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Abstract

For a given source and lens pair, there is a thin on-axis tube volume behind the lens in which the radiation flux is greatly increased due to gravitational lensing. Any objects (such as dust grains) which pass through such a thin tube will experience strong bursts of radiation, i.e., Extreme Gravitational Lensing Events (EGLE). We study the physics and statistics of EGLE for the case in which finite source size is more important than shear. EGLE may have significant astrophysical effects.

1. Introduction

Gravitational lensing has been studied almost exclusively in the context of direct observation of lensed sources from the Earth or spacecraft (Schneider et al. 1992). However, every astrophysical object receives the light from sources lensed by intervening massive objects. The most powerful gravitational lensing events occur when the source, lens, and a target object are nearly on-axis. Such events are extreme in magnification, and rare in occurrence for any given target. By considering arbitrarily located targets, we can study the statistics of Extreme Gravitational Lensing Events (EGLE). EGLE can have a significant effect on certain objects, especially fragile components of the interstella medium, such as molecules or dust grains.

For a pair of source and lens, an EGLE occurs when a moving target object crosses the source-lens line behind the lens. The maximum magnification of the source seen by the target can be extremely large, limited only by the source size and the shear on the lens. As the target moves away from the line connecting the source and the lens, the magnification of the source decreases. The duration of an EGLE depends on the velocity of the target and the size of the high flux region. A slowly moving target in the neighborhood of a pair of small-size source and slightly-sheared lens can experience a strong burst of radiation due to the lensing of the source, which may be sufficient to affect the target's properties.

If the target moves a distance d away from the line connecting the source and the lens, it is equivalent to the source moving an angular distance of y from the optical axis (the line connecting the lens and the target). Measuring y in units of the angular Einstein radius, we have

$$y \simeq \left(\frac{D_{\rm ds}}{D_{\rm d}}\right) \frac{d}{D_{\rm s}\theta_{\rm E}}, \tag{1}$$

where $D_{\rm ds}$, $D_{\rm s}$, and $D_{\rm d}$ are angular diameter distances between the lens and source, target and source, target and lens respectively. $\theta_{\rm E} = \sqrt{2R_{\rm S}D_{\rm ds}/(D_{\rm d}D_{\rm s})}$ is the angular Einstein radius. $R_\mathrm{S} = 2GM$ is the Schwartzschild radius of the lens with mass M. Eq.(1) is valid for sufficiently distant lens or source.

We can write θ_E as

$$\theta_\mathrm{E} = 10^{-6} \times \sqrt{\left(\frac{M}{5\times 10^6 M_\odot}\right)\left(\frac{1\,\mathrm{Mpc}}{D_\mathrm{d}}\right)\frac{D_\mathrm{ds}}{D_\mathrm{s}}}. \tag{2}$$

The dimensionless radius of a source with physical radius ρ is defined as

$$R \equiv \frac{\rho}{D_\mathrm{s}\theta_\mathrm{E}} = \left(\frac{\rho}{3.09\times 10^{18}\,\mathrm{cm}}\right)\left(\frac{10^{-6}}{\theta_\mathrm{E}}\right)\left(\frac{1\,\mathrm{Mpc}}{D_\mathrm{s}}\right). \tag{3}$$

For a given pair of lens and source, the shear γ on the lens due to other lensing objects near the line-of-sight is the same order of magnitude as the optical depth for microlensing, τ, the probability that the source is lensed. We find

$$\gamma = \sqrt{2}\,\zeta\left(\frac{3}{2}\right)\tau \simeq 3.7\,\tau. \tag{4}$$

$\zeta(x)$ is the Riemann zeta function. Not surprisingly, the statistics of EGLE is much more complicated for sheared lenses than for isolated lenses ($\gamma = 0$). Since $\tau = \Omega_\mathrm{L} z_\mathrm{Q}^2/4$ (Ω_L is the critical density fraction in lenses and z_Q is the redshift of small sources), the shear γ is probably small in the low redshift Universe. (Turner 1980, Turner et al. 1984) For $\gamma \ll 1$, the caustic is a astroid shaped curve with four cusps. For a finite source with dimensionless radius R which crosses the optical axis, Figure 1 shows the typical lightcurves for $\gamma = 0$ (solid line), $R/2$ (dotted line), R (short dashed line), $5R$ (long dashed line), and $10R$ (dot-dashed line) respectively; Figure 2 shows the corresponding cross-sections of magnification. Clearly, shear is not important for $\gamma < R \ll 1$.

In this paper, we study the statistics of EGLE for sources with small dimensionless radius R and isolated lenses ($\gamma = 0$). We generally follow the notation and conventions of Schneider et al. (1992). We will discuss the statistics of EGLE for lenses with small shear γ elsewhere (Wang and Turner, in preparation).

2. Basic statistics

In this section, we discuss mean and rms magnifications, as well as integrated excess flux (IEF) seen by a target, for finite sources and isolated lenses (i.e. no shear).

The magnification of a source with dimensionless radius R is given by

$$\mu_{\rm e}(y,R) \simeq \begin{cases} \frac{1}{R}\,\zeta\left(\frac{y}{R}\right) & \text{for } y \lesssim 5R \\ \\ \mu_{\rm p}(y) = \frac{(y^2+2)}{y\sqrt{y^2+4}} & \text{for } y \gtrsim 5R \end{cases} \tag{5}$$

where

$$\zeta(w) = \frac{2}{\pi}\int_0^1 {\rm d}x\, x \int_0^\pi \frac{{\rm d}\phi}{\sqrt{w^2+x^2+2wx\cos\phi}}. \tag{6}$$

$\zeta(0)=2$, $\zeta(w>0)$ can be easily integrated numerically.

Using Eq.(1), we can define the half-widths of observables in the source plane. Let us define the half-width of the light curve seen by the target to be $y^{\rm w}_{\rm HM}$, the source's distance to the optical axis when its magnification goes down to 1/2 its maximum $\mu_{\rm max}$. It's straightforward to find:

$$\mu_{\rm max} = \frac{2}{R}, \qquad y^{\rm w}_{\rm HM} \equiv y(\mu = 0.5\,\mu_{\rm max}) \simeq 1.145R. \tag{7}$$

The mean, root mean square, and root variance maginifications are given by

$$\langle A\rangle\,(y \le y^{\rm w}_{\rm HM}) \simeq \frac{1.5583}{R}, \qquad \sqrt{\langle A^2\rangle}\,(y \le y^{\rm w}_{\rm HM}) \simeq \frac{1.588}{R},$$
$$\sqrt{\langle (A-\langle A\rangle)^2\rangle}\,(y \le y^{\rm w}_{\rm HM}) \simeq \frac{0.3}{R}. \tag{8}$$

We are also interested in the integrated excess flux F seen by a moving target. Let us define $t=0$ to be the moment when the moving target crosses the line connecting the source and lens. For a target moving at constant velocity v, its distance from the line connecting the source and lens is $d = vt$. Using Eq.(1), we have

$$\begin{aligned} F(t) &\equiv \int_0^t \mathrm{d}t \; [\mu_{\rm e}(t,R) - 1] \\ &= \frac{D_{\rm d}}{D_{\rm ds}} \frac{D_{\rm s}\theta_{\rm E}}{v} \int_0^y \mathrm{d}y \; [\mu_{\rm e}(y,R) - 1] \equiv \frac{D_{\rm d}}{D_{\rm ds}} \frac{D_{\rm s}\theta_{\rm E}}{v} \overline{F}(y). \end{aligned} \tag{9}$$

Using Eqs.(5), we find

$$\overline{F}_{\rm total} \equiv \int_0^\infty \mathrm{d}y \; [\mu_{\rm e}(y,R) - 1] \simeq 1.27 - \ln R, \tag{10}$$

for $R \lesssim 0.05$. We define the half-width of the integrated excess flux (IEF) to be $y^{\rm w}_{\rm IEF}$, the source's distance from the optical axis when the IEF seen by the target is half the total IEF, i.e., $\overline{F}(y^{\rm w}_{\rm IEF}) = \overline{F}_{\rm total}/2$. We find

$$y^{\rm w}_{\rm IEF}(R) \simeq 0.287\sqrt{R}. \tag{11}$$

The corresponding mean, root mean square, and root variance maginifications are

$$\begin{aligned} &\langle A \rangle \, (y \leq y^{\rm w}_{\rm IEF}) \simeq \frac{6.97}{\sqrt{R}}, \qquad \sqrt{\langle A^2 \rangle} \; (y \leq y^{\rm w}_{\rm IEF}) \simeq \sqrt{\frac{24.28}{R} \, \ln\left(\frac{1.466}{\sqrt{R}}\right)}, \\ &\sqrt{\langle (A - \langle A \rangle)^2 \rangle} \; (y \leq y^{\rm w}_{\rm IEF}) \simeq \frac{4.93}{\sqrt{R}} \sqrt{\ln\left(\frac{0.2}{\sqrt{R}}\right)}. \end{aligned} \tag{12}$$

It is useful to consider only the $y < 1$ regime, the half-width of microlensing events. The corresponding mean, root mean square, and root variance maginifications are

$$\begin{aligned} &\langle A \rangle \, (y \leq 1) \simeq 2.236 - 0.06\, R, \qquad \sqrt{\langle A^2 \rangle} \; (y \leq 1) \simeq \sqrt{2 \, \ln\left(\frac{7.531}{R}\right)}, \\ &\sqrt{\langle (A - \langle A \rangle)^2 \rangle} \; (y \leq 1) \simeq \sqrt{2 \, \ln\left(\frac{0.6182}{R}\right)}. \end{aligned} \tag{13}$$

Generally, for $y \gtrsim 5R$, we have

$$\begin{aligned} &\langle A \rangle \, (\leq y) \simeq \frac{\sqrt{y^2+4}}{y} - \frac{0.06R}{y^2}, \\ &\langle A^2 \rangle \; (\leq y) \simeq 1 + \frac{2}{y^2} \left[\ln\left(\frac{10.2135}{R}\right) - \ln\left(\frac{\sqrt{y^2+4}}{y}\right) \right]. \end{aligned} \tag{14}$$

For $y \to \infty$, $\langle A \rangle$=1, as required by flux conservation, and $\langle A^2 \rangle$=1. However, it is not physical to average over y from 0 to infinity. For a given source, there is a natural cut-off $y_{\max}$ which is given by the source's distance to the nearest lens, i.e., $y_{\max} \sim 1/\sqrt{\tau}$, where τ is the optical depth.

3. EGLE volume statistics for a point source

Let us consider a point source S with luminosity $L_{\rm S}$, being lensed by a lens L with Schwartzschild radius $R_{\rm S}$ (mass M) at a distance $D_{\rm ds}$. Let Q denote the magnification or flux of the source seen by the target. In a narrow tube-shaped volume $V_{\rm SL}$ behind the lens, which extends from the lens and tapers off to infinity, Q exceeds some value q. The cross section of the tube is

$$\sigma(q) = \pi d^2 = \pi \left[\left(\frac{D_{\rm d}}{D_{\rm ds}} \right)^2 D_{\rm s}^2 \theta_{\rm E}^2 \right] y^2(q), \tag{15}$$

where we have used Eq.(1). $y(q)$ is the source's dimensionless distance from the optical axis when Q equals q. Hence

$$V_{\rm SL}(q) = \int_0^{D_{\rm d}(q)} {\rm d}D_{\rm d}\, \sigma(q). \tag{16}$$

Summing over S gives the total volume $V_{\rm L}$ in which Q exceeds q for a given lens L; further summing over L gives the total volume $V_{\rm tot}(> q)$. We use $D_{\rm s} = D_{\rm d} + D_{\rm ds}$ for simplicity in our calculations.

Let us consider the volume $V_{\rm SL}(f)$ behind the lens in which the flux from the source exceeds f. In the absence of magnification, the flux from the source is $f_0 = L_{\rm S}/(4\pi D_{\rm s}^2)$. The magnified flux $f = \mu f_0$. Since we are only interested in high magnification events, we use $y(f) \simeq 1/\mu = f_0/f$ in calculating the cross-section $\sigma(f)$ of $V_{\rm SL}(f)$. We find

$$\sigma(f, D_{\rm d}) = \frac{2\pi R_{\rm S}}{D_{\rm ds}} \cdot \frac{D_{\rm d}}{D_{\rm s}^3} \left(\frac{L_{\rm S}}{4\pi f} \right)^2. \tag{17}$$

$\sigma(f, D_{\rm d})$ is maximum at $D_{\rm d} = D_{\rm ds}/2$. We find

$$\sigma_{\rm p}^{\rm max}(f) = \frac{8\pi R_{\rm S}}{27 D_{\rm ds}^3}\left(\frac{L_{\rm S}}{4\pi f}\right)^2. \tag{18}$$

We can define $\sigma_{\rm p}^{\rm max}(f)$ to be the characteristic cross section of the high-flux ($> f$) tube. The lens which is closest to the source has the thicknest high-flux tube behind it.

Using Eq.(17), we obtain

$$V_{\rm SL}(f) \simeq \frac{\pi R_{\rm S}}{D_{\rm ds}^2}\left(\frac{L_{\rm S}}{4\pi f}\right)^2, \qquad V_{\rm L}(f) = 4\pi^2 n_{\rm S} R_{\rm S} D_{\rm c}\left(\frac{L_{\rm S}}{4\pi f}\right)^2, \tag{19}$$

where $n_{\rm S}$ is the number density of sources, and $D_{\rm c}$ is the size of the system, in effect the maximum distance between a lens and a source.

Let $\mathcal{F}_{\rm L}(f)$ be the volume fraction of space in which the flux from the source exceeds f due to gravitational lensing. $\mathcal{F}_{\rm L}(f)$ should be compared with the volume fraction of space $\mathcal{F}_{\rm S}(f)$ in which the flux from the source exceeds f due to being close to the source. We have

$$\begin{aligned} &\mathcal{F}_{\rm L}(f) = n_{\rm L} V_{\rm L}(f) = \frac{3}{2}\tau N_{\rm S}\left(\frac{f}{f_{\rm min}}\right)^{-2}, \qquad \mathcal{F}_{\rm S}(f) = N_{\rm S}\left(\frac{f}{f_{\rm min}}\right)^{-3/2}, \\ &\frac{\mathcal{F}_{\rm L}(f)}{\mathcal{F}_{\rm S}(f)} = \frac{3}{2}\tau\left(\frac{f}{f_{\rm min}}\right)^{-1/2}, \end{aligned} \tag{20}$$

where τ is the optical depth, $N_{\rm S}$ is the total number of sources, and $f_{\rm min} = L_{\rm S}/(4\pi D_{\rm c}^2)$. Note that the volume weighted rms flux due to lensing diverges logarithmically.

The average flux from the general population of sources is $n_{\rm S} D_{\rm c} L_{\rm S}$. Let us define relative flux

$$f' = \frac{f}{n_{\rm S} D_{\rm c} L_{\rm S}}. \tag{21}$$

Let $\mathcal{F}_{\rm L}(f')$ and $\mathcal{F}_{\rm S}(f')$ be the volume fractions of space in which the relative flux from the source exceeds f' due to lensing and due to being close to a source respectively. We have

$$\begin{aligned} &\mathcal{F}_{\rm L}(f', \rho = 0) = n_{\rm L} V_{\rm L}(f', \rho = 0) = \frac{\tau}{6 N_{\rm S} f'^2}, \\ &\frac{\mathcal{F}_{\rm L}(f', \rho = 0)}{\mathcal{F}_{\rm S}(f')} = \frac{\tau}{2}\left(\frac{3}{N_{\rm S}}\right)^{1/2} f'^{-1/2}. \end{aligned} \tag{22}$$

4. EGLE volume statistics for a finite source

Now let us consider a source S with physical radius ρ and luminosity $L_{\rm S}$, being lensed by a lens L with Schwartzschild radius $R_{\rm S}$ (mass M) at a distance $D_{\rm ds}$. The tube-shaped volume $V_{\rm SL}(f,\rho)$ behind the lens in which the flux from the source exceeds f has finite length $D_{\rm d}^{\rm m}(f,\rho)$, because of the finite size of the source.

For a finite source with dimensionless radius R, $\mu_{\rm max}=2/R$. Let us define a parameter $q_0(f)$ which measures the maximum magnification of the source relative to the flux f,

$$q_0 \equiv \frac{8R_{\rm S}D_{\rm c}}{\rho^2}\left(\frac{L_{\rm S}}{4\pi D_{\rm c}^2 f}\right)^2. \tag{23}$$

The tube volume $V_{\rm L}(f,\rho)$ has the cross-section $\sigma(f,\rho,D_{\rm d})$ which vanishes at $D_{\rm d}=0$, $D_{\rm d}^{\rm m}$. To calculate the cross-section $\sigma(f,\rho)$, we need to know $y(f)$ (see Eq.(15)), which can be found by inverting $\mu(y)=f/f_0$ numerically. $\sigma(f,\rho)$ can be written as

$$\sigma(f,\rho,D_{\rm d}) = \pi\rho^2\times d^2/\rho^2 = \pi\rho^2\,\overline{\sigma}(q_0,D_{\rm d}), \tag{24}$$

for given $D_{\rm ds}$. Figure 3 shows the cross-section $\sigma(f,\rho,D_{\rm d})$ with $q_0(f)=4$, for $D_{\rm ds}=0.2\,D_{\rm c}$ (solid line), $0.5\,D_{\rm c}$ (long dashed line). The lens which is closest to the source has the thickest tube of high flux behind it, as in the point source case.

To simplify the calculation for the volume fractions, let us make the approximation

$$\mu_{\rm e}(y,R) \simeq \begin{cases} \mu_{\rm p}(y) & \text{for } \mu<\mu_{\rm max} \\ \\ \mu_{\rm max} & \text{elsewhere} \end{cases} \tag{25}$$

where $\mu_{\rm p}(y)$ is the point source magnification, and $\mu_{\rm max}=2/R$. Eq.(25) is reasonably good for $\mu \lesssim 1/R$. Figure 3 shows the approximate cross-sections obtained by using Eq.(25) for $D_{\rm ds}=0.2\,D_{\rm c}$ (short dashed line), $0.5\,D_{\rm c}$ (dot-dashed line). The dotted lines indicate $D_{\rm d}^{\rm m}$.

This approximate cross-section always *under-estimates* the true cross-section; the difference increases with decreasing $q_0(f)$ (large ρ or f), but it is negligible for our purpose.

Now let us derive the length of the tube-volume $V_{\rm SL}(f,\rho)$. Note that $f=\mu f_0 \le \mu_{\rm max} f_0$. Let $f=\mu_{\rm max} f_0$ at $D_{\rm d}(f)=D_{\rm d}^{\rm m}(f)$, i.e., the flux is equal to f on the line SL connecting the source and lens. For given $D_{\rm d}$, the flux decreases away from line SL, hence the volume in which the flux exceeds f converges to a point at $D_{\rm d}(f)=D_{\rm d}^{\rm m}(f)$. For $D_{\rm d}>D_{\rm d}^{\rm m}$, the volume in which the flux exceeds f is zero. For a given pair of source and lens, $D_{\rm d}^{\rm m}(f)$ gives the length of the tube volume in which the flux exceeds f. To find $D_{\rm d}^{\rm m}(f)$, we write $f=\mu_{\rm max} f_0$ as

$$\frac{D_{\rm d}^{\rm m}}{D_{\rm c}}\left(\frac{D_{\rm d}^{\rm m}}{D_{\rm c}}+\frac{D_{\rm ds}}{D_{\rm c}}\right)^3=\frac{8R_{\rm S}D_{\rm ds}}{\rho^2}\left(\frac{L_{\rm S}}{4\pi D_{\rm c}^2 f}\right)^2=q_0\left(\frac{D_{\rm ds}}{D_{\rm c}}\right). \tag{26}$$

The above equation can be solved analytically for $D_{\rm d}^{\rm m}(f)$. Let us define

$$x\equiv\frac{D_{\rm ds}}{D_{\rm c}}, \qquad\qquad \omega\equiv\frac{x}{q_0^{1/3}}. \tag{27}$$

We find

$$\frac{D_{\rm d}^{\rm m}(f)}{D_{\rm ds}}=\frac{1}{2}\left\{\frac{[\omega\, u(\omega)]^{3/2}}{2}+\sqrt{\frac{1}{2}\left[1+[\omega\, u(\omega)]^{-3/2}\right]+u(\omega)}-\frac{3}{2}\right\}\equiv g(\omega), \tag{28}$$

where

$$u(\omega)=\frac{3}{4}\left[b(\omega)+\frac{1}{b(\omega)}+1\right]^{-1}, \qquad b(\omega)=\frac{3\omega}{4}\left[\frac{1}{2}+\sqrt{\frac{1}{4}+\left(\frac{4}{3\omega}\right)^3}\,\right]^{2/3}. \tag{29}$$

$g(\omega)=D_{\rm d}^{\rm m}(f)/D_{\rm ds}$ is shown in Figure 4. Given the separation between the source and the lens, $x=D_{\rm ds}/D_{\rm c}$, the length of the tube volume behind the lens in which the flux exceeds f is given by $D_{\rm d}^{\rm m}(f)/D_{\rm c}=x\,g(\omega)$, where $\omega=xq_0^{-1/3}$. The tube length is of order $D_{\rm c}$ for q_0 of order 1.

The function $g(\omega)$ has the following asymptotic behavior:

$$g(\omega)=\begin{cases}\omega^{-3} & \text{for } \omega\gg 1\\ \omega^{-3/4} & \text{for } \omega\ll 1\end{cases} \tag{30}$$

Thus we have

$$\frac{D_{\rm d}^{\rm m}(f)}{D_{\rm c}} = \begin{cases} q_0(f)/x^2 & \text{for } q_0^{1/3} \ll x \\ [q_0(f)\, x]^{1/4} & \text{for } q_0^{1/3} \gg x \end{cases} \tag{31}$$

Substitution of Eq.(28) into Eq.(16) gives $V_{\rm SL}(f,\rho)$. Using Eq.(25) and $\mu_{\rm p}(y) \simeq 1/y$ (for high magnification events) in computing $\sigma(f,\rho)$, we find

$$V_{\rm L}(f,\rho) = 4\pi^2 n_{\rm S} R_{\rm S} D_{\rm c} \left(\frac{L_{\rm S}}{4\pi f}\right)^2 I(q_0), \tag{32}$$

where

$$I(q_0) = q_0^{1/3} \int_0^{q_0^{-1/3}} \frac{{\rm d}\omega}{\left[1+1/g(\omega)\right]^2}. \tag{33}$$

For high flux, $q_0(f) \ll 1$, $I(q_0) = 0.3583\, q_0^{1/3}$. Note that $I(q_0) = V_{\rm L}(f,\rho)/V_{\rm L}(f,\rho=0)$. In the point source limit, $q_0 \gg 1$, $I(q_0) = 1$. We show $I(q_0)$ in Figure 5.

Given $I(q_0) = A q_0^{\alpha}$, with $\mathcal{F}_{\rm L}(f)$ and $\mathcal{F}_{\rm S}(f)$ denoting the volume fractions of space in which the flux from the source exceeds f due to lensing and due to being close to the source respectively, we find

$$\begin{aligned} \mathcal{F}_{\rm L}(f,\rho) &= \frac{A}{4\cdot 2^{\alpha}\pi^{2\alpha}}\, n_{\rm S} n_{\rm L} R_{\rm S}^{1+\alpha} L_{\rm S}^{2+2\alpha} f^{-(2+2\alpha)} D_{\rm c}^{1-3\alpha} \rho^{-2\alpha}, \\ \frac{\mathcal{F}_{\rm L}(f,\rho)}{\mathcal{F}_{\rm S}(f)} &= \frac{3\pi^{1/2-2\alpha} A}{2^{1+\alpha}}\, n_{\rm L} R_{\rm S}^{1+\alpha} L_{\rm S}^{1/2+2\alpha} f^{-(1/2+2\alpha)} D_{\rm c}^{1-3\alpha} \rho^{-2\alpha}. \end{aligned} \tag{34}$$

5. Possible astrophysical effects

For a population of sources (with number density $n_{\rm S}$) lensed by a population of lenses (with number density $n_{\rm L}$), the physical picture for EGLE is a complex network of thin high-flux tubes, at each knot sits a lens, and each tube line points away from a source. In other words, a given lens has one high-flux tube coming out of it because of each source, and a given source induces one high-flux tube behind each lens.

To roughly survey possible astrophysical effects of EGLE, we construct tables of possible source and lens populations. Table 1 lists a few types of small sources with high luminosity. Note that the space density n_s associated with transient sources such as γ-ray bursts and supernovae includes the finite lifetime factor; in other words, it is the density of sources shining at a particular moment. Table 2 gives two possible lens populations.

In astrophysical units, we write

$$\log(q_0) = -5.62 + 0.8\, m_{\rm bol} + 2\,\log\left(\frac{L_{\rm S}}{L_\odot}\right) + \log\left(\frac{M_{\rm L}}{M_\odot}\right) - 2\,\log\left(\frac{\rho}{R_\odot}\right) -3\,\log\left(\frac{D_{\rm c}}{1\,{\rm kpc}}\right) \tag{35}$$

where the minimum flux f is measured by $m_{\rm bol}$. For Galactic supernovae lensed by stars, $\log(q_0) = 5.38 + 0.8\, m_{\rm bol}$. For QSO's (X-ray) lensed by giant black holes, $\log(q_0) = 4 + 0.8\, m_{\rm bol}$. For gamma-ray bursts, $\log(q_0) = 26 + 0.8\, m_{\rm bol}$ (lensed by stars), and $\log(q_0) = 34 + 0.8\, m_{\rm bol}$ (lensed by giant black holes). q_0 measures the maximum magnification of the source relative to the flux f [see Eq.(23)]. Fig.6 shows q_0 versus $m_{\rm bol}$ for lensing by stars, the sources are gamma-ray bursts (solid line), QSO (X-ray) (dotted line), QSO (UV-opt) (short dashed line), Galactic supernovae (long dashed line), neutron stars (dot-short dashed line), hot O stars (dot-long dashed line), and hot B stars (short dash-long dashed line) respectively.

First, let us consider the physical dimensions of the tube volume behind the lens in which the flux exceeds f (measured by $m_{\rm bol}$). Using Eq.(28), we find

$$\log\left(\frac{D_{\rm d}^{\rm m}}{\rm cm}\right) = 21.49 + \log\left(\frac{D_{\rm ds}}{\rm kpc}\right) + \log g(\omega), \tag{36}$$

where $\omega = (D_{\rm ds}/D_{\rm c})q_0^{-1/3}$. $g(\omega)$ is shown in Fig.4. For $q_0 \ll 1$, $\omega \gg 1$ at a given $D_{\rm ds}$, $g(\omega) \simeq \omega^{-3}$; i.e., the length of the tube volume decreases sharply for small q_0. In Fig.7(a), we show $D_{\rm d}^{\rm m}$ versus $m_{\rm bol}$ for the same lenses and sources as in Fig.6 (with the same line types), for $D_{\rm ds} = D_{\rm c}/2$.

Since the cross-section of the tube volume for a finite source does not deviate significantly from that of a point source (see Fig.3), let us define the characteristic thickness a of the tube volume to be the maximum thickness (along the optical axis) for a point source (see Eq.(17)). We find

$$\log\left(\frac{a}{\mathrm{cm}}\right) = 7.38 + 0.4\, m_{\mathrm{bol}} + \log\left(\frac{L_{\mathrm{S}}}{L_{\odot}}\right) + \frac{1}{2}\log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) - \frac{3}{2}\log\left(\frac{D_{\mathrm{ds}}}{\mathrm{kpc}}\right). \tag{37}$$

In Fig.7(b), we show a versus m_{bol} for the same lenses and sources as in Fig.6 (with the same line types), for $D_{\mathrm{ds}} = D_{\mathrm{c}}/2$.

Next, we compute the volume fractions in which the flux from the source exceeds f (measured by m_{bol}) for the sources and lenses in Tables 1 and 2. For $q_0 \ll 1$, $I(q_0) = 0.3583\, q_0^{1/3}$, we find

$$\begin{aligned}
\log \mathcal{F}_{\mathrm{L}} &= -10.55 + \frac{8}{3}\log\left(\frac{L_{\mathrm{S}}}{L_{\odot}}\right) + \frac{4}{3}\log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) + \log\left(\frac{n_{\mathrm{L}}}{1\mathrm{pc}^{-3}}\right) + \log\left(\frac{n_{\mathrm{S}}}{1\mathrm{pc}^{-3}}\right) \\
&\quad -\frac{2}{3}\log\left(\frac{\rho}{R_{\odot}}\right) + 1.067 m_{\mathrm{bol}} \\
\log\left(\frac{\mathcal{F}_{\mathrm{L}}}{\mathcal{F}_{\mathrm{S}}}\right) &= -11.32 + 0.467 m_{\mathrm{bol}} + \frac{7}{6}\log\left(\frac{L_{\mathrm{S}}}{L_{\odot}}\right) + \frac{4}{3}\log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) + \log\left(\frac{n_{\mathrm{L}}}{1\mathrm{pc}^{-3}}\right) \\
&\quad -\frac{2}{3}\log\left(\frac{\rho}{R_{\odot}}\right) \\
&= -17.88 + 0.467 m_{\mathrm{bol}} + \frac{7}{6}\log\left(\frac{L_{\mathrm{S}}}{L_{\odot}}\right) + \log\Omega_{\mathrm{L}} + \frac{1}{3}\log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) \\
&\quad -\frac{2}{3}\log\left(\frac{\rho}{R_{\odot}}\right) + 2\log h_{100} \\
&= -3.88 + 0.467 m_{\mathrm{bol}} + \frac{7}{6}\log\left(\frac{L_{\mathrm{S}}}{10^{12} L_{\odot}}\right) + \frac{1}{3}\log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) - \frac{2}{3}\log\left(\frac{\rho}{R_{\odot}}\right) \\
&\quad + \log\Omega_{\mathrm{L}} + 2\log h_{100}
\end{aligned} \tag{38}$$

For $q_0 \gg 1$, $I(q_0) \simeq 1$, we find

$$\begin{aligned}
\log \mathcal{F}_{\mathrm{L}} &= -8.23 + 2\log\left(\frac{L_{\mathrm{S}}}{L_{\odot}}\right) + \log\left(\frac{M_{\mathrm{L}}}{M_{\odot}}\right) + \log\left(\frac{n_{\mathrm{L}}}{1\mathrm{pc}^{-3}}\right) + \log\left(\frac{n_{\mathrm{S}}}{1\mathrm{pc}^{-3}}\right) \\
&\quad + \log\left(\frac{D_{\mathrm{c}}}{1\mathrm{kpc}}\right) + 0.8 m_{\mathrm{bol}}
\end{aligned}$$

$$
\begin{aligned}
&= -14.79 + 0.8m_{\rm bol} + 2\log\left(\frac{L_{\rm S}}{L_\odot}\right) + \log\Omega_{\rm L} + \log\left(\frac{n_{\rm S}}{1{\rm pc}^{-3}}\right) + \log\left(\frac{D_{\rm c}}{1{\rm kpc}}\right) \\
&\quad +2\log h_{100} \\
\log\left(\frac{\mathcal{F}_{\rm L}}{\mathcal{F}_{\rm S}}\right) &= -9.00 + 0.2m_{\rm bol} + \frac{1}{2}\log\left(\frac{L_{\rm S}}{L_\odot}\right) + \log\left(\frac{M_{\rm L}}{M_\odot}\right) + \log\left(\frac{n_{\rm L}}{1{\rm pc}^{-3}}\right) \\
&\quad + \log\left(\frac{D_{\rm c}}{1{\rm kpc}}\right) \\
&= -15.56 + 0.2m_{\rm bol} + \frac{1}{2}\log\left(\frac{L_{\rm S}}{L_\odot}\right) + \log\Omega_{\rm L} + \log\left(\frac{D_{\rm c}}{1{\rm kpc}}\right) + 2\log h_{100} \\
&= -3.56 + 0.2m_{\rm bol} + \frac{1}{2}\log\left(\frac{L_{\rm S}}{10^{12}L_\odot}\right) + \log\left(\frac{D_{\rm c}}{1{\rm Gpc}}\right) + \log\Omega_{\rm L} + 2\log h_{100}
\end{aligned}
\tag{39}
$$

The volume fraction of space in which the flux from the source exceeds f has a bolometric magnitude less than $m_{\rm bol}$. In Fig.8(a), we plot $\log(\mathcal{F}_{\rm L}/\mathcal{F}_{\rm S})$ for lensing by stars again, for the same sources as in Figure 6 (with the same line types). Fig.8(b) shows the corresponding $\log\mathcal{F}_{\rm L}$. Fig.9(a) and (b) show the $\log(\mathcal{F}_{\rm L}/\mathcal{F}_{\rm S})$ and $\log(\mathcal{F}_{\rm L})$ for lensing by giant black holes for the same sources as in Figures 6-9 (with the same line types). Only rough order of magnitude properties are used for the sources and lenses in Fig.9. The largest effect comes from gamma-ray bursts lensed by stars in our simple model (zero shear).

We can convert the flux from the source into the local effective blackbody temperature, $T = (f/4\sigma)^{1/4}$, where σ is the Stefan-Boltzmann constant. $m_{\rm bol}$ is related to T by

$$
m_{\rm bol} = -10\,\log\left(\frac{T}{0.56{\rm K}}\right). \tag{40}
$$

Temperatures as low as a few hundred degrees will have dramatic effects on some dust grain populations, for example.

Finally, we note that although the volume fractions of high flux due to lensing are small, the corresponding absolute volumes can be large. Further, since materials move across the high flux tubes constantly, the fraction of material affected by EGLE is much higher than the static volume fractions.

A thorough investigation of the astrophysical effects of EGLE will require far more detailed and elaborate calculations which are beyond the scope of the present paper.

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Table 1: List of possible EGLE sources

	ρ	$L_{\rm S}$	$D_{\rm c}$	$n_{\rm S}$
QSO (X-ray)	10^{13}cm	$10^{12} L_\odot$	1 Gpc	3×10^{-4} (Mpc)$^{-3}$
QSO (UV-opt)	10^{15}cm	$10^{13} L_\odot$	1 Gpc	3×10^{-4} (Mpc)$^{-3}$
gamma-ray bursts	10^{6}-10^{10}cm	$10^{18-20} L_\odot$	1 Gpc	10^{-4} (Gpc)$^{-3}$
Galactic supernovae	$10^{3}\ R_\odot$	$10^{10} L_\odot$	10kpc	10^{-5} (kpc)$^{-3}$
neutron stars	10^{6}cm	$10^{5}\ L_\odot$ (X-ray) $10^{6}\ L_\odot$ (radio)	10pc-1kpc	1 (kpc)$^{-3}$
hot stars O	$10\ R_\odot$	$10^{5}\ L_\odot$	1kpc	10^{-8} pc^{-3}
hot stars B	$4\ R_\odot$	$10^{3}\ L_\odot$	1kpc	10^{-4} pc^{-3}

Table 2: List of possible EGLE lenses

	$M_{\rm L}$	$n_{\rm L}$
giant black holes	$10^{6-8} M_\odot$	$10^3/(\rm Mpc)^3$
stars	0.1-1 $M_\odot$	$0.1/(\rm pc)^3$

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Fig. 1.— Typical lightcurves for $R \ll 1$, and $\gamma = 0$ (solid line), $R/2$ (dotted line), R (short dashed line), $5R$ (long dashed line), and $10R$ (dot-dashed line).

Fig. 2.— Cross-sections of magnification for lightcurves in Figure 1.

Fig. 3.— Cross-section $\sigma(f, \rho, D_{\rm d})$ with $q_0(f) = 4$, for $D_{\rm ds} = 0.2\,D_{\rm c}$ (solid line), $0.5\,D_{\rm c}$ (long dashed line).

Fig. 4.— $g(\omega) = D_{\rm d}^{\rm m}(f)/D_{\rm ds}$ as a function of $\omega = D_{\rm ds}/D_{\rm c}\,q_0^{-1/3}(f)$.

Fig. 5.— $I(q_0) = V_{\rm L}(f, \rho)/V_{\rm L}(f, \rho = 0)$.

Fig. 6.— q_0 versus $m_{\rm bol}$ of lensing by stars for various sources.

Fig. 7.— For the same lens and sources as in Fig.6 (with the same line types), (a) $D_{\rm d}^{\rm m}$ versus $m_{\rm bol}$; (b) a versus $m_{\rm bol}$.

Fig. 8.— Lensing by stars of the same sources as in Fig.6 (with the same line types), (a) $\log(\mathcal{F}_{\rm L}/\mathcal{F}_{\rm S})$; (b) $\log \mathcal{F}_{\rm L}$ corresponding to (a).

Fig. 9.— Lensing by giant black holes of the same sources as in Figure 6 (with the same line types), (a) $\log(\mathcal{F}_{\rm L}/\mathcal{F}_{\rm S})$; (b) $\log(\mathcal{F}_{\rm L})$.