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The Nuclear Impact on Cosmology: The H_0-Ω Diagram

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Abstract

The H_0-Ω diagram is resurrected to dramatically illustrate the nature of the key problems in physical cosmology today and the role that nuclear physics plays in many of them. In particular it is noted that the constraints on Ω_{baryon} from big bang nucleosynthesis do not overlap with the constraints on Ω_{vis} nor have significant overlap with the lower bound on Ω from cluster studies. The former implies that the bulk of the baryons are dark and the later is the principle argument for non-baryonic dark matter. A comparison with hot x-ray emitting gas in clusters is also made. The lower bound on the age of the universe from globular cluster ages (hydrogen burning in low mass stars) and from nucleocosmochronology also illustrates the Hubble constant requirement $H_0 \leq 66$ km/sec/Mpc for $\Omega_0 = 1$. It is also noted that high values of H_0 (~ 80 km/sec/Mpc) even more strongly require the presence of non-baryonic dark matter. The lower limit on H_0 (≥ 38 km/sec/Mpc) from carbon detonation driven type Ia supernova constrains long ages and only marginally allows Ω_{baryon} to overlap with Ω_{cluster}. Diagrams of H_0-Ω for $\Lambda_0 = 0$ and $\Lambda_0 \neq 0$ are presented to show that the need for non-baryonic dark matter is independent of Λ.

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1 Introduction

In 1974, Gott, Gunn, Schramm, and Tinsley[1] (hereafter, GGST) showed that a plot of the Hubble constant, H_0, verse the the dimensionless density parameter,

$$\Omega \equiv \frac{\rho}{\rho_{\rm crit}}, \tag{1}$$

where ρ is the mass density and

$$\rho_{\rm crit} = \frac{3H_0^2}{8\pi G} \tag{2}$$

is the critical cosmological density, well illustrated the issues in physical cosmology, particularly for models with cosmological constant $\Lambda = 0$. Twenty years later we again use the H_0-Ω diagram and show that the constraints of GGST have not changed significantly but the interpretation now illustrates the critical issues in physical cosmology today, namely the dark matter and age problems. As nuclear/particle astrophysicists we note with pride (or fear) how many of the most significant lines on the H_0-Ω diagram have their origin in nuclear physics arguments.

It will be shown how the H_0-Ω diagram dramatically illustrates that there are at least two dark matter problems, namely the bulk of the baryons are dark and the bulk of the matter in the universe is non-baryonic. It will also be shown that these two dark matter problems persist regardless of the value of H_0. These arguments center on the big bang nucleosynthesis (BBN) constraints on Ω in baryons,[2] $\Omega_{\rm baryon}$. We will also review the age constraints from globular cluster ages[3] and from nucleocosmochronology.[4] We will show with the H_0-Ω diagram that $\Omega_0 = 1$ and the cosmological constant $\Lambda_0 = 0$ requires $H_0 \leq 66$ km/sec/Mpc. Variations in age-H_0-Ω relationships for non-zero Λ are also discussed. A lower bound on $H_0 \geq 38$ km/sec/Mpc from carbon-detonation powered type Ia supernova[5] is also plotted. For comparison, the information on the baryon content of hot x-ray emitting gas in clusters based on ROSAT measurements[6] is also discussed. This paper will now go through each of the constraints in turn and generate appropriate H_0-Ω diagrams.

2 Cosmological Model

The Friedmann-Robertson-Walker (FRW) cosmological model provides a simple physical and mathematical model for describing the large scale structure of the universe by assuming the universe is isotropic and homogeneous. The smoothness of the background radiation is striking confirmation that the universe is isotropic at a level of 1 part in 10^5 (COBE). The assumption of homogeneity is less straight forward to confirm, however, measurements of peculiar velocities of galaxies on very large scales[7] as well as radio source count studies[8] seem to indicate that it is valid. Within the FRW model the distance and time scales can be related to the Hubble constant,

$$H_0 = \frac{\dot{R}(t_0)}{R(t_0)}, \tag{3}$$

where $R(t)$ is the scale factor of the universe and t_0 is the present age. As we shall see the value of the Hubble constant is still quite uncertain. Thus in practice it is useful to introduce a dimensionless factor

$$h = \frac{H_0}{100\ \text{km/sec/Mpc}} \tag{4}$$

to express this uncertainty. The geometry of the universe is encoded in Ω (and Λ); for $\Omega < 1$ ($\Omega > 1$) and $\Lambda = 0$ the universe is opened and hyperbolic (closed and spherical) and for $\Omega = 1$ it is opened and flat. For the most part we shall assume the vacuum has no density nor pressure, in other words we assume $\Lambda = 0$ unless explicitly stated to the contrary.

In the FRW, $\Lambda = 0$ model the present age of the universe is

$$t_0 = \frac{f(\Omega_0)}{H_0}, \tag{5}$$

where

$$f(\Omega_0) = \begin{cases} \dfrac{\Omega_0}{2(\Omega_0 - 1)^{3/2}} \left[\cos^{-1}(2/\Omega_0 - 1) - \dfrac{2}{\Omega_0}\sqrt{\Omega_0 - 1}\right], & \Omega_0 > 1 \\ \dfrac{2}{3}, & \Omega_0 = 1 \\ \dfrac{\Omega_0}{2(1 - \Omega_0)^{3/2}} \left[\dfrac{2}{\Omega_0}\sqrt{1 - \Omega_0} - \cosh^{-1}(2/\Omega_0 - 1)\right], & \Omega_0 < 1 \end{cases}, \tag{6}$$

and Ω_0 is the present value of Ω.

3 The Age of the Universe

The most stringent limits on the age of the universe come from the age of globular clusters and nucleocosmochronology. The age of globular clusters is essentially determined by the rate of hydrogen burning in low mass, metal poor stars. When the core of the star has been completely converted to helium the star changes its structure and no longer lies on the main sequence of a Hertzsprung-Russel luminosity-temperature plot. The main sequence turnoff age is dependent on numerous aspects of the globular clusters, such as metallicity, helium diffusion in the stars, and the initial helium abundance.[3] While many groups have calculated globular cluster ages in the 14–16 Gyr range,[9] Shi[10] has shown that reasonable systematic assumptions in the calculations could lower the ages to 12 ± 2 with a firm lower bound of $t_0 \geq 10$ Gyr. This is also consistent with an independent study by Chaboyer.[11] This lower bound can be obtained trivially by noting that globular clusters should be burning hydrogen less rapidly than the sun since they have lower metallicity and a slightly lower mass. Our sun will exhaust the hydrogen in its core in about 10 Gyr (based on the hydrogen burning rate verified by the calibrated GALLEX experiment[12]). Thus globular clusters must have a lower bound on their age of $t_0 \geq 10$ Gyr as quoted above.

Nucleocosmochronology provides information about time scales over which the elements in the solar system were formed. This method couples knowledge of present day abundance ratios, production ratios, and lifetimes of long lived radioactive nuclides. The standard methods of determining nucleochronometric ages rely heavily on the adopted galactic evolution model. This can lead to large errors in the deduced galactic age. An alternative approach is to employ less restrictive, model-independent nucleocosmochronology which studies the constraints that can be made without specific reference to galactic nucleosynthesis models. When using radioactive decay alone only a strict lower bound is possible.[13] In particular the mean age of the longest lived chronometer,* ^{232}Th relative to ^{238}U, can be used to give an extremely conservative lower bound[4,15] of ~ 8 Gyr. Since this bound assumes all nucleosynthesis occurs in a single event, it is obviously too extreme. We know that ^{235}U, ^{244}Pu, and ^{129}I all existed in measurable abundances when the solar system formed 4.6 Gyr ago and free decay from a single production event several Gyr earlier would not be consistent (for example, ^{129}I has a half-life of only 17 Myr and ^{244}Pu only 82 Myr). Thus we

*Although ^{187}Re is longer lived in its ground state, its lifetime is dependent on its thermal environment so is not useful for a lower bound. It does, however, constrain the maximum mean age.[14]

know the production was more spread out than a single event. Meyer and Schramm[4] quantified this spreading out to show that the lower bound from chronology was $\gtrsim 9$ Gyr and subsequent analysis by Schramm[15] using improved limits on the production ratios pushed the bound up to $t_{NC} \geq 9.7$ Gyr.

The results from globular cluster and nucleocosmochronology provide a consistent lower bound for the age of the universe. We note that globular cluster ages and nucleocosmochronology do not provide a strong upper bound to the age of the universe since one could in principle add several Gyr of formation time to any such age determination. The lower bound does not have these problems since the extreme limit is globular cluster formation on a Kelvin-Helmholtz collapse time scale, $t \sim 10^7$ yr at recombination, $t \sim 10^5$ yr which yields formation times $\ll 1$ Gyr after the big bang. It is not possible to form globular clusters earlier than this time. Based on the above results the age of the universe is constrained to be

$$t_0 \geq 10 \text{ Gyr}. \tag{7}$$

The resulting excluded region in the H_0-Ω plane is shown in figure 1 for $\Lambda = 0$, figure 2 for $\Omega_\Lambda = 0.4$, and figure 3 for $\Omega_\Lambda = 0.8$. Here Ω_Λ is defined as above (1) with $\rho_\Lambda = \Lambda/8\pi G$.

4 Big Bang Nucleosynthesis Limits

Standard homogeneous BBN accurately predicts the primordial abundances of the light elements over 9 orders of magnitude in terms of a single parameter, the density of baryons, ρ_{baryon}. For the constraints on Ω_{baryon} we adopt the recent determination by Copi, Schramm, and Turner,[2]

$$0.01 \leq \Omega_{\text{baryon}} h^2 \leq 0.02. \tag{8}$$

Note that ρ_{baryon} is independent of the Hubble constant, thus the Hubble constant enters into Ω_{baryon} only through ρ_{crit}. The curves defined by this choice and the excluded regions are shown in figure 1. Attempts to by-pass these constraints with inhomogeneous models have been shown to fail in that the constraints on the light element abundances yield essentially the same constraints on Ω_{baryon}.[16] Recently Tytler and Fann[17] have observed deuterium in a quasar absorption system that, if confirmed, restricts the baryon density to an extremely tight range near our quoted upper limit.[18]

It might be noted that one significant difference between our H_0-Ω diagram and that of GGST is that in 1974 BBN only provided an upper bound to ρ_{baryon} from deuterium,[19] whereas now we also have a lower bound on ρ_{baryon} from ^{3}He plus deuterium arguments and we have the strict lithium constraints adding consistency to the picture.

5 Direct Measurements of Ω

5.1 Visible Matter

The most straight forward method of estimating Ω is to measure the luminosity of stars in galaxies and estimate the mass to light ratio, M/L, in a particular wave band. The mass density of visible objects is then given by

$$\rho_{\text{vis}} = \frac{M}{L}\mathcal{L}. \tag{9}$$

For example for blue light $\mathscr{L} = (1.6 \pm 0.2)h \times 10^8 L_\odot$ Mpc^{-3} is the average luminosity density[20] and for our Galaxy[21]

$$\frac{M}{L} = (6 \pm 3)\frac{M_\odot}{L_\odot}. \tag{10}$$

Here $M_\odot$ is the mass of the sun and $L_\odot$ is the luminosity of the sun. An alternative method of determining $\Omega_{\rm vis}$ employed by GGST is to measure $\mathscr{L}$ and M/L for the visible part of many different types of galaxies. This method relies on dynamical observations to determine M/L and produces $\Omega_{\rm vis}$ independent of H_0. Recent work following this method has produced values in excellent agreement with the value for our Galaxy.[22,23] The combination of these two methods produces a range consistent with

$$0.002 \leq \Omega_{\rm vis}h \leq 0.006. \tag{11}$$

The curves defined by this range and the excluded regions are shown in figure 1.

5.2 Dynamical Measurements

Numerous methods for dynamically measuring the density of the universe have been developed all of which give complimentary results. A review of many of these methods can be found in Peebles.[22] We shall highlight a few of these methods.

The simplest means of measuring mass inside a radius, r, is via Kepler's third law

$$GM(r) \approx v^2 r. \tag{12}$$

If the mass were solely associated with the light then we would expect $v \propto r^{-1/2}$ for some object outside of the core of the galaxy. Instead it is observed that $v \approx$ constant for objects far from the center of the galaxy. This indicates that dark matter exists in a halo around the galaxy. Typical estimates of the mass in halos from this method gives $\Omega_{\rm halo} \approx 0.05$. This dark-matter could in principle be dark baryons; however, see the discussion on MACHOS below. Note that estimates of the mass density from dynamics scale with h^2 as does $\rho_{\rm crit}$, thus Ω is independent of H_0.

Measurements of average velocity dispersion and average separation of galaxies in clusters provide a means of assessing the amount of matter associated with clusters. It is generally observed that the velocity of galaxies approach a constant value for large distances from the cluster core. As noted above this indicates the presence of significantly more mass than is visible in the galaxies themselves. A detailed statistical analysis of galaxy dynamics[24] yields

$$\Omega_{\rm cluster} = 0.15 \pm 0.06. \tag{13}$$

An independent method of verifying this value of $\Omega_{\rm cluster}$ is due to the observation of giant luminous arcs by Lynds and Petrosian.[25] These arcs are the image of a bright background object that falls in the line of sight of a cluster core. The mass of the cluster serves as a gravitational lens of this background object producing the arc.[26] Although the modeling of the mass distribution in the cluster can be quite complex, the general prediction of $\Omega_{\rm cluster} \sim 0.2$ is in good agreement with the above value.

Finally we note that many of these methods can be applied on even larger scales. For example, the peculiar velocities of clusters of galaxies can be studied similar to what has been done for galaxies.[7] The result of these types of studies is a consistent bound of $\Omega > 0.3$. To be conservative we shall adopt

$$\Omega_{\rm cluster} > 0.1. \tag{14}$$

This limit is shown in figure 1.

5.3 Baryon Content of Clusters

In addition to providing a measure of Ω, clusters also provide a means of estimating $\Omega_{\rm baryon}$. The three main mass components of clusters of galaxies are star in galaxies, hot intracluster gas, and dark matter. The first two are comprised solely of baryons. Optical observations provide an estimate of the baryon mass in stars and x-ray studies provide an estimate of the baryons in hot gas. These two quantities together provide an estimate to the total baryon mass in the cluster. The total mass of the cluster is more difficult to determine. It is sensitive to numerous assumptions. In particular the cluster is typically assumed to be spherical and in dynamical equilibrium (virialized). If either of these assumptions are not valid the derived total mass could be incorrect. Frequently structure formation models are employed to remove some of this sensitivity. White, Navarro, Evrard, and Frenk[6] employed a "standard" cold dark matter (CDM) model ($\Omega_0 = 1$) coupled with optical and x-ray studies to deduce a baryon fraction for the Coma cluster of

$$\Omega_{\rm B,C} = (0.009 \pm 0.002) + (0.05 \pm 0.01)h^{-3/2}. \tag{15}$$

Here the first term is due to baryons in stars and the second to baryons in hot gas. Note that $\Omega_{\rm B,C}$ is defined by

$$\Omega_{\rm B,C} = \frac{M_{\rm baryon}}{M_{\rm tot}}, \tag{16}$$

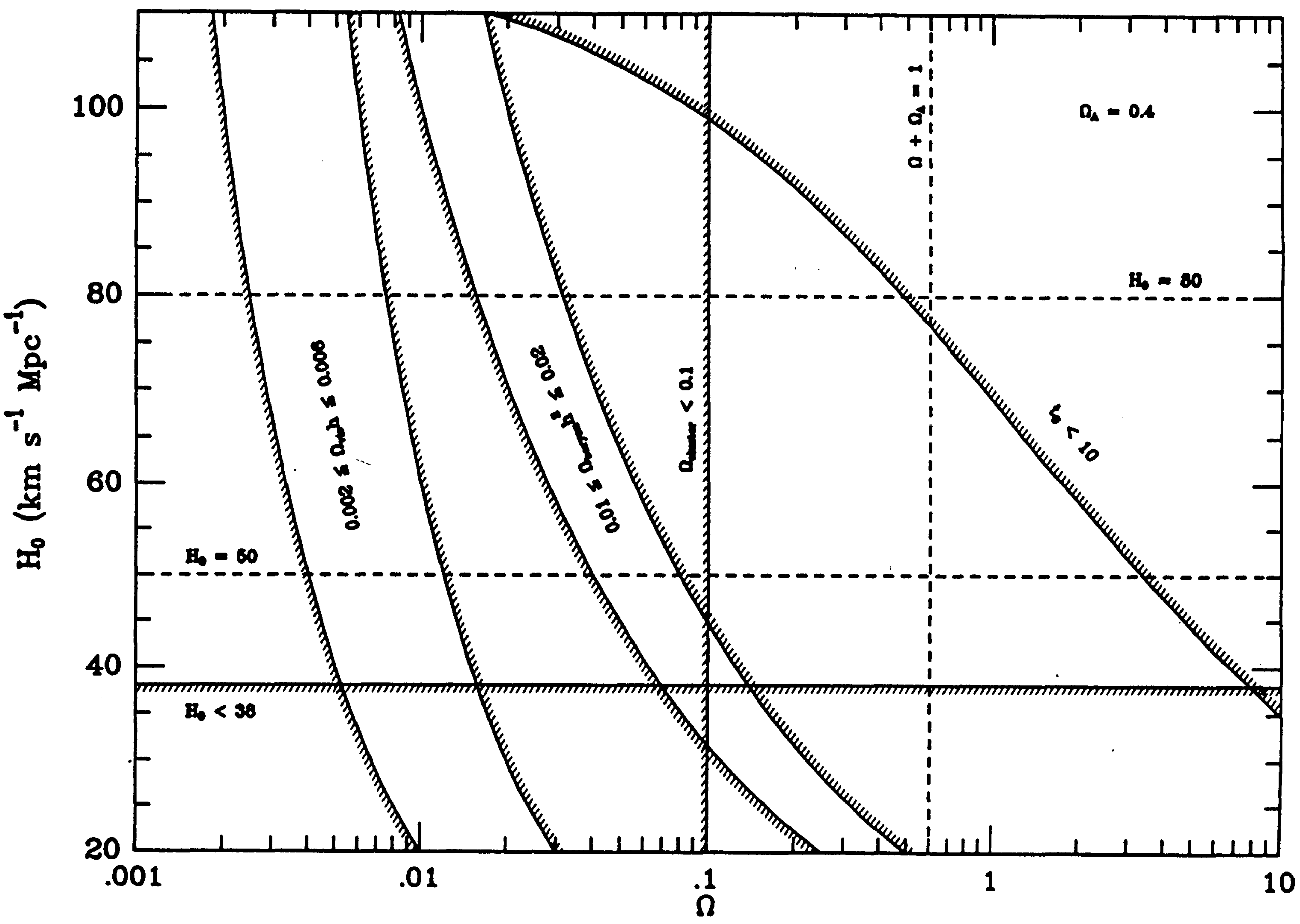
where $M_{\rm baryon}$ is the mass in baryons of the cluster and $M_{\rm tot}$ is its total mass. The region defined by these limits is shown in figure 4.

If clusters are a fair sample of the universe then we expect $\Omega_{\rm baryon} = \Omega_{\rm B,C}/\,\Omega_0$ which is clearly not the case if $\Omega_0 = 1$. The question of whether galaxies in clusters trace the dark matter is still an open one. Babul and Katz[27] found that baryons in an $\Omega_0 = 1$ CDM model are more strongly concentrated than the dark matter. Thus $\Omega_{\rm B,C} > \Omega_{\rm baryon}$ and there is no inconsistency in the results. Alternate models with some admixture of hot dark matter also yield $\Omega_{\rm B,C} > \Omega_{\rm baryon}$. At present there are still a number of difficulties to be worked out in the interpretation of the x-ray gas in clusters result. It is clear, however, that this observation provides important constraints on cluster formation models.

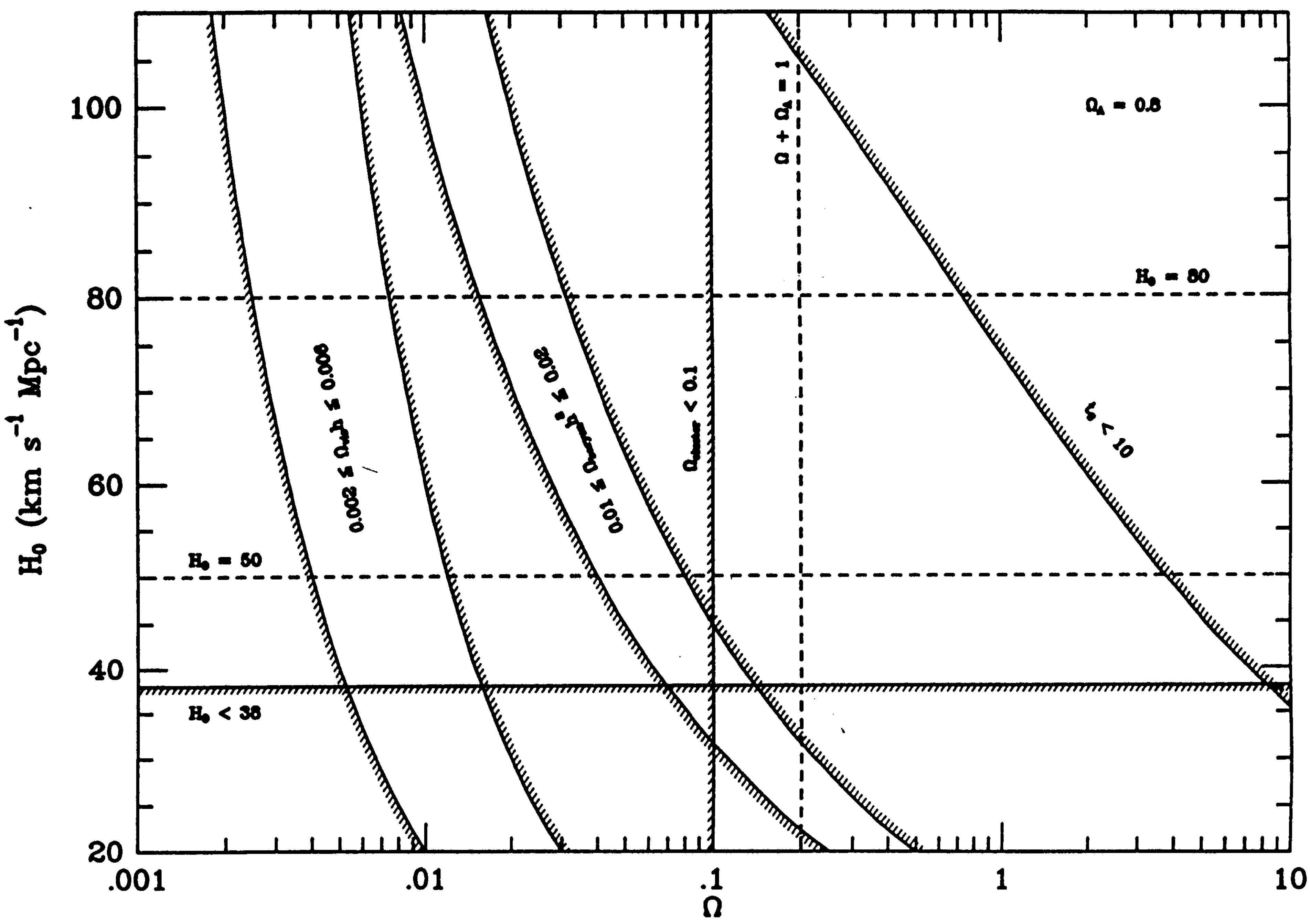
5.4 $\Omega_0 = 1$?

A well known feature of FRW cosmologies is at an epoch t if $\Omega < 1$ ($\Omega > 1$) the universe evolves towards $\Omega = 0$ ($\Omega = \infty$) on a time scale $\sim 1/H(t)$ (see ref. 28). Notice that $\Omega = 1$ is an unstable equilibrium point. At early times $R(t)$ was changing rapidly and $H(t)$ was large. Thus all evolutionary changes occurred on much shorter time scales. Since the universe is clearly not more than an order of magnitude away from $\Omega = 1$ today it must have been unity to high accuracy in all earlier epochs. In particular we have a good understanding of the physics of the universe at the beginning of BBN ($t \sim 1$ sec). If we trace Ω to the epoch of BBN we find that Ω must have been unity to ~ 17 decimal places at that time. The extreme amount of tuning required to satisfy this is an *initial condition* within the standard big bang model.

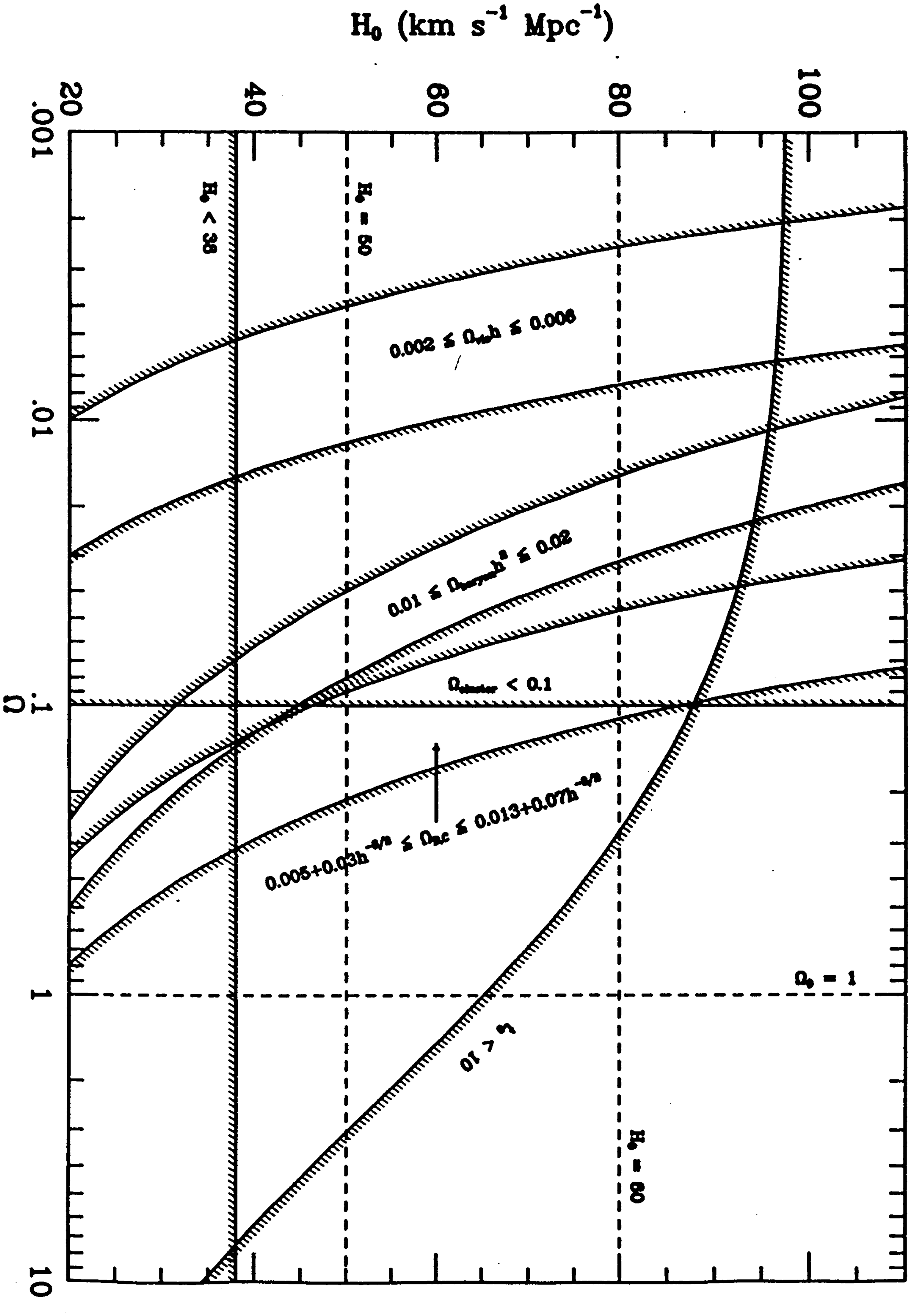
The theory of inflation succinctly explains this fine tuning, the homogeneity and isotropy of the universe, and other initial conditions with physics motivated by particle physics theory. Most models of inflation require the universe to be perfectly flat, $\Omega_0 = 1$, or at least $\Omega_{\rm matter} + \Omega_\Lambda = 1$. Though this is not a measurement and is an untested theory it provides a compelling theoretical argument for $\Omega_0 = 1$. Furthermore, recent measurements on the largest scales[7] indicate that $\Omega_0 \approx 1$. Though there is no firm evidence, we show our bias towards $\Omega_0 = 1$ by plotting this value as a dashed line in figure 1 and 4 and the line $\Omega_{\rm matter} + \Omega_\Lambda = 1$ in figures 2 and 3.



H_0 (km s^{-1} Mpc^{-1})
100
80
60
40
20
.001
.01
.1
1
10
Ω
$\Omega + \Omega_\Lambda = 1$
$\Omega_\Lambda = 0.4$
$H_0 = 80$
$H_0 = 50$
$H_0 < 38$
$\xi < 10$
$\Omega_{cluster} < 0.1$
$0.01 \le \Omega_{baryon}h^2 \le 0.02$



H_0 (km s^{-1} Mpc^{-1})
20
40
60
80
100
.001
.01
.1
1
10
Ω
$H_0 < 38$
$H_0 = 50$
$H_0 = 80$
$\Omega + \Omega_\Lambda = 1$
$\Omega_\Lambda = 0.8$
$\xi < 10$
$\Omega_{cluster} < 0.1$
$0.01 \le \Omega_{baryon} h^2 \le 0.02$



H0 (km s-1 Mpc-1)
20
40
60
80
100
.001
.01
.1
1
10
Ω
H0 < 38
H0 = 50
H0 = 80
Ωcluster < 0.1
Ω0 = 1