside a self-bunching bucket can also be lost. For example, a particle at point A will travel to point B. The time taken can be computed. Let A be at the point where the phase is $\pi/2$ in unit of the disturbance harmonic and momentum spread zero. From the Hamiltonian of Eq. (1.1) with synchronous phase $\phi_s = 0$ or π, it is easy to obtain the number of turns required:

$$\Delta N = \frac{\Delta t}{T_0} = \frac{\eta_1^2}{n\eta_0^3}\int_0^{\pi/2}\frac{d\Delta\phi}{p^2+p} , \tag{3.6}$$

where

$$p = \frac{\eta_1\delta}{\eta_0} \tag{3.7}$$

is a normalized momentum spread and depends on $\Delta\phi$ through

$$p^3 + \frac{3p^2}{2} = \frac{1}{4} + \frac{3eI_p|Z_\parallel/n|}{4\pi\beta^2 E}\frac{\eta_1^2}{\eta_0^3}\cos\Delta\phi . \tag{3.8}$$

In Eq. (3.6), T_0 is the revolution period. Therefore, the left side ΔN is the number of turns, and its inverse represents a loss rate, which is proportional to the harmonic n of the disturbance. The integral cannot be performed analytically. However when $I_p|Z_\parallel/n|$ is very large, or more exactly when the left side of Eq. (3.8) is very much larger than $\frac{1}{2}$, the integral can be estimated to obtain

$$\frac{1}{\Delta N} = 2n|\eta_0|\left(\frac{3eI_p|Z_\parallel/n|}{4\pi\beta^2 E\eta_1}\right)^{2/3} . \tag{3.9}$$

With $|\eta_0| = 1 \times 10^{-6}$, $|\eta_0/\eta_1| = 3\delta_{\rm max} = 0.009$, and a bunch intensity of 2×10^{12} muons, the integral in Eq. (3.6) is evaluated numerically, and the loss time ΔN is plotted in Fig. 5 for different values of the impedance. Also plotted is the loss rate $1/\Delta N$, which is not far from the dotted curve, which

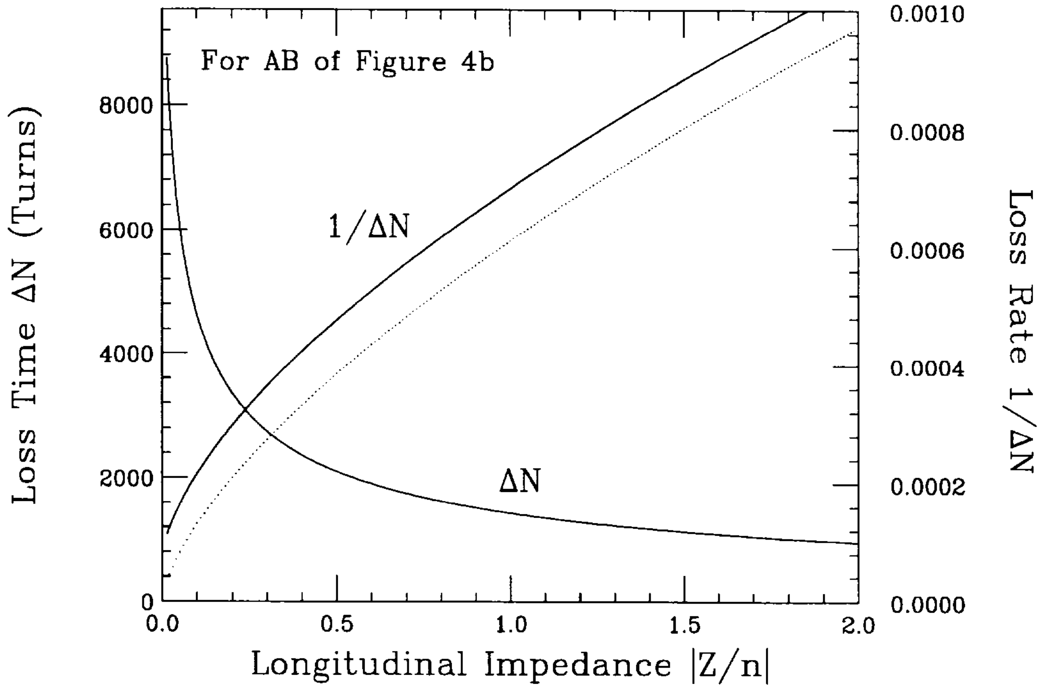

FIGURE 5. The growth time and growth rate for a bunch inside an α-like bucket, when the self-bunching buckets are also α-like. The dotted curve is an analytic estimate.

is the estimate given by Eq. (3.9). We see that this loss time is around 1500 turns even when $|Z_{\|}/n|$ =1 Ohm, which is very much longer than the 9.36 turn loss time due to synchrotron oscillations inside a self-bunching α-like bucket. Such slow rate may come from the fact that the trajectory AB is situation between two sets of separatrices.

IV CONCLUSION

The problem of microwave instability in an α-like bucket has been investigated. Using the concept of self-bunching, a stability criterion similar to the Boussard-modified Keil-Schnell criterion has been derived. We found that the stability limit for this bucket is only $|Z_{\|}/n| \lesssim 0.00055$ Ohm. We also argued that it is reasonable to believe the broad-band microwave driving force centered $\gtrsim 15$ GHz, can have a peak value of $|Z_{\|}/n| \approx 0.1$ to 1 Ohm. Thus, the growth time can be as short as 38 to 12 turns. So microwave instability can become really serious for storage rings with α-like buckets, even for a muon collider that has a storage time of only ~ 1000 turns. We also found that in most cases the self-bunch buckets for the muon in the α-like bucket will also be α-like. In that case, particle loss will be inevitable.

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