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Acceleration to Collisions for the $\mu^+ - \mu^-$ Collider

David V. Neuffer

*Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510*

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Acceleration to Collisions for the μ^+ - μ^- Collider

David V. Neuffer,[†] Fermilab, P. O. Box 500, Batavia IL 60510

Abstract. We discuss the problem of transforming muon beam bunches from a low-energy cooled state ($E_\mu \sim 1$ GeV) to short, high-energy bunches matched to high-energy collision conditions ($E_\mu \sim 2$ TeV). In this process the beam energy must increase by \sim three orders of magnitude, while the bunch length must be reduced by \sim two orders of magnitude (to ~ 3 mm), while beam emittance dilution and beam losses, particularly through decay, must be minimized. From general considerations, we discuss possible acceleration scenarios including rapid-cycling synchrotron and recirculating linac options. The presently favored choice is a multi-stage recirculating linac system, which is discussed, and initial simulations of possible scenarios are presented. Future directions for development are discussed.

INTRODUCTION

The possibility of muon (μ^+ - μ^-) colliders has been introduced by Skrinsky et al.¹ and Neuffer². More recently, intensified investigations with the goal of a practical design for a high-energy high-luminosity $\mu^+\mu^-$ collider have increased the level of conceptual development,^{3, 4, 5, 6} and this effort includes the present workshop.⁷ A candidate scenario for a collider, with an energy of $E_{cm} = 2E_\mu = 4$ TeV, and a luminosity of $L \sim 10^{35} \text{cm}^{-2}\text{s}^{-1}$, has been developed.⁸ Table 1 shows parameters for the candidate design. The design consists of a muon source, a muon collection, cooling and compression system, a recirculating linac (or rapid-cycling) system for acceleration, and a full-energy collider with detectors for multiturn high-luminosity collisions.

In this paper we concentrate on the portion of this scenario in which the muons are accelerated from the output of the cooling system to full energy and transferred to storage in the 2 TeV collider ring. Thus, we assume that the muons have been cooled and collected into moderately compact μ^+ and μ^- bunches at $E_\mu \sim 1$ GeV. Studies of the cooling system indicate that an energy spread of $\sim 1\%$ at a bunch length of ~ 30 cm at ~ 1 GeV are reasonable design goals, and we use these as reference initial parameters. The accelerator must accelerate these bunches to 2 TeV and transfer them into the collider, with a final energy spread of $\sim 0.1\%$ and a bunch length reduced to ~ 0.3 cm.

A critical requirement is that the muons must be accelerated before they decay. This sets severe constraints on the acceleration system and these are first discussed

[†] on assignment from CEBAF, 12000 Jefferson Avenue, Newport News VA 23606

in detail. We then describe potential acceleration systems, including full-energy linac, rapid-cycling synchrotron and recirculating-linac options. Our currently favored acceleration choice is a sequence or cascade of recirculating linacs, each of which increases beam energy by \sim an order of magnitude and accommodates bunch length reductions by almost as much. We discuss constraints and present candidate scenarios. Particle tracking in a candidate choice provides a “proof of principle” of this general approach. Optimization considerations and possible variations are discussed. We also discuss directions for further development of these acceleration and transport systems.

MUON LIFETIME CONSIDERATIONS

The central difficulty in a $\mu^+\mu^-$ collider is that muons decay with a mean lifetime of $\tau_\mu = 2.2 \mu\text{s}$ (in the μ rest frame), and the muons must be collected, cooled, accelerated, and collided within that lifetime. In the lab frame the lifetime is increased by the relativistic factor $\gamma = E_\mu/m_\mu$, where E_μ is the μ energy and m_μ is the mass ($m_\mu = 0.10566 \text{ GeV}$). The muon decay rate along the beam path length s can be written as:

$$\frac{dN}{ds} = -\frac{1}{L_\mu \gamma} N, \text{ where } L_\mu = c \tau_\mu \cong 660 \text{ m.}$$

and where we have used the relativistic approximation $v/c \cong 1$.

In a non-accelerating transport, this implies the usual exponential beam loss:

$$N = N_o e^{-\frac{s}{L_\mu \gamma}}$$

In an accelerating section, γ is not constant:

$$\gamma = \gamma_o + \gamma' s = \gamma_o + \frac{e V_{rf}'}{m_\mu c^2} s,$$

where $e V_{rf}'$ is the accelerating gradient. Using this in the decay equation obtains the solution :

$$N(s) = N_o \left(\frac{\gamma_o}{\gamma_o + \gamma' s} \right)^{\frac{1}{L_\mu \gamma'}} \quad \text{or} \quad \frac{N(s)}{N_o} = \left(\frac{E_o}{E_{\text{final}}} \right)^{\frac{1}{L_\mu \gamma'}} \quad (1)$$

Low losses imply that the exponential factor must be small, which implies that:
 $L_{\mu} \gamma' \gg 1$. This can be rewritten as:

$$L_{\mu} \frac{eV_{rf}'}{m_{\mu} c^2} \gg 1$$

which means $eV_{rf}' \gg 0.16 \text{ MeV/m}$ is required. This general rule must be followed throughout the entire muon system. For example, beam-cooling and reacceleration must occur in systems whose averaged accelerating gradients (including loss and transport elements) are much greater than 0.16 MeV/m to avoid large decay losses.

All previous acceleration systems have not been concerned with μ decay. However, we can compare existing accelerator and transport systems with this guideline to obtain some sense of the changes necessary in transforming to a μ accelerating system:

LEP II synchrotron: 2 GeV/ 27 km \Rightarrow 0.074 MV/m

CEBAF recirculating linac: 0.8 GeV/ 1.3 km \Rightarrow 0.6 MV/m

SLAC linac: 50 GeV/ 3 km \Rightarrow 17 MV/m

In these examples, the SLAC linac easily meets the gradient requirement by two orders of magnitude, and any linac-based system should have adequate gradient. The CEBAF recirculating linac barely meets the criterion; however, a recirculating linac with somewhat improved gradient per total transport length would also be adequate. Almost all existing synchrotrons do not have adequate gradient, and a synchrotron-based scenario would have to be greatly changed to be acceptable.

For a multiturn μ accelerator, the gradient criterion can be rewritten as:

$$E' \rightarrow \frac{E_{\text{final}}}{N_{\text{turns}} 2\pi R} \gg 0.16 \text{ MeV/m},$$

where R can be written in terms of the mean bending field B and the magnetic rigidity $B\rho$ as $R = B\rho/B \approx 0.00334 E_{\text{final}}(\text{MeV})/B$, and N_{turns} is the total number of acceleration turns. Inserting this into the previous equation obtains the criterion for any multi-turn accelerator:

$$\frac{N_{\text{turns}}}{B(\text{T})} \ll 300.$$

In this expression B refers to the average bending field in the highest energy turn (including straight sections, rf sections, and other non-bending elements).

ACCELERATION OPTIONS AND SCENARIOS

From these constraints, we can develop possible acceleration scenarios. A single-pass linac can easily meet the gradient constraint. However single-pass rf structures are prohibitively expensive and do not exploit a primary advantage in muons: our ability to bend them into multipass devices, enabling multipass use of the accelerating structures. We thus consider two forms of multipass acceleration: rapid-cycling synchrotrons and recirculating linac. These are shown in schematic form in figure 2.

Rapid-Cycling Synchrotrons

A synchrotron consists of rf accelerating structures within a circular magnetic beam transport, and the magnetic fields are increased from low-field to high-field while the beam is accelerated from low to high energies, passing many times through the same transport system. The magnetic fields must change rapidly to follow the beam transport, and with current technology only conventional magnets ($B < 2$ T) can cycle rapidly. The multiturn acceleration criterion can be met (barely) for $N_{\text{turn}} < \sim 100$ and mean bending field $B \sim 1$ T. As an example, we can consider a scenario with a final multiturn rapid-cycling cycle in which the beam is accelerated from 100 to 2000 GeV in a ring with $R = 5$ km ($B = 1.33$ T). This would require a 19 GV/turn rf system (1 km of 19 MV/m rf) for a 100-turn cycle, and would have an acceleration cycle of ~ 12 ms. 46.2% of initial μ 's would survive the cycle. We note that this cycle time is reasonably well matched to an ~ 30 Hz driver, and that the ring circumference is remarkably similar to that of the CERN LEP tunnel.

From equation (1), we can write an expression for beam survival in a multi-turn system:

$$\frac{N(s)}{N_0} \cong \left(\frac{E_0}{E_{\text{final}}} \right)^{\frac{2\pi R N_{\text{turn}} m_\mu}{L_\mu E_{\text{final}}}}$$

We can improve survival by increasing acceleration rate (decreasing N_{turn}). For example, reducing N_{turn} to 50 turns improves survival to 68%.

Recirculating Linacs

Another multiturn approach is the use of recirculating linacs, similar to CEBAF, which accelerates electrons to 4 GeV in a 5-pass system. In a recirculating linac (RLA), the beam is accelerated and returned for several passes of acceleration in the same linac, but a separate return path is provided for each pass. At the end of the linac, the beam passes through dipoles, which sort the beam by energy, directing it to an energy-matched return arc. (A pulsed kicker magnet system may also be used.) The various energy transports are then recombined at the end of the arc for further acceleration, until full energy is reached, when the beam is transferred to another linac or the collider. Thus the magnets are at fixed-field and the beam passes through each transport only once.

Since the beam passes through a separate transport on each turn, the magnets can be at fixed-field, allowing superconducting magnets, and simplified designs. However the requirement for a separate transport on each turn limits the total number of turns that could be practical, to ~ 10 — 20 turns. This is very compatible with the lifetime constraint: $N_{\text{turns}}B \ll 300$, which then can be met with relatively modest field magnets, and typically beam-survivals of $\sim 95\%$ are obtained in μ RLA's. High-field magnets are not required. The RLA is rather ideally matched to μ -acceleration constraints.

Because of the independence of each return transport, there is an enormous flexibility in RLA design, with only the rf acceleration frequency and voltage remaining constant from linac pass to linac pass. Since return path lengths are independent, the synchronous phase ϕ_s can be changed arbitrarily from pass to pass. Also the chromicity, $M_{56} = \partial z / \partial(\delta p/p)$, where z is particle position within the bunch, can be changed from turn to turn, by fitting the transport. At CEBAF,⁹ an isochronous transport ($M_{56} = 0$) was used, but for the μ -collider some bunching is required and non-zero M_{56} will be needed in some of the transport. Higher order chromicity control (M_{566}) with sextupoles is also possible, and one can consider adding higher-harmonic rf and additional compressor arcs, if needed.

The same RLA system could be used to accelerate both μ^+ and μ^- bunches. The oppositely charged bunches would propagate around the RLAs in opposite directions. If the bunches are injected into opposite sides of each RLA at the beginning of the separate linacs, then energy match of the beams in each arc is obtained, as well as phase matching across the arcs. Separate (but symmetric) transport lines into the higher-energy RLA's and into the collider would be needed.

RLA ACCELERATION SCENARIOS

From the previous discussion, RLA scenarios are currently the preferred μ -acceleration option. In this section we develop in more detail explicit acceleration scenarios for the 2 TeV collider, and then discuss possible variations.

Following cooling and initial bunch compression to $\sim 1\text{--}3\text{m}$ bunch lengths at $\sim\text{GeV}$ energies, the beams are accelerated to full energy (2 TeV). In this process, the μ -bunches must be compressed, to a length of $\sim 0.003\text{m}$ at full energy. A factor of 1000 energy increase in a single RLA is probably not optimum. A sequence of RLAs (i. e., 1–10, 10–100 and 100–2000 GeV), with rf frequency increasing as bunch length decreases, may be used. A factor of ~ 10 energy increase per stage is a plausible first approximation, before detailed optimization. It is important to obtain the acceleration and bunch compression with minimal phase space dilution, in order to avoid energy-spread blowup and beam losses. The RLA flexibility permits many possible compression scenarios; however, it is also quite easy to obtain very badly matched schemes within that broad tuneability.

Sample scenario - simulation results

As a simplified first example, which we use as a proof of principle, we consider in detail the scenario displayed in Table 2. This is a modularized 3-stage case, and a schematic view of a 3-stage RLA accelerator is displayed in figure 3. In each stage the energy is increased by a factor of 10 (2 to 20 to 200 to 2000 GeV). The rf frequency is also changed by a factor of 4 from RLA to RLA, from 100 to 400 to 1600 MHz. Each RLA consists of two linacs (each at 1 to 10 to 100 GeV) with recirculating arcs connecting them, and a total of ~ 10 turns in each stage. In this simplified format it is straightforward to scale the design from stage to stage.

We have developed the 1-D program μRLA to simulate the RLA longitudinal motion. In that program particle energy and position offsets are calculated from turn to turn. On each passage through a linac, particle energies change following:

$$\Delta E \rightarrow \Delta E + eV_{\text{rf}}(\cos \phi - \cos \phi_s),$$

while the synchronous energy increases by $eV_{\text{rf}} \cos \phi_s$. On each pass through an arc, particle phases change by:

$$\phi \rightarrow \phi + M_{56} \frac{2\pi}{\lambda} \frac{\Delta E}{E} + M_{566} \frac{2\pi}{\lambda} \left(\frac{\Delta E}{E} \right)^2 + \dots$$

where we have included first and second order chromaticities M_{56} and M_{566} . Note that ϕ_s , M_{56} , and M_{566} can be changed from turn to turn.

In this initial scenario, the beam is bunched within the injection transport for each RLA, while within the body of the RLA the synchronous phase is kept constant and M_{56} changes to maintain matched bucket conditions for fixed bunch-length. The matching conditions are set by varying ϕ_s and M_{56} to obtain a stable phase-space bucket matched to the beam-phase space area, and maintaining a constant area bucket. We approximate that bucket shape from synchrotron formulae. The matched energy spread of the rf bucket is:

$$\frac{\Delta E}{E} = \pm \sqrt{\frac{eV_{\text{rf}}\lambda}{EM_{56}}} \sqrt{\frac{2(\sin \phi_s - \phi_s \cos \phi_s)}{\pi}}$$

Maintaining a matched energy spread for fixed bunch length requires $\Delta E/E$ to decrease as $1/E$, which therefore implies that M_{56} must increase linearly with E . That condition was used in our initial simulations. (We note that the small number of turns in an RLA makes the synchrotron motion approximation somewhat inaccurate.) This matching minimizes bunch lengths within the RLAs, which reduces amplitude-dependent nonlinearities and also reduces bunch length oscillations, both of which can cause phase-space dilution. A similar matching condition on M_{56} occurs naturally in microtron design.

We have simulated this initial scenario using μ RLA, and some results are summarized in Table 2, and displayed in figure 4. Some phase-space dilution and mismatch does occur, particularly in transfers between RLAs. However the rms emittance dilution is $< \sim 5\%$ per RLA or 15% over the entire system. Particle loss through the beam dynamics is less than 1%. Particle loss through μ -decay is somewhat larger, but less than $\sim 5\%$ per RLA or $\sim 12\%$ over the entire system. (We have assumed gradients of up to 19 MV/m in the linacs, and mean bending fields of $\sim 5\text{T}$ in the highest-energy arcs.) Bunch compression to $\sigma < 0.003$ m is obtained through rebunching and matching with the frequency increase from RLA to RLA, and is acceptable. Thus the simulation demonstrates that a cascade of RLAs can provide acceptable acceleration with bunching for a $\mu^+\mu^-$ collider, with minimal dynamic and decay beam loss and emittance dilution.

This scenario sets a ‘‘proof of principle’’ baseline for the exploration of acceleration scenarios. It is certainly unoptimized, and does not exploit the full degrees of freedom possible in the RLA scenarios. As initially formulated, it requires a separate rf system for bunching at the entrance of each RLA (0.2 GV of 100 MHz rf before RLA 1, 1.25 GV of 400 MHz rf before RLA 2, and 6 GV of 1600 MHz rf before RLA 3). In future development, these will be integrated with the acceleration rf, perhaps within a more gradual bunching scenario.

Another scenario, presented by Palmer in July 1995,¹⁰ is displayed in Table 3, and gives some impression of the possible variations in design. Beam is accelerated from 1 GeV to 2 TeV using 4 RLA steps, with top energies of 8, 75, 250 and 2000 GeV. The 250 GeV step is a suitable accelerator for a 250×250 GeV collider. Similar performance to the initial baseline is obtained, but with slightly larger losses and dilution due to the additional RLA. Beam loss through decay is ~19%. A complete bunching and acceleration sequence for this scenario is not yet developed, however.

COMMENTS ON SCENARIO OPTIMIZATION

In a multiturn RLA system there is a balance between rf acceleration and beam transport cost/requirements. Increasing the number of turns per RLA directly reduces the linac lengths and therefore linac costs, but it also increases the total amount of beam transport, adding cost and complexity. We have not yet developed cost estimates that are adequate to obtain an accurate optimum. In this section we discuss some of the considerations which must be included in developing an optimum design.

rf Considerations

We need a separate rf linac system for each RLA, with lower frequencies for the initial lower-energy RLAs, where the beam has a relatively long bunch length and higher frequencies for the high energy end, where the bunches are shortened, since higher-frequencies are expected to be less expensive. We have not determined whether separate bunching rf systems are desirable.

Very high-gradient is not essential in the acceleration, but rather minimal cost is. The Table 2 scenario requires ~200 GV of rf acceleration, while the Table 3 scenario requires ~100GV; these are both quite large and would require ~5—10km at 20 MV/m.

The rf cavities must sustain field throughout the multipass acceleration time, which is ~1ms in the 2 TeV RLA. That implies SRF cavities should be used in the higher-energy RLAs, although we do not have clear guidelines for optimum parameters. We have used TESLA¹¹ and CEBAF parameters (~1.5GHz) in this study. TESLA is actually designed for 1ms cycles, repeating at 5Hz; these parameters are very close to our requirements. These use low-temperature(~2K) materials; higher temperature alternatives (4K or ??) should also be studied.

Transport Considerations

The beam transports for the recirculation arcs are relatively straightforward, but are nontrivial, since they require good transverse matching throughout the system to avoid

emittance dilution. Each transport must be achromatic (matched to zero dispersion), and also must have a chromaticity M_{56} matched to the bunching requirements. A transport modeled on the CEBAF RLA could be used. High field is not required, and even conventional fields ($B < 2T$) are adequate.

Since the beam passes through a different return arc on each turn, the total amount of beam transport is relatively large (~85km of arcs in the Table 2 scenario, and 160km for Table 3). The transport can easily become **very** expensive, so cost-saving designs are needed. Multiple-aperture magnets, in which several passes go through separate (different field) apertures in the same magnetic structure are possible. S. Kahn, G. Morgan and E. Willen¹² have proposed 9 and 18 aperture dipoles with this purpose. Other “low-cost” technologies could be used (permanent magnets, super-ferric, etc.).

Hybrid magnets, in which rapid-cycling and high-field magnetic elements are mixed and pulsed so that several passes can go through the same transport, would be a very attractive technology in this application, and could permit more passes. (A scenario requiring only 20 GeV of rf, using an injector and three RLA stages with 200-turn rapid-cycling in the last stage, has been developed.)

Note that at the beginning and end of the arcs beam-separation and beam-recombination transports for all passes must be inserted, and this adds considerable complication. CEBAF has a 5-pass separation and recombination system with carefully matched transports, and it is easy to imagine a 10-pass extrapolation of that system to our case. However many more passes (20?) may lead to impractically congested designs.

There will be some μ -decay in the transport, which will deposit electrons with an average of 1/3 of the μ energy throughout the system. Since the decay rate decreases as the energy increases, the mean beam energy deposition (per μ) per meter is a constant :

$$\frac{dE}{ds} = \frac{m_{\mu} c^2}{3L_{\mu}} \text{ per } \mu.$$

This comes to ~0.25 watts/m with a beam of 10^{12} μ 's at 30Hz; and this level seems tolerable even in superconducting structures.

COMMENTS AND CONCLUSIONS

We have presented a candidate scenario for a high-energy $\mu^+ - \mu^-$ accelerator. That scenario includes a first proof-of-principle calculation of the design concept. Much further optimization and design and concept development is needed.

The bunch-compression and acceleration scenario must be optimized and further simulated. Variations such as rapid-cycling should be considered. Complete lattices are needed, with designs for the transport arcs, including beam separation and

recombination. An accurate cost algorithm for rf and beam transport components is needed to obtain an optimal scenario. rf acceleration development would also be desirable, both in the low-frequency rf systems needed in the first stages and in the high-frequency SRF needed in the high-energy accelerators.

In this paper, we have concentrated on a 2 TeV accelerator. We can obtain a first muon collider (FμC at ~250 GeV) accelerator by stopping with the penultimate RLA. Total transport and rf requirements (now 10—20 GV) are naturally an order of magnitude less. However the rapid-cycling variations we are also considering apply primarily to the last RLA stage (2 TeV). The FμC would require rapid-cycling at an order of magnitude larger frequency.

Acknowledgments

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Table 1: Parameter list for a 4 TeV $\mu^+\text{-}\mu^-$ Collider

<u>Parameter</u>	<u>Symbol</u>	<u>Value</u>
Energy per beam	E_μ	2 TeV
Luminosity	$L=f_0n_s n_b N_\mu^2/4\pi\sigma^2$	$10^{35} \text{ cm}^{-2}\text{s}^{-1}$
Source Parameters		
Proton energy	E_p	30 GeV
Protons/pulse	N_p	$2 \times 3 \times 10^{13}$
Pulse rate	f_0	15 Hz
μ -production acceptance	μ/p	.2
μ -survival allowance	N_μ/N_{source}	.33
Collider Parameters		
Number of μ /bunch	$N_{\mu\pm}$	2×10^{12}
Number of bunches	n_B	1
Storage turns	n_s	1000
Normalized emittance	ϵ_N	$3 \times 10^{-5} \text{ m-rad}$
μ -beam emittance	$\epsilon_t = \epsilon_N/\gamma$	$1.5 \times 10^{-9} \text{ m-rad}$
Interaction focus	β_0	0.3 cm
Beam size at interaction	$\sigma = (\epsilon_t \beta_0)^{1/2}$	2.1 μm

Table 2: Parameters for an idealized 3-RLA acceleration scenario
(The Bi are bunchers; RLAI are multipass recirculating linacs)

Cycle	Energy (GeV)	rf frequency	Bunch length σ	$\delta E/E$	passes	Time (μs)
B1	2	100 MHz	25→7	1→4%		
RLA 1	2→20	100 MHz	7 cm	4→0.4 %	9	8
B2	20	400 MHz	7→1.5	0.4→2%		
RLA 2	20→200	400 MHz	1.5 cm	2→0.2%	10	65
B3	200	1.6 GHz	1.5→0.3	0.2→1.0%		
RLA 3	200→2000	1.6 GHz	0.3 cm	0.2→0.1%	10	585

Table 3: Parameters for a 4-RLA acceleration scenario

Cycle	Energy (GeV)	rf frequency	Bunch length σ	$\delta E/E$	passes	Time (μs)
B1	1	100 MHz	25→7.5	1.5→ 5%		
RLA 1	1→8	100 MHz	7.5→5.0	5.0→1 %	8	5.6
B2	8	400 MHz	5→1.6	1→4.5%		
RLA 2	8→75	400 MHz	1.6cm	4.5→0.5%	12	30
B3	75	1.3 GHz	1.6→0.5	0.5→1.5%		
RLA 3	75→250	1.3 GHz	0.5	1.5→0.5%	18	96
B4	250	2.0 GHz	0.5→0.3	0.5→0.8%		
RLA 4	250→2000	2.0 GHz	0.3cm	0.8→0.1%	18	662

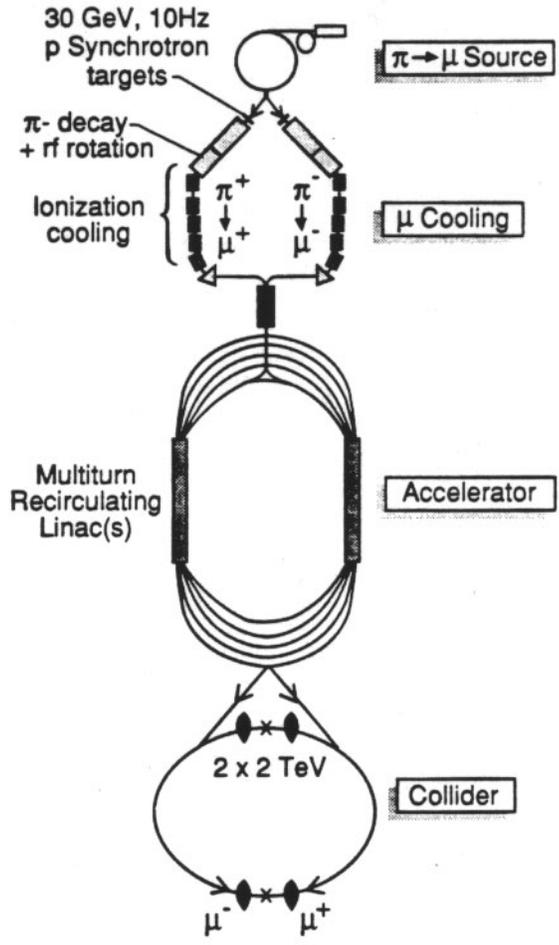
Figure 1: Overview of the $\mu^+\mu^-$ collider system, showing a muon (μ) source based on a high-intensity rapid-cycling proton synchrotron, with the protons producing pions (π 's) in a target, and the μ 's collected from subsequent π decay. The source is followed by a μ -cooling system, and an accelerating system of recirculating linac(s) and/or rapid-cycling synchrotron(s), feeding μ^+ and μ^- bunches into a superconducting storage-ring collider for multiturn high-energy collisions. The entire process cycles at 15 Hz.

Figure 2. Schematic views of a rapid-cycling synchrotron (RCS) and a recirculating linac (RLA). In the RCS, the beam is accelerated for many turns through the rf, while the magnetic fields in the ring cycle from low-field to high-field, following the beam energy; the beam passes through the same transport on each turn. In the RLA, the beam is accelerated through several passes of the linacs. On each return arc, the beam passes through a different transport path, matched to the increasing beam energy. Magnetic fields are fixed, and the number of return transports (per arc) equals the number of linac passes. (Hybrid schemes, with several return passes, but with some cycling magnets in each pass which track the increasing beam energy, keeping the beam for several passes through the same transport, are also possible.)

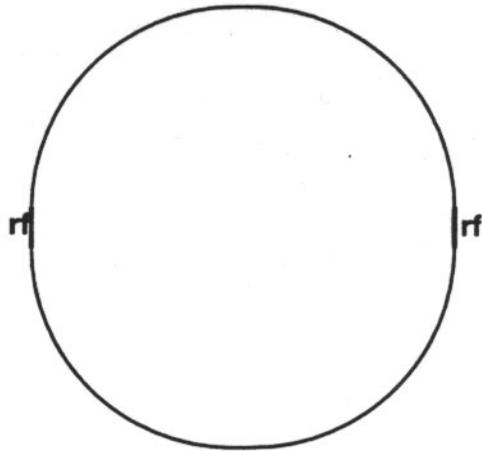
Figure 3. Conceptual view of an RLA-based accelerator, showing a linac feeding beams into a sequence of 3 recirculating linacs (RLA1, RLA2, RLA3) followed by a collider ring. Note that the drawing is not to scale (size change from RLA to RLA would be greater), and the separation between lines in the arcs is exaggerated in this sketch. (There will also be more arc beam lines than displayed, and the separations could be vertical.)

Figure 4. Some simulation results from μ RLA. In these simulations a beam is accelerated from 2GeV to 2000 GeV through the three cascaded RLAs of table 2, with bunching at the beginning of each linac. An initially bunched beam for RLA 1 is shown in Fig. 4A, and beam phase-space distributions at the end of RLAs 1, 2, and 3 are shown in 4B, 4C, 4D. The vertical and horizontal scales are $\delta E/E$ and $\delta\phi$, respectively. Note that rf frequency increases from 100 to 400 to 1600 MHz from RLA to RLA. The beam is accelerated with very little loss from beam dynamics acceptance and with a longitudinal emittance dilution of $\sim 12\%$. Beam loss from decay would be $\sim 12\%$.

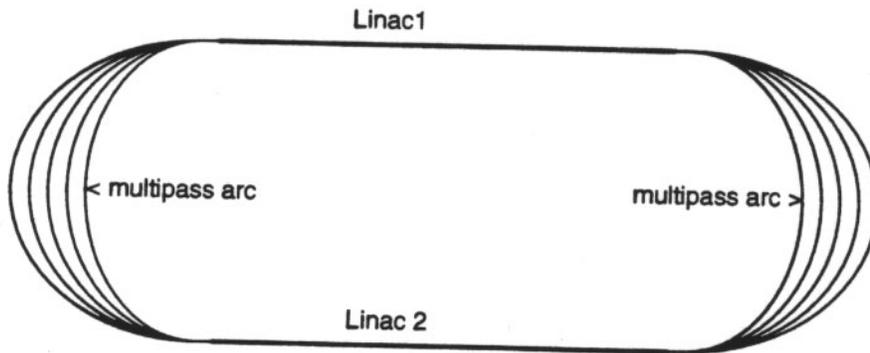
A $\mu^+ \mu^-$ COLLIDER SYSTEM



Rapid-Cycling Synchrotron



Recirculating Linac



$\mu^+\mu^-$ Accelerator and Collider System

