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## THE COSMOLOGICAL CONSTANT IS BACK

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### SUMMARY

A diverse set of observations now compellingly suggest that Universe possesses a nonzero cosmological constant. In the context of quantum-field theory a cosmological constant corresponds to the energy density of the vacuum, and the wanted value for the cosmological constant corresponds to a very tiny vacuum energy density. We discuss future observational tests for a cosmological constant as well as the fundamental theoretical challenges—and opportunities—that this poses for particle physics and for extending our understanding of the evolution of the Universe back to the earliest moments.



In the early history of modern cosmology the cosmological constant was invoked twice. First by Einstein to obtain static models of the Universe.<sup>1</sup> Next by Bondi and Gold and by Hoyle to resolve an age crisis and to construct a Universe that satisfied the "Perfect Cosmological Principle," i.e., one that appears the same at all times and places. In both instances the motivating crisis passed and the cosmological constant was put aside.

While Einstein called the cosmological constant his biggest blunder and attempted to put the genie back in the bottle, he failed. The cosmological constant remains a focal point of cosmology (see e.g., Refs. [2]) and of particle theory (see e.g., Ref. [3]). The former because today a wide range of observations seem to call for a cosmological constant. The latter because in the context of quantum-field theory a cosmological constant corresponds to the energy density associated with the vacuum and no known principle demands that it vanish.

As we shall discuss, the observational case for a cosmological constant is so compelling today that it merits consideration in spite of its checkered history. On the theoretical side the value of the cosmological constant remains extremely puzzling, and it just could be that cosmology will provide a crucial clue. Fortunately, there are observations that should settle the issue sooner rather than later.

What then are the data that cry out for a cosmological constant? They include the age of the Universe once again, the formation of large-scale structure (galaxies, clusters of galaxies, superclusters, voids and great walls), and the matter content of the Universe as constrained by dynamical estimates, Big Bang Nucleosynthesis and X-Ray observations of clusters of galaxies. They also relate to a bold attempt to extend the highly successful hot big-bang model by adding a very early epoch of rapid expansion known as Inflation. Inflation addresses squarely the outstanding problems in cosmology: the nature of the ubiquitous dark matter and the origin of the flatness and smoothness of the Universe as well as that of the inhomogeneity needed to seed structure. Inflation, which itself is based upon changes in the energy of the vacuum,<sup>2</sup>

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<sup>1</sup>Einstein's motivation went beyond obtaining a static solution; it was also to insure that an empty universe satisfied Mach's principle (see Ref. [1]).

<sup>2</sup>Changes in the vacuum energy are well understood in modern particle theory; it is the absolute

predicts a spatially flat Universe and a nearly scale-invariant spectrum of density perturbations.<sup>3</sup> Since big-bang nucleosynthesis precludes ordinary matter (baryons) from contributing the mass density needed for a flat universe, inflation requires exotic dark matter, and this has profound implications for structure formation. The most promising possibility is that the bulk of the exotic dark matter is in the form of slowly moving elementary particles left over from the earliest moments, which leads to “cold dark matter” models for structure formation.

Perhaps the most pressing piece of data mentioned above which motivates a re-consideration of the cosmological constant involves the present estimate of the age of the Universe. The expansion age of the Universe (the extrapolated time back to the bang) must necessarily be greater than the age of any object within it. Without a cosmological constant, the expansion age is  $\frac{2}{3}H_0^{-1}$  for a flat (critical density) Universe and  $H_0^{-1}$  for an empty Universe. While the present expansion rate (i.e., Hubble constant  $H_0$ ) is still not known with precision, a variety of techniques are converging on a value in the range  $80 \pm 5 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [4]; this received important support from the Hubble Space Telescope measurement of the distance to a Virgo Cluster galaxy using Cepheid variable stars which yielded a value of  $H_0 = 80 \pm 17 \text{ km s}^{-1} \text{ Mpc}^{-1}$  [5]. The expansion age for a Hubble constant of  $80 \text{ km s}^{-1} \text{ Mpc}^{-1}$  is 8.2 Gyr for the theoretically favored flat Universe. Even taking a conservative lower bound to the fraction of critical density in matter,  $\Omega_{\text{matter}} \gtrsim 0.2$ , leads to an expansion age of only 10.4 Gyr.

Therein lies the problem; the ages of the oldest globular clusters are estimated to be  $16 \pm 3 \text{ Gyr}$  [6], and it is likely that a Gyr or so elapsed before the formation of these stars. The globular-cluster age estimate receives support from other methods. For example, studies of the cooling of white-dwarf stars in the disk of galaxy leads to a disk age of  $9.3 \pm 2 \text{ Gyr}$  [7] (the disk is believed to be considerably younger than the galaxy).

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scale of vacuum energy that is poorly understood.

<sup>3</sup>Scale invariance refers to the fact that fluctuations in the gravitational potential are independent of scale.

The age problem is more acute than ever before. A cosmological constant helps because for a given matter content and Hubble constant the expansion age is larger.<sup>4</sup> For example, for a flat universe with  $\Omega_{\text{matter}} = 0.2$  and  $\Omega_{\Lambda} = 0.8$ , the expansion age is  $1.1H_0^{-1} = 13.2 \text{ Gyr}$  for a Hubble constant of  $80 \text{ km s}^{-1} \text{ Mpc}^{-1}$ .

Next, consider the formation of structure in the Universe. The COBE detection of temperature variations in the cosmic background radiation (CBR) of about  $30 \mu\text{K}$  on the  $10^\circ$  angular scale provided striking confirmation for the idea that structure evolved through the gravitational amplification (Jeans' instability) of small primeval density perturbations (variations in the density of around  $10^{-5}$ ). Subsequent detections of CBR anisotropy on angular scales from about  $0.5^\circ$  to  $90^\circ$  by other experiments have begun to reveal the spectrum of primeval inhomogeneity on very-large scales<sup>5</sup> (greater than about 100 Mpc), and this spectrum is consistent with that predicted by inflation [8].

The spectrum of density perturbations today is not scale invariant because the Universe evolved from an early radiation-dominated phase to a more recent matter-dominated phase; this imposes a scale which depends upon  $\Omega_{\text{matter}}$ , the Hubble constant, and the amount of radiation (in the standard scenario, the CBR and three massless neutrino species). Through this scale, the extrapolation from very-large scales to galaxy scales depends upon  $\Omega_{\text{matter}}$  and  $H_0$ . The distribution of galaxies in the Universe today can probe the spectrum of inhomogeneity on small scales (from roughly 1 Mpc to 300 Mpc). The agreement between the extrapolated, COBE-normalized spectrum and data on small-scale inhomogeneity, including the abundance of rich clusters, the cluster-cluster correlation function and pairwise velocities of galaxies, is very good when  $\Gamma = \Omega_{\text{matter}}(H_0/80 \text{ km s}^{-1} \text{ Mpc}^{-1})$  is  $0.3 \pm 0.06$  [9]. This can be accomplished with a Hubble constant of around  $70 - 80 \text{ km s}^{-1} \text{ Mpc}^{-1}$  provided  $\Omega_{\text{matter}}$  is around 0.3 - 0.4. Other variants of COBE-normalized cold dark matter

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<sup>4</sup>In a universe with only matter the expansion slows due to gravity so that  $1/H_0$  is an overestimate for the time back to the bang; a cosmological constant corresponds to a repulsive force so that the expansion decreases more slowly and eventually increases, leading to a larger expansion age.

<sup>5</sup>For reference, the scale of 1 Mpc corresponds to galaxies, 10 Mpc to clusters, 30 Mpc to voids, and 100 Mpc to the great walls

that fit the data well include a very low Hubble constant (around  $30 \text{ km s}^{-1} \text{ Mpc}^{-1}$ ) and  $\Omega_{\text{matter}} = 1.0$ , a significant increase in the radiation level in the Universe, or the addition of a small admixture of hot dark matter (in the form of a neutrino species of mass  $5 \text{ eV}$ ). With the exception of the very low Hubble constant variant, which is very much in conflict with current measurements, *none of these other scenarios* are consistent with the measured age of the Universe.

Finally, consider the mass density of the Universe. An accurate “inventory” of matter in the Universe is still lacking. It is known that: (1) most of the matter is dark, its presence being revealed only by its gravitational influence; (2) the fraction of critical density contributed by ordinary matter is constrained by big-bang nucleosynthesis to be between  $0.015(H_0/80 \text{ km s}^{-1} \text{ Mpc}^{-1})^{-2}$  and  $0.035(H_0/80 \text{ km s}^{-1} \text{ Mpc}^{-1})^{-2}$  [10, 11]; (3) dynamical estimates, e.g., virial masses of clusters of galaxies, our infall to the Virgo cluster, and peculiar velocities of galaxies, indicate that the *clustered* mass density is probably at least 20% of critical density and perhaps as large as the critical density. The apparent discrepancy between the total mass density and what ordinary matter can contribute provides the case for exotic dark matter; the fact that few estimates indicate the clustered mass density is as large as the critical density suggests that if the Universe is flat, there must be an unclustered component of energy density, like a cosmological constant.

Several authors have recently emphasized how measurements of x-rays from rich clusters of galaxies (like Coma which contains several thousand galaxies) together with the nucleosynthesis estimate for  $\Omega_{\text{baryon}}$  can be used to estimate  $\Omega_{\text{matter}}$  [12]. Assuming that a rich cluster provides a “fair sample” of the universal mix of matter,  $\Omega_{\text{matter}}$  is given by the ratio of total mass to baryon mass times  $\Omega_{\text{baryon}}$ . Most of the baryons in a rich cluster are in the hot, x-ray emitting gas (as opposed to the galaxies); the x-ray flux can be used to determine the baryonic mass, and assuming that the gas is in virial equilibrium, the temperature distribution of the gas determines total mass. Using this technique one obtains the following estimate for the matter density:  $\Omega_{\text{matter}}(H_0/80 \text{ km s}^{-1} \text{ Mpc}^{-1})^{1/2} = 0.1 - 0.4$ . A matter-dominated flat universe is only possible in this case if the Hubble constant is extremely small, around

$30 \text{ km s}^{-1} \text{ Mpc}^{-1}$ , or if a cosmological constant contributes the bulk of the critical density today.

Cosmological observations thus together imply that the “best-fit” model consists of matter accounting for 30%-40% of critical density, a cosmological constant accounting for around 60%-70% of critical density, and a Hubble constant of  $70-80 \text{ km s}^{-1} \text{ Mpc}^{-1}$  (summarized in the Figure). We emphasize that we are driven to this solution by simultaneously satisfying a number of independent constraints. Most important in this analysis is the fact that merely violating one of the constraints is not sufficient to allow a zero value of the cosmological constant. *Unless at least two of the fundamental observations described here are incorrect a cosmological constant is required by the data.*

While a model with a cosmological constant may lead to the only allowed fit to the data, and can extend our understanding of the evolution of the Universe back the earliest moments of the Big Bang, it also raises a host of fundamental concerns. Not the least of these is the fact that a cosmological constant implies a special epoch (today!) when for the first time since inflation, its role in the dynamics of the Universe becomes dominant. In the context of quantum-field theory there is the fact that a nonzero cosmological constant corresponds to a vacuum energy density, and particle theorists have yet to successfully constrain its value, even to within 50 orders of magnitude of the observational upper limit.

The energy density of the quantum vacuum receives contributions from quantum fluctuations of arbitrarily high frequency (and energy) and is formally infinite. It is generally believed high-frequency fluctuations are cutoff at the Planck scale,  $m_{\text{Pl}} \approx 10^{19} \text{ GeV}$ , and if current ideas about supersymmetry are correct, perhaps as low as the weak scale,  $1/\sqrt{G_F} \approx 300 \text{ GeV}$ . The second possibility would lead to a vacuum energy density of around  $10^{10} \text{ GeV}^4$ , and the first to a vacuum energy density of around  $10^{76} \text{ GeV}^4$ . Compare these estimates to the energy density that corresponds to the desired cosmological constant, about  $10^{-46} \text{ GeV}^4$ , and to the maximum value permitted by present data, only a factor of a few higher. The dilemma is apparent.

The enormity of the vacuum-energy problem has led many to conclude that there

must be some kind of cancellation mechanism at work which zeros out the ultimate value of the vacuum energy, or that quantum-cosmological considerations favor a zero value [13]. However, no symmetry principle has yet been found that guarantees a zero value for the vacuum energy, and quantum-cosmological arguments currently rely on the shaky foundations of Euclidean quantum gravity. It could be then that whatever mechanism does diminish the cosmological constant below one's naive estimates does not involve an exact symmetry and leaves a small vacuum energy. It has been noted that the desired value is close to a factor of  $\exp(-2/\alpha_{EM})$  less than  $m_{PI}^4$ . (Imperfect cancellation mechanisms are not unknown; Peccei-Quinn symmetry, which provides most attractive solution to the strong- $CP$  problem, reduces the electric-dipole moment of the neutron by about 20 orders of magnitude.)

Perhaps the most intriguing possibility is that the energy of the quantum vacuum is indeed zero, but we are currently in the midst of a phase transition where the Universe is hung up in the false-vacuum (a mild period of inflation). The energy scale of this transition would correspond to  $(10^{-46} \text{ GeV}^4)^{1/4} \approx 0.003 \text{ eV}$ , which is close to neutrino masses postulated in some models as well as is suggested in the solution to the solar neutrino problem. Indeed, a model for a late-time phase transition involving neutrino masses has been previously discussed in another context [14].

What are the possibilities for detecting a cosmological constant? Indirectly, as we have indicated, a definitive measurement of  $H_0 \gtrsim 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$  would necessitate a cosmological constant or the abandonment of big-bang cosmology, and the Hubble Space Telescope Key Project to determine  $H_0$  to an accuracy of 5% is well on its way. More directly, one might hope to measure the geometry of the Universe. In particular, for a flat Universe with a cosmological constant the distance to an object of given redshift is much larger.<sup>6</sup> A host of geometrical tests, including gravitational lensing [15], galaxy number counts and angular size [16], offer the possibility of detecting this difference. It may be that the best hope lies in CBR measurements. In particular, for a model with a cosmological constant the distribution of the spherical-harmonic

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<sup>6</sup>This is quantified by the deceleration parameter which takes the value  $q_0 = 0.5 - 1.5\Omega_\Lambda \sim -0.6$  rather than 0.5 for a matter-dominated flat model.

multipoles that characterize CBR anisotropy is distinctive, and measurements of sufficient accuracy are likely to be made within the next decade [17].

We are currently facing a crisis in cosmology that is once again driving us to consider the possibility that the cosmological constant is nonzero and dominates the energy density of the Universe today. The challenge this poses for fundamental physics is dramatic. If, in their third attempt to invoke a cosmological constant, cosmologists are finally correct, the impact for our understanding of both the Universe and of fundamental physics will be profound.

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## Figure Caption

**Figure 1:** Constraints to the matter density ( $\Omega_{\text{matter}}$  in a flat universe as a function of the Hubble constant  $H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1}$ ; shaded regions are allowed. Region (a) comes from combining the BBN limit to the baryon density with x-ray observations of clusters; (b) arises from the constraint to  $\Gamma = \Omega_{\text{matter}} H_0$  based upon small-scale inhomogeneity; (c) is based on age determinations of the Universe; (d) is a lower limit to  $\Omega_{\text{matter}}$  based upon dynamical estimates of the mean matter density. The horizontal line is a one-sigma lower limit to the Hubble constant based upon the recent HST measurement. The diagonal dashed lines indicate the allowed region based upon the CDM fluctuation spectrum and the  $\sigma_8$  normalization. The darkest region indicates the parameters allowed by all constraints.

