



# Fermi National Accelerator Laboratory

CERN-TH/96-230  
FERMILAB-CONF-96/306-T

## HIGHER ORDER QCD CORRECTIONS TO TAGGED PRODUCTION PROCESSES <sup>1</sup>

STEPHANE KELLER

*Fermilab, MS 106  
Batavia, IL 60510, USA*

and

ERIC LAENEN

*CERN TH-Division  
1211-CH, Geneva 23, Switzerland*

### Abstract

We extend the phase space slicing method to allow for heavy quarks and fragmentation functions. The method can be used to calculate differential cross section in which any particular particle (massive or massless) is tagged.

CERN-TH/96-230  
FERMILAB-CONF-96/306-T

---

<sup>1</sup>Talk given by S. K. at the DPF96 Conference, Minneapolis, MN, August 10-15, 1996, to appear in the Proceedings



## **Disclaimer**

*This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process or service by trade name, trademark, manufacturer or otherwise, does not necessarily constitute or imply its endorsement, recommendation or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.*

## **Distribution**

*Approved for public release: further dissemination unlimited.*

A next-to-leading order calculation has well known advantages compared to a leading order calculation: it reduces the normalization uncertainty, it starts to reconstruct the parton shower, and it tests the convergence of the QCD perturbative expansion. Over the last few years the emphasis has been on constructing Monte-Carlo programs that include all  $\alpha_s$  corrections and are fully differential in the final state particle momenta, such that any experimental cuts can be imposed. One such method is the so-called “phase space slicing” method. The formalism for this method developed by Giele, Glover and Kosower [1] introduced a high degree of automation by using the following ingredients: decomposition of the amplitude according to the color structure into colorless subamplitudes, factorization of the phase space and of the colorless subamplitudes in the soft and collinear region, and the generalization of crossing to NLO. We have extended this particular formalism to allow for heavy quarks and fragmentation functions. In this short contribution, we briefly review the different aspects of the method, full details will be presented elsewhere [2]. The method has been used in Ref. 3 to calculate the QCD corrections to  $W$ + heavy quark production at the Tevatron.

The QCD corrections consist of virtual and real corrections. The virtual corrections are the interference between the LO and all the one loop diagrams and must be calculated in  $n = 4 + \epsilon$  dimension in order to regularize the singularities. Coupling constant and mass renormalization take care of the ultraviolet singularities, and require the introduction of the renormalization scale. At the end, some collinear and soft singularities remain as  $1/\epsilon$  and  $1/\epsilon^2$  poles. The real corrections are those contributions with one more parton than the LO and have soft and/or collinear singularities. One begins by considering the processes where all the quarks and gluons are in the final state, e.g.,  $V \rightarrow q\bar{q} + n$  gluons, where  $V$  stands for an electroweak gauge boson, such that all the singularities cancel without having to do mass factorization. The basic idea of the phase space slicing method is to separate the phase space in two regions using the invariants  $S_{ij} = 2 P_i \cdot P_j$ , where the  $P_i$  are the momentum of the final state particles. The hard region is defined so that all the  $S_{ij}$  are bigger than a theoretical cut-off  $S_{min}$ . In this case, the calculation can be done numerically in  $n = 4$  dimensions. The collinear and soft region is defined such that one or two  $S_{ij}$  are smaller than  $S_{min}$ . In this case, the calculation must be done analytically in  $n = 4 + \epsilon$  dimensions. If  $S_{min}$  is small enough, the soft and collinear approximation can be used such that the integration in  $n$  dimensions is greatly simplified. We have generalized the soft approximation to the case where the particles involved are massive. In the collinear region, the mass regularizes the singularities and the calculation can be done numerically. The  $1/\epsilon$  and  $1/\epsilon^2$  poles that remain after the integration over the soft and collinear region cancel with the corresponding poles of the virtual contributions. At the end we are left with a “K” factor, proportional to the born cross section and dependent on  $S_{min}$ , the finite part of the virtual contribution, and the real corrections in the hard region, that also depends on  $S_{min}$ . An important numerical test is that any observables should

be independent of  $S_{min}$ .

The generalization of crossing to NLO is done through the use of the so-called “crossing functions”. Let us consider the process:  $pp \rightarrow V + n$  jets. The basic idea here is that we do not want to redo the cancellation of all the poles, but rather use the “K” factor already derived for  $V \rightarrow (n + 2)$  jets. First, the usual crossing for all the matrix elements is done, along with the crossing of the finite terms of the virtual piece and the crossing of the “K” factor with appropriate analytical continuation. Then two corrections must be applied in  $n = 4 + \epsilon$  dimensions: 1) subtraction of some collinear singularities included in “K” that are not present when the born is crossed, 2) add some initial state collinear singularities not yet included in “K”. Along with mass factorization, all these corrections give terms that are proportional to the Born cross section:

$$\alpha_s \sum_{a,b} \int dx_1 \int dx_2 C_a^{H,scheme}(x_1) f_b^{H_2}(x_2) d\sigma_{ab}^{LO}(x_1, x_2). \quad (1)$$

$f_a^H$  is the distribution function of parton a inside of the hadron H, and the  $C_a^H$  are the factorization scheme dependent crossing functions:

$$C_a^{H,scheme}(x, \mu_F^2) = A_a^H(x) \ln\left(\frac{S_{min}}{\mu_F^2}\right) + B_a^{H,scheme}(x), \quad (2)$$

where  $\mu_F$  is the factorization scale. The crossing function are universal and only need to be calculated once for a given set of parton distribution functions.  $A_a^H$  and  $B_a^H$  are convolution integrals of splitting-like functions with different parton distribution functions.

To add fragmentation functions to the formalism we adopted the same idea as in the crossing case: we want to use the “K” factor already calculated. Let us consider the process:  $V \rightarrow H + (n - 1)$  jets. First the NLO calculation for  $V \rightarrow n$  jets is convoluted with all the appropriate fragmentation functions. Then, corrections must be applied in  $n = 4 + \epsilon$  dimensions. When the collinear contribution from a parton  $h$  splitting to parton  $i$  and  $j$  is calculated for the “K” factor, the phase space is not only integrated over  $S_{ij}$  up to  $S_{min}$ , but also over the momentum fraction of  $i$  compared to  $h$ . This last information is needed to properly add the fragmentation functions and should not be integrated over. Furthermore, this contribution is convoluted with the parton  $h$  fragmentation function, instead of  $i$  or  $j$ . The corrections for both of these effects are, along with mass factorization, proportional to the Born cross section:

$$\alpha_s \sum_h \int dz d\sigma_h^{LO} T_h^{H,scheme}(z). \quad (3)$$

The  $T_a^H$  are the factorization scheme dependent “tagging functions”, they have the same properties and functional  $S_{min}$  dependence as the crossing functions. In the massive case, there is no collinear contribution included in the “K” factor

of the calculation without the fragmentation functions. We derived the heavy quark tagging and crossing functions that implement the variable flavor number scheme [4] (it defines the heavy quark factorization scheme). This takes care of large logarithms involving the heavy quark mass, leads to collinear safe quantities in the sense that when the mass of the heavy quark tends to zero the massless result is recovered, and makes our formalism applicable at any transverse energy.

## References

- [1] W.T. Giele and E.W.N. Glover, Phys. Rev. **D46** (1992) 1980. W.T. Giele, E.W.N. Glover, and D.A. Kosower, Nucl. Phys. **B403** (1993) 633.
- [2] S. Keller and E. Laenen, in preparation.
- [3] W.T. Giele, S. Keller and E. Laenen, Phys. Lett. **B372** (1996) 141.
- [4] J.C. Collins and W-K. Tung, Nucl. Phys. **B278** (1986) 934. M.A.G. Aivazis, J.C. Collins, F.I. Olness and W-K. Tung, Phys. Rev. **D50** (1994) 3102.