Chapter 12

APPENDIX

12.1 Final Stage Electron Cooling for Muons

A preliminary examination of final stage electron cooling of negative muons with intense electron beams is presented here. This scheme seems feasible with the present day cutting edge technology. Furthermore, a low density Z pinch combined with an intense relativistic electron beam may be a viable cooling scheme for positive muons as well.

12.1.1 High Intensity Electron Beams

Electron beam currents that are in the mega-Ampere range have been generated by diodes. Although most of these diodes operate with pulses that are in the nsec range, some diodes have operated with pulse lengths of up to 2 microseconds. More conventional electron guns (some with plasma cathodes) can also be stacked up (or even scaled up) to yield hundreds of kA to 1 MA of current. Although in most practical applications (pulse lengths of 100 microseconds or longer), current densities in beam forming gun structures are limited to $100A/cm^2$ due to voltage breakdown effects, much higher current densities are possible in devices operating with pulse lengths that are sufficiently short (no more than a few microseconds, i.e., shorter than an arc propagation time). A hybrid system in which an electron beam is launched and is propagated through a plasma channel can be a very attractive option, since it is possible that neither technique may need to be "pushed" to its technological limit to reach resultant axial currents exceeding 10 MA that are about 1 meter long. Hollow-beam electron guns may be particularly suitable for such an application due to their larger perveance and enhanced stability in addition to the obvious advantage of their hollow structure. Another component of the presented scheme – the so called Z pinch – involves a sudden compression of a low-density plasma by means of a large discharge current that lasts for a few micro-seconds. Its fill pressure is below a milli-Torr. First, a low-density, low-temperature plasma is created by rf, lasers or exploding wires. Second, a large voltage is applied to the end plates that drives a very large axial current that compresses the plasma due to an inward acceleration of a surface current shell. At first glance a Z pinch seems to be a poor option due to its minuscule radial dimension, nevertheless, discharge currents of 10 MA over a few centimeters have been reached in a rather expensive system [1]. In a series of experiments with magnetized Z pinches, 2 MA were reached for a length of 0.8 meters with an axial magnetic field of 1.5 Tesla [2]. Various pion and muon focusing options with spark channels, Z-pinches and electron beams were explored [3]. These devices are described in detail elsewhere [3].

12.1.2 Electron Cooling Effects

A low thermal spread electron beam moving at the same velocity with a hotter charged particle beam will have a cooling effect on that beam. Electron beams, low density Z pinches, or hybrid systems can be designed to have electrons moving at the same velocity as pions and muons during the discharge. Pions and muons focused into such a discharge channel will be cooled by the electrons. At first glance, this idea does not seem very feasible since pions and muons are not trapped; consequently, cooling must be on a time scale much shorter than a second (which is typical for electron beam coolers). However, if the parameters from LEAR are scaled up, this idea seems more interesting. Using calculations from Poth's CERN report, [4] a 1 Ampere electron beam will cool antiprotons in 0.03 seconds if exposed continuously to the electron beam (since they are subjected to the cooling effects of the electrons for only 1/50 of their orbit, cooling occurs in 1.5 seconds). Theoretically, the thermal equilibration time is given by [5]

$$\tau = 5.56 \times 10^{18} \frac{(m_h T_c + m_c T_h)^{3/2}}{(m_c m_h)^{1/2} Z_h^2 Z_c^2 \lambda n_c} [sec.] \tag{12.1}$$

where subscripts c and h refer to cold and hot particles, respectively and λ is the Coulomb logarithm. It is clear from Eq. 12.1 that equilibration time is proportional to the density of the lower temperature particles and for electron beams with equal cross section (and velocity), the electron density is proportional to the current. To scale up from LEAR, consider a 3 meter long 1 MA electron beam channel. The transit time of a pion or a muon through that distance is 10 nsec; therefore, to compensate for this, shorter cooling period, the electron current (density) must be raised, 1 MA to make up six orders of magnitude. An additional gain is made by the fact that electrons equilibrate faster with lighter particles (pions and muons here versus antiprotons in LEAR). Since the energy equipartion time is proportional to the square root of the mass ratio, cooling time is reduced by a factor of 2.6 for pions and about 3 for muons. Thus, the cooling properties of such an electron beam channel for pions and muons and the electron cooler for antiprotons are not too far apart. Furthermore, these cooling channels can be stacked. However, Eq. 12.1 shows a very strong temperature dependence. Those pions and muons whose temperature is not too far off the electron temperature can indeed be cooled in such a channel. At LEAR, electron cooling of 308.6 MeV/c antiprotons with an initial momentum spread of 2×10^{-3} was performed [6]. To cool 2 GeV/c pions or muons with a thermal spread of about 200 MeV, cooler parameters need to be increased by close to four orders of magnitude. This can be accomplished by increasing the total current to 10 MA and by stacking channels to a total length of more than 3 km. Such a cooler is not very feasible due to its cost. However, if pions are initially cooled by other means, electron cooling can be used as a final stage cooler. Consider a muon beam that was cooled and slowed down to a momentum p = 300 MeV/c and a momentum spread of

$$\frac{\Delta p}{p} = 0.04 \tag{12.2}$$

To calculate the cooling time, Poth's formula [4] is used

$$\tau = \frac{\sigma_\mu^3 + \Delta_e^3}{6\pi Z^2 R_e^2 R_\mu^2 n_c L_c} \tag{12.3}$$

where L_c is the Coulomb logarithm, R is the classical radius, σ_μ and Δ_e are velocity spreads of muons and electrons, respectively (in MKS units). For the electron velocity to match that of 300 MeV/c muons, 0.516 MeV electron are needed. Therefore, a 10 MA electron beam or in a very low density Z pinch with a 1 cm radius (to match the radius of the muon beam) will have a density of $n_e = 4.77 \times 10^{15} \mathrm{electrons}/cm^3$ and we choose for the electrons to have a thermal spread of 3.48 keV to match the velocity spread of the muons (and $L_c = 15$). Using Eq. 12.3 to calculate the cooling time, yields 6.48×10^{-9} sec. Hence, since 300 MeV/c muons travel a distance of 1.68 meters during this time (their velocity is 2.6×10^8 m/sec), a cooling channel of 1.68 meters is needed. At the end of this cooling channel, $\Delta p/p = 2.86 \times 10^{-3}$. An additional stage with much colder electrons $T_e = 0.1$ eV can be added. In this stage, cooling occurs according to Eq. 12.3 in 2.16 nsec (in a 10 MA channel). At this current, a cooling channel length of 56 cm is needed (or the current can be reduced in a longer channel) and $\Delta p/p$ can be reduced to

$$\frac{\Delta p}{p} = 1.53 \times 10^{-5} \tag{12.4}$$

These preliminary calculations indicate that two channels, containing 10 MA 516 keV electron beams, with a total length of 2.24 meters can serve as a very effective final cooling stage for the muons. Furthermore, it can reduce their momentum spread by more than three orders of magnitude. We notice in passing that, 10 MA Z pinches and the electron beams, discussed above, have been achieved in practice. In cooling μ^-, the magnetic field generated by the co-moving electrons focuses μ^-. But, such a magnetic field defocuses μ^+. To cool μ^+ particles, no net axial current should exist in a channel. One possibility is to shoot the 10 MA electron beam through a 10 MA Z pinch such that the two currents cancel each other. Confinement can then be provided by a multiple magnetic field. For a Z pinch with parameters achieved earlier [1], the confining magnetic field can be calculated from $B^2/2\mu_0 = nkT$ to be 6.34 Tesla, or the required confinement can be achieved if the Z pinch current is greater than 10 MA, with an axial magnetic field of about 2 Tesla. In such a channel scattering by the Z pinch particles can be shown not to be a problem, since electron-electron scattering time is 320 nsec, while electron-muon scattering time is 3.6 msec, both of which are much longer than the total cooling time of 8.48 nsec. This configuration is expected to be stable for the time scale of interest (nsec.) to muon colliders. Although such a velocity space configuration is potentially micro-unstable, the resulting instabilities, if they occur, will have growth rates that are slower than the cooling time. This configuration resembles that of reverse field pinches, which are more stable than conventional Z pinches (since a number of macro-instabilities have a slower growth rate).

12.1.3 Conclusions

A hybrid system of electron beams with Z pinches, is an interesting idea to pursue, as a final stage cooler for the muon collider. The scheme proposed for cooling positive muons involves a more complex configuration, which is also worth looking into.

12.2 Ionization Energy Loss in a Crystal Channel

A possible method for high energy muon cooling via ionization energy loss in a focusing crystal channel is outlined here. We show that starting with an initially 'cool' muon beam, e.g. coming from a 25 GeV photo-production source, with the normalized emittance of $\epsilon_N = 10^{-5}$ m rad, one can decrease the transverse emittance to less than $\epsilon_N = 10^{-7}$ m rad by passing muons through total of 280 meters of the crystal absorber. For a practical implementation, we suggest a storage ring configuration, where sections of crystal absorber are alternated with a conventional high gradient re-acceleration (20 MeV/m) inserts. The necessary circular confinement (bending) would be provided by additional sections of bent crystals – employing powerful steering features of bent Silicon crystal demonstrated by recent experiments. Required cooling length of 280 meters constitutes only about 2×10^{-3} of the muon life-time in the laboratory frame (about 90 turns in our model 'cooling ring'). Dominant heating process (due to multiple scattering on electron gas inside a crystal channel) limits the minimum achievable emittance to about $\epsilon_N = 10^{-9}$ m rad, while the characteristic ionization cooling damping length is about 62.5 m. Feasibility of effective ionization cooling rests on the ultra high fundamental crystal fields available in a solid state environment.

12.2.1 Introduction

We outline here a possible method for high energy muon cooling via ionization energy loss in a focusing crystal channel. Recent experiments in the U.S., Europe and Russia have shown impressive progress in high-efficiency steering of charged particle beams by means of the bent crystal channeling [11], [12]. The scope of these experimental studies has been focused on variety of possible applications of crystal components for high energy accelerators. A model calculation, presented here, shows that starting with an initially 'cool' muon beam (e.g. coming from a photo-production source, providing 25 GeV $\mu^{\pm}$ pairs with the normalized emittance of $\epsilon_N = 10^{-5}$ m rad) one can decrease emittance to less than $\epsilon_N = 10^{-7}$ m rad by multiple passage of the muon beam through a sequence of Silicon crystal absorbers followed by a conventional rf re-acceleration module. Appropriate bending and circular confinement of the muon beams should allow multiple passage through the crystal absorber to decrease the transverse emittance below $\epsilon_N = 10^{-9}$ m rad. Experimental demonstration of high efficiency muon channeling promised by the high focusing fields in a crystal [13], [14] is under way; e.g. an experiment at TRIUMF [15]. Dominant heating process (due to multiple scattering on electron gas inside a crystal channel) is also taken into account. Derived here, transverse emittance 'cooling equation' shows that the minimum achievable emittance (equilibrium cooling limit) is of the order of $\epsilon_N^{min} = 10^{-9}$ m rad, while the characteristic transverse emittance damping length is about 62.5 m. To reach the final emittance of $\epsilon_N = 10^{-7}$ m rad one would have to pass muons through total of 62.5×2 log10 meters = 280 meters of the focusing crystal channel. Both processes, ionization cooling and re-acceleration, require ultra high fundamental crystal fields available in a solid state environment. There are at least two effective ways to provide $\mu^{\pm}$ for a muon collider. First, through π production with hadron beams and subsequent π decay [7]. Second, by photo-production, or electro-production [16]. The advantage of the later is that the bunches are very short, compared to any hadronic source, due to the bunch structure of the linac. Furthermore, the $\mu^{\pm}$ can be produced with very low transverse emittance [16]. The disadvantage is a relatively low intensity. Nevertheless, the final luminosity of a $\mu^+ \mu^-$ collider will depend on the emittance – a method that produces very low $\mu^{\pm}$ emittance might be comparable with a hadronic source [8].

12.2.2 Crystal Channeling

We explore unique properties of relativistic channeling of charged particles in a bent crystal as a technique for particle beam steering [11], [12]. Particularly we are interested in the circular confinement of muon beams in a crystal with a strain imposed curvature as a possible functional element of a high energy storage ring. The transverse motion of a relativistic ($\gamma >> 1$) muon of mas m_μ, channeling through a crystal is described, in linear approximation, by the following equation [13]

$$m_\mu \gamma \frac{d^2 x}{dt^2} + U'(x) = 0, \qquad (12.5)$$

where x is the distance from the centerline between the atomic planes and U(x) is the averaged planar continuum electrostatic potential energy at the distance x. For a positive particle the continuum potential well is given to a good approximation by

$$U(x) = \frac{1}{2}\phi x^2. \qquad (12.6)$$

The focusing strength of a crystal channel for [110] planar channeling in Silicon has been experimentally measured and has a value of [14] $\phi = 6 \times 10^{12}\mathrm{GeVm^{-2}}$. As one can see from Eq. 12.5 and Eq. 12.6 the beam dynamics of charged particles channeling through a straight crystal channel corresponds to a transverse harmonic oscillator moving relativistically in the longitudinal direction – the crystal channel plays the role a strongly focusing transfer line characterized by the beta function, β, (or alternatively by the betatron wavelength, λ) both expressed by the following formula

$$\beta = \frac{\lambda}{2\pi} = \sqrt{\frac{E_\mu}{\phi}}. \qquad (12.7)$$

Assuming 25 GeV muons channeling in Silicon the corresponding beta function has the following value $\beta = 2 \times 10^{-6} m$. Following Tsyganov [17], we consider motion of planar channeled particles in a crystal, which is bent elastically in a direction perpendicular to the particle velocity and to the channeling planes. The effect of bending introduces a centrifugal force into the equation of transverse motion, Eq. 12.5. The modification of the crystal continuum potential due to the bending curvature, ρ, may be described as follows [13]

$$U(x) \rightarrow \frac{1}{2}\phi x^2 - \frac{m_\mu \gamma c^2}{\rho} x, \quad m_\mu \gamma c^2 = E_\mu. \qquad (12.8)$$

Adding a linear (centrifugal) piece to the crystal potential is equivalent to lowering one side of the continuum potential well and raising the other. One can see from Eq. 12.5 and Eq. 12.8 that the equilibrium planar trajectory will move away from the midpoint of the planar channel toward the plane on the convex side of the curved planar channel. However, such shift would cause some fraction of the channeled particles to leave the potential well (dechannel). The critical curvature at which no particle can remain channeled is reached when the equilibrium point of planar channeled motion is shifted to the position of the planar wall on the outside of the curve. This critical radius of curvature, known as the Tsyganov radius [17] is defined by the following equilibrium condition, $U'(a/2) = 0$, where a $= 2.2 \times 10^{-10}$ m is the distance between adjacent atomic planes and U(x) is given by Eq. 12.8. This translates into the following explicit expression for the Tsyganov radius, ρ_T

$$\rho_T = \frac{2E_\mu}{\phi a}. \tag{12.9}$$

Using simple formula, which links the equivalent magnetic bending field, B, with the particle trajectory's curvature, ρ, one can get the maximum available equivalent bending field corresponding to the Tsyganov curvature, expressed as follows

$$B_T[Tesla] = 3.34 \times \frac{1}{2}\phi a. \tag{12.10}$$

Its numerical value for Silicon is evaluated as follows: $B_T = 2 \times 10^3$ Tesla. We notice in passing, that the maximum bending field is energy independent. Assuming 25 GeV muons channeling through a 2 cm- long section of a Silicon crystal the maximum bending angle, θ_T, derived from Eq. 12.9, is equal to enormous value of about 0.5 rad. This value of the critical bending angle will be used later in the paper.

12.2.3 Ionization Energy Loss

Here we propose a fast muon cooling scheme based on the ionization energy loss [18] experienced by high energy muons (25 GeV) channeling through a Silicon crystal. Applying classical theory of ionization energy loss [19] to a relativistic muons passing through a Silicon crystal of length, ΔL, yields the total energy loss, ΔE_μ, experienced by muons, which is expressed in the following simple form

$$\Delta E_\mu[MeV] = 4 \times 10^2 \times \Delta L[m]. \tag{12.11}$$

The following useful quantity, Λ

$$\frac{1}{\Lambda} = \frac{1}{E_\mu}\frac{\Delta E_\mu}{\Delta L}, \tag{12.12}$$

describes a characteristic damping length – over which particle looses all its energy. Relativistic muons passing though the crystal loose energy uniformly in both the transverse and longitudinal directions according to Eq. 12.11. After passing through a short section of a crystal ($\Delta L << \Lambda$) muons are re-accelerated longitudinally to compensate for the lost longitudinal energy. Combining both processes (ionization energy loss and re-acceleration) leads to the transverse emittance shrinkage. Introducing the normalized transverse emittance, ϵ_N, in the following standard way

$$\epsilon_N = \gamma \sigma_x \sigma_{x'}, \tag{12.13}$$

one can write down the normalized emittance budget in the form of the following cooling/heating equation

$$\frac{d\epsilon_N}{dL} = -\frac{\epsilon_N}{\Lambda} + \left(\frac{\Delta\epsilon_N}{\Delta L}\right)_{scatt}. \quad (12.14)$$

The last term in the above equation accounts for the transverse heating processes (Coulomb scattering) increasing the beam divergence according to the following relationship

$$\left(\frac{\Delta\epsilon_N}{\Delta L}\right)_{scatt} = \frac{1}{2}\gamma\beta\frac{\Delta\langle\theta^2\rangle_{scatt}}{\Delta L}. \quad (12.15)$$

Here β is the beta function of a focusing crystal channel, defined by Eq. 12.7, which has enormously small value of 2×10^{-6} m, for 25 GeV muons channeling through a Silicon crystal.

One has to distinguish between the emittance of the macroscopic beam outside the crystal and the emittance of 'beamlets' in the individual channels. Assuming channeling condition – the beam divergence is equal or smaller than the critical channeling angle, $\theta_c = 34$ micro radians – a macroscopic muon beam with energy of 25 GeV and normalized emittance of 10^{-5} m rad will have a spot size of about 1 mm. Only part of the entering beam will be accepted into channels, since particles that impinge too near crystal planes will be scattered to large angles even if their initial pitch angles are less than the critical angle, θ_c. The emittance cooling equation, Eq. 12.14, applies to the channeled 'beamlets'. Crystal channel cooling, discussed here, occurs only for those muons that enter channels successfully stage after stage. All other unchanneled muons simply experience normal ionization cooling with heating due to Coulomb scattering off atomic nuclei. The macroscopic emittance of the final beam is determined by all muons including the micro-emittance of the channeled beam mixed in with empty phase-space due to crystal plane blocking.

For muon channeling in a dielectric crystal the dominant scattering process comes from the elastic (Rutherford) muon scattering off the conduction electrons, which are present in the channel. One can integrate the Rutherford cross section over the solid angle, which yields the following formula

$$\frac{\Delta\langle\theta^2\rangle_{scatt}}{\Delta L} = 16\pi n_c\frac{r_\mu^2}{\gamma^2}log\left(\frac{\theta_{max}}{\theta_{min}}\right), \quad (12.16)$$

where

$$log\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx 5. \quad (12.17)$$

Here, $r_\mu = 1.4 \times 10^{-17}$ m, n_c is the concentration of the conduction electron gas in a crystal channel. The average electron gas concentration in Silicon, n can be estimated as follows

$$n = \frac{S}{a^3} = 6 \times 10^{29} m^{-3}, \quad (12.18)$$

where S = 6 is the coordination number for the basic crystallographic cell for Silicon crystal (cubic face centered) and $a = 2.2 \times 10^{-10}$ m is the distance between two neighboring [110] planes. One expects the channel electron density, n_c, to be less then the average density, n, since electrons tend to be concentrated around nuclei. From the critical angle, $\theta_c = 34$ micro radians, and the dechanneling length, $l_d = 2$ cm, for 25 GeV/c momentum, one can estimate the electron channel density, n_c, as $6 \times 10^{28} m^{-3}$, with the multiple scattering formula, Eq. 12.16 and the following identity

$$\frac{d\langle\theta^2\rangle_{scatt}}{dx} \approx \frac{\theta_c^2}{l_d}. \tag{12.19}$$

Now, one can summarize balance between ionization energy loss (cooling) and multiple scattering (heating) in the derived cooling/heating equation, Eq. 12.14, in terms of the following two quantities

$$\Lambda = 62.5\,\mathrm{m}, \tag{12.20}$$

and

$$\alpha = \left(\frac{\Delta\epsilon_N}{\Delta L}\right)_{scatt} = 40\pi n_c \frac{r_\mu^2}{\gamma}\beta = 1.2 \times 10^{-11}\,\mathrm{rad}. \tag{12.21}$$

Integrating the cooling equation, Eq. 12.14, one obtains the following compact solution in terms of the normalized transverse emittance evolution

$$\epsilon_N = \epsilon_N^0 e^{-\frac{L}{\Lambda}} + \Lambda\alpha\left(1 - e^{-\frac{L}{\Lambda}}\right). \tag{12.22}$$

The second term in Eq. 12.22 sets the equilibrium cooling limit of

$$\epsilon_N^{min} = \Lambda\alpha,\ L \to \infty. \tag{12.23}$$

Assuming 25 GeV muons one gets the equilibrium limit of the normalized emittance of

$$\epsilon_N^{min} = 0.75 \times 10^{-9}\mathrm{m\,rad}. \tag{12.24}$$

Practical realization of muon cooling at 25 GeV could be done in a compact 'cooling ring', where one would employ powerful steering properties of bent crystals (see previous subsection) to provide circular confinement of the muon beam. Projecting experimental results for proton channeling in a bent Silicon crystal, one can assume that 25 GeV muons channeling through a 2 cm-long crystal should follow (without significant dechanneling effects) a bend of $\theta = 2\pi \times 10^{-2}$ rad (compare with the critical bending angle of $\theta_T = 5 \times 10^{-1}$ rad. calculated in the previous section). Assuming bending angle per cell of, $\theta = 4\pi \times 10^{-2}$ rad, only fifty ($50 \times \theta = 2\pi$) of the functional bending cells would be needed to complete the entire cooling ring. Its effective circumference would be equivalent to 3 meters of Silicon crystal. Assuming characteristic damping length, L, of 62.5 meters, the energy loss suffered by the muon beam after passing through a 2 cm - long section of a Silicon crystal, is equal to 8 MeV. In principle, a conventional high gradient (20 MeV/m) acceleration inserts (40 cm - long rf insert following every 2 cm - long crystal absorber) could be used to replenish the suffered energy loss ($0.4m \times 20MeV/m = 8MeV$). The proposed cooling ring of fifty-fold symmetry is illustrated schematically in Fig. 12.1 It has a nominal circumference of 63 meters! Our goal is to start with the initial muon phase-space of the normalized emittance of 10^{-5} m rad and cool it down to the final emittance of 10^{-7} m rad. One can see from Eq. 12.21 that to achieve this goal muons have to pass through the total Silicon crystal length of $L = 2\log 10 \times \Lambda = 280$ m. In the proposed cooling cell architecture the total cooling medium (Silicon) length of L = 280 m is equivalent to about 90 turns of the beam circulation in the ring. The lost

Figure 12.1: Layout of a 'cooler ring' consisting of fifty bending-focussing-acceleration multi-functional cells. A straight piece of Silicon crystal rotated by 90° separating two sections of bent crystals provides vertical focusing which, maintains betatron phase stability in the proposed lattice. A conventional rf, 40 cm - long inserts (20 MeV/m) follow every 2 cm - long section of Silicon crystal absorber.

energy is replenished every $\Delta L = 2$ cm, which easily satisfies the adiabatic re-acceleration condition ($\Delta L << \Lambda = 62.5$ m). To go beyond the above simple analytic calculation, we are planning to carry out a realistic computer simulations of planar channeling in bent crystals. One should tracks a charged particle through the distorted crystal lattice with the use of a realistic continuous potential approximation and taking into account the processes of both single and multiple scattering on electrons, nuclei as well as on various defects and imperfections of the crystal lattice.

12.2.4 Conclusions

We pointed out that initially cool muons obtained from a photo-production source could be used as a starting point for a high energy $\mu^+\mu^-$ collider complex, providing that an effective cooling scheme is available. We suggest employing ionization energy loss in an alternating focusing crystal channel as a cooling mechanism, since initially small muon phase space allows for efficient channeling through long sections of Silicon crystal. The ultra-strong focusing in a crystal channel results in the ultra small beta function. Derived here cooling equation shows that it is quite feasible to decrease the transverse emittance of a muon beam by two orders of magnitude. Our model calculation done for 25 GeV muons shows that final emittances as low as 10^{-9} m rad could be achieved, limited only by multiple scattering off the conduction electrons in the crystal. We conclude our study with the following observation: the proposed ionization crystal cooling could be used at some later stages of the collider scheme, e.g. for the final cooling, due to a 'favorable' energy scaling of the relevant cooling characteristics, ϵ_N^{min} and Λ. Their energy scaling can be summarized as follows

$$\epsilon_N^{min} \sim \gamma^{-3/2}. \tag{12.25}$$

$$\Lambda \sim \log\gamma. \tag{12.26}$$

Therefore, the proposed cooling mechanism scaled to higher energies looks even more attractive.

12.3 Frictional Cooling – Recent Experimental Results

Frictional cooling – that is cooling a beam of very low energetic charged particles by moderation in matter and simultaneous acceleration in an electrostatic field – has been shown to be feasible during our experiments in 1994-1995 at PSI. In agreement with our previous closed form and Monte-Carlo calculations we found a significant increase in spectral density and a decrease in the angular spread in the case of a beam of negative muons.

12.3.1 Introduction

Without any doubt many experiments in muon physics become feasible only when intense sources of muons with low energy and, sometimes even more important, with small energy spread and divergence are available. This includes experiments where slow muons, both positive and negative, are used as probes in surface and thin film physics and experiments with gas targets, e.g. in muon-catalyzed fusion research [20]. High quality muon beams are also needed to set-up a high luminosity $\mu^+\mu^-$ collider [7]. Slow muons are usually produced by moderation of the high energetic muons from pion decay. New developments are the extraction of slowed down muons from an anticyclotron [21], [22] and the conversion of muons via muon-catalyzed dt-fusion [23]. Unfortunately all these methods yield a divergent muon beam with a wide energy distribution and poor density in phase space. One way to enhance the the quality of the beam is the method of frictional cooling [24]. It relies on the fact that at very low energies (for muons below roughly 10 keV) the stopping power increases with increasing energy, as shown in Fig. 12.1. The application of an electric field along the flight path of the muons in a moderator lets muons of a certain (equilibrium) energy [25], [26] T_{eq} unaffected in velocity and accelerates lower energy muons as they gain more energy from the electrostatic field than they lose due to their interaction with the moderator. Muons with higher energy are decelerated as long as they lose more energy than they gain, this, naturally, only up to the point where energy loss and gain are equal again (higher energy muons are accelerated and lost for cooling). In addition the frictional force acts in a direction opposite to the muon motion while the electrostatic force accelerates the muons in beam direction only. Therefore the beam divergence diminishes too. The two cooling

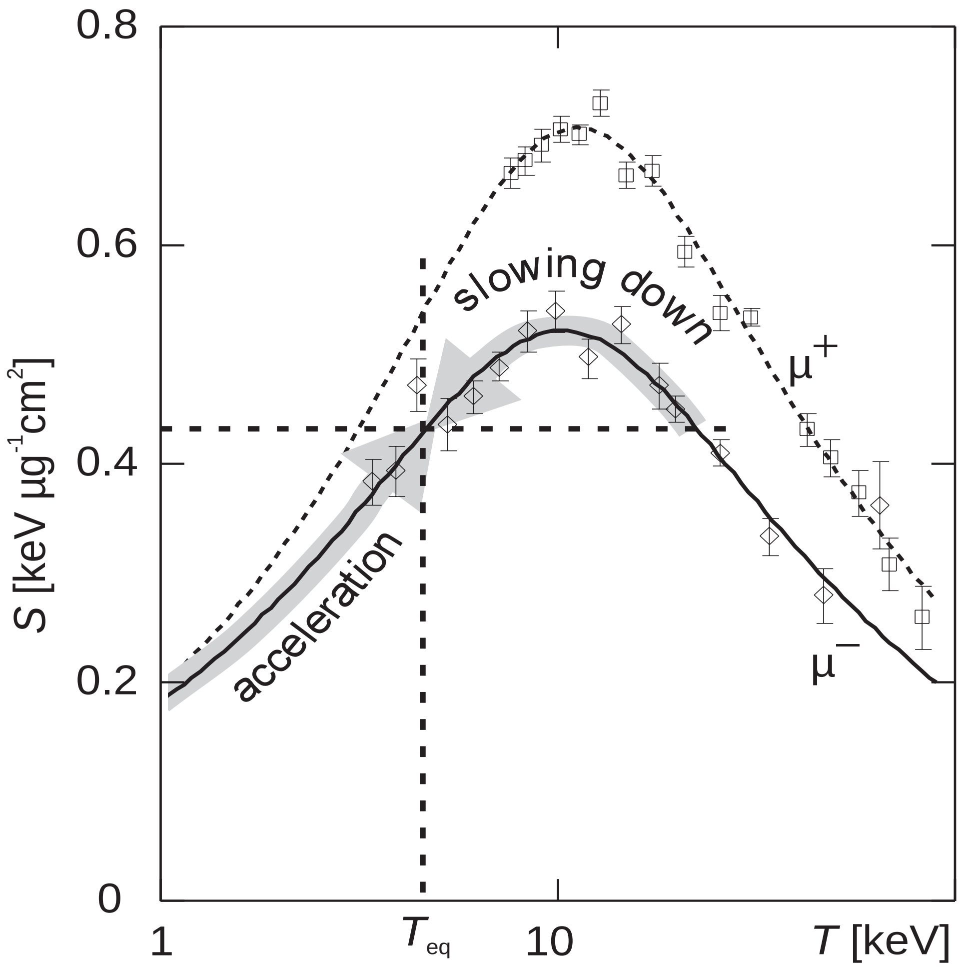

Figure 12.2: Average stopping power S(T) of carbon for both positive muons and negative muons as a function of energy T; T_{eq} denotes the equilibrium energy; the energy region where cooling takes place is marked by the arrows labeled “acceleration” and “slowing down”.

effects just described are limited by multiple scattering and straggling. Nevertheless our closed-form and Monte-Carlo calculations [28], including straggling and scattering, verified the method of frictional cooling to be feasible and efficient. The experimental proof was found during our experiment at Paul Scherrer Institute (PSI) in the Summer of 1994. More detailed studies on the frictional cooling have been done within our beam time in the Spring of 1995.

12.3.2 Experimental Arrangement

The experiments take place in the pE5 Area at PSI which provides the most intense beam for experiments using slow muons. As the momentum of the muon beam has to be kept low $p_\mu \sim 10\,\mathrm{MeV/c}$ the electron contamination is quite large. Therefore a Wien filter is used to clean the beam. The experimental set-up shown in Fig. 12.3 is placed inside the superconducting solenoid which is part of the existing phase space compression apparatus [29]. Frictional cooling is achieved with a stack of thin graphite foils mounted on stainless steel rings. In front and behind the foils we placed some additional rings not covered with foils. These extra rings allow a smooth high voltage variation. To this end the first and last rings are kept on ground voltage whereas the respective upstream and downstream foils are put on a voltage of U_{up} = -10 to -20 kV and U_{down} = -3 kV, respectively. All rings are connected to a resistive voltage-divider chain. This gives a voltage difference ΔU between two adjacent foils of about 1 to 2 kV. With this set-up we are able to run with negative muons only. In the case of positive muons we have to change the sign of the upstream voltage. This will build up a trap for the secondary electrons knocked out from the foils by the muons and some charge will be accumulated in the stack region leading to a high-voltage break through. A more elaborate arrangement might make frictional cooling feasable also for μ^+. To measure the effect of the frictional cooling a time-of-flight (TOF) technique is used.

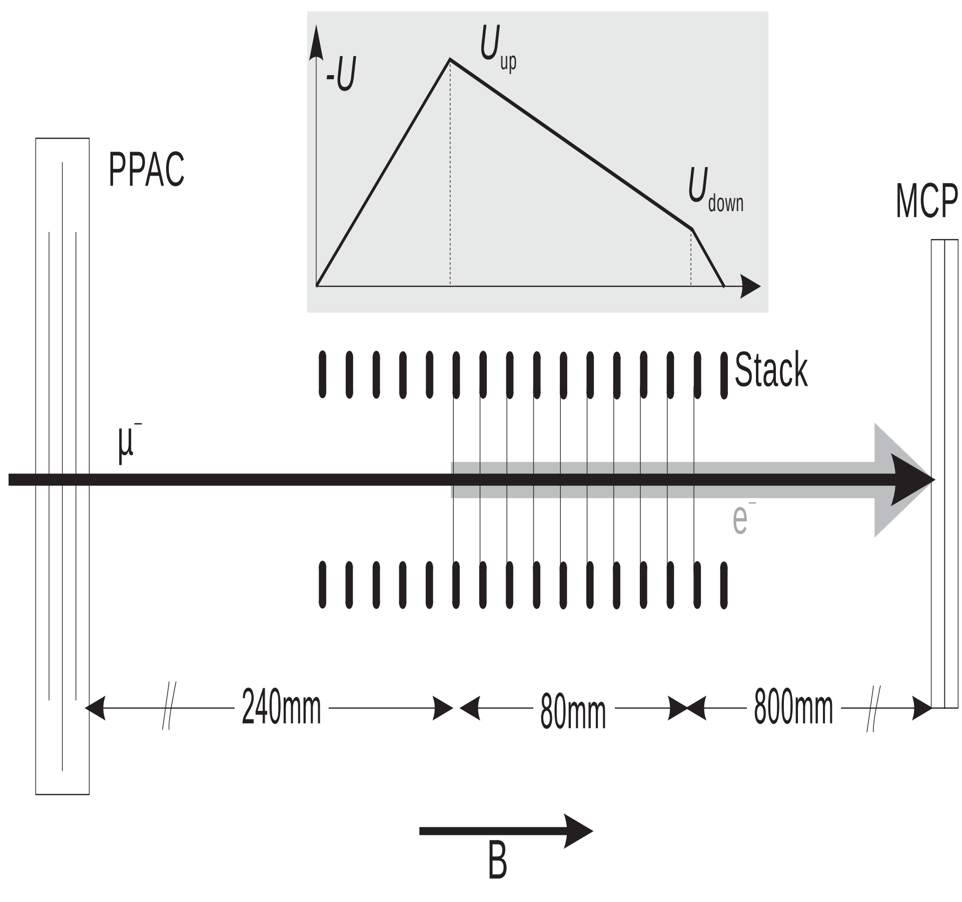

Figure 12.3: Schematic view of the experimental setup as it is housed by the superconducting solenoid. This solenoid generating a high magnetic field in beam direction is not shown in this sketch. The small insert shows the voltage distribution in the region of the foil stack.

A muon entering the apparatus is slowed down from approximately 0.5 MeV to energies in the 10 keV range and detected at time t_1 in the entrance detector, a parallel plate avalanche counter (PPAC). A strong magnetic field guides the muon escaping from the PPAC on a spiral orbit along the field lines through the experimental arrangement. The muon loses energy according to the stack voltage U_{up} and enters the foil stack. While crossing one of the graphite foils the muon knocks out secondary electrons. These electrons are accelerated downstream as well as the muon and are able to produce secondary electrons from the next foils. This leads to a pulse of near to one hundred secondary electrons moving downstream with energies up to approximately 10 keV. These electrons are detected at time t_1 in the microchannel plate (MCP) detector at the very end of our apparatus. While the muon passes the stack it is being slowed down in the foils and accelerated in between. Finally the muon leaves the stack as a particle within a cooled beam, follows the magnetic field lines and hits the MCP detector at time t_3. The times the muon enters or leaves the stack needs to be calculated from t_2. Therefore we have to make some corrections. The main contributions are the energy spread of the electrons released from the stack and the time the muon spends inside the stack. These effects will be studied in detail in the near future but at the moment we have only a preliminary knowledge of these numbers. From the TOF values and the length of the flight paths we obtain the energies T_1 and T_2 of the muon in front and behind the stack, respectively. As mentioned before the strong magnetic field of approximately 5 Tesla of the superconducting solenoid surrounding the set-up makes the diverging muons spiral around the field lines and guides them from the PPAC to the MCP. The spiral radius is a function of the transverse momentum of the muon. In order to study the influence of the frictional cooling on the beam divergence we placed a collimator behind the stack. This collimator consists of a stack of parallel plastic foils (thickness of 0.2 mm) with a spacing of 1.5 mm or 1 mm and a length of 50 mm in beam direction. The comparison of the count rates under different conditions gives a measure of the beam divergence.

12.3.3 Results

Fig. 12.4 shows the energy spectra of the outgoing muons as a function of the energy of the incident muons. The energy distribution of the incident muons is not flat at all. Therefore each spectrum is multiplied by a certain factor to correct the differences in count rate. No background subtraction is done so far. If we concentrate on the incident muons at low energies we can see a clear peak of cooled muons with a width of less than 2 keV. Position and width of the peak are in good agreement with our previous calculations. This peak vanishes when we select higher incident energies T_1. We find no such peak when we turn off the cooling by setting $\Delta U = 0$ kV.

12.3.4 Summary and Outlook

This first evaluation of part of our data shows the feasibility of frictional cooling in practice. We see a significant increase in spectral density and a decrease in the angular spread in the case of a beam of negative low-energy muons according to the predictions of our closed-form and Monte-Carlo calculations. In addition frictional cooling gives a sharp pulse of a large number of secondary electrons at energies up to 10 keV. These electrons provide a 100 %

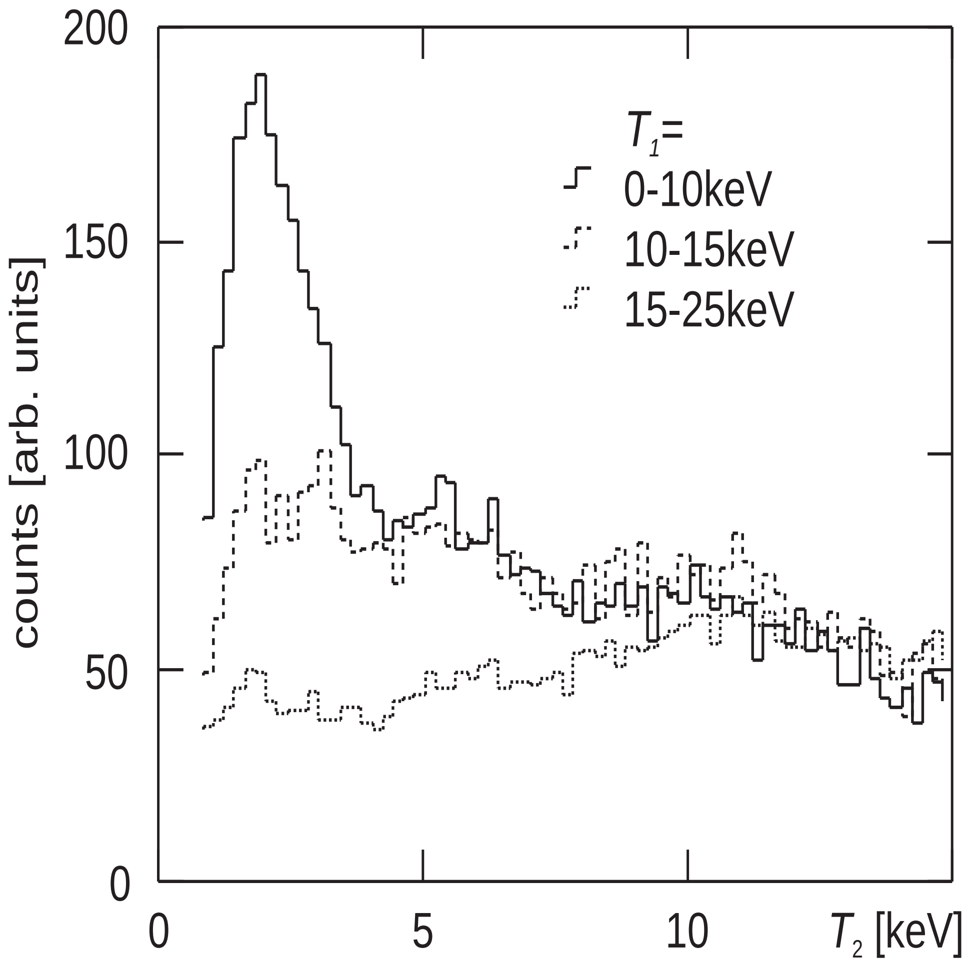

Figure 12.4: Energy spectra of the outgoing muons as a function of the energy T_1 of the incident muons 10 foils 4.3 mg/cm^2 (Carbon), each, U_{up} = -18 kV, U_{down} = -3 kV and ΔU = 1.7 kV. The energy calibration is preliminary.

efficient muon trigger and allows the muon to be detected even with a scintillation counter. Frictional cooling found its first application in our measurement of the pm kinetic energy in a low-pressure hydrogen gas target 1 (mbar and below) [26]. The frictional cooling is used together with other techniques to stop the muons in this low density target and to provide a muon trigger with help of the secondary electrons created. A further development is the so-called frictional accumulation [30]. With this technique based on frictional cooling we should be able to convert intermediate-energy negative muons into a low-energy muon beam. First results from a Monte-Carlo simulation show that for a divergent muon beam entering the accumulation stage at energies up to 150 keV the conversion efficiency into a beam of a few keV is about 30 %.

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