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Vacuum Decay along Supersymmetric Flat Directions *

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Abstract

It has been recently realized that within the Minimal Supersymmetric Standard Model, for certain patterns of superpartner masses, consistent with all the present experimental constraints, the scalar potential may develop at some scale Q_0 unbounded color/charge breaking directions involving the sfermion fields, and that these patterns are then excluded unless some new physics is invoked at or below the scale Q_0. We reanalyze this observation and point out that such patterns of superpartner masses at the weak scale are *not* ruled out when taking into account the probability of decay for the metastable color conserving minimum along these color breaking unbounded directions. It turns out that the color conserving minimum, although metastable, has a lifetime longer than the present age of the Universe and can survive both quantum tunneling and the effects of high temperatures in the early Universe, causing the color/charge breaking effects to be in practice not dangerous.

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1. Introduction

There are many good reasons to believe that the Standard Model is not the ultimate theory of nature since it is unable to answer many fundamental questions. One of them, why and how the electroweak scale and the Planck scale are so hierarchically separated has motivated the Minimal Supersymmetric extension of the Standard Model (MSSM) as the underlying theory at scales of order 1 TeV [1]. A huge number of new parameters appear when considering the MSSM. Some of these new parameters are constrained by the unsuccessful searches of new particles at accelerators. Others may receive severe bounds from the requirement of avoiding large flavour-changing neutral currents. Moreover, constraints on the parameter space, which mostly involve the soft supersymmetry breaking trilinear terms A's, arise from the existence of charge and/or color breaking minima in the scalar sector when looking at some particular directions with nonvanishing vacuum expectation values (VEV's) of the Higgs fields [2].

By considering directions in the field space which do not interest the VEV's of the Higgs fields, it has been recently pointed out that certain mass patterns for the superpartners cannot arise at low energy unless there is new physics beyond the MSSM and below the grand unified (GUT) scale [3]. Indeed, there are many flat directions in the field space of the MSSM and it may happen that some combination of the squark and/or slepton mass-squared parameters get negative at some scale Q_0 below the GUT scale when running through the Renormalization Group Equations (RGE's) from the weak scale up. This leads either to the appearance of unacceptable color breaking minima or to unbounded from below directions in the effective potential for the squark and/or slepton fields ϕ's, making the color conserving minimum $\phi = 0$ metastable. In such a case, it has been argued in ref. [3] that the corresponding region of the parameter space is either ruled out or there must exist some new physics below the scale Q_0 whose effects can change the evolution of the mass-squared parameters with the scale Q [3].

The situation here is fairly analogous to what happens for the effective potential of the SM Higgs field H: for a top quark mass large compared to the Higgs and gauge bosons masses, the one–loop top quark contribution to the effective potential will dominate the others and drive the coefficient of the quartic term H^4 negative for very large values of

H, thus destabilizing the effective potential and making our vacuum at $\langle H \rangle \simeq 250$ GeV a local, but not global, minimum [4]. Nonetheless, the electroweak vacuum need not be absolutely stable. For certain top quark mass M_t and Higgs boson mass M_H it may just be instead metastable, as long as its lifetime exceeds the present age of the Universe [5]. The decay of the electroweak vacuum may be driven at low temperatures by quantum tunneling or at high temperatures by thermal excitations. Even if the requirement that our vacuum survives the high temperatures of the early Universe places strong constraints from vacuum stability on the (M_t, M_H) parameter space, still values of the top quark and Higgs boson masses for which our vacuum is metastable, but with a lifetime larger than the present age of the Universe, are allowed.

The purpose of the present Letter is to reanalyze the constraints on supersymmetric models discussed in ref. [3], involving some combination of the sparticle masses at the weak scale and imposed to avoid large color breaking VEV's $\phi \neq 0$ or destabilized effective potentials along some squark and/or slepton directions, and to point out that such limits may be weakened by considerations about the survival of the color conserving minimum. Even if such a minimum may be metastable in large regions of the parameter space of the superpartner masses, its lifetime turns out to be almost everywhere longer than the present age of the Universe. This means that the regions of the parameter space ruled out (unless some new physics appears before a certain scale Q_0) in ref. [3] may be indeed permitted without any need of new physics.

The paper is organized as follows: In Section **2** we shall briefly review the effective potential along some particular directions of the squark fields showing why it may be destabilized and which are the consequent constraints on the superpartner mass patterns [3]. In Section **3** we discuss the color conserving $\phi = 0$ decay rate at zero temperature, leaving the finite temperature case to Section **4**. Finally, Section **5** presents our conclusions.

2. The effective potential and flat directions

Let us consider the same flat direction in the squark fields analysed in ref. [3]: $\tilde{u}^r_R =$

$\tilde{s}_R^g = \tilde{b}_R^b \equiv \phi/\sqrt{2}$. Along this particular direction the coefficient $\lambda(Q)$ of the quartic term ϕ^4 is vanishing for all scales Q, and the one–loop effective potential reads

$$V(\phi) = \frac{1}{2} m^2(Q)\phi^2(Q) + V_{1-\text{loop}}(\phi), \tag{1}$$

where $m^2 \equiv m^2_{\tilde{u}_R} + m^2_{\tilde{s}_R} + m^2_{\tilde{b}_R}$ and $V_{1-\text{loop}}$ is the one–loop correction to the effective potential (in the $\overline{DR}$-scheme)

$$V_{1-\text{loop}}(\phi) = \frac{1}{64\,\pi^2} \,\text{Str}\, \mathcal{M}^4(\phi) \left[\ln\left(\frac{\mathcal{M}^2(\phi)}{Q^2}\right) - \frac{3}{2}\right]. \tag{2}$$

where Str counts properly all the degrees of freedom, summing over all the mass eigenstates which get mass in the ϕ-background field.

Since the one–loop potential behaves as $\propto \ln(g_3^2\phi^2/Q^2)$ for large values of ϕ (g_3 is the $SU(3)$ gauge coupling constant), in order that the approximation of neglecting loop effects be safe one should adopt in the renormalization group improved tree level potential a scale $Q \simeq g_3\phi$ to make the logarithms small. In practice, we will adopt $Q = Q_\phi \equiv \sqrt{g_3^2\phi^2 + M_3^2}$, so as to stop the running below $Q \simeq M_3$, and use for the effective potential

$$V(\phi) = \frac{1}{2} m^2(Q_\phi)\phi^2. \tag{3}$$

The prescription is trivial: just evaluate the mass parameter m^2 at the scale $Q \simeq Q_\phi$.

The RGE for m^2 is given by

$$Q\frac{dm^2}{dQ} = \frac{1}{8\,\pi^2}\left[-16\, g_3^2\, M_3^2 - \frac{8}{3}\, g_1^2\, M_3^2 + 2\, h_b^2\, \left(m^2_{\tilde{q}_L} + m^2_{\tilde{b}_R} + m^2_{H_1} + A_b^2\right)\right], \tag{4}$$

where g_1 is the standard $U(1)$ coupling, M_i are gaugino masses, h_b is the bottom quark Yukawa coupling and A_b is the bottom quark trilinear mixing parameter, and all parameters are running. If $\tan\beta$ (the ratio of the two Higgs VEV's) is not too large, h_b is small and the term proportional to h_b^2 in the above Equation can be neglected, leading to the solution [3]

$$\begin{aligned} m^2(Q) = m^2 &- \frac{2}{\pi^2} g_3^2(M_3) M_3^2 \ln(Q/M_3) \left\{ \frac{1 + 3g_3^2(M_3)\ln(Q/M_3)/(16\pi^2)}{[1 + 3g_3^2(M_3)\ln(Q/M_3)/(8\pi^2)]^2} \right\} \\ &- \frac{1}{3\pi^2} g_1^2(M_1) M_1^2 \ln(Q/M_1) \left\{ \frac{1 - 11g_1^2(M_1)\ln(Q/M_1)/(16\pi^2)}{[1 - 11g_1^2(M_1)\ln(Q/M_1)/(8\pi^2)]^2} \right\}, \end{aligned} \tag{5}$$

where all masses on the right-hand side are physical (propagator pole) masses and the only undertermined factors are $g_1^2(M_1)$ and $g_3^2(M_3)$, which depend on the full spectrum of the superpartner masses and the initial condition $\alpha_3(M_Z) = 0.12$. If one takes $M_1 \leq M_3$, as usually results in GUT models, the effects of the term proportional to g_1^2 in Eq. (5) is negligible. To obtain $g_3(M_3)$ we have assumed a common supersymmetric threshold at M_3, but the results are almost unsensitive to this simplifying assumption.

In Fig. 1 we show the contours of $m^2(Q_0) = 0$ for different values of Q_0 in the $(m/\sqrt{3}, M_3)$–plane. Portions of the parameter space lying to the right of each contour (for a given scale Q_0) are characterized by an effective potential unbounded from below and by a metastable color conserving minimum $\phi = 0$, unless some new physics capable of modifying this situation is present between the soft supersymetry breaking scale (~ 1 TeV) and Q_0 [1]. It is easy to see, from Eq. (5), that for $m \leq M_3$ one has that the value of the field ϕ_0 for which $V(\phi_0) = 0$ is

$$\phi_0 \simeq M_3 \exp\left[\frac{\pi^2 m^2}{2 g_3^2 M_3^2}\right]. \tag{6}$$

In Fig. 2 we draw the running scalar mass $m^2(Q_\phi)$, normalized to its value m^2 for $\phi = 0$, as a function of the field ϕ, assuming that $m/\sqrt{3} = 500$ GeV and for values of M_3 of 1500, 1000, 700 and 600 GeV. The effective potential is just obtained by multiplying the result by $m^2\phi^2/2$. Notice that a barrier will be present in $V(\phi)$ separating the metastable vacuum $\phi = 0$ from the region of very large values of ϕ, where $m^2(Q_\phi) < 0$ and the potential becomes negative and unbounded from below.

From the analysis of the effective potential one can then conclude that the sparticle masses at the weak scale must satisfy severe relations among each other to avoid the

[1] The contours of Fig. 1 are slightly different from those presented in ref. [3] since there the authors have introduced a non-renormalizable operator $(1/6)\phi^6/M^2$ which lifts the unbounded direction and results in a VEV $\phi(Q) = M^{1/2}(-m^2(Q))^{1/2}$. Then they have plotted the self-consistency condition $\phi(Q) = Q$ and found that for $m/\sqrt{3} \lesssim 0.7\, M_3$ some new physics below M_{Pl} is necessary because of the existence of large color breaking VEV's.

color conserving minimum $\phi = 0$ becoming a local minimum and the potential to develop unbounded directions for large values of the field ϕ.

In the next Sections we shall show that these relations may be relaxed, since in most of the region in the parameter space $(m/\sqrt{3}, M_3)$ where the minimum $\phi = 0$ is metastable, it has however a lifetime longer then the age of the Universe, so that color breaking effects are in practice not dangerous.

3. Nucleation by quantum tunneling

The decay of the metastable color conserving minimum $\phi = 0$ may occur by the nucleation of bubbles of the unstable phase. If the bubble is too small, it collapses under its surface tension. If the bubble is large enough, it expands classically, eventually absorbing all the metastable phase. At zero temperature, the vacuum can decay only by quantum tunneling through the barrier separating the metastable vacuum from the region of negative values of $V(\phi)$. The WKB amplitude for false-vacuum decay by tunneling may be found expanding the Euclidean path integral about the *bounce* solution to the Euclidean equation of motion [7]

$$\partial^2 \phi = \frac{dV(\phi)}{d\phi}. \tag{7}$$

The bounce solution is an $O(4)$ rotationally symmetric solution and solves

$$\left(\partial_s^2 + \frac{3}{s}\partial_s\right)\phi = \frac{dV(\phi)}{d\phi}, \tag{8}$$

where $s = (t_E^2 + \mathbf{r}^2)^{1/2}$. It takes some convenient value $\phi(0)$ at $s = 0$ (with $\phi'(0) = 0$), probing the unstable region of the potential ($\phi(0) > \phi_0$, where $V(\phi_0) = 0$), and falls to the false vacuum $\phi = 0$ as $s \to \infty$. When viewed as a function of the Euclidean time t_E, the bounce solution interpolates between the false vacuum $\phi(t_E \to -\infty, \mathbf{r}) = 0$ and an unstable bubble $\phi(t_E = 0, \mathbf{r})$, which is just large enough to expand on its own classically. The Euclidean action S_4 of the bounce solution yields the exponential suppression of the rate for the false vacuum decay per unit volume

$$\Gamma_4/V = D_4 \exp\left[-S_4\right], \tag{9}$$

where D_4 is a coefficient calculated from fluctuations around the bubble.

The rate for the decay of the metastable color conserving minimum $\phi = 0$ at zero temperature may be found simply by solving Equation (8) numerically, using the effective potential (3) discussed earlier, and computing the Euclidean action. Some qualitative flavour of the results can be obtained by noting that, if $\phi_0 \gg m$ and M_3, the bounce is mainly determined by the behaviour of V at large values of ϕ, near ϕ_0. Since $dV/d\phi|_{\phi_0} \propto M_3^2\phi_0$, we see from Eq. (6) that the radius of the bounce is $\sim M_3^{-1}$, and for instance the kinetic energy contribution to S_4 results $\sim \phi_0^2/M_3^2 \simeq \exp\left(\pi^2 m^2/(2g_3^2 M_3^2)\right)$, which is quite large unless $m \ll M_3$.

To decide whether the metastable vacuum $\phi = 0$ would survive the age of the present Universe $\sim 10^{10}$ yr, one has to multiply Eq. (9) by the space-time volume of the past light cone of the observable Universe. The condition that the lifetime of the metastable state $\phi = 0$ is longer than the present age of the Universe translates into (neglecting the very weak dependence upon the prefactor D_4)

$$S_4 \gtrsim 400. \tag{10}$$

Fig. 3 shows the contours in the $(m/\sqrt{3}, M_3)$–plane for different values of $\log_{10}(S_4)$, the critical value $S_4 = 400$ corresponding to the contour labelled by 2.6. Regions to the right of each contour are characterized by an Euclidean action smaller than the value at the contour. A comparison between Figs. 1 and 3 shows that the color symmetric vacuum $\phi = 0$, although metastable, has indeed a lifetime which exceeds the present age of the Universe, in all the regions of the $(m/\sqrt{3}, M_3)$–plane, with the exception of a very small wedge at small values of m. From Fig. 1, one sees that in this wedge the vacuum $\phi = 0$ could become stable only by the introduction of new physics at a scale below $\sim 10^4$ GeV.

From the analysis of the metastable vacuum decay by quantum tunneling at zero temperature we may therefore conclude that apparently dangerous flat and unbounded directions in the squark ϕ-field space are indeed safe: the barrier separating the metastable vacuum from the regions of negative values of $V(\phi)$ inhibits the formation of sufficiently large bubbles and the color conserving vacuum $\phi = 0$ survives the quantum tunneling.

In the next Section we shall analyze the fate of the color conserving minimum when considering the thermal effects present in the early Universe.

4. Nucleation by thermal excitation

Another source of energy to cross the barrier is the high temperature of the early Universe. In the high-temperature plasma, thermal fluctuations may excite a bubble sufficiently large so as to be able to induce the transition, and the probability of a thermal fluctuation of associated energy E_b to cross the barrier is simply given by a Maxwell-Boltzmann suppression factor $\exp(-E_b/T)$. One may think that the rate will not be exponentially suppressed at high temperatures, large compared to the barrier energy, but however the effective potential, and therefore E_b, also depends on the temperature making the barrier higher. At high temperatures, therefore, there is a thermal energy to cross the barrier, but the barrier is higher.

We now need to find the energy barrier for the phase transition. For this we need a bubble corresponding to a static, unstable solution of the classical equation of motion [8]

$$\left(\partial_r^2 + \frac{2}{r}\partial_r\right)\phi = \frac{dV(\phi,T)}{d\phi}, \tag{11}$$

where $V(\phi,T) = V(\phi) + \Delta V_T(\phi)$ is obtained by adding to the zero temperature effective potential the one–loop finite–temperature correction [9] $\Delta V_T(\phi) \equiv V_T(\phi) - V_T(0)$, with

$$V_T(\phi) = \frac{T^4}{2\pi^2}\sum_i \pm n_i \int_0^\infty dq\, q^2 \ln\left[1 \mp \exp\left(-\sqrt{q^2 + m_i^2(\phi)/T^2}\right)\right]. \tag{12}$$

Here the upper sign is valid for bosons and the lower one for fermions, and n_i are the corresponding degrees of freedom. Notice that substracting $V_T(0)$ from the effective potential is necessary for computing the bounce action.

It is easy to see that $V_T(0) = -\pi^2 g_* T^4/90$, with $g_* = n_b + (7/8)n_F$ counting the degrees of freedom lighter than T. Hence, for $\phi \gg T$, $\Delta V_T(\phi) = \pi^2 \bar{g}_* T^4/90$, where $\bar{g}_*$ counts the particles heavier than T in the presence of the field ϕ and lighter than T in $\phi = 0$. In our case, $g_* = 60$ for $T \gg m$ and M_3. On the other hand, one has for $T \gg \phi$,

$$\Delta V_T(\phi) \simeq \frac{T^2}{24}\sum\left[n_B \Delta m_B^2 + \frac{n_F}{2}\Delta m_F^2\right], \tag{13}$$

where $\Delta m^2 \equiv m^2(\phi) - m^2(0)$, so that only the ϕ dependent mass terms contribute. In our case $\Delta V_T(\phi) \simeq 2g_3^2 T^2 \phi^2$ for $T \gg \phi$, getting its dominant contributions from the gluons (via seagull terms), the gluinos and the u_R^r, s_R^g and b_R^b quarks (from the $\tilde{g}q\tilde{q}$ couplings) and the corresponding squarks (via the D–terms), with the exception of the flat direction [2].

Again, we have solved Eq. (11) numerically searching for a solution which probes unstable values of ϕ at $r = 0$ and falls off to the metastable vacuum $\phi = 0$ as $r \to \infty$. The probability of tunneling per unit time per unit volume is given by

$$\Gamma_3/V = D_3 \exp\left[-S_3/T\right], \tag{14}$$

where D_3 is the determinant factor and S_3 is the three-dimensional action of the solution of Eq. (11). As discussed by Anderson in ref. [5] we should now multiply by the volume our current horizon had when at temperature T, which is $V(T) \sim (10^{10}\,\mathrm{yr})^3 \times (3\,\mathrm{K}/T)^3$, and by the amount of time the Universe spent at temperatures T, which is $t \sim M_{\mathrm{Pl}}/T^2$. Putting this together, one finds that the metastable vacuum $\phi = 0$ has survived the high temperatures of the early Universe if

$$S_3/T \gtrsim 230. \tag{15}$$

For a given choice of masses m and M_3, we have computed the minimum of S_3/T as a function of temperature.

The resulting minimum values are shown as contours in the $(m/\sqrt{3}, M_3)$–plane in Fig. 4. The interpretation of these results is the following: in the regions where for $T = 0$ the action S_4 was very large, corresponding to very large values of $\phi(0)$ in the bounce solution, also the finite temperature bounce action is hoplessly large due to the exponentially large values of the bounce at the origin (since the finite T correction ΔV_T is positive, the value of $\phi(0)$ is even larger in the finite T case). In the remaining region of small scalar masses, where the $T = 0$ bounce had smaller values of $\phi(0)$ ($\sim 10^3$–10^5 GeV), the following is found: for $T \gg m$ and M_3, the barrier has a height $\propto \phi^2 T^2$ for $\phi < T$, while it behaves as $\bar{g}_* \pi^2 T^4/90 + m^2(Q_\phi)\phi^2/2$ for $T > \phi$, and hence the potential can become negative only for values $\phi \propto T^2/\sqrt{|m^2(Q_\phi)|} \gg T$. The width of the bounce

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[2] we have neglected the small contribution from the photon and photino.

is now $\sim |m^2(Q_\phi)|^{-1/2}$, so that $S_3/T \propto (T/\sqrt{|m^2(Q_\phi)|})^3$. This implies that at large temperatures, it becomes harder to jump over the barrier. In fact, we usually find that the optimum temperature for the transition is $\mathcal{O}(m)$. As can be seen in Fig. 4, this considerations imply that the lifetime of the metastable vacuum is again larger than the age of the Universe with the exception of the small wedge corresponding to $m \ll M_3$ (to the right of the contour labeled 2.4), similarly as in the $T = 0$ case.

Notice that so far we have been assuming as an initial condition that the scalar field ϕ is always sitting at the origin $\phi = 0$ at high temperatures. Although this is the thermal equilibrium state at large temperatures, it is however also possible that the scalar field ϕ is left initially far from the origin, *e.g.* at an early epoch near the end of inflation, and may the roll towards the unbounded direction before reaching the $\phi = 0$ minimum. Due to the uncertainties connected with the hidden sector and the form of the gravitational couplings at the Planck scale, it is difficult to determine the (model–dependent) form of the potential for ϕ during inflation [10], specially for large values of the field, although some mechanisms which could adjust the initial ϕ values close to the origin have been suggested [11]. In this paper we have then assumed that, when the temperature of the Universe decreases to the point in which the unbounded direction first appears in $V(\phi, T)$, the scalar field is already sitting close to its color conserving minimum[3].

5. Conclusions

In this paper we have investigated the constraints involving some combination of the sparticle masses at the weak scale arising from the requirement that the metastable and color conserving ground state $\phi = 0$ is stable along flat directions where the effective potential $V(\phi)$ becomes unbounded from below for very large values of the scalar field. We have concentrated our attention in the direction in field space analysed in ref. [3], and a similar analysis may be applied to other dangerous directions considered in the literature [2].

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[3]We thank E. Kolb for enlightening discussions about this point.

In spite of the presence of large regions of the $(m/\sqrt{3}, M_3)$–parameter space where the effective potential becomes unbounded unless some new physics appears between the MSSM scale and the GUT scale [3], we have pointed out that such constraints are significantly weakened when considering the decay probability for the metastable state $\phi = 0$ along this unbounded direction. It turns out that the lifetime of this state is longer than the present age of the Universe and that it can survive both quantum tunneling, occuring at zero temperature, and the thermal excitations present in the early Universe. We then suggest that no severe relations among the superpartner masses should be imposed to avoid disastrous color breaking: the metastable color conserving minimum is protected against jumping towards dangerous squark unbounded directions and color breaking effects are in practice not present.

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Figure Captions

Figure 1: Contours of $m^2(Q_0) = 0$ for different choices of the scale Q_0, below which new physics is then required, in the $(m/\sqrt{3}, M_3)$–plane.

Figure 2: The plot of $m^2(Q_\phi)/m^2$ as a function of ϕ for the particular choice of $m/\sqrt{3} = 500$ GeV and $M_3 = 600$, 700, 1000, 1500 GeV.

Figure 3: Contours in the $(m/\sqrt{3}, M_3)$–plane for different values of $\log_{10}(S_4)$, the critical value $S_4 = 400$ corresponding to the contour labelled by 2.6.

Figure 4: Contours of the minimum (as a function of the temperature) of S_3/T in the $(m/\sqrt{3}, M_3)$–plane. The critical value $S_3/T = 230$ corresponding to the contour labelled by 2.4.

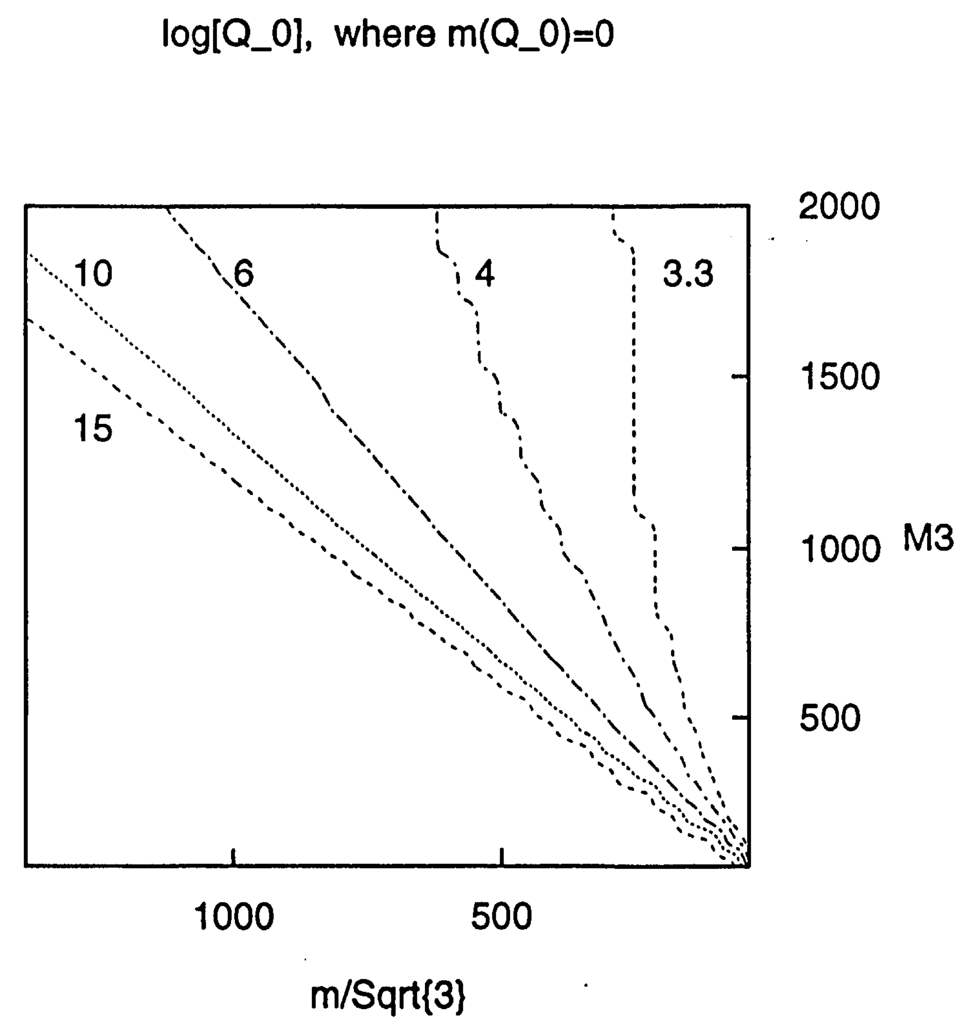
log[Q_0], where m(Q_0)=0
10
6
4
3.3
15
2000
1500
1000
500
M3
1000
500
m/Sqrt{3}

Figure 1

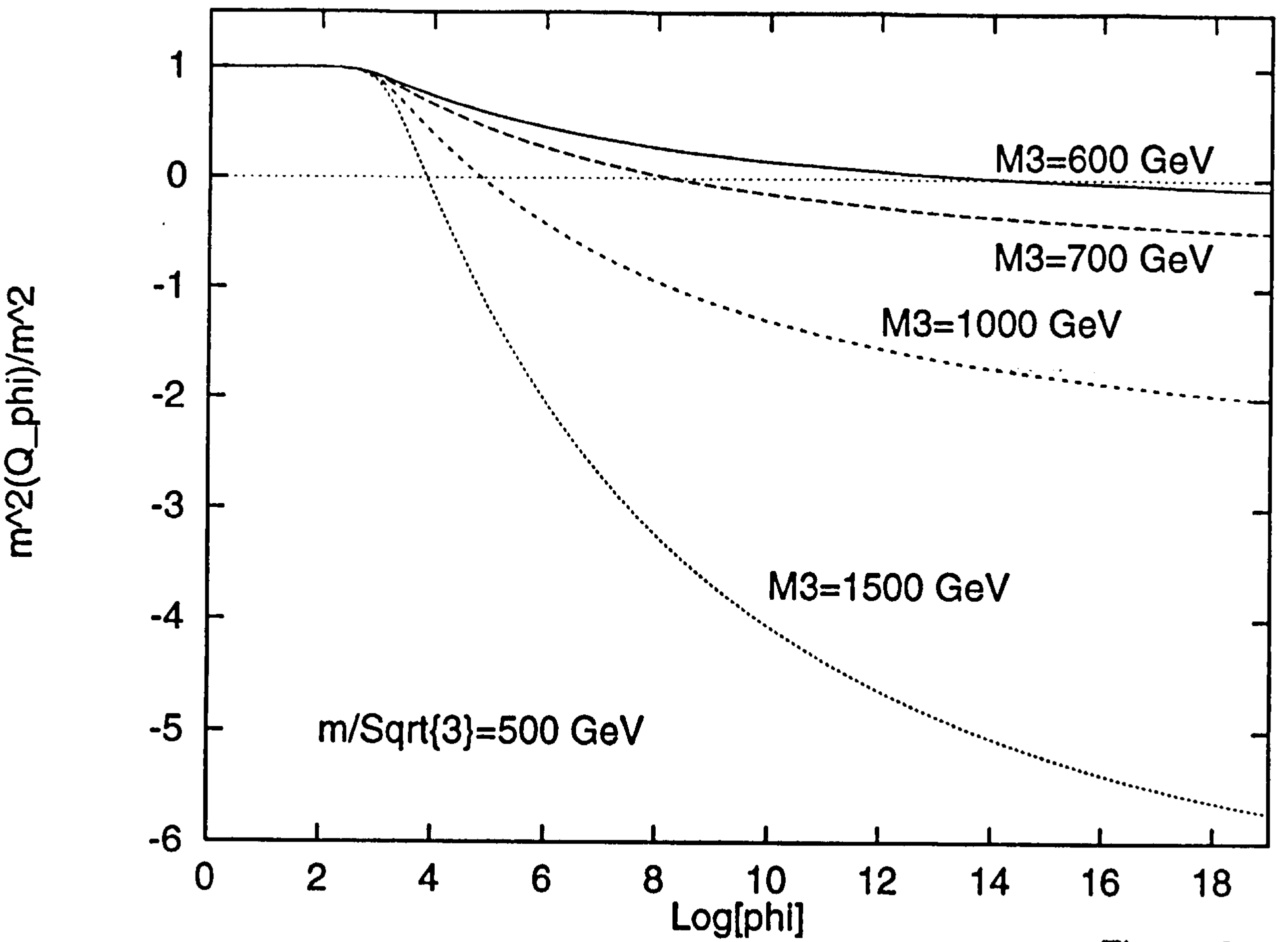
M3=600 GeV
M3=700 GeV
M3=1000 GeV
M3=1500 GeV
m/Sqrt{3}=500 GeV
m^2(Q_phi)/m^2
Log[phi]
1
0
-1
-2
-3
-4
-5
-6
0
2
4
6
8
10
12
14
16
18

Figure 2

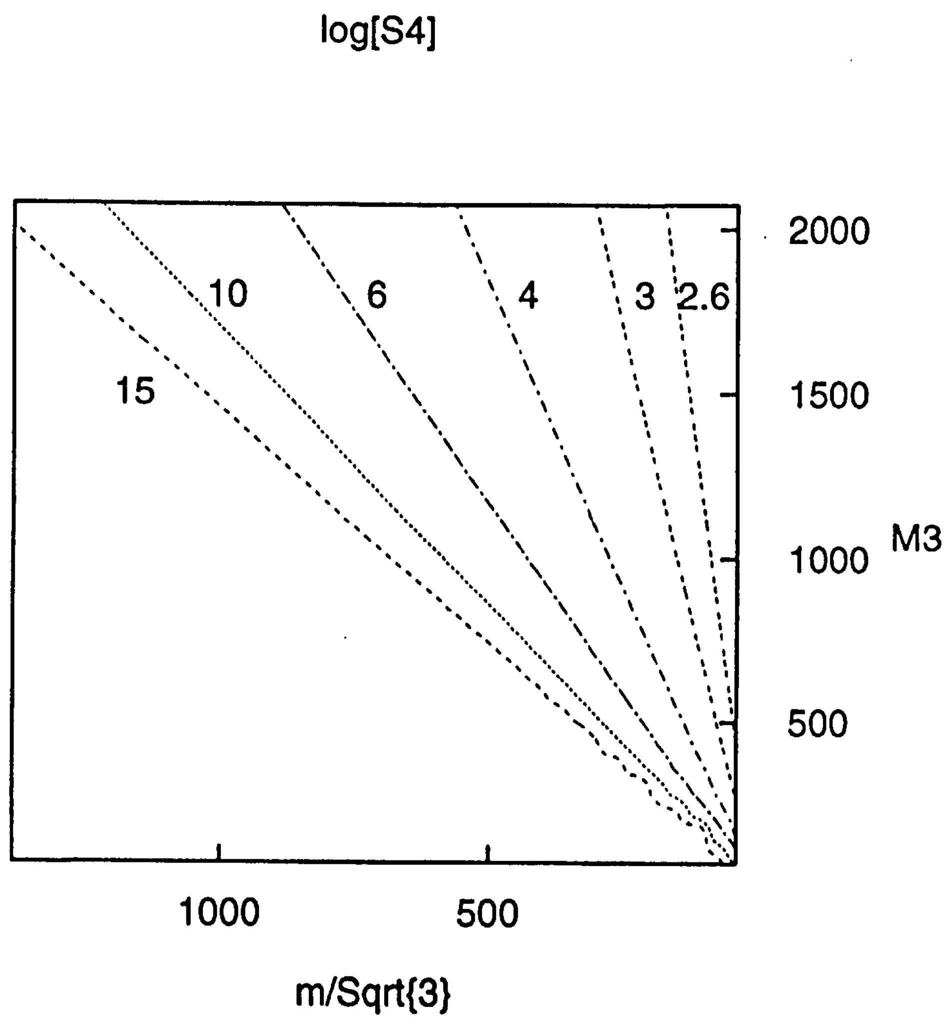

Figure 3

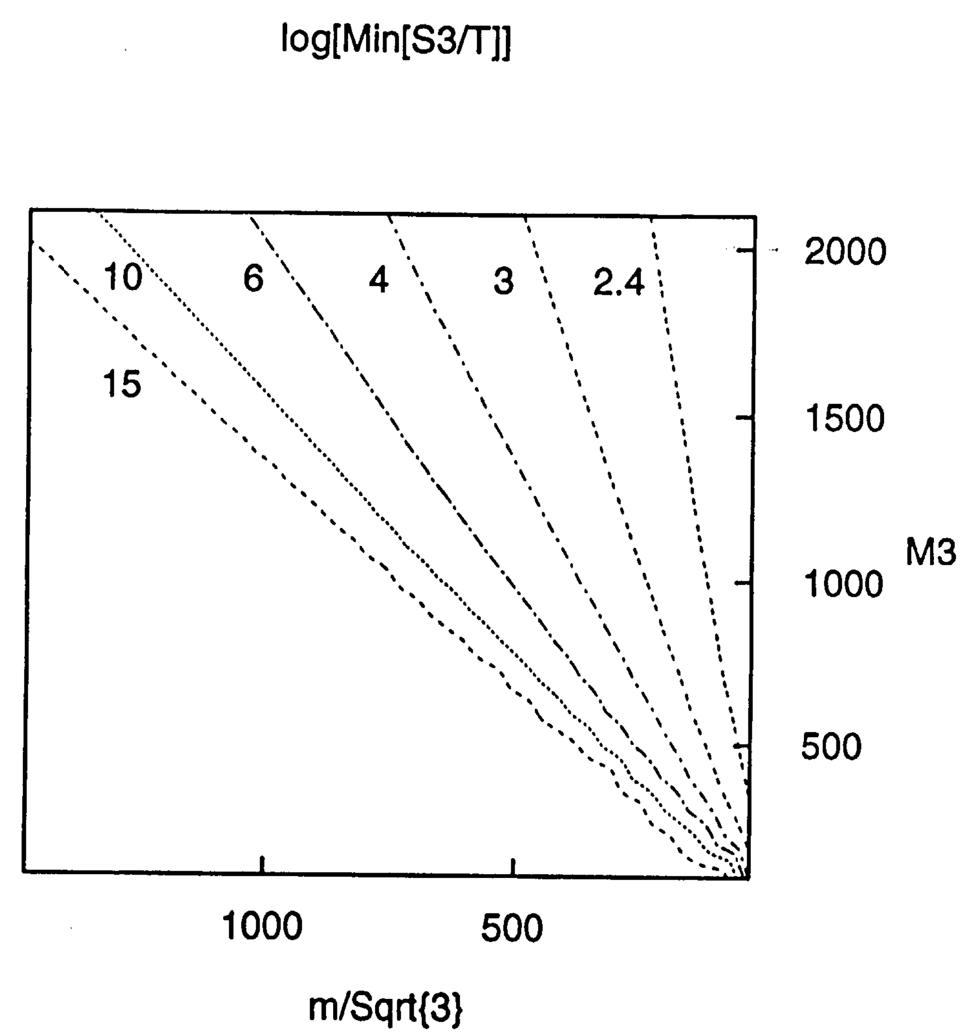

Figure 4