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## Mathematical Modeling of a Fermilab Helium Liquefier Coldbox

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A mathematical model was developed to describe the thermo- and gas-dynamic processes for the equipment included in the Fermilab Central Helium Liquefier coldboxes. The model is based on a finite element approach, opposite to a global variables approach, thus providing for higher accuracy and conversion stability. Though the coefficients used in equations are unique for a given coldbox, the general approach, the equations, the methods of computations, and most of the subroutines written in FORTRAN can be readily applied to different coldboxes. The simulation results are compared against actual operating data to demonstrate applicability of the model.

### INTRODUCTION

The Fermilab Central Helium Liquefier (CHL) facility supplies liquid helium at 4.6°K for the Fermilab Tevatron superconducting proton-antiproton collider ring and recovers warm return gases. The CHL includes two independent helium cold boxes rated at 4000 liters/hour and 5400 liters/hour with LN<sub>2</sub> precool [1]. Both coldboxes have heat exchangers manufactured by ALTEK International, Inc. Coldbox-I has three oil bearing turbo-expanders manufactured by Sulzer Brothers, Ltd. Coldbox-II has three oil bearing turbo-expanders manufactured by Atlas Copco Rotoflow Corp. The Tevatron cryogenics demand for higher helium supply from CHL was the driving force to investigate an installation of an expansion engine in place of the Joule-Thompson valve. This solution is thermodynamically viable, though needs to be evaluated in terms of overall cycle efficiency and production increase versus required capital investment. Some helium liquefier facilities have reported this type of successful conversion in the past [2]. To evaluate this conversion, as well as to have a reliable simulation tool in the future, the authors have developed the mathematical model and implemented it in FORTRAN code. The model describes mathematically the thermodynamic processes in the coldbox equipment and final subcooler at steady-state conditions. The model is a set of non-linear algebraic equations, which describe the links between thermodynamic variables within the coldbox's flow paths. The FORTRAN code of the HEPAK of Cryodata Inc. is used in the model to calculate helium properties. The heat transfer in the coldbox's heat exchangers is computed by using the ALTEC plate-fin geometric data and a customized finite element analysis (FEA) method. In this method every heat exchanger is considered as a set of sections of equal length with a known geometrical data and calculated mass and heat transfer properties. Each section is calculated separately, and then all sections are laced together through "in-out" variables. The gas expansion in the turbo-expanders is computed by using relationships for each turbine's efficiency versus its speed and flow, as per data provided by turbines' manufacturer. The model allows for stable conversion to new conditions at 100% step change of any of the input variables. The overall heat balance of the coldbox is checked to determine the accuracy of conversion. The number of iterations to provide for accurate conversion is less than 10,000. The full description of the model and FORTRAN code can be found in Fermilab technical memo [3].

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## COLDBOX LAYOUT AND MODEL ASSUMPTIONS

Coldboxes' layout and the graphical representation of the model are shown on Fig.1 below. One more 3-path heat exchanger is not shown on Figure 1 and located on the warm end. This is a  $LN_2$ -to-GHe precool heat exchanger. Though the pressure drops are the most significant for this heat exchanger, its helium gas exit temperature stays at the preset level due to fine regulation via temperature control loop. Therefore we excluded this heat exchanger from the coldbox model, but instead shifted the input variables one heat exchanger down the coldbox.

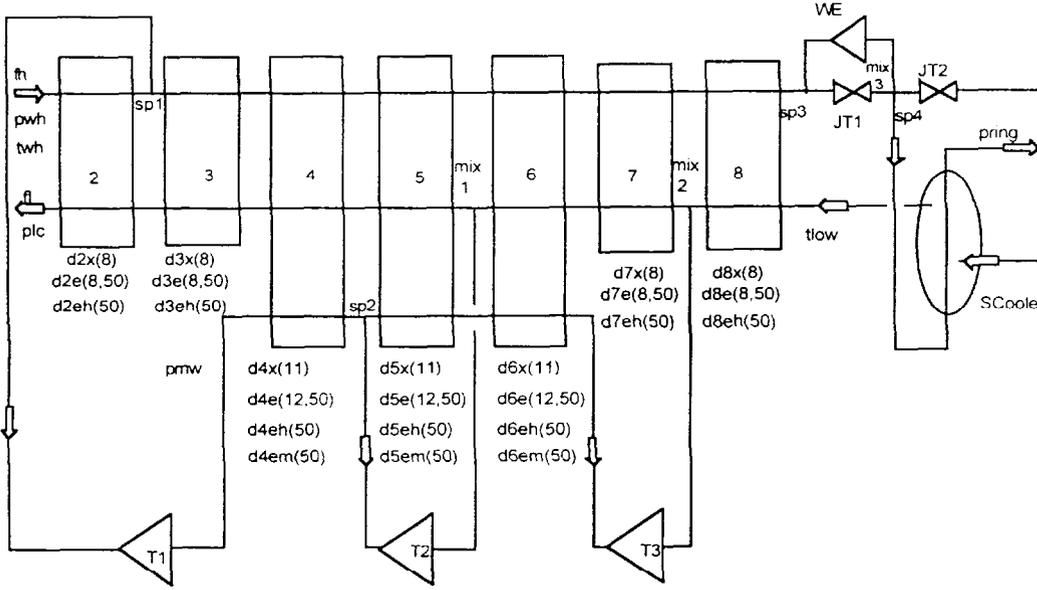


Figure 1 Layout of the Fermilab CHL Coldbox

The model includes 7 heat exchangers, numbered as  $n$  from 2 to 8. The heat exchangers 2,3,7,8 are 2-path exchangers, and 4,5,6 are the 3-path ones. A heat exchanger  $n$  consists of  $k$  sections and is represented with one-dimensional array  $d_{n,x}$  of geometrical, flow and heat loss parameters, and 2-dimensional arrays  $d_{n,e}$ ,  $d_{n,eh}$ ,  $d_{n,em}$  of process parameters and fin data. The number of cross-sections  $k$  for each heat exchanger can be set independently. Iteration results have shown that 50 cross-sections are sufficient for most heat exchangers, though the model allows to set that number up to 100. The following parameters are defined as inputs and can be read from an external file for every run of the simulator:  $fh=d_{n_2x}(1)$ , high path flow;  $sp1=f_1/f_h$ , ratio of turbine 1 flow to the coldbox flow;  $sp2=f_2/f_1$ , ratio of turbine 2 flow to turbine 1 flow;  $sp3=f_{we}/(1-sp1)$ , ratio of wet engine flow to coldbox high side flow;  $phw=d_{2e}(1,1)$ ,  $thw=d_{2e}(2,1)$ , parameters of the high path warm end;  $pmw=d_{4e}(3,1)$ , midline pressure;  $plw=d_{2e}(3,1)$ , pressure of the coldbox low path warm end (compressor suction);  $pring$ , pressure of the subcooler tube-side flow to the Ring (demand pressure);  $tlow$ , temperature of the subcooler tube side flow to the Ring (demand temperature);  $effwe$ , efficiency of wet engine.

Notations are: h - high, l - low, m - middle pressure path; w - warm, c - cold end. For example,  $thw$  - temperature at the high pressure path warm end of a heat exchanger, or its section.

The following nomenclature is used:  $t$  - temperature,  $^{\circ}K$ ;  $p$  - pressure, Pa;  $n$  - rpm;  $h$  - enthalpy, J/kg;  $s$  - entropy, J/kg-K;  $f$  - mass flow, kg/s;  $\rho$  - density, kg/m<sup>3</sup>;  $\mu$  - viscosity, Pa-s;  $c_p$  - specific heat, J/kg- $^{\circ}K$ ;  $q$  - quality;  $qh$ ,  $ql$ ,  $qm$  - heat transferred from high, middle, and to the low pressure paths, watt;  $ac$  - cross section area of flow, m<sup>2</sup>;  $aw$  - wetted fin area, m<sup>2</sup>;  $flen$  - effective length of the heat exchanger section and the plate fins, m;  $d_e$  - equivalent diameter,  $d_e = 4 \cdot ac \cdot flen \div aw$ , m;  $q_{loss}$  - heat load from outside, watt;  $t_{fin}$  - temperatures of a fin.  $^{\circ}K$ ;  $hh$ ,  $hm$ ,  $hlh$ ,  $hlm$  - heat transfer coefficients between bulk temperatures in the high path and  $t_{finh}$ , mid path and  $t_{finm}$ , low path and  $t_{finh}$ , and low path and  $t_{finm}$ .

Heat transfer from middle to high path is negligible. The  $ql = qh + qm$  as shown on Fig. 2. The heat transfer coefficients  $h$  and the pressure drops  $\Delta p$  for the section of the length  $dx$  of the given heat exchanger are calculated with the following equations:

$$h = j_i \cdot c_p \cdot G \cdot (\text{Pr})^{-2/3}$$

$$\Delta p = f_i \cdot (dx / d_e) \cdot (G^2 / \rho)$$

In the above equations:  $G = \dot{m}/ac$  is a mass flow per unit of area, and  $\text{Pr} = c_p \mu / k$ ,  $k$  is thermal conductivity. The coefficients  $j_i$  and  $f_i$  are functions of Reynolds number, where  $\text{Re} = d_e G / \mu$ . The numerical correlation between  $j_i$  and  $f_i$  vs.  $\text{Re}$  are defined per manufacturer test data. In case of the Fermilab coldbox heat exchangers ALTEC provided the authors with the tabulated data for  $j_i$  and  $f_i$  vs. Reynolds numbers ranging from 40 to 40,000 for each type of its brazed aluminum plate-fin heat exchangers. Then the authors plotted the tabulated data and fitted it with non-linear fits.

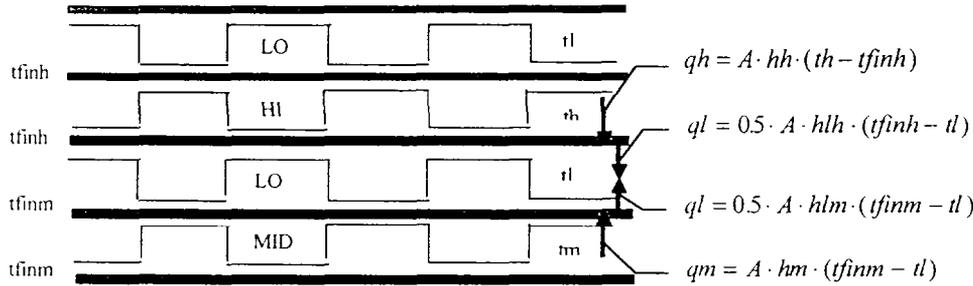


Figure 2 Model of heat transfer in the 3-path plate-fin heat exchanger

To obtain isentropic efficiency of the turbo-expander the authors have used the set of equations and graphical correlation between efficiency, speed and flow provided by Atlas Copco Rotoflow Corp. The equations are:

$$\eta_{tot} = \eta_u - \eta_v$$

$$n_{actual} = n_{design} \cdot \left[ \frac{Power^{actual}}{Power^{design}} \right]^{1/2.8} = n_{design} \cdot \left[ \frac{(f \cdot \Delta h \cdot \eta_{tot})^{actual}}{(f \cdot \Delta h \cdot \eta_{tot})^{design}} \right]^{1/2.8}$$

In these equations:  $\eta_u$  is a function of  $U/C_0$  and  $\eta_v$  is a function of  $(\text{flow}/n)^{actual} / (\text{flow}/n)^{design}$ , where  $U = \pi \cdot n \cdot d_{wheel} / 60$  is a peripheral blade tip speed, and  $C_0 = \sqrt{2 \cdot (h_1 - h_2)}$  is an ideal expansion speed. Then the authors have fitted the graphical dependencies for  $\eta_u$  and  $\eta_v$  for each turbo-expander with 3rd order polynomial equations.

## STRUCTURE OF THE FORTRAN CODE

The model uses the geometry of each heat exchanger to calculate the heat transfer at each point in the heat exchanger, unlike the programs which use an overall UA and a log mean temperature difference formulation. This is why it is necessary to go to a relaxation method, because the number of unknowns and equations becomes two times the number of steps or elements, typically 50 to 500. The intention of the authors was to make a program that uses the high pass temperature at the warm end and the low pass temperature at the cold end as inputs. That generates a logical and stable calculation to use the incoming

flow parameters as inputs. Each element is stored and solved, using the above parameters as input parameters. Afterwards, all the elements are relaxed by replacing the low path, middle path, and high path process parameters of the i-th section with the corresponding path process parameters of the adjacent upstream (i-1)-th or (i+1)-th section. Similarly, the same approach is used for the whole heat exchangers train by propagating calculation results as inputs for next heat exchangers after each step of iteration. The simulator reads in eight data files, one for each heat exchanger with geometrical data and guess values of process parameters for each section, plus a data file for the rest of the input data discussed above. The guess values for the each section process parameters are generated with a separate FORTRAN code based on the known heat exchanger experimental/design global process data, number of sections, and liner distribution. The structure of the input data files read into the simulator is identical to the structure of the output data files of the code.

## RESULTS OF THE SIMULATION

The simulator generates eight data files, one for each heat exchanger with geometrical data and final values of process parameters for each section, plus a data file for the coldbox global input data. The simulation model has been checked upon several sets of experimentally known process conditions for 2-compressors and 3-compressors flows. The model has proven to be accurate within 5% when allowed to converge with sufficient number of iterations (normally greater than 7500). Then multiple runs have been performed to evaluate the coldbox liquid helium production to the ring at different inlet conditions. Inlet pressure, midline pressure, exit pressure, and inlet flow are the limiting factors of the CHL coldbox. These limiting factors reduce the overall potential increase of production resulting from JT valve replacement with an expander. The pressure drop across the expansion engine in the CHL case is only 54% of that reported in [2]. Also, it must be understood that the potential increase of production due to JT valve replacement with an expansion engine is a relative number, which is dependent upon the base conditions, e.g., how close the coldbox thermodynamic efficiency to the Carnot efficiency. If a coldbox is designed and operated far below its optimum for a given set of inlet/outlet conditions, then such a replacement would produce a greater relative increase of production than for an optimized coldbox. Therefore the increase of production as much as 28% reported in [2] should be understood in this context. In case of the CHL coldbox, the potential increase of production due to wet expander installation is calculated as 10.8%.

## CONCLUSION

A mathematical model based on a finite element approach has been developed to describe the thermo- and gas-dynamic processes for the equipment included in a helium coldbox. The model provides for high accuracy and conversion stability. Though the coefficients used in thermo- and gas-dynamic equations are unique for a given coldbox, the general approach, the equations, the methods of computations, and most of the subroutines written in FORTRAN77 can be readily applied to different coldboxes.

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