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WEAK INTERACTIONS IN SUPERNOVA CORES AND SATURATION OF NUCLEON SPIN FLUCTUATIONS

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ABSTRACT

Extrapolation of perturbative nucleon spin fluctuation rates seems to suggest a strong suppression of weak interactions in supernova cores. We derive a new sum rule for the dynamical spin-density structure function which relates the spin fluctuation rate to the average nuclear interaction energy. For a bremsstrahlung like structure function profile we show that instead of strongly decreasing, the neutrino scattering cross section is roughly density independent and axion emission rates increase somewhat slower than the lowest order emissivities towards the center of a hot supernova core.

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1 Introduction

The cooling history of a newly born neutron star in the center of a supernova (SN) is mainly determined by neutrino diffusion. Numerical simulations employing the lowest order neutrino interaction rates calculated within the Glashow-Salam-Weinberg theory predict a cooling time scale which agrees remarkably well with the neutrino signal observed from SN 1987 A [1]. The emission of novel weakly interacting particles like axions [2] could change the cooling time scale substantially which in turn allows to derive constraints on the properties of such particles [3].

Within linear response theory weak interaction rates with a medium of nonrelativistic nucleons are determined, apart from the weak phase space, by only two dynamical structure functions, one for the density and one for the nucleon spin-density [4, 5]. Some work has been devoted to their calculation but either the Landau theory of quasiparticles was applied assuming a "cold" nuclear medium [4, 6] or the authors focused on quasielastic scattering studying static structure functions [4, 7, 8]. Interactions of neutrinos and axions with a nonrelativistic nuclear medium are mainly governed by the local nucleon spin-density and its fluctuations. To lowest nontrivial order in the spin dependent nucleon-nucleon interactions causing these fluctuations, the relevant weak processes are of the nucleon bremsstrahlung type. Due to the Landau Pomeranchuk Migdal (LPM) effect [9] which accounts for multiple nucleon scattering the inelasticity of these processes depends on the nucleon spin flip rate. In addition, once this rate becomes considerably larger than the medium temperature T, the total weak interaction rates tend to be suppressed [10]. Since perturbative estimates for the nucleon spin flip rate can be as high as $\simeq 50T$ around nuclear densities, this could have profound implications for SN core physics [5, 10, 11, 12]. By dramatically reducing the predicted SN cooling time scale it would spoil the agreement between theory and the observed neutrino pulse from SN 1987 A [11]. On these phenomenological grounds it has been suggested that axial-vector neutrino scattering cross sections might be roughly density independent [12] instead of being suppressed at high densities by the LPM effect.

In this letter we derive a new sum rule for the dynamical spin-density structure function (SSF) which provides an independent theoretical argument supporting this conjecture. It also predicts that emissivities for weakly interacting particles should increase somewhat slower than the lowest order rates at high densities.

2 The Spin-Density Structure Function

In terms of the nucleon field operator in the nonrelativistic limit, $\psi(x)$, the spin-density operator is given by $\boldsymbol{\sigma}(x) = \frac{1}{2}\psi^\dagger(x)\boldsymbol{\tau}\psi(x)$ where $\boldsymbol{\tau}$ are the Pauli matrices. In the following we denote the momentum, coordinate, and spin operators for a single nucleon by $\mathbf{p}_i$, $\mathbf{r}_i$, and $\boldsymbol{\sigma}_i$, respectively, where $i = 1, \cdots, N_b$ runs over N_b nucleons. Then, for a normalization volume V, we can define the Fourier transform

$$\boldsymbol{\sigma}(t,\mathbf{k}) = \frac{1}{V}\int d^3\mathbf{r}e^{-i\mathbf{k}\cdot\mathbf{r}}\boldsymbol{\sigma}(t,\mathbf{r}) = \frac{1}{V}\sum_{i=1}^{N_b} e^{-i\mathbf{k}\cdot\mathbf{r}_i}\boldsymbol{\sigma}_i\,. \tag{1}$$

In terms of these operators and the baryon density n_b the SSF is defined as [5, 12]

$$S_\sigma(\omega,\mathbf{k}) = \frac{4}{3n_b}\int_{-\infty}^{+\infty} dte^{i\omega t}\left\langle \boldsymbol{\sigma}(t,\mathbf{k})\cdot\boldsymbol{\sigma}(0,-\mathbf{k})\right\rangle\,, \tag{2}$$

where $(\omega,\mathbf{k})$ is the four-momentum transfer to the medium. The expectation value $\langle\cdots\rangle$ in Eq. (2) is taken over a thermal ensemble.

The contribution of S_σ to the neutrino scattering rate (per final state density) from four momentum $(\omega_1,\mathbf{k}_1)$ to $(\omega_2,\mathbf{k}_2)$ can be written as $\frac{1}{4}G_{\rm F}^2C_A^2n_b(3-\cos\,\theta)S_\sigma(\omega_1-\omega_2,\mathbf{k}_1-\mathbf{k}_2)$ with $G_{\rm F}$ the Fermi constant, C_A the relevant axial-vector charge, and θ the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$. Similarly, the rate for pair production would read $\frac{1}{4}G_{\rm F}^2C_A^2n_b(3+\cos\,\theta)S_\sigma(-\omega_1-\omega_2,-\mathbf{k}_1-\mathbf{k}_2)$ [5]. The axion emission rate per volume, Q_a, is governed by the same structure function [in an isotropic medium $S_\sigma(\omega,\mathbf{k}) = S_\sigma(\omega,k)$ only depends on $k=|\mathbf{k}|$]:

$$Q_a = \frac{C_N^2 n_b}{(4\pi)^2 f_a^2}\int_0^\infty d\omega\,\omega^4 S_\sigma(-\omega,\omega)\,. \tag{3}$$

Here, f_a is the Peccei-Quinn scale and the numerical factor C_N depends on the specific axion model [2]. Neutrino opacities and axion emissivities are therfore mainly determined by the SSF at thermal energies $\omega\simeq k\lesssim T$.

Eq. (2) implies

$$\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}\,\omega S_\sigma(\omega,\mathbf{k}) = -\frac{4}{3n_b}\left\langle [H,\boldsymbol{\sigma}(0,\mathbf{k})]\cdot\boldsymbol{\sigma}(0,-\mathbf{k})\right\rangle\,, \tag{4}$$

where H is the Hamiltonian of the system of interacting nucleons for which we assume the following form:

$$H = H_0 + H_{\rm int} = \sum_{i=1}^{N_b}\frac{\mathbf{p}_i^2}{2M} + \frac{1}{2}\sum_{i\neq j}^{N_b} V(\mathbf{r}_{ij},\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)\,. \tag{5}$$

Here, $\mathbf{r}_{ij}=\mathbf{r}_i-\mathbf{r}_j$, M is the free nucleon mass, and $V(\mathbf{r}_{ij},\boldsymbol{\sigma}_i,\boldsymbol{\sigma}_j)$ is the spin dependent two nucleon interaction potential. For notational simplicity we restrict ourselves to only one nucleon species for the moment; the general case will be discussed further below.

For free nucleons one gets $\int_{-\infty}^{+\infty}(d\omega/2\pi)\,\omega S_\sigma(\omega,\mathbf{k}) = \mathbf{k}^2/2M$, in analogy to the well known f sum rule for the dynamical density structure function. In the latter case nucleon number conservation ensures that the f sum rule even holds in the presence of velocity independent interactions. In contrast, the f sum for the SSF is modified in the presence of spin dependent interactions since the nucleon spin is in general not conserved.

For one nucleon species the most general two nucleon interaction potential is of the form [13]

$$V(\mathbf{r},\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2) = U(r) + U_S(r)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 + U_T(r)\left(3\boldsymbol{\sigma}_1\cdot\hat{\mathbf{r}}\,\boldsymbol{\sigma}_2\cdot\hat{\mathbf{r}} - \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\right)\,, \tag{6}$$

where $\mathbf{r} = \mathbf{r}_{12}$, $r = |\mathbf{r}|$, and $\hat{\mathbf{r}} = \mathbf{r}/r$. We denote the spin dependent terms by $V_{ij}^S = U_S(r_{ij})\boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j$ (the "scalar force") and $V_{ij}^T = U_T(r_{ij})\,(3\boldsymbol{\sigma}_i \cdot \hat{\mathbf{r}}_{ij}\,\boldsymbol{\sigma}_j \cdot \hat{\mathbf{r}}_{ij} - \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)$ (the "tensor force"). In order to calculate the additional commutator in Eq. (4) from Eqs. (1), (5) and (6) we make use of the commutation relations $\left[\sigma_i^a, \sigma_j^b\right] = i\delta_{ij}\epsilon^{abc}\sigma_i^c$, where $i, j = 1, \cdots, N_b$ and ϵ^{abc} is the total antisymmetric tensor in the spatial indices a, b, c. After some algebra and using the symmetry properties of the Hamiltonian the modified sum rule reads

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \omega S_\sigma(\omega, \mathbf{k}) = \frac{\mathbf{k}^2}{2M} - \frac{4}{3N_b}\sum_{i \neq j}^{N_b} \left\langle V_{ij}^S + V_{ij}^T + \cos\, \mathbf{k} \cdot \mathbf{r}_{ij}\left(\frac{1}{2}V_{ij}^T - V_{ij}^S\right)\right\rangle . \qquad (7)$$

The kinetic nucleon recoil term is in general negligible compared to the V-dependent terms which govern the inelasticity of axial-vector interactions. If $V(\mathbf{r}, \boldsymbol{\sigma}_1, \boldsymbol{\sigma}_2) \gtrsim -\alpha/r^s$ with $\alpha > 0$ and $s < 2$ the eigenvalues of H are bounded from below and the r.h.s. of Eq. (7) is finite as long as $U_S(r)$ and $U_T(r)$ are integrable. This is the case for typical meson exchange potentials with hard core repulsion [13, 14, 15]. Assuming the three terms in Eq. (6) to be of similar size the r.h.s. of Eq. (7) is roughly proportional to the average interaction energy per nucleon W. At zero temperature and for SN core densities and compositions, $W \simeq 30\,\mathrm{MeV}$ corresponding to an average binding energy of about 10 MeV per nucleon. For $T \gtrsim 10\,\mathrm{MeV}$ nucleons are bound more weakly and W should be considerably smaller. We can therefore write

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \omega S_\sigma(\omega, \mathbf{k}) \simeq 4W \lesssim 100\,\mathrm{MeV}\,, \qquad (8)$$

where the inequality is a conservative bound reflecting our poor knowledge about the equation of state for hot nuclear matter. Since it involves bound state energies, Eq. (8) is a nonperturbative result and will play an important role for the high density behavior of weak interaction rates below.

The dependence on the momentum transfer $\mathbf{k}$ in Eq. (7) is expected to be only modest. In fact, for $k \lesssim T \lesssim 50\,\mathrm{MeV}$, we have $|\mathbf{k} \cdot \mathbf{r}| \ll 1$ within the range of the potential $r_s \simeq 1/m_\pi$ which is determined by the pion mass $m_\pi \simeq 140\,\mathrm{MeV}$. We can thus go to the long wavelength limit [4, 5, 7, 11, 12], $\mathbf{k} \to 0$, using $S_\sigma(\omega) \equiv S_\sigma(\omega, \mathbf{k} \to 0)$. Eq. (7) then simplifies to

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi}\, \omega S_\sigma(\omega) = -\frac{4}{N_b}\langle H_T\rangle\,, \qquad (9)$$

where $H_T = \frac{1}{2}\sum_{i \neq j}^{N_b} V_{ij}^T$.

First, note that the scalar force does not contribute to Eq. (9) because it conserves the total nucleon spin $\boldsymbol{\sigma}(0, \mathbf{k} \to 0)$ [see Eq. (1)]. Below nuclear densities the nucleon-nucleon (NN) interaction is dominated by one-pion exchange (OPE) leading to a tensor force. This contribution induces a spin orbit coupling and does therefore not conserve the total nucleon spin. Thus only the tensor force contributes to Eq. (4) in the long wavelength limit. This agrees with the lowest order bremsstrahlung calculation for $\mathbf{k} \to 0$ [15]. Finally, note that the r.h.s. of Eq. (9) is positive as it should be since the interaction induced correlations reduce $\langle H_T\rangle$ below the value for free nucleons, $\langle H_T\rangle = 0$.

An additional sum rule [5, 11, 12] can be obtained by integrating Eq. (2) and using Eq. (1) in the long wavelength limit:

$$\int_{-\infty}^{+\infty} \frac{d\omega}{2\pi} S_\sigma(\omega) = 1 + \frac{4}{3N_b} \left\langle \sum_{i \neq j}^{N_b} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right\rangle . \tag{10}$$

Note that for free nucleons Eqs. (9) and (10) yield $S_\sigma(\omega) = 2\pi\delta(\omega)$, whence only elastic scattering on the medium is possible in the absence of NN interactions.

3 Dilute Medium Limit

At low densities, i.e. for large average inter-nucleon spacing, the interaction energy W in Eq. (8) is much smaller than the kinetic terms from the free Hamiltonian. In case of the long wavelength limit, Eq. (9), we can therefore treat H_T as a small perturbation and write to lowest non-trivial order in H_T:

$$\langle H_T \rangle = \frac{2}{Z} \sum_n \exp\left(-E_n^0/T\right) \mathrm{Re}\left[{}_1\langle n| H_T |n\rangle_0\right] . \tag{11}$$

Here, E_n^0, $|n\rangle_0$ are the eigenvalues and eigenstates of the free Hamiltonian H_0, respectively, $|n\rangle_1$ are the eigenvectors of $H_0 + H_T$ to first order in H_T, and $Z = \sum_n \exp\left(-E_n^0/T\right)$ is the normalization factor. Assuming nondegenerate eigenstates for simplicity and applying standard first order perturbation theory for $|n\rangle_1$ we can express everything in terms of zeroth order quantities. Dropping the index 0 from now on, Eq. (11) reduces to the negative definite expression

$$\langle H_T \rangle = \frac{1}{Z} \sum_{n \neq m} \frac{e^{-E_n/T} - e^{-E_m/T}}{E_n - E_m} \left|(H_T)_{mn}\right|^2 , \tag{12}$$

where $(H_T)_{mn} = {}_0\langle m| H_T |n\rangle_0$. This matrix element is expected to vary in $E_m - E_n$ over a scale $\gtrsim 3m_\pi^2/M \simeq 50\,\mathrm{MeV}$ where m_π is a typical momentum scale in the NN interaction potential. Therefore, for $T \lesssim 50\,\mathrm{MeV}$ the thermal factor in Eq. (12) can be approximated by $\delta(E_m - E_n)$. Converting the sum over m into an integral over E_m Fermi's golden rule finally gives $W \simeq -\langle H_T \rangle / N_b = \Gamma_\sigma/(2\pi)$. Here, Γ_σ is the average perturbative NN scattering rate mediated by H_T which is a measure for the spin fluctuation rate. The spins fluctuate on a time scale given by the inverse energy scale of the tensor force which causes the spin fluctuations. At high densities we use $\Gamma_\sigma \equiv -2\pi\langle H_T \rangle / N_b$ as an effective spin flip rate.

4 Saturation of Spin Fluctuation Rates

We now use the sum rules Eqs. (9) and (10) to determine the qualitative form of $S_\sigma(\omega, \mathbf{k})$ in the long wavelength limit. To this end let us introduce the dimensionless quantity $\tilde{S}_\sigma(x) \equiv T S_\sigma(xT)$ with $x = \omega/T$ as in Ref. [11]. Due to the principle of detailed balance, $S_\sigma(\omega, \mathbf{k}) = S_\sigma(-\omega, -\mathbf{k}) e^{\omega/T}$, it is sufficient to specify $\tilde{S}_\sigma(x)$ for $x > 0$ only.

Introducing the dimensionless effective spin flip rate $\gamma_\sigma = \Gamma_\sigma/T$, we can write the sum rule Eq. (9) as

$$\int_0^{+\infty} \frac{dx}{2\pi} x\tilde{S}_\sigma(x)\left(1-e^{-x}\right) = \frac{2\gamma_\sigma}{\pi} \simeq \frac{4W}{T}, \tag{13}$$

where in a newly born neutron star γ_σ does not increase beyond a few.

Furthermore, since in a hot SN core the thermal energies are expected to be considerably higher than the interaction energy W, within a first approximation we can neglect spin correlations in the second sum rule Eq. (10) and write

$$\int_0^{+\infty} \frac{dx}{2\pi} \tilde{S}_\sigma(x)\left(1+e^{-x}\right) \simeq 1. \tag{14}$$

For the following discussion we consider the general case of an ensemble of neutrons and protons with fractional number densities Y_n and Y_p. Introducing the isospin operators $\boldsymbol{\tau}_i$ for nucleon i, $\boldsymbol{\sigma}_i$ in the definition of S_σ [see Eqs. (2) and (1)] has to be multiplied by $[1+(\tau_i)_3]\,C_{A,p}/2 + [1-(\tau_i)_3]\,C_{A,n}/2$. Here, $C_{A,p}$ and $C_{A,n}$ are the relevant proton and neutron axial-vector charges. Moreover, there will be additional terms proportional to $\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j$ in the interaction potential Eq. (6). However, this leaves our discussion qualitatively unchanged since the additional isospin operators appearing under the expectation values only lead to additional factors of order unity. If correlations among different nucleons are absent the r.h.s. of the sum rules Eqs. (13) and (14) get multiplied by $(Y_pC_{A,p}^2 + Y_nC_{A,n}^2)/(C_{A,p}^2 + C_{A,n}^2)$.

Parametrizing the high ω behavior of S_σ by $S_\sigma(\omega) \propto \omega^{-n}$, classical collisions would lead to $n = 2$. On the quantum mechanical level the deviation of $S_\sigma(\omega)$ from $2\pi\delta(\omega)$ is to lowest order in the strong interactions given by nucleon bremsstrahlung. Using a dipole like OPE potential without a hard core cutoff yields $n = 5/2$ and $n = 3/2$ in the case of one and two nucleon species, respectively [5, 15, 16, 17]. The non-existence of the f sum Eq. (13) in the latter case stems from the unphysical r^{-3} divergence of this potential at $r = 0$. Except for s waves this divergence indeed leads to an infinite $\langle H_T\rangle$. If one regularizes the potential by a hard core repulsion f sum integrability is restored.

This motivates the following representative ansatz:

$$\tilde{S}_\sigma(x) = \frac{a}{x^{5/2}+b} \quad \text{for } x > 0, \tag{15}$$

where a and b are positive constants. The sum rule Eq. (13) is sensitive to the high energy behavior and therefore mainly to a. In contrast, the sum rule Eq. (14) probes the "infrared" regime which is sensitive to b. Eq. (15) is of the form expected from nucleon bremsstrahlung where b accounts for the LPM effect.

We can now pick a number for a, determine the corresponding value of b numerically from Eq. (14) and compute the f sum Eq. (13). The result is plotted in Fig. 1 as a function of a. Most importantly, from the expected density and temperature dependence of W we expect the f sum to increase monotonically towards the SN core before saturating at a value of order unity. As a consequence, the thermally averaged axial-vector neutrino scattering cross section $\langle\sigma_A\rangle$ which dominates the neutrino opacity should roughly scale as T^2 being density independent as naively expected (see Fig. 1). Furthermore, the axion emission rate from Eq. (3) approximately scales as $n_b\Gamma_\sigma T^3$. The lowest order axion emissivities should therefore be multiplied by $\Gamma_\sigma/\Gamma'_\sigma$ whenever this ratio is smaller than 1. Here, Γ'_σ is the lowest order spin flip rate extrapolated from the dilute medium limit. For example, $\Gamma'_\sigma \simeq 32\,\mathrm{MeV}\rho_{14}T_{10}^{1/2}$ for the standard OPE calculations [15, 16, 17], where ρ_{14} is the mass density in $10^{14}\,\mathrm{gcm}^{-3}$ and $T_{10} = T/10\,\mathrm{MeV}$. A turn over in $\langle\sigma_A\rangle/T^2$ and $Q_a/(n_bT^3)$ typically only occurs at $\gamma_\sigma \gtrsim 10$ and is the less pronounced the stronger $S_\sigma(\omega)$ falls off at large ω. The absence of a decrease of these quantities at high density is therefore rather independent of uncertainties in the exact saturation value for γ_σ.

5 Summary

Neutrino opacities and axion emissivities are governed mainly by the SSF. We have derived a new sum rule for the SSF which corresponds to the f sum rule for the density structure function but depends on the nucleon spin flip interactions. Our treatment so far assumes absence of possible pion and kaon condensates. Employing an infrared regularized bremsstrahlung spectrum for the functional form of the SSF we have shown that the effective spin fluctuation rate Γ_σ must saturate somewhere below $\simeq 150\,\mathrm{MeV}$ which is within factors of a few of SN core temperatures. Neutrino scattering cross sections should therefore exhibit the naive T^2 scaling whereas axion emissivities should increase somewhat slower than the lowest order rates at high densities. There is no turnover of weak interaction rates towards the SN core. These results have an important impact on SN cooling simulations and their application to the derivation of axion mass bounds. They are also relevant for the rates for URCA processes and emission of right handed neutrinos.

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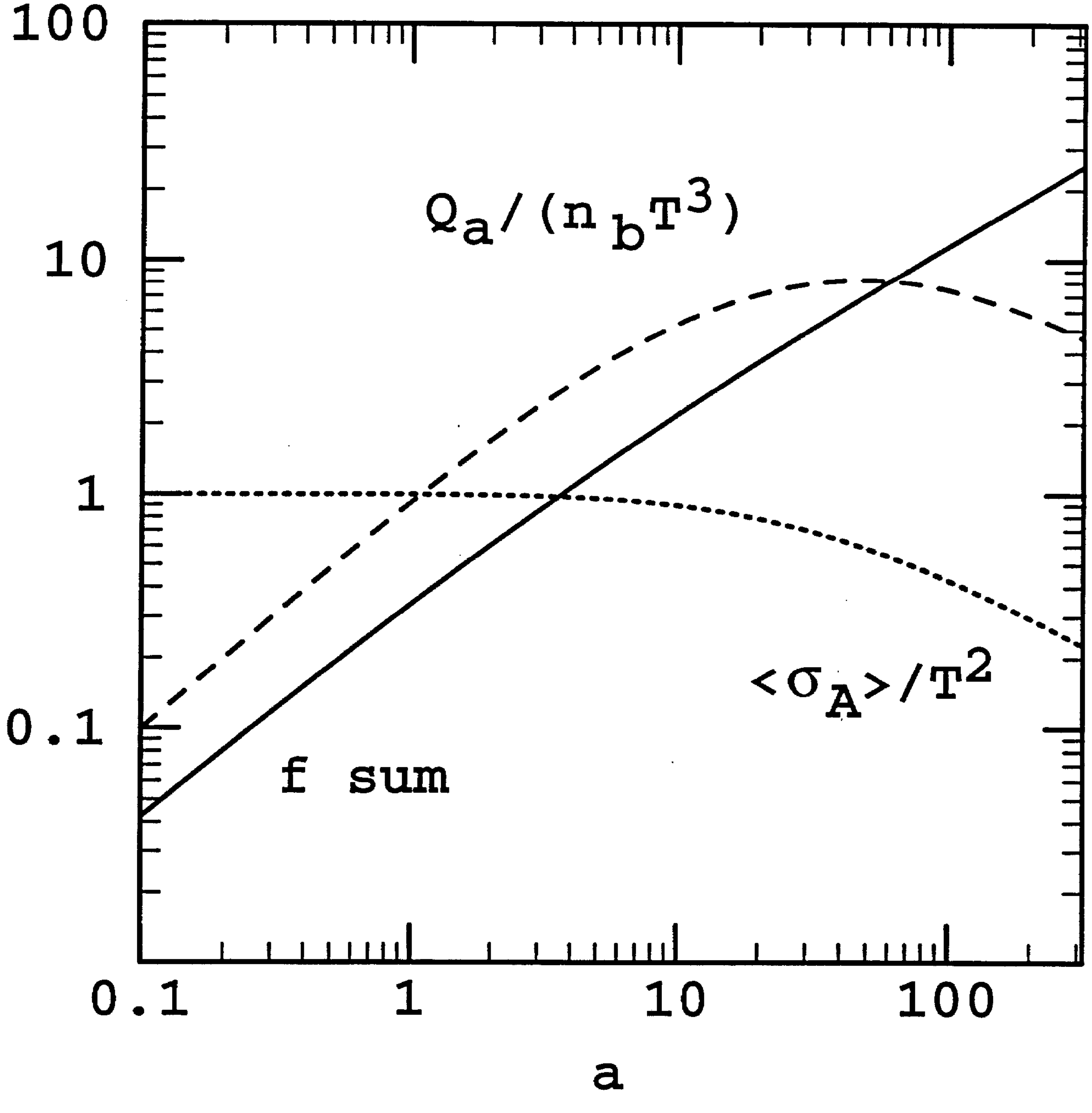
Figure Captions

Fig. 1. The f sum Eq. (13) characterized by γ_σ as a function of the parameter a (solid line). Also shown in arbitrary units are the axion emission rate per baryon Q_a/n_b (dashed line) and the thermal axial-vector neutrino scattering cross section $\langle\sigma_A\rangle$ (dotted line) normalized to a fixed temperature. The physical range is where the f sum is smaller than a few.



100
10
1
0.1
0.1
1
10
100
a
$Q_a/(n_bT^3)$
$\langle\sigma_A\rangle/T^2$
f sum