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# FOUR-DIMENSIONAL SUPERSTRING MODELS†

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ABSTRACT

These five lectures give an elementary introduction to perturbative superstring theory, superstring phenomenology, and the fermionic construction of perturbative string models. These lectures assume no prior knowledge of string theory.

## 1. Introduction and Outline

Superstring theory is a unified description of gravity, gauge bosons, and chiral matter. Superstring theory is free of ultraviolet infinities. It is also exactly solvable (at least) in perturbation theory around a large class of backgrounds. Very recent developments seem to indicate that there is only one consistent superstring theory, which appears in many avatars.

Superstring pheneomenology is the study of how superstring theory makes contact with physics at accessible energy and length scales. This field is still in its infancy: our idea of what constitutes superstring phenomenology has evolved remarkably in the past

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ten years, and is likely to be revised in profound ways as we get a better handle on nonperturbative string effects.

There are *no* quantitative predictions, as yet, from superstring theory. There are however a number of important qualitative predictions and insights. Some of these are listed below.

1. The existence of new particles: a dilaton, axion, and perhaps other scalar moduli.
2. Gauge coupling unification should occur, at a scale not wildly different from the Planck scale, independently of whether or not there is grand unification.
3. Global continuous symmetries, in the effective low energy theory that we see, are "accidental".
4. Hidden sectors appear naturally. This makes hidden sector dynamical supersymmetry breaking seem more appealing. New SUSY breaking scenarios involving the dilaton and other moduli fields are also possible.
5. Gauged discrete and continuous flavor symmetries appear naturally. This makes various schemes for explaining the fermion mass and mixing hierarchies seem more appealing.
6. Having precisely three light generations of standard model fermions is somehow special in string theory. Furthermore the number of generations in a particular string solution is correlated with all of the other low energy properties of that solution. Thus in superstring theory we can hope to eventually relate the number of generations to, e.g., other features of the standard model.

This incomplete list is rather impressive, I would say. However one does have to worry about the robustness of even these qualitative statements, given our still rather poor understanding of the true nature of string theory. Many string theorists would advance the notion that it is premature to even think about superstring phenomenology in any serious way, either because it will turn out that our current perturbative methods are a poor approximation of anything, or because we are on the verge of solving string theory completely in one fell swoop.

I have a different view. My guess is that we are on the verge of achieving with string theory the situation we currently have with respect to QCD. For QCD we have a powerful qualitative understanding of nonperturbative effects, via instantons, flux tubes, monopole condensates, etc. (I will ignore the lattice in making this analogy); this gives us confidence that we understand the basic nature of the theory from first principles. To compare numbers with experiment what we then do is to parametrize our ignorance, using structure functions, etc., apply a few tricks like heavy quark symmetry, then do a really good job on

the perturbation theory. I expect that current developments in string theory will similarly give us confidence that we understand the basic nature of the theory, and furthermore allow us to at least parametrize our ignorance of nonperturbative effects. Then it will be crucial to do a very good and thorough job on string perturbation theory, supplemented by some "tricks" that allow us to include certain nonperturbative information in a controlled fashion.

If this scenario is correct we should be able to extract a wealth of detailed information and insights from superstring phenomenology, even though we cannot compute from first principles how the string determines its own vacuum state.

The outline of these lectures is as follows:

LECTURE 1:

– Perturbative string theory

LECTURE 2:

– Supersymmetry on the worldsheet

– The heterotic superstring in 10 dimensions and 4 dimensions

LECTURE 3:

– Gauge coupling unification

– Supersymmetry breaking

– Fermion masses, fractional charge, proton decay

LECTURE 4:

– Introduction to the fermionic construction of four-dimensional superstring vacua

LECTURE 5:

– A semi-realistic example: flipped $SU(5)$

– A three generation $SU(5)$ GUT

## 2. Perturbative String Theory

We do not yet know what string theory really is. This is because we do not have an adequate nonperturbative formulation of the theory. What we have at the moment is a first-quantized perturbative description of string theory as dynamics on two-dimensional world-sheets. From a spacetime point of view, perturbative string theory is simply a prescription for computing S-matrix elements in a theory with a finite number of massless particles and an infinite number of massive particles.

In perturbative string theory the world-sheet dynamics must involve a set of bosonic fields $X^\mu(\sigma^0, \sigma^1)$, where $\sigma^0$, $\sigma^1$ are the world-sheet proper time and string coordinate, respectively. The superscript $\mu$ is a spacetime vector index, $\mu$=0,1,...$D-1$; these bosonic fields are thus a mapping from the world-sheet into a target space which is some $D$-dimensional spacetime. These fields are necessary to achieve Poincaré invariance of the spacetime amplitudes. A priori there may be other world-sheet fields as well.

We will be interested in closed strings, and thus apply periodic boundary conditions to the $X^\mu$:

$$X^\mu(\sigma^0, 0) = X^\mu(\sigma^0, \pi) \tag{2.1}$$

What should we write down as a world-sheet action for the $X^\mu$? The simplest assumption is that they are free fields. We will also assume that the world-sheet action should be invariant under two-dimensional general coordinate transformations; it is hard to imagine a sensible theory otherwise. These two considerations already suggest the form of the action:

$$S = \frac{1}{4\pi\alpha'} \int_{\Lambda_g} d^2\sigma \, \sqrt{-\mathrm{det} g_{\alpha\beta}} \, g^{\alpha\beta} \, \partial_\alpha X^\mu \partial_\beta X^\nu \eta_{\mu\nu} \tag{2.2}$$

Here $\alpha$, $\beta = 0$, 1 are worldsheet indices, while $\mu$, $\nu = 0$,...D$-1$ are spacetime indices. $\eta_{\mu\nu}$ is the flat Minkowski metric in spacetime, while $g_{\alpha\beta}$ is a world-sheet metric. $\Lambda_g$ represents a two-dimensional surface of genus $g$. $1/(2\pi\alpha')$ is the string tension; the parameter $\alpha'$ has units of length squared.

The world-sheet metric $g_{\alpha\beta}$ is a world-sheet field, but it is an auxiliary field with no dynamics. We could attempt to introduce some dynamics for $g_{\alpha\beta}$ by adding to the action above an Einstein-Hilbert term. However in two dimensions this is a topological term:

$$\int d^2\sigma \sqrt{-\mathrm{det} g} \; R = 4\pi\chi \tag{2.3}$$

where $\chi$ is the Euler characteristic of $\Lambda_g$.

Since $g_{\alpha\beta}$ is an auxiliary field we can, if we wish, integrate it out. This then gives the Nambu-Goto form of the action:

$$S = -\frac{1}{2\pi\alpha'} \big(\text{area of the worldsheet}\big) \tag{2.4}$$

which should not be surprising since the area of world-sheet is the obvious nontrivial invariant under world-sheet general coordinate transformations.

The action (2.2) has another local symmetry in addition to two-dimensional general coordinate invariance. This is Weyl invariance under a coordinate-dependent overall rescaling of the world-sheet metric:

$$g_{\alpha\beta}(\sigma^0,\sigma^1) \to \mathrm{e}^{f(\sigma^0,\sigma^1)}\, g_{\alpha\beta}(\sigma^0,\sigma^1) \tag{2.5}$$

We thus have three local world-sheet symmetries to gauge-fix (two coordinate transformations + Weyl rescaling). We can therefore fix three independent components of $g_{\alpha\beta}$. For example, let us us go over to complex coordinates:

$$z = \sigma^0 + i\sigma^1 \quad ; \quad \bar{z} = \sigma^0 - i\sigma^1 \tag{2.6}$$

then we may write:

$$\begin{aligned} g_{zz} &= g_{\bar{z}\bar{z}} = 0 \\ g_{z\bar{z}} &= g_{\bar{z}z} = \frac{1}{2}\mathrm{e}^{\phi(z,\bar{z})} \end{aligned} \tag{2.7}$$

and the Liouville mode $\phi(z,\bar{z})$ drops out of the action, classically, due to Weyl invariance.

There is a large residual coordinate invariance after this gauge-fixing. This is invariance under conformal transformations, i.e., two-dimensional conformal mappings:

$$z \to f(z) \quad ; \quad \bar{z} \to f(\bar{z}) \tag{2.8}$$

Thus the "flat-gauge" action is a two-dimensional conformal field theory of $D$ free bosons:

$$S = \frac{1}{4\pi\alpha'}\int_{\Lambda_g} dz d\bar{z}\; \partial_z X^\mu \partial_{\bar{z}} X_\mu \tag{2.9}$$

The equations of motion

$$\partial_z \partial_{\bar{z}} X^\mu(z,\bar{z}) = 0 \tag{2.10}$$

indicate that $\partial_z X^\mu$ is an analytic function of $z$, while $\partial_{\bar{z}} X^\mu$ is an antianalytic function, i.e., a function of $\bar{z}$. We often say that $X^\mu$ splits into separate left-moving and right-moving degrees of freedom.

If we now define normal mode operators $\alpha^\mu_n$, $\tilde{\alpha}^\mu_n$ by

$$\partial_z X^\mu \sim \sum_n z^{-n-1}\, \alpha_n^\mu$$
$$\partial_{\bar z} X^\mu \sim \sum_n \bar z^{-n-1}\, \tilde\alpha_n^\mu \tag{2.11}$$

we can then define a Hilbert space of states.

Let us pause for a moment. At this point what we are calling perturbative string theory seems absolutely trivial, and furthermore has no apparent relation to real physics. These objections will disappear once we quantize the theory. To properly quantize a theory with local symmetries, we must either introduce Fadeev-Popov ghosts on the the world-sheet, or perform a canonical quantization with constraints on the physical states. Let us sketch the canonical approach.

The three local world-sheet symmetries imply three independent first-class constraints, namely, that the three independent components of the two-dimensional symmetric energy-momentum tensor $T_{ab}$ must annihilate all physical states. Actually, since $T_{z\bar z}$ vanishes by the equation of motion, we only have to worry about two contraint operators:

$$T(z) \equiv T_{zz} = -\frac{1}{2}\partial_z X^\mu \partial_z X_\mu$$
$$\bar T(\bar z) \equiv T_{\bar z\bar z} = -\frac{1}{2}\partial_{\bar z} X^\mu \partial_{\bar z} X_\mu \tag{2.12}$$

A mode expansion of these defines the Virasoro mode operators $L_n$, $\bar L_n$:

$$T(z) = \sum_n z^{-n-2}\, L_n \tag{2.13}$$

Upon canonically quantizing this system, we encounter a surprise: the commutator $[T(z),T(z')]$ has a Schwinger term, indicative of a possible anomaly. In terms of modes

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{c}{12} n(n^2-1)\delta_{n,-m} \tag{2.14}$$

This Virasoro algebra is, more generally, always a subalgebra of the constraint algebra for any two-dimensional conformal field theory. The second term is the Schwinger or "central" term; the coefficient $c$, which equals $D$ in the case at hand, is called the central charge.

Using (2.11),(2.12), and (2.13) we can express the Virasoro constraint operators in terms of normal mode operators, up to a normal ordering ambiguity in the definition of

$L_0$. We thus write the constraints on physical states as

$$\begin{aligned} L_n|\text{phys}\rangle &= 0 \quad ; \quad \text{for all } n>0 \\ (L_0 - a)\,|\text{phys}\rangle &= 0 \\ \bar{L}_n|\text{phys}\rangle &= 0 \quad ; \quad \text{for all } n>0 \\ \left(\bar{L}_0 - a\right)|\text{phys}\rangle &= 0 \\ \left(L_0 - \bar{L}_0\right)|\text{phys}\rangle &= 0 \end{aligned} \tag{2.15}$$

where $a$ represents the normal ordering ambiguity.

To get the full spectrum of physical states, one would then fix the residual conformal symmetry by the light-cone gauge condition: $X^+(\sigma^0,\sigma^1)\propto\sigma^0$. This leaves only the $D-2$ transverse components of $X$. The physical states are then all states which can be constructed using the transverse mode operators $\alpha^i_n$, $\tilde{\alpha}^i_n$, and satisfying the constraints (2.15). This is clearly not a trivial theory.

For general values of $D$ the physical state spectrum will turn out to contain states with negative norm. This is an indication that the theory is not unitary, due to the presence of an anomaly. The anomaly in this case is in the Weyl symmetry: the Liouville mode which decouples in the classical action does not in general decouple in the quantum theory.

Fortunately for precisely the values $D$=26, $a$=1, all is well: the spectrum is unitary and the anomaly absent. We say that $D$=26 is the critical dimension which defines a consistent bosonic string theory. One can also see from the spacetime point of view that precisely for these values the theory respects Lorentz invariance at the quantum level.

Let us now compute the mass and spin of some of the low-lying physical states. We note first that spacetime four-momentum for string states is defined from the center of mass momentum of the string. This in turn corresponds to the zero mode operators

$$\alpha^i_0 = \tilde{\alpha}^i_0 = \sqrt{\frac{\alpha'}{2}}\,p^i \tag{2.16}$$

where the superscript $i$ denotes transverse components.

We then recognize the constraint equations in (2.15) involving $L_0$, $\bar{L}_0$ as mass-shell conditions:

$$\begin{aligned} \alpha' M^2|\text{phys}\rangle &\equiv -\alpha' p^\mu p_\mu|\text{phys}\rangle \\ &= \Big[2(N-1) + 2(\tilde{N}-1)\Big]\,|\text{phys}\rangle \end{aligned} \tag{2.17}$$

where $N$, $\tilde{N}$ are occupation numbers:

$$\begin{aligned} N &= \sum_{n=1}^{\infty} \alpha^i_{-n}\alpha^i_n \\ \tilde{N} &= \sum_{n=1}^{\infty} \tilde{\alpha}^i_{-n}\tilde{\alpha}^i_n \\ (N-\tilde{N})|\text{phys}\rangle &= 0 \end{aligned} \tag{2.18}$$

The simplest physical state has $N$=$\tilde{N}$=0; let us call this state $|0,p^\mu\rangle$. We may think of $|0,p^\mu\rangle$ as being created from the conformal field theory vacuum by a vertex operator:

$$|0,p^\mu\rangle =: \mathrm{e}^{ip_\mu X^\mu(0,0)} : |0\rangle \tag{2.19}$$

where "::" denotes normal ordering. The vertex operator is a local operator which creates a physical state at a point on the world-sheet.

The mass-shell condition on this state is:

$$\alpha' M^2|0,p^\mu\rangle = -4|0,p^\mu\rangle \tag{2.20}$$

indicating that this state represents an on-shell scalar tachyon!

This is a little alarming, but let us continue. The next simplest physical state has $N$=$\tilde{N}$=1. We can express this state in terms of normal mode operators acting on $|0,p^\mu\rangle$:

$$|\Omega^{ij},p^\mu\rangle = \alpha^i_{-1}\tilde{\alpha}^j_{-1}|0,p^\mu\rangle \tag{2.21}$$

and the mass-shell condition is:

$$\alpha' M^2|\Omega^{ij},p^\mu\rangle = 0 \tag{2.22}$$

Thus we have $(D-2)^2$ on-shell massless states. The states corresponding to the symmetric part of the second rank tensor $\Omega^{ij}$ are the transverse components of a massless spin 2 particle, i.e., a graviton! The trace part is a massless scalar "dilaton", and the antisymmetric parts represent a massless antisymmetric tensor field (in $D$=4 this would just be a massless pseudoscalar "axion").

Thus the bosonic string is a quantum theory of gravity plus matter. To be consistent we should fix $\alpha'$ in terms of the Planck scale (or equivalently Newton's constant):

$$\alpha' \sim \frac{1}{M_{pl}^2} \tag{2.23}$$

*2.1. Green Functions in String Theory*

As already indicated, we use vertex operators to create on-shell physical states at points on worldsheet surfaces. Thus the external states of an on-shell n-point function are represented by n vertex operator insertions at n points on a closed two-dimensional surface. Since the world-sheet metric was gauged away, the only distinct two-dimensional surfaces are those differing either topologically or by some other *global* parameters. The topology of closed two-dimensional manifolds is completely classified as the Riemann surfaces of genus $g$. Genus $g$=0 is the sphere, $g$=1 is the torus, $g$=2 is the double torus, etc..

An n-point function is then *defined* to be a simple sum of path-integrals over the distinct world-sheet surfaces, with vertex operator insertions at n points $z_i$, and integrals over the locations of these points:

$$\sum_g \mathrm{e}^{(n+2g-2)\phi_0} \int [dX\, dg]\, \mathrm{e}^{-S} \prod_{i=1}^{n} \int d^2 z_i\; V_i(z_i) \tag{2.24}$$

The first exponential occurs because we have been careful to include an Einstein-Hilbert term in the action with coefficient $\phi_0/4\pi$; using (2.3) we then obtain the exponential by using the following expression for the Euler characteristic of a Riemann surface punctured at n points:

$$\chi = 2 - 2g - n \tag{2.25}$$

It is possible to show that the parameter $\phi_0$ has a physical interpretation: it is the vacuum expectation value (vev) of the dilaton field. We note further that the sum over genus in (2.24) can be interpreted as a perturbation series in an effective coupling constant $\exp(2\phi_0)$. We therefore define the string coupling $g_s$ by

$$\frac{g_s^2}{4\pi} \equiv \mathrm{e}^{2\phi_0} \tag{2.26}$$

This is what we mean by saying that string theory defined as conformal field theory on two-dimensional closed surfaces is really just *perturbative* string theory.

Note that in (2.24) we denoted an integral over world-sheet metrics, despite the fact that this metric can be gauged away. This is because the world-sheet metric can really only be gauged away *locally*; for genus 1 or larger there are in fact additional global parameters which we must integrate over appropriate domains. For example with the torus there is a complex parameter $\tau$ which distinguishes inequivalent tori. We want two-dimensional general coordinate invariance to apply with respect to these global parameters – this is

referred to as "modular invariance". Thus we need to impose additional restrictions on the quantum theory to ensure modular invariant Green functions. Although modular invariance sounds rather abstract, it is related to two very physical properties of tree-level string theory, namely, s-t-u duality of the tree-level 4-point functions, and the fact that the vertex operators form a mutually local closed associative algebra.

## 3. Superstrings

### *3.1. Supersymmetry on the world-sheet*

The spectrum of the $D$=26 perturbative closed bosonic string contains a tachyon. This is not a satisfactory situation. Can we reformulate perturbative string theory in such a way as to remove the tachyon while keeping the graviton?

One way to do this is to incorporate spacetime supersymmetry into string theory (recall that we have already implemented spacetime Poincaré invariance). If the string spectrum were spacetime supersymmetric, the scalar tachyon would have a superpartner – a fermionic tachyon. However since the Dirac equation does not possess tachyonic solutions, what should happen instead is that the tachyon is simply removed from the physical spectrum by additional physical state constraints.

As far as we know, the only way to implement spacetime supersymmetry in string theory is to first implement *world-sheet* supersymmetry. This is rather straightforward to accomplish. Suppose we want $N$=1 world-sheet supersymmetry; clearly we should introduce superpartners for the $X^\mu$:

$$X^\mu \Rightarrow X^\mu,\ \psi^\mu \tag{3.1}$$

where $\psi^\mu(z,\bar{z})$ are $D$ Majorana world-sheet fermions. Note that these *world-sheet fermions* carry a *spacetime vector* index $\mu$.

It is easy to see how to modify the flat-gauge action (2.9) :

$$\int d^2\sigma \left[\partial_\alpha X^\mu \partial^\alpha X_\mu - i\bar{\psi}^\mu \gamma^\alpha \partial_\alpha \psi^\mu\right] \tag{3.2}$$

where the $\gamma^\alpha$ are two-dimensional gamma matrices. This gauge-fixed action now exhibits *global* world-sheet SUSY; with more work one can improve the ungauge-fixed action (2.2) to exhibit *local* world-sheet SUSY.

The $\psi^\mu$ contribute to $T(z)$, $\bar{T}(\bar{z})$ and thus to the central charge:

$$c = D + \frac{D}{2} = \frac{3D}{2} \tag{3.3}$$

In addition, local worldsheet SUSY implies new constraint operators (the superpartners of $T(z)$, $\bar{T}(\bar{z})$): $T_F(z)$, $\bar{T}_F(\bar{z})$. These promote the Virasoro algebra to an $N$=1 superconformal algebra. The explicit form of these constraint operators is

$$\begin{aligned} T_F(z) &\sim \partial_z X^\mu \psi_\mu \\ \bar{T}_F(\bar{z}) &\sim \partial_{\bar{z}} X^\mu \bar{\psi}_\mu \end{aligned} \tag{3.4}$$

where we have broken up the Majorana fermions into their left and right-moving Majorana-Weyl components:

$$\psi^\mu(z,\bar{z}) = \begin{pmatrix} \psi^\mu(z) \\ \bar{\psi}^\mu(\bar{z}) \end{pmatrix} \tag{3.5}$$

Because we have added new degrees of freedom and new local symmetries on the world-sheet, we must reexamine the question of the Weyl anomaly. This is easier to discuss in the language of covariant quantization, where the condition for anomaly freedom is that the total central charge, *including* contributions from world-sheet ghosts, should vanish. For the original bosonic string the world-sheet ghosts associated with conformal invariance contribute central charge $c$=$-26$, hence the statement that the critical dimension for $D$ is 26. When we promote conformal invariance to superconformal invariance, we must introduce another set of ghosts; the total ghost contribution to the central charge then becomes $-15$. Each Majorana worldsheet fermion contributes $c$=1/2; thus anomaly freedom requires:

$$D + \frac{D}{2} - 15 = 0 \tag{3.6}$$

Thus $D$=10 is the critical dimension of the superstring.

Also, the normal ordering constant in the mass-shell conditions changes:

$$(L_0 - 1)\,|\text{phys}\rangle = 0 \Rightarrow \left(L_0 - \tfrac{1}{2}\right)|\text{phys}\rangle = 0 \tag{3.7}$$

Notice, however, that there is still the danger of obtaining a tachyon in the physical spectrum.

*3.2. Boundary Conditions*

We imposed periodic boundary conditions for the $X^\mu$; for the Majorana fermions $\psi^\mu$ we may choose either periodic or antiperiodic. We will refer to world-sheet fermions with periodic boundary conditions as Ramond or R, and refer to fermions with antiperiodic boundary conditions as Neveu-Schwarz or NS. The R/NS notation is superior because the notion of periodic/antiperiodic can be interchanged by a conformal mapping.

What is the difference between R and NS boundary conditions from the point of view of two-dimensional conformal field theory? There is a unique conformal field theory vacuum state $|0\rangle$; this can be considered the Fock vacuum of the NS modes of the worldsheet Majorana-Weyl fermions $\psi^\mu(z)$, $\bar{\psi}^\mu(\bar{z})$. The "Neveu-Schwarz sector" of the conformal field theory corresponds to the states built upon this vacuum with the NS modes.

The conformal field theory also contains $4*D$ additional local operators called *twist fields*: one set $\sigma(z)$, $\mu(z)$, $\bar{\sigma}(\bar{z})$, $\bar{\mu}(\bar{z})$, for each $\psi^\mu$, $\bar{\psi}^\mu$. These operators are necessary for the consistency of the conformal field theory. (One might reasonably ask why there are no twist fields in the action, but answering that question completely requires a fairly nasty detour.) The states $\sigma(0)|0\rangle$, $\mu(0)|0\rangle$ define a doubly degenerate left-moving *Ramond vacuum* (and similarly for barred fields). The "Ramond sector" of the conformal field theory corresponds to the states built upon the Ramond vacuum with the R modes of $\psi^\mu$, $\bar{\psi}^\mu$. Note that from the conformal field theory point of view there is only one type of Majorana fermion field, not two; the existence of both NS and R modes coming from the same fermion field is a result of the fact that the operator product of a fermion field with a twist field is nonlocal.

*3.3. Spacetime Supersymmetry*

So far we have seen that the Hilbert space of a superconformal field theory factorizes into Neveu-Schwarz sectors and Ramond sectors. We may also wonder what other role the twist fields play in the theory. For simplicity, suppose that $D$ is even and that we have paired up the $D-2$ transverse left-moving Majorana-Weyl world-sheet fermions $\psi^\mu(z)$ to make $(D-2)/2$ left-moving Weyl fermions (and similarly for the right-movers). There are a pair of left-moving twist fields associated with each left-moving Weyl fermion; thus the total left-moving Ramond vacuum degeneracy is $2^{(D-2)/2}$. Futhermore, we can define a local operator $S_\alpha(z)$, called a *spin field*, which is the product of the $(D-2)/2$ left-moving twist fields. The index $\alpha$ takes $2^{(D-2)/2}$ values; this can be interpreted as *a spacetime*

*spinor index*. So we see that the other role of twist fields in the perturbative superstring is that they allow us to construct states which are spacetime fermions!

Now we have enough formalism to discuss spacetime supersymmetry. In a first-quantized formalism like perturbative string theory, the presence of (local) spacetime SUSY is most easily identified by looking for a massless spin 3/2 particle – a gravitino – in the physical spectrum. This means there must be vertex operator which creates the gravitino. The form of this vertex operator is:

$$V_\alpha^\mu(z,\bar{z}) \sim S_\alpha(z)\ \Sigma(z)\ \partial_{\bar{z}} X^\mu\ \mathrm{e}^{ip\cdot X} \tag{3.8}$$

where I have suppressed ghost field dependence, and where $\Sigma(z)$ is an analytic field with some known conformal field theory properties. In fact, the existence of $\Sigma(z)$ in the conformal field theory can be shown to imply the existence of an abelian current, $J(z)$, which extends the local $N$=1 superconformal world-sheet symmetry to a global $N$=2 superconformal world-sheet symmetry:

$$T(z),\, T_F(z) \Rightarrow T(z),\, T_F^+(z),\, T_F^-(z),\, J(z) \tag{3.9}$$

This is a beautiful example of an explicit relation between spacetime and world-sheet symmetries.

### *3.4. GSO Projections*

So far we have discussed vertex operators for three kinds of particles: the tachyon, the graviton, and the gravitino. The states created by these vertex operators do satisfy the superconformal contraint equations for physical states. However we have already mentioned the fact that the physical spectrum must be consistent with additional constraints from modular invariance. In particular we mentioned the tree-level modular invariance constraint that all of the physical vertex operators must be mutually local. In general we have to perform additional projections on the physical states to some subset that obeys the additional constraints. Such projections are known as *GSO projections*.

The locality properties of the operator products of our three vertex operators are given below:

$$\begin{aligned} \text{tachyon} * \text{graviton}: &\ \textit{local} \\ \text{graviton} * \text{gravitino}: &\ \textit{local} \\ \text{tachyon} * \text{gravitino}: &\ \textit{nonlocal} \end{aligned} \tag{3.10}$$

Thus with regard to these three states, we see that a GSO projection is indeed required. Furthermore there are two possible choices of projection: the first one removes the gravitino from the physical spectrum, while the second choice removes the tachyon from the physical spectrum. In the first case we will end up with a superstring whose spectrum contains a tachyon; in the second case we end up with a *tachyon-free* superstring which exhibits spacetime supersymmetry.

### *3.5. The 10-dimensional Heterotic Superstring*

The name "heterotic" refers to the following trick: as we have seen, the world-sheet Majorana fermions $\psi^\mu(z,\bar{z})$ split up into separate left-moving and right-moving Majorana-Weyl fermions $\psi^\mu(z)$, $\bar{\psi}^\mu(\bar{z})$. Thus we can in principle implement $N$=1 superconformal invariance on, e.g., just the right-movers. In this case our world-sheet degrees of freedom will be

$$X^\mu(z,\bar{z}), \quad \bar{\psi}^\mu(\bar{z}); \quad \mu = 0,\ldots 9 \tag{3.11}$$

The right-moving contributions to the Weyl anomaly will cancel, however for the left-moving part we need additional central charge 26−10=16 to remove the anomaly. The simplest choice of additional world-sheet fields to accomplish this is 32 left-moving Majorana-Weyl free fermions: $\lambda^i(z)$, $i$=1, ... 32.

The flat gauge action of the 10-dimensional heterotic superstring is thus

$$\int d^2z \left[\partial_z X^\mu \partial_{\bar{z}} X_\mu - 2i\bar{\psi}^\mu(\bar{z})\partial_{\bar{z}}\bar{\psi}_\mu(\bar{z}) - 2i\lambda^i(z)\partial_z\lambda^i(z)\right] \tag{3.12}$$

The $\lambda^i(z)$ do not carry a spacetime index; they can be regarded as an "internal" part of the conformal field theory. However these new worldsheet fermions have an important effect on the physical spectrum. This is because the normal ordered product

$$:\lambda^i(z)\lambda^j(z): \tag{3.13}$$

for any $i\neq j$ is a world-sheet current. These 496 currents define an operator product current algebra usually called a Kac-Moody algebra:

$$J^A(z_1)J^B(z_2) = \frac{k\delta^{AB}}{(z_1-z_2)^2} + \frac{if^{ABC}J^C(z_2)}{(z_1-z_2)} + \ \ldots \tag{3.14}$$

where $f^{ABC}$ are structure constants of $SO(32)$, and $k$ is the Kac-Moody level, which equals 1 in this case. We have assumed all NS boundary conditions for the $\lambda^i$ in this discussion;

later we will see that by being more careful with the NS and R sectors we can also realize an $E_8 \times E_8$ heterotic superstring.

These currents allow us to construct vertex operators that make gauge bosons:

$$V^{\mu A}(z,\bar{z}) \sim \frac{1}{\sqrt{k}} J^A(z) \bar{\psi}^\mu(\bar{z}) \mathrm{e}^{ip\cdot X} \tag{3.15}$$

Thus the massless spectrum of the 10-dimensional heterotic superstring consists of a graviton, dilaton, antisymmetric tensor, gauge bosons of $SO(32)$ or $E_8 \times E_8$, plus superpartners for all of the above.

### *3.6. The 4-dimensional Heterotic Superstring*

Our construction of the 10-dimensional heterotic superstring suggests an obvious 4-dimensional counterpart. Suppose we take the same world-sheet degrees of freedom as for the 10-dimensional heterotic string, but let the spacetime index $\mu$ run over only 0–3. We then would require additional left-moving central charge 6, and right-moving central charge 9, to cancel the Weyl anomaly. We can do this very simply by adding 6 left-moving and 9 right-moving free world-sheet Weyl fermions:

$$\begin{aligned} &\chi^a(z), \qquad a = 1, \ldots 6 \\ &\bar{\chi}^\alpha(\bar{z}), \qquad \alpha = 1, \ldots 9 \end{aligned} \tag{3.16}$$

If we align the boundary conditions of these new Weyl fermions, we can indeed construct a consistent modular invariant 4-dimensional heterotic superstring in this way. Since we live in a 4-dimensional world, we may therefore ask: who needs 10-dimensional superstrings? The answer is that the 4-dimensional heterotic superstring just described is not a new superstring – it is the precisely the same string expanded around a different background. To be precise, what we called the 10-dimensional heterotic superstring should more properly be called the heterotic superstring expanded around 10-dimensional Minkowski space. What we called the 4-dimensional heterotic superstring should more properly be called the heterotic superstring expanded around 4-dimensional Minkowski space $\times$ a 6-dimensional torus whose radii (in string units) are all unity. The fact that we can represent this torus at a certain radius by Weyl fermions is an example of the well-known phenomenon of *bosonization* in two-dimensional field theories.

Our current belief is that there is only one heterotic superstring (in fact, only one superstring). Conformal field theory solutions can exhibit this superstring as a perturbative expansion around a huge variety of backgrounds (also called string vacua). These different solutions are often referred to as "compactifications", although this terminology is only meaningful when the world-sheet degrees of freedom have an obvious geometrical interpretation, which in general they don't.

### *3.7. Moduli*

In the example above of the heterotic string compactified on a six-torus, the radii of the torus obviously define a continuously connected family of (perturbatively) consistent string vacua. Since all of these vacua respect spacetime supersymmetry, they are exactly degenerate. From the point of view of the effective spacetime field theory, there must be scalar fields, called *moduli*, whose potentials have flat directions. Turning on vevs for these moduli fields then corresponds to changing the radii of the torus –i.e., moving around in a space of continuously connected string solutions, called *moduli space*.

Obviously a key problem in string theory is to map out and characterize the full moduli space. The real world, perhaps, would correspond to one point in that moduli space, but string perturbation theory gives us no clue as to how this huge degeneracy of degenerate vacua is lifted, or even as to whether or not it *is* lifted.

The appearance of moduli with undetermined vevs is a serious problem for superstring phenomenology. Moduli vevs, like the Higgs vev in the standard model, contribute to coupling constants in the low energy effective field theory. Thus the low energy physics is completely dependent on where we are in moduli space. Furthermore the dilaton field is a modulus; its vev, undetermined in string perturbation theory, fixes the effective string coupling $g_s$ that defines the perturbative string expansion. It is easy to find regions in moduli space where the string is strongly coupled, but in such regions our perturbative string apparatus is presumably breaking down. One hopes that nonperturbative effects will stabilize the dilaton vev and the other moduli vevs without otherwise doing too much damage to the picture provided by perturbative calculations, but this is not at all obvious.

Fermionic string constructions (which is our focus in these lectures) always sample special points in moduli space, while more geometrical constructions like orbifolds sample entire regions of moduli space. Thus it is useful to think about fermionic solutions in terms of an equivalent orbifold description. This is not always possible, however, as the fermionic construction samples points in the full moduli space which cannot be reached by the orbifold construction.

## 4. Superstring Phenomenology

As long as we are restricted to perturbative string theory, we probably do not have enough information to determine if any string solution corresponds precisely to the real

world. The problem is compounded by our ignorance of what "the real world" looks like at energy scales close to the Planck scale.

Nevertheless there is much to be learned by examining string solutions which share at least some of the features of the standard model, and of the minimal supersymmetric standard model (MSSM). For some purposes it may be sufficient to look at perturbative string vacua which share only one or two features of the MSSM. However it is clearly important to focus on models which share *many* of the key features of the MSSM; this is because in string theory *all* phenomenological properties of the effective low energy theory are *related* by world-sheet symmetries. To understand superstring physics we need to understand these relationships.

It is useful therefore to define the concept of a "semi-realistic" superstring model. I will define this to be a perturbative string solution which satisfies the following criteria:
– Four uncompactified dimensions.
– N=1 spacetime supersymmetry (perturbatively, at the string scale).
– three light generations of standard model fermions.
– the gauge group includes $SU(3)_c \times SU(2)_L \times U(1)_Y$, or embeds it in a larger group like $SU(5)$ while providing some mechanism (e.g. Higgs) to break the larger group.

These may seem like rather weak criteria for "semi-realistic", but they will suffice for two reasons. The first is that, despite a decade of effort, string theorists have constructed only about two dozen (depending on how you count) superstring models that meet these criteria. The second reason is that it turns out that models which meet this criteria often display a number of other desirable phenomenological properties.

We will discuss next two such properties which show up rather generally in semi-realistic string models: gauge coupling unification, and mechanisms for dynamical SUSY breaking.

### *4.1. Gauge Coupling Unification*

Recall that perturbative string theory contains only two fundamental parameters: $\alpha'$ which has units of length squared, and the effective string coupling, $g_s$, which is dimensionless. Suppose I now consider, at string tree-level, the four-point amplitude for on-shell gauge boson scattering. This amplitude is of course proportional to $g^2/4\pi$, where $g$ is the appropriate gauge-coupling defined at some scale. We can therefore relate this field theory coupling to string parameters:

$$\frac{g^2}{4\pi} = \frac{g_s^2}{4\pi}(\alpha')^3 \frac{1}{V_6} \tag{4.1}$$

where $V_6$ is some function, with dimension $(\text{length})^6$, of the moduli vevs. In string models with a geometric interpretation $V_6$ is roughly the volume of the compactified space.

This relationship is of limited use since we don't know how to compute either $g_s$ or $V_6$. We can obtain a much cleaner relationship by computing a *ratio*, i.e. the ratio of the graviton exchange contribution to this amplitude over the gauge boson exchange contribution. Consider first graviton exchange in, say, the s-channel, in the limit as $s \to 0$. There is a factor of $\sqrt{8\pi}/M_{pl}$, field theoretically, from each gauge-gauge-graviton vertex. Thus in field theory this diagram goes like $8\pi/M_{pl}^2$, compared to the gauge boson exchange contribution which goes like $g^2$. In string theory the ratio of these contributions is just $k\alpha'$. The explicit factor of the Kac-Moody level appears for the following reason: in the graviton exchange diagram we contract two pairs of currents with Kronecker deltas, introducing (by (3.14)) two factors of $k$; for the gauge boson exchange diagram we contract two pairs of currents to make two new currents times structure constants (see the second term in (3.14)), then contract those currents to produce a single factor of $k$.

We can now equate the field theory ratio with the string theory ratio (being careful that we have not left out any numerical factors):

$$\frac{16\pi}{g^2 M_{pl}^2} = k\alpha' \tag{4.2}$$

Furthermore, this relation holds for all of the gauge couplings! Thus, for example, for a string model which contains $SU(3)_c \times SU(2)_L \times U(1)_Y$, we have the string tree-level relation:

$$k_3 g_3^2 = k_2 g_2^2 = k_1 g_1^2 = \frac{16\pi}{\alpha' M_{pl}^2} \tag{4.3}$$

at some appropriate running scale called *the string scale*. This scale, defined by minimizing threshold effects, turns out to be approximately $5\times10^{17}$ GeV. We also see from (4.3) that, for $kg^2 \sim 1$, the fundamental dimensionful unit of string theory is approximately $10^{18}$ GeV, in energy units.

We have thus discovered a very general prediction of semi-realistic superstring models with $k_3 = k_2 = k_1$: gauge coupling unification occurs (modulo threshold corrections) at a scale of roughly $5\times10^{17}$ GeV. This is a remarkable prediction for two reasons. The first is that no mention was made here of grand unification, which is the only known method in field theory of unifying gauge couplings. The second is that the predicted unification scale, $5\times10^{17}$ GeV, is only a factor of 10 - 20 larger than the scale suggested by the MSSM

and low energy data: $3\times10^{16}$ GeV. Put another way, there is less than 10% disagreement in the exponents.

There has been much effort to improve this prediction by explaining the remaining discrepancy. Some possible explanations:

– The Kac-Moody levels are not all equal. The problem with this idea is that for nonabelian groups the Kac-Moody levels are integers; thus it is difficult to tune the ratio $k_3/k_2$ without going to quite large levels (which then has other unfortunate consequences). However $k_1$ is adjustable, since for abelian groups the "Kac-Moody level" is really just a model-dependent normalization factor.

– The string threshold corrections may be large. This corresponds to a modulus vev getting a large value or perhaps several moduli vevs getting moderate values. In the semi-realistic models constructed so far this doesn't appear to happen, but this is still an attractive solution.

– There is a GUT, e.g. $SU(5)$, which is then broken dynamically or by adjoint Higgs at $3\times10^{16}$ GeV. This seems a little ugly, but does have the advantage of introducing a small parameter into the low energy theory, namely, the ratio of the GUT scale to the string scale.

– There is exotic matter beyond the MSSM content, which thus effects the running of the standard model gauge couplings between $M_Z$ and the string scale. This possiblity is currently being investigated in some known semi-realistic models. The problem with this solution is ugliness; in particular, the contribution of any not-too-exotic extra matter on the gauge coupling running is rather large, requiring rather large cancelling effects to get the right result.

### *4.2. Supersymmetry Breaking*

Supersymmetry must be broken, at an effective scale of 100 GeV - 1 TeV, to agree with the real world. If the SUSY breaking sector of the theory is "hidden" from the sector that contains the MSSM –e.g. they are coupled only gravitationally– then the SUSY breaking scale can be very large $\sim10^{13}$-$10^{14}$ GeV. In string theory there are many possibilities for how this happens:

– SUSY breaking is field theoretic, i.e., in the effective supergravity field theory below the string scale, SUSY is broken spontaneously or dynamically. The scale is set either by

moduli vevs or by a gauge coupling getting strong. SUSY breaking in field theory means giving a vev to either the F auxiliary field of a chiral superfield

$$\Phi = \varphi + \theta\psi + \theta\theta F \tag{4.4}$$

or to the D term of a vector superfield.

– SUSY breaking is inherently stringy. This can happen in string perturbation theory when a GSO projection associated with keeping the radius moduli of a compactification *in* the spectrum, projects the gravitino *out* of the spectrum. In this case the scale of SUSY breaking is related to the compactification scale. However we see from (4.1) that the compactification scale is naturally close to $10^{18}$ GeV when the string coupling is reasonably weak – which we have assumed is the case in order to apply string perturbation theory! So this does not seem to be a promising mechanism. String nonperturbative effects may contribute to SUSY breaking, but at the moment there isn't much that we can say about this.

*4.3. Hidden Sector Gaugino Condensate*

Much of the work on SUSY breaking in string theory has focused on the idea that SUSY breaking is triggered by the formation of a gaugino condensate in a hidden sector. If the hidden sector contains a fairly large nonabelian group and the hidden matter content is such that the hidden gauge coupling is asymptotically free, then gaugino condensation may occur at a high scale $M \sim 10^{13}$ GeV. The known semi-realistic string models at least come close to meeting these conditions, making this scenario seem rather natural in perturbative string theory. This should be regarded as a nontrivial success of string theory, since these ingredients are in no way built into our definition of "semi-realistic".

However the hidden gaugino condensate

$$\langle \lambda\lambda \rangle \sim M^3 \tag{4.5}$$

does not itself break supersymmetry, because $\lambda\lambda$ is not an F term. Thus gaugino condensation is supposed to trigger SUSY breaking indirectly through its effect on other fields: either moduli fields (including the dilaton) or hidden sector chiral matter.

In the visible sector we would see SUSY-breaking only via gravitational effects, or more generally via Planck-mass-suppressed effects. The effective scale of SUSY breaking in the visible sector is set by the gravitino mass:

$$m_{3/2} \sim \frac{M^3}{M_{pl}^2} \tag{4.6}$$

Not too surprisingly, it is very difficult to come up with actual models (field theoretic or string theoretic) which break SUSY while simultaneously stabilizing the dilaton and keeping the cosmological constant zero. Despite a lot of effort very few unambiguous results have yet been achieved.

*4.4. Dilaton Dominated SUSY breaking*

In the effective supergravity theory of a four-dimensional superstring model, the dilaton field can be regarded as forming a chiral supermultiplet with the axion, dilatino, and axino:

$$S = \mathrm{e}^{-2\phi} + ia + \theta(\text{dilatino} + \text{axino}) + \theta\theta F_S \tag{4.7}$$

Since we know that the dilaton vev $\langle\phi\rangle$ is nonzero it is tempting to imagine that $\langle F_S\rangle$ is also nonzero, breaking supersymmetry.

If a vev of $F_S$ is the dominant SUSY breaking effect, we can obtain *model independent* predictions for the effective soft SUSY breaking terms in the visible sector. This is because, *in string perturbation theory*, the effective supergravity theory depends on the dilaton superfield $S$ in a simple, universal way. The superpotential is in fact independent of $S$ to all orders (due to a Peccei-Quinn symmetry of the axion), while the Kähler potential and gauge kinetic functions have the following dependence:

$$\begin{aligned} K &= -log(S + S^*) \\ f_a &= k_a S \end{aligned} \tag{4.8}$$

where $k_a$ is the Kac-Moody level.

This dilaton dominated scenario has the great virtue of making model independent predictions of the soft SUSY breaking couplings, modulo threshold effects:

$$\begin{aligned} m_{gauginos} &= \sqrt{3} m_{3/2} \\ m^2_{scalars} &= m^2_{3/2}\delta_{ij} \\ A_{ijk} &= -\sqrt{3} m_{3/2} y_{ijk} \end{aligned} \tag{4.9}$$

where the $y_{ijk}$ are Yukawa couplings.

Unfortunately this attractive scenario has two important difficulties;

– At string loop level, or if there are significant F term vevs for other moduli, then you get flavor-changing neutral current problems due to nonuniversal scalar masses.

– Nonperturbative string corrections to (at least) the Kähler potential are probably not small, which makes the predictions suspect.

Of course, if supersymmetry is discovered and the relations (4.9) appear to hold, this will be declared a triumphant verification of superstring theory.

*4.5. Anomalous U(1)*

In the previous section we did not consider the possibility of D term SUSY breaking involving the dilaton. This is in fact an important topic, not for SUSY breaking per se, but for other phenomenological issues. Many tree level conformal field theory solutions to string theory, including almost all known semi-realistic models, contain a U(1) gauge factor which is anomalous. When this occurs, the Green-Schwarz mechanism breaks the anomalous $U(1)$, at the expense of generating a Fayet-Iliopoulos $D$ term proportional to

$$D_A \;=\; \sum_i Q_i^A |\chi_i|^2 + \frac{g^2}{192\pi^2} e^{\phi}\; TrQ^A \quad , \tag{4.10}$$

where $\phi$ is the dilaton, $TrQ^A$ is the trace over the U(1) charges which gives the mixed gauge-gravitational anomaly, and the $\chi_i$ are scalar fields with anomalous charge $Q_i^A$. This term will break supersymmetry and destabilize the vacuum. The vacuum becomes stable and supersymmetry is restored when one or more of the scalar fields which carry nonzero anomalous charge acquire a vev such that the right-hand side of (4.10) vanishes. Supersymmetry is then restored provided that this vacuum shift is in a direction which is F-flat and also D-flat with respect to all non-anomalous $U(1)$'s. If we let $\chi_i$ now denote the scalar *vevs* which cause (4.10) to vanish, then the additional D and F flatness constraints are

$$D_a \;=\; \sum_i Q_i^a |\chi_i|^2 \;=\; 0, \quad < \frac{\partial W}{\partial \phi_j} > = \; 0 \quad . \tag{4.11}$$

where $a$ labels the nonanomalous $U(1)$'s, and the $\phi_j$ are all the chiral superfields, not just those whose scalar components get vevs.

Note that the shifted vacuum is no longer a classical string vacuum, but does correspond to a consistent perturbative quantum string vacuum. Thus conformal field theory solutions which contain an anomalous $U(1)$ in some sense access a much larger class of perturbative string vacua than those that do not.

Note also that because the D term cancellation in (4.10) involves the one-loop generated anomaly, the scale of vevs in (4.10) is naturally (depending on the value of the anomaly) smaller than the string scale, by an order of magnitude or so. Since the scalars whose vevs $\chi_i$ contribute to (4.10) often carry a variety of other abelian and nonabelian quantum numbers, the vacuum shift generically breaks the original gauge group to one of smaller rank. This rank reduction is variable and can be quite large. It may be possible to perform this vacuum shift without breaking the standard model gauge group, although

there is no fundamental reason why this should *always* be the case. In fact, in many solutions there is considerable freedom in choosing the flat directions involved in the shift.

After the vacuum shift a number of previously massless fields will acquire masses, of order $(\alpha_{str})^n M_{str}$ for some $n$, via coupling to scalar vevs. The spectrum of light fields, particularly light exotics, is often much reduced. In addition, the scalar vevs also tend to induce a number of effective Yukawa interactions for the MSSM quarks and leptons, with Yukawa couplings that are naturally suppressed by powers of $\alpha_{str}$. The fact that this combination of favorable outcomes occurs automatically in the known semi-realistic orbifold models and free fermionic models is quite remarkable!

### *4.6. Quark and Lepton Masses*

A major challenge for any unified model is to reproduce, even qualitatively, the many observed hierarchies of masses and mixings for quarks and leptons. In known semi-realistic string models the numerical values of the couplings in the effective superpotential are order one, and this is likely to be true rather generally in perturbative string vacua. Thus, small Yukawa couplings in the MSSM may originate from scalar vevs or fermion condensates which take values at scales other than $M_{str}$. Nonrenormalizable couplings of quarks and leptons to these vevs or condensates can then generate effective Yukawa couplings which are small. A beautiful property of the known string models is that such a mechanism does indeed occur: the vacuum shift associated with the anomalous $U(1)$. Even so it appears unlikely that any one mechanism will explain all of the observed hierarchies.

Some semi-realistic string models can produce a top quark Yukawa which is order one, while all the other effective Yukawas are suppressed by an order of magnitude or more. This is again encouraging, since such a feature is in not built into the construction of these models, except for the intriguing fact that the requirement of precisely three generations is realized *in these models* in a way which also implies restricted and flavor-dependent Yukawas. While one cannot take such model dependent perturbative results very seriously, it is tempting to speculate that string theory may ultimately relate the heaviness of the top quark to the number of generations!

*4.7. Fractional Charge*

All $SU(3)_c \times SU(2)_L \times U(1)_Y$ conformal field theory string solutions with $k_3 = k_2 = 1$ must contain exotics with fractional electric charge. This is because, if all physical states in a string vacuum obey charge quantization, there exists a certain conformal operator which is mutually local with respect to all the physical fields. This operator must thus itself correspond to a physical field, which leads to a contradiction unless the standard model gauge group is promoted to unbroken $SU(5)$ at level one.

This argument does not determine whether or not there are any *massless* string states with fractional charge; it may be possible to arrange for fractional charges to occur only in the *massive* modes of the string, and thus be superheavy. However in all of the known semi-realistic models, fractionally charged exotics do occur at the massless level. Some or all of these may become superheavy after the vacuum shift associated with the anomalous $U(1)$; others may be confined by nonabelian hidden sector gauge interactions. It may be possible to avoid fractionally charged exotics entirely in three generation string models with higher Kac-Moody levels, but this has never been demonstrated.

The lightest fractionally charged particle will be stable. This can create conflicts with experimental bounds from direct searches, as well as rather severe cosmological and astrophysical bounds. For example, the lightest fractionally charged particle will completely dominate the energy density in the universe if its mass is greater than a few hundred Gev. If there is an inflationary epoch and subsequent reheating, we can probably tolerate a lightest fractionally charged particle with mass greater than the reheating temperature.

In the known semi-realistic string models, a variety of fractionally charged exotics are seen to occur. They can be $SU(3)_c \times SU(2)_L$ singlets with hypercharges less than $\pm 2$, and they can be color triplets or Higgs with nonstandard hypercharge. These exotics have important effects on the RG running of the couplings.

*4.8. Rapid proton decay*

String models typically violate matter parity, allowing for the appearance of B and L violating terms in the cubic part of the effective superpotential. In particular, terms of the form

$$Q\, L\, d^c + u^c\, d^c\, d^c \tag{4.12}$$

where $Q$ denotes a quark doublet, $L$ a lepton doublet, and $u^c$, $d^c$ the conjugates of the right-handed up and down quarks, would lead to instantaneous proton decay. In addition to these

cubic terms, there is also the possibility of quartic terms which can lead to unacceptably rapid proton decay.

To check a particular string model for the absence of such dangerous terms, it is insufficient to compute the effective superpotential to quartic order. This is because the dangerous B violating terms may be generated at *any* order via nonrenormalizable terms which are unsuppressed due to string scale vevs. One simple solution to this problem is to gauge $U(1)_{B-L}$; other possibilities that have been considered in the known models are a combination of B-L and custodial $SU(2)$ along with other flavor symmetries which distinguish quarks from leptons.

## 5. The Fermionic Construction of Four-Dimensional Superstring Vacua

From now on we will specialize to the properties of four-dimensional perturbative heterotic superstring models obtained from the fermionic construction. As we saw previously, this can be regarded as a straightforward extension of the free field conformal field theory approach used to construct the 10-dimensional heterotic superstring.

Consider again the four-dimensional free field construction of section 3.6. Let us break up each of the free Weyl fermions introduced in (3.16) into a pair of free Majorana-Weyl fermions, e.g.

$$\chi^a(z) = \chi_1^a(z) + i\chi_2^a(z) \tag{5.1}$$

Then the complete set of world-sheet degrees of freedom in the light cone gauge consists of the following set of free bosons and free Majorana-Weyl fermions:

$$\begin{aligned} &X^i(z,\bar{z}) \quad && i = 1,2 \\ &\bar{\psi}^i(\bar{z}) && i = 1,2 \\ &\lambda^\alpha(\bar{z}) && \alpha = 1,2,\ldots 18 \\ &\lambda^a(z) && a = 1,2,\ldots 44 \end{aligned} \tag{5.2}$$

where $i$ is a transverse spacetime vector index, while $\alpha$ and $a$ are "internal" indices.

These are in fact the world-sheet degrees of freedom for *any* four-dimensional perturbative heterotic superstring model in the fermionic construction. As we will see shortly, such models differ only in choices of boundary conditions (more precisely, spin structures) and the choice of certain phases which appear in the partition function.

Modular invariance severely constrains these choices in two ways:

– A fermionic string model cannot be modular invariant with any *one* choice of boundary conditions R/NS for the 20+44 Majorana-Weyl fermions. The Hilbert space from which we construct the physical states must in fact be a product of Fock spaces corresponding to different boundary condition choices, called spin structures, for the various fermions. There is a nontrivial set of rules for how to combine spin structures to produce modular invariants.
– Within this product space modular invariance also puts constraints on the physical states –GSO projections– beyond those of world-sheet superconformal invariance.

To understand how this works we need to consider string theory at one-loop, i.e. on the torus. Let us consider at one-loop the one-point function of the identity operator, i.e. the one-loop vacuum-to-vacuum amplitude. This has a physical interpretation as a partition function. By imposing modular invariance on this amplitude we can identify and count the physical states which survive the GSO projections.

Start with the simplest case: the partition function of a single Majorana fermion (i.e. one left-moving Majorana-Weyl fermion and one right-moving Majorana-Weyl fermion. We can in fact construct a modular invariant conformal field theory for this case: it is not string theory but rather describes the critical behavior of the two-dimensional Ising model.

Now not all tori can be mapped into each other by a conformal transformation. There is a complex parameter, $\tau$, which parametrizes the inequivalent tori. So one constraint of modular invariance is that we must compute the partition function for fixed $\tau$, then integrate in $\tau$ over some appropriate domain:

$$Z = \int_{\substack{\text{Fundamental}\\ \text{domain}}} \frac{d^2\tau}{\tau_2} \left[\eta(\tau)\bar{\eta}(\bar{\tau})\sqrt{\tau_2}\right]^{-2} Z(\tau) \tag{5.3}$$

where $\tau{=}\tau_1{+}i\tau_2$ and $\eta(\tau)$ is the Dedekind eta function.

$Z(\tau)$ is just the path-integral of the action over the torus parametrized by $\tau$:

$$Z(\tau) = \int_{\text{torus}} d^2z\, \mathrm{e}^{-S} = tr\ \mathrm{e}^{2\pi i\tau_1 P}\, \mathrm{e}^{-2\pi\tau_2 H} \tag{5.4}$$

the second expression is in Hamiltonian operator form, where

$$\begin{aligned} H &= L_0 + \bar{L}_0 - \frac{1}{24} \\ P &= L_0 - \bar{L}_0 \end{aligned} \tag{5.5}$$

Thus we can write

$$Z(\tau) = q^{-1/48}\bar{q}^{-1/48} tr\ q^{L_0}\bar{q}^{\bar{L}_0} \tag{5.6}$$

where $q$=exp$(2\pi i\tau)$. Since we know how $L_0$ acts on states in the Fock space, we can compute this. Furthermore, we can compute it separately for the left and right-moving pieces.

We have to specify two boundary conditions for each fermion, corresponding to the two independent cycles of the torus. If you like you may think of these as "space" and "time", each compactified on a circle. The "space" choice determines the mode expansion of $\psi(z)$ to be either R or NS, i.e. either periodic or antiperiodic modes. If the "time" boundary condition is NS, then the partition function is just given by the naive trace with $L_0$ acting on the appropriate R or NS Fock space:

$$Z_{NS}^{NS}(\tau) = tr_{NS}\ q^{L_0-1/48} \quad ; \quad Z_R^{NS}(\tau) = tr_R\ q^{L_0-1/48} \tag{5.7}$$

When the "time" boundary condition is Ramond the definition of the trace is modified:

$$Z_{NS}^{R}(\tau) = tr_{NS}\ (-1)^F q^{L_0-1/48} \quad ; \quad Z_R^{R}(\tau) = tr_R\ (-1)^F q^{L_0-1/48} \tag{5.8}$$

where $F$ is the fermion number operator, defined by the relations

$$\begin{aligned} \left\{(-1)^F, \psi_n\right\} &= 0, \quad \forall \text{ modes} \\ F\,|0\rangle_{NS} &= 0 \\ F\,|\uparrow\rangle_R &= 0 \\ F\,|\downarrow\rangle_R &= 1 \end{aligned} \tag{5.9}$$

where $|\uparrow\rangle$ and $|\downarrow\rangle$ are the doubly degenerate Ramond vacua.

$Z_{NS}^{NS}$, $Z_R^{NS}$, $Z_{NS}^{R}$, $Z_R^{R}$ define the four possible spin structures for a single Majorana-Weyl fermion on the torus. Modular invariance now requires that we find combinations of these four functions and their right-moving analogs which are invariant under the torus modular transformations. These transformations are generated by:

$$\begin{aligned} \tau &\to \tau + 1 \\ \tau &\to -1/\tau \end{aligned} \tag{5.10}$$

After some fiddling around with the Poisson resummation formula, one sees that under these transformations

$$\begin{aligned} \tau \to \tau + 1: \qquad Z_{NS}^{NS}(\tau) &\to \mathrm{e}^{-\frac{\pi i}{24}} Z_{NS}^{R}(\tau) \\ Z_{NS}^{R}(\tau) &\to \mathrm{e}^{-\frac{\pi i}{24}} Z_{NS}^{NS}(\tau) \\ Z_{R}^{NS}(\tau) &\to \mathrm{e}^{\frac{\pi i}{12}} Z_{R}^{NS}(\tau) \\ Z_{R}^{R}(\tau) &\to Z_{R}^{R}(\tau) \end{aligned} \tag{5.11}$$

$$\tau \to -1/\tau : \qquad \begin{aligned} Z_{NS}^{NS}(\tau) &\to Z_{NS}^{NS}(\tau) \\ Z_{NS}^{R}(\tau) &\to Z_{R}^{NS}(\tau) \\ Z_{R}^{NS}(\tau) &\to Z_{NS}^{R}(\tau) \\ Z_{R}^{R}(\tau) &\to Z_{R}^{R}(\tau) \end{aligned} \tag{5.12}$$

The right-moving analogs transform in precisely the same way, with the phases complex conjugated.

At first sight it appears that $Z_R^R(\tau)$ is modular invariant all by itself. This is true, however $Z_R^R(\tau)$ evaluates to zero, due to the Ramond zero mode which makes the Ramond vacuum doubly degenerate. So I can make a modular invariant partition function this way, but it vanishes.

So now we see that indeed we need a nontrivial *combination* of spin structures to get a modular invariant partition function. We must combine left-movers with right-movers and sum over different spin structures. In our example:

$$Z_{NS}^{NS}(\tau)\bar{Z}_{NS}^{NS}(\bar{\tau}) + Z_{NS}^{R}(\tau)\bar{Z}_{NS}^{R}(\bar{\tau}) + Z_{R}^{NS}(\tau)\bar{Z}_{R}^{NS}(\bar{\tau}) + Z_{R}^{R}(\tau)\bar{Z}_{R}^{R}(\bar{\tau}) \tag{5.13}$$

is the correct modular invariant partition function for the critical Ising model.

How do we count states in this partition function? There are two *sectors*, the Neveu-Schwarz and the Ramond. In each sector there is a GSO projection, because e.g. in the NS sector

$$Z_{NS}^{NS} + Z_{NS}^{R} = tr\left(1 + (-1)^F\right) q^{L_0 - 1/48} \tag{5.14}$$

so states with odd fermion number are projected out of the trace. Thus, if this were full-fledged string theory, I should construct all the Virasoro physical states and then *remove* those with odd fermion number to get the true physical spectrum from the NS sector.

We can immediately generalize these results to handle four-dimensional heterotic superstring models. We always have 20 right-moving Majorana-Weyl fermions, two of which carry the transverse spacetime index, and we always have 44 left-moving Majorana-Weyl fermions. The left-moving part of the fermionic contribution to the partition function will have the form:

$$Z(\tau) = \sum_{\substack{\text{spin structures} \\ \{\vec{\alpha}_i, \vec{\beta}_i\}}} C_{\beta_i}^{\alpha_i} Z_{\beta_i}^{\alpha_i} \tag{5.15}$$

where $\vec{\alpha}_i$ and $\vec{\beta}_i$ are a set of 44 component vectors denoting spin structures. The $Z_\beta^\alpha$ are just the corresponding product of the single fermion spin structures $Z_{NS}^{NS}(\tau)$, etc., while the $C_\beta^\alpha$ are phases.

Antoniadis, Bachas, and Kounnas, and independently Kawai, Lewellen, and Tye, solved the modular invariance constraints *in general* for such models. Only a few additional constraints are then required to ensure the full modular invariance of the theory, not just of the one-loop vacuum-to-vacuum amplitude. You can rederive most of their result yourself using (5.11) and (5.12).

This general solution for which choices of spin structures give modular invariant conformal field theory solutions can be expressed as some simple rules about Ramond-Ramond overlaps of the vectors $\vec{\alpha}_i$ and $\vec{\beta}_i$, and the choice of the phases $C^{\alpha}_{\beta}$. We will use the following simple notation to denote the components of $\vec{\alpha}_i$ and $\vec{\beta}_i$, i.e., the individual boundary conditions of the Majorana-Weyl fermions:

$$\begin{aligned} 1 &= \text{Ramond} \\ 0 &= \text{Neveu} - \text{Schwarz} \end{aligned} \tag{5.16}$$

One of the things modular invariance requires is that, if $\vec{\alpha}_i$ and $\vec{\alpha}_j$ occur in the sum over spin structures, then $\vec{\alpha}_i + \vec{\alpha}_j$ must also occur. Here "addition" of boundary condition vectors in the notation of (5.16) is defined mod 2, i.e.

$$\begin{aligned} \text{NS} + \text{NS} &= \text{NS} \\ \text{NS} + \text{R} &= \text{R} \\ \text{R} + \text{R} &= \text{NS} \end{aligned} \tag{5.17}$$

It is also the case that if some $\vec{\alpha}_i$ occurs in the sum over spin structures, then an equivalent $\vec{\beta}$ must also occur. These two facts taken together imply that any allowed set of spin structures is completely specified by some set of *basis vectors* $\vec{V}_i$. The complete set of $\vec{\alpha}_i$ and $\vec{\beta}_i$ then consists of all possible distinct linear combinations of the basis vectors with positive integer coefficients (there are a finite number of these since "addition" is mod 2).

Modular invariance also requires that every model contains at least the following two sectors:

– the all-Neveu-Schwarz or "untwisted" sector

$$\left(0^{20} \| 0^{44}\right) \tag{5.18}$$

where the double vertical line separates the boundary conditions of the 20 right-movers from those of the 44 left-movers.

– the all-Ramond sector

$$\left(1^{20}\|1^{44}\right) \tag{5.19}$$

Since the untwisted sector is obtained as twice the all-Ramond sector, we only need the latter as a basis vector. Conventionally this is always labelled $V_0$:

$$V_0 = \left(1^{20}\|1^{44}\right) \tag{5.20}$$

Given a set of basis vectors consistent with modular invariance we first have to identify if there are any pairs of left-left Majorana-Weyl fermions (or pairs of right-right Majorana-Weyl fermions) whose boundary conditions *match* in the entire set of spin structures (equivalently, in all the basis vectors). Any such pairs are actually Weyl fermions, and we must treat them separately since bilinears of them are *currents* (recall the 16 Weyl pairs in the 10-dimensional heterotic string which generated $SO(32)$).

We should then identify any right-left pairs whose boundary conditons match in all the basis vectors. These pairs make copies of the Ising model discussed aboved; we thus call them Ising fermions.

Whatever is left are by definition Majorana-Weyl fermions whose boundary conditions *don't* pair up with any other Majorana-Weyl fermion. These unpaired Majorana-Weyl fermions have been called "real fermions" or "chiral Ising" fermions in the literature.

A partition function will be a sum over sectors, labelled by $\vec{\beta}_i$. The contribution of each sector is itself a sum over $\vec{\alpha}_i$, which has the effect of performing the GSO projections. The number of independent GSO projections performed for each sector is approximately equal to the number of basis vectors. For semi-realistic models which have hundreds of sectors and perhaps ten basis vectors, you obviously need a computer to extract the physical spectrum.

### *5.1. $SO(32)$ Versus $E_8 \times E_8$ in 10 Dimensions*

We now know enough formalism to understand the difference between the $SO(32)$ and $E_8 \times E_8$ heterotic strings in ten dimensions. The modular invariance rules for ten dimensions are the same as in four dimensions. The only difference in the construction is the counting of Majorana-Weyl fermions: there are now only 8 right-movers, all of which carry a transverse spacetime index, and there are only 32 left-movers.

The $SO(32)$ heterotic string can be defined (there are many equivalent definitions) by the following two basis vectors:

$$\begin{aligned} V_0 &= \left(1^8 \| 1^{32}\right) \\ V_1 &= \left(1^8 \| 0^{32}\right) \end{aligned} \tag{5.21}$$

Let us locate all the massless physical states. The untwisted sector contains the graviton, dilaton, antisymmetric tensor, and the 496 gauge bosons of $SO(32)$, obtained from currents which are bilinears of the 16 left-moving Weyl fermions. This gauge group is realized at Kac-Moody level one, as is always the case when the currents are all fermion bilinears. Sector $V_1$ contains the gravitino, dilatino, tensorino, and the gauginos –recall from section 3.3 that to get *spacetime* fermions we need a spin field, and thus we need a sector which is Ramond in the right-moving slots whose associated fermion carries a spacetime index. There are two other sectors in this model, $V_0$ and $V_0$+$V_1$, but neither of these contains any massless physical states.

The $E_8 \times E_8$ heterotic string can be defined by the following three basis vectors:

$$\begin{aligned} V_0 &= \left(1^8 \| 1^{32}\right) \\ V_1 &= \left(1^8 \| 0^{32}\right) \\ V_2 &= \left(0^8 \| 1^{16} 0^{16}\right) \end{aligned} \tag{5.22}$$

Let us locate all the massless physical states. The untwisted sector contains the graviton, dilaton, antisymmetric tensor, and the 240 gauge bosons of $SO(16) \times SO(16)$, obtained from currents which are bilinears of either the first 8 left-moving Weyl fermions, or of the second 8 left-moving Weyl fermions. Sector $V_2$ contains another 128 gauge bosons, obtained from currents which are composites of left-moving *twist fields*, not fermion bilinears. To be precise, there is a pair of twist fields $\sigma^\alpha(z)$, $\alpha$=1, 2 for each of the 8 left-moving Weyl fermions in $V_2$ that have Ramond boundary conditions. The vertex operators of the gauge bosons coming from $V_2$ have the form

$$\bar{\psi}^\mu(\bar{z}) \sigma_1^{\alpha_1} \sigma_2^{\alpha_2} \sigma_3^{\alpha_3} \sigma_4^{\alpha_4} \sigma_5^{\alpha_5} \sigma_6^{\alpha_6} \sigma_7^{\alpha_7} \sigma_8^{\alpha_8} \mathrm{e}^{ip \cdot X} \tag{5.23}$$

There are $2^8$ such states (not counting helicities) before the GSO projections. $V_1$ provides no projection on these states, since $V_1$ and $V_2$ have no Ramond-Ramond overlap. $V_0$ and $V_2$ provide the *same* GSO projection, which removes half the states. Thus we get $2^7$=128 gauge bosons from sector $V_2$. Sector $V_0$+$V_1$+$V_2$ also gives another 128 gauge bosons,

filling out the 240+128+128=496 gauge bosons of $E_8 \times E_8$, at Kac-Moody level one. The corresponding gauginos come from sectors $V_1$, $V_1+V_2$, and $V_0+V_2$.

Note that the $E_8 \times E_8$ heterotic string was obtained by simply adding one additional basis vector, $V_2$, to the set that gave the $SO(32)$ heterotic string. This additional basis vector had two distinct effects on the massless physical spectrum. The first effect was to provide an additional GSO projection on the untwisted sector and sector $V_1$, which removed the 248 gauge bosons and gauginos of $SO(32)$ not in $SO(16) \times SO(16)$. The second effect was to provide new massless states, in sector $V_2$ itself and in sector $V_0+V_1+V_2$.

This is the general pattern for model-building in the fermionic construction in any number of dimensions. Each additional basis vector (usually) provides both new massless states and new GSO projections which eliminate some of the previous set of massless states. Because these two effects are highly correlated, and constrained by the modular invariance rules, it becomes something of an art to construct sets of basis vectors that will produce a desired massless spectrum and gauge group.

## 6. Examples of Semi-Realistic Models

### *6.1. The Flipped $SU(5)$ Model*

The flipped $SU(5)$ model was the first semi-realistic model to be obtained in the fermionic construction. It has a number of simplifying features, e.g. it contains only Weyl and Ising fermions, and uses a very simple gauge embedding of $SU(5)$.

There a several slightly different versions of the flipped $SU(5)$ model; we will discuss the "search" version, so-called because it appears in a paper with that word in the title. While it is a priori unlikely that this model corresponds to the real world, it is not obviously wrong. As discussed already in section 4, a number of happy circumstances seem to conspire to make the model look much more like the real world than one would have expected from the weak criteria of "semi-realistic".

The model is defined by 8 basis vectors – I will suppress in this discussion the additional phases $C^{V_i}_{V_j}$ which must also be specified. It will be more illuminating to first discuss the model defined by just the first 5 of these basis vectors:

$$\begin{aligned}
V_0 &= \big((1^2)(111111)\ \ (1^{12})\ \ \|\,(1^{10})(111111)\ \ (1^{12})\ \ (1^{16})\big)\\
V_1 &= \big((1^2)(111111)\ \ (0^{12})\ \ \|\,(0^{10})(000000)\ \ (0^{12})\ \ (0^{16})\big)\\
V_2 &= \big((1^2)(110000)\ \ (1^4 0^8)\ \ \|\,(1^{10})(110000)\ \ (1^4 0^8)\ \ (0^{16})\big)\\
V_3 &= \big((1^2)(001100)\,(0^4 1^4 0^4)\ \|\,(1^{10})(001100)\,(0^4 1^4 0^4)\,(0^{16})\big)\\
V_4 &= \big((1^2)(000011)\ \ (0^8 1^4)\ \ \|\,(1^{10})(000011)\ \ (0^8 1^4)\ \ (0^{16})\big)
\end{aligned} \tag{6.1}$$

Note that all of the Majorana-Weyl fermions in this model pair up into Weyl fermions. The model defined by just the first two of these basis vectors is the 10-dimensional $SO(32)$ heterotic string compactified on a six-torus, with each circle of the torus fixed at the fermionic radius. The gauge group of such a solution is $SO(32)\times[U(1)]^{12}$ at generic values of the radii, but at the fermionic radii it is enhanced to $SO(44)\times[U(1)]^6$, with Kac-Moody level one. The model defined by just the first two basis vectors also has $N$=4 spacetime supersymmetry, i.e. there are 4 gravitinos in the massless spectrum.

Each additional basis vector $V_2$, $V_3$, or $V_4$ adds GSO projections which remove half of the gravitinos; however the projection from $V_4$ is equivalent to the projection from $V_0$+$V_2$+$V_3$, and thus one gravitino survives. This means that the model defined by (6.1) has $N$=1 spacetime supersymmetry. The resulting model can be thought of as a symmetric orbifold of the torus model. The 22 left-moving Weyl fermions no longer contribute the currents of $SO(44)$ realized as fermion bilinears in the untwisted sector. Rather, from (6.1) it is clear that only the subgroup $SO(10)\times[SO(6)]^3\times SO(16)$ arises from bilinears in the untwisted sector. In addition, there are $2^8$ new currents coming from twist fields in sector $V_0$+$V_2$+$V_3$+$V_4$; there is only one distinct GSO projection on these, leaving 128 additional gauge bosons. This promotes the $SO(16)$ to an $E_8$, still at Kac-Moody level one.

Once we have identified the full gauge group, and precisely how this group is embedded into quantum numbers of the fermionic Fock space, we can determine how all of the massless states transform under the group. We are guaranteed (by unitarity or modular invariance) that all of the massless states assemble into a set of complete irreducible representations of the gauge group.

In addition to the gauge bosons, the gravity multiplet, and their superpartners, we get massless fermions from sectors $V_2$, $V_3$, and $V_4$ (their scalar superpartners come from $V_1$+$V_2$, $V_1$+$V_3$, and $V_1$+$V_4$). Let us count the number of massless fermion states coming from, e.g. sector $V_2$. The massless states correspond to all of the Weyl fermions being in the vacuum state; for each Ramond Weyl fermion there are two degenerate vacuum states, as we have already discussed. The first two right-mover slots of $V_2$ correspond to the two Majorana-Weyl fermions which carry the transverse spacetime index; the fact that these are Ramond for $V_2$ tells us that the massless states are spacetime fermions, with two possible helicities. The total number of massless states from $V_2$ before the GSO projections is 2 helicities times a $2^3$ right-mover Ramond degeneracy times a $2^5$ degeneracy in the left-mover $SO(10)$ slots times a $2^3$ degeneracy in the left-mover $SO(6)$ slots. For fixed helicity, these states correspond to 8 copies of a $(16,4)$+$(\bar{16},4)$+$(16,\bar{4})$+$(\bar{16},\bar{4})$ of $SO(10)\times SO(6)$.

The GSO projection from $V_3$ makes these fermions *chiral* with respect to $SO(10)$. This is clear from (6.1), where we see that the Ramond-Ramond overlaps of $V_2$ with $V_3$ occur only in the first two right-movers and the $SO(10)$ slots of the left-movers. After this projection the *positive helicity* states consist of 8 copies of a $(16,4)+(16,\bar{4})$; their CPT conjugates are negative helicity states in a $(\bar{16},4)+(\bar{16},\bar{4})$. Continuing, the GSO projections from $V_1$ and from $V_2$ itself reduce the 8 copies down to 2 copies; $V_0$ and $V_4$ do not provide any new projections. Thus we find that after all the projections $V_2$ contibutes 2 copies of chiral fermions in a $(16,4)+(16,\bar{4})$ of $SO(10)\times SO(6)$.

If we think of this $SO(6)$ as a gauged flavor symmetry, and recall that a chiral 16 of $SO(10)$ contains precisely one generation of standard model fermions plus a right-handed neutrino, then we may say that $V_2$ contributes 16 light generations. $V_3$ and $V_4$ similarly contribute 16 generations each, for a total of 48 generations.

This is very encouraging, since we want 3 generations for a semi-realistic model. Clearly all we need now is to introduce 4 additional GSO projections on $V_2$, $V_3$, and $V_4$, to cut the number of states down to one generation each.

We must also do something about the gauge group. Without worrying yet about the "flavor" group $[SO(6)]^3$ or the "hidden" group $E_8$, there is a problem with leaving $SO(10)$ unbroken. The problem is that there are no massless adjoint Higgs chiral supermultiplets in the spectrum to break $SO(10)$ to $SU(3)_c\times SU(2)_L\times U(1)_Y$ as in a standard field theoretic GUT.

Let us examine why this is. We know that we have a set of left-moving currents in the adjoint representation of $SO(10)$, formed from Weyl fermion bilinears. So why can't we make massless adjoint Higgs from these? The vertex operator for such adjoint Higgs has the form

$$\lambda^\alpha(\bar{z})\lambda^a(z)\lambda^b(z)\mathrm{e}^{ip\cdot X} \tag{6.2}$$

Here $a$ and $b$ label the bilinears that make the $SO(10)$ currents, while $\alpha$=1, 2, ... 9 labels any one of the "internal" right-moving Weyl fermions. These vertex operators will indeed create adjoint Higgs in untwisted sector, before the GSO projections. The GSO projection from $V_1$, however, only allows the states where $\alpha$ labels one of the 3 right-mover Weyl fermions which are Ramond in $V_1$; let us call these $\alpha$=1, 2, 3. Furthermore, this model has $N$=1 spacetime SUSY, not $N$=4. This implies that there is at least one GSO projection in the untwisted sector which distinguishes the spacetime helicity slots from each $\alpha$=1, 2, 3. These projections *either* remove $SO(10)$ gauge bosons, or $SO(10)$ adjoint Higgs. Since by

assumption they do not remove any $SO(10)$ gauge bosons, all of the $SO(10)$ adjoint Higgs are projected out.

We have proven that we cannot make adjoint Higgs from currents, but perhaps there are other left-moving local conformal operators in the adjoint representation of $SO(10)$ that we could use to make massless adjoint Higgs. Such operators, to make massless physical states, have to be what are called *primary* conformal operators with respect to the Kac-Moody algebra. However for any Kac-Moody algebra at level one there are no primary conformal operators in the adjoint.

More generally, for $SO(10)$ at level one the only massless chiral supermultiplets which can occur are the singlet, the 10, the 16, and the $\bar{16}$. For $SO(10)$ at level two we can have in addition the 45 and the 54. Similarly, for $SU(5)$ at level one we can only have the singlet, the 5, $\bar{5}$, 10, and $\bar{10}$. For $SU(5)$ at level two we can have in addition the 24, 40, $\bar{40}$, 45, and $\bar{45}$.

This long detour has taught us that, at the same time as we cut the number of generations in (6.1) from 48 to 3, we must break $SO(10)$ to either $SU(3)_c \times SU(2)_L \times U(1)_Y$ or to flipped $SU(5) \times U(1)$. Flipped $SU(5)$ avoids the necessity of adjoint Higgs by embedding $U(1)_Y$ in $SU(5) \times U(1)$ differently than the standard $SU(5)$ embedding; this has the result that the 10 and $\bar{10}$ of $SU(5)$ can now get vevs which are standard model singlets, and accomplish the desired symmetry breaking.

Now we can write down the final three basis vectors which, together with (6.1), define the flipped $SU(5)$ model:

$$\begin{aligned}
V_5 &= \big((1^2)(110000)(1^2 0^2 1 0^3 1 0^3) \,\|\, (1^{10})(110000)(1^2 0^2 1 0^3 1 0^3)(0^{16})\big) \\
V_6 &= \big((1^2)(001100)(1 0^3 1^2 0^3 1 0^2) \,\|\, (1^{10})(001100)(1 0^3 1^2 0^3 1 0^2)(0^{16})\big) \\
V_7 &= \big((1^2)(000000) \quad (1 0^5 1 0^5) \quad \| \, (\tfrac{1}{2}{}^{10})(\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}\tfrac{1}{2}) \quad (1 0^5 1 0^5) \quad (\tfrac{1}{2}{}^8 1^4 0^4)\big)
\end{aligned} \tag{6.3}$$

It is evident that not all of the Majorana-Weyl fermions pair up into Weyl fermions – in fact there are now 8 Ising fermions. The notation $\frac{1}{2}$ in $V_7$ is defined as follows. As we pointed out previously, each Weyl fermion may be regarded as a complexified pair of Majorana-Weyl fermions:

$$\lambda(z) = \lambda_1(z) + i\lambda_2(z) \tag{6.4}$$

We therefore may choose from a more general set of boundary conditions for Weyl fermions, corresponding to rotating $\lambda_1$, $\lambda_2$ into each other:

$$\begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \rightarrow \begin{pmatrix} cos\pi\theta & sin\pi\theta \\ -sin\pi\theta & cos\pi\theta \end{pmatrix} \begin{pmatrix} \lambda_1 \\ \lambda_2 \end{pmatrix} \tag{6.5}$$

In this language $\theta$=0, 1 correspond to NS, R boundary conditions, while $\theta$=1/2 is the new boundary condition we have used in $V_7$.

The GSO projections from $V_5$, $V_6$, and $V_7$ cut the number of generations from 48 to 24 to 12 to 6. In addition, there is a independent GSO projection associated with $2*V_7$,

$$2 * V_7 = \left(0^{20} \parallel 1^{16} 0^{12} 1^8 0^8\right) \tag{6.6}$$

which cuts the number of generations down to three, one from each of $V_2$, $V_3$, and $V_4$. Of course since we have introduced new sectors we must also worry that new generations appear in the new sectors. Indeed in this model there is new $SU(5)$ matter in the sectors, but it is all vectorlike; thus the net number of generations is still three.

The full gauge group is

$$[SU(5) \times U(1)]_{\text{flipped}} \times [U(1)]^5_{\text{flavor}} \times [SO(10) \times SO(6)]_{\text{hidden}} \tag{6.7}$$

where "flavor" indicates that the standard model fermions carry nonzero flavor dependent charges under these five extra $U(1)$s. One combination of these $U(1)$s is anomalous; we must therefore perform a vacuum shift as described in section 4.5. The analysis of all the F and D flat directions is too complicated to go into here. The vacuum shift will in general break some or all of the extra $U(1)$s at a scale of about $10^{16}$ GeV.

Here is a complete list of the massless chiral superfields in this model:
– five 10's of $SU(5)$ (with various extra $U(1)$ charges).
– two $\bar{10}$'s which combine with two combinations of the five 10's above to make two vectorlike pairs. One of these pairs is presumably the Higgs which break flipped $SU(5)$ down to $SU(3)_c \times SU(2)_L \times U(1)_Y$, although the perturbative superpotential of the effective field theory below the string scale gives no indication of this. The other pair may be superheavy after the vacuum shift.
– three $\bar{5}$'s.
– four vectorlike pairs of $5+\bar{5}$'s. One linear combination of these can be the MSSM Higgs; the others are exotics.
– 27 $SU(5) \times SO(10) \times SO(6)$ singlets carrying various $U(1)$ charges.
– three 10's of the hidden $SO(10)$.
– seven 6's and six $4+\bar{4}$ pairs of the hidden $SO(6)$.

By computing n-point functions in this conformal field theory of Weyl and Ising fermions we can determine the effective field theory below the string scale. The effective superpotential has no linear or quadratic terms (because of conformal invariance),

53 cubic terms, 15 quadratic terms, and over 1000 quintic terms. The proliferation of terms at quintic and higher order is a generic feature of semi-realistic models; fortunately only a limited number of terms at any order represent new physically important contact interactions – the rest are either physically uninteresting or represent diagrams which are composites of many lower order vertices. However to do a complete analysis of F and D flat directions for such an effective superpotential is still a daunting task even with a computer.

This model has Yukawa terms in the cubic superpotential, as well as effective Yukawas in the quartic and higher terms which arise when various $SU(5)$ singlet fields get vevs in the vacuum shift. The cubic Yukawa couplings are order one, while the higher order effective Yukawas are suppressed by powers of a parameter which is approximately 1/10. Without committing ourselves to a specific vacuum shift there is not much more that we can say in detail. However one can immediately read off from the cubic superpotential that at most one of the 3 up-type quarks (u,c,t), and at most one of the down-type quarks (d,s,b), and at most one of the charged leptons (e,$\mu$,$\tau$) has an unsuppressed Yukawa coupling. Thus this model at the very least has a natural hierarchy between the third generation and the two light generations.

Clearly a lot more detailed analysis is needed to make a hard comparision between this model and the real world. This has been accomplished to a large extent for two other classes of semi-realistic models: the $Z_3$ orbifold models, and the $SU(3)_c \times SU(2)_L \times U(1)_Y$ fermionic models of Faraggi – which are based on the same construction as flipped $SU(5)$. Flipped $SU(5)$ has an additional ambiguity relative to these models, because the details of the $SU(5)$ breaking are not determined in string perturbation theory. However all three classes of models are otherwise very similar in character. They all have potential difficulties with gauge coupling unification, fractionally charged particles, masses and mixings of the lighter generations, and providing an appropriate SUSY breaking mechanism. But as a first pass at realistic superstring phenomenology, they succeed remarkably well.

### *6.2. A Three Generation $SU(5)$ Model*

We saw in the previous subsection that you cannot have a semi-realistic string model which resembles a conventional $SU(5)$ GUT unless the $SU(5)$ current algebra is realized at Kac-Moody level greater than one. In the fermionic construction it is only possible to achieve Kac-Moody levels 1, 2, 4, and 8. Going to level two or larger allows adjoint

Higgs in the massless spectrum, but also allows the possibility of new exotics in higher dimensional irreps of the GUT group.

We will begin with a construction which produces $SO(16)$ at Kac-Moody level two. This can be done with the following six basis vectors:

$$
\begin{aligned}
V_0 &= \big((1^2)(111111)\ (1^{12})\ \|\,(1111111111111111)(1111111111111111)(0^{12})\big)\\
V_1 &= \big((1^2)(111111)\ (0^{12})\ \|\,(0000000000000000)(0000000000000000)(0^{12})\big)\\
V_2 &= \big((0^2)(000000)\ (0^{12})\ \|\,(1111111100000000)(1111111100000000)(0^{12})\big)\\
V_3 &= \big((0^2)(000000)\ (0^{12})\ \|\,(1111000011110000)(1111000011110000)(0^{12})\big)\\
V_4 &= \big((0^2)(000000)\ (0^{12})\ \|\,(1100110011001100)(1100110011001100)(0^{12})\big)\\
V_5 &= \big((1^2)(110000)(1^4 0^8)\ \|\,(\text{++++}-\!-\!-\!-\!-\!-\!-\!-\!-\text{++++})(10101010101010101010)(0^{12})\big)
\end{aligned}
\tag{6.8}
$$

where the notation "+" and "−" in $V_5$ denotes $+1/2$ and $-1/2$.

Currents from fermion bilinears always give a level one Kac-Moody algebra, unless they are abelian currents. Thus to make $SO(16)$ at level two only the 8 Cartan currents, at most, can come from fermion bilinears in the untwisted sector. In (6.8) we have basically started with two level one $SO(16)$s coming from fermion bilinears in the untwisted sector, then introduced projections to remove all but the 8+8 Cartan currents. The final basis vector, $V_5$, then removes the second 8 of these Cartan currents. Simultaneously, the new sectors $V_2$, $V_3$, $V_4$, and $2{*}V_5$ themselves contain new currents constructed entirely from twist fields. The end result is a single $SO(16)$ at level two. There is also an $SO(13)$ at level one associated with the last 13 of the 44 left-movers.

Note that in this model the first 16 left-movers make 8 Weyl fermions, and the last 12 left-movers make another 6 Weyl fermions. However the remaining 16 left-movers are *unpaired Majorana-Weyl fermions*. The presence of unpaired Majorana-Weyl fermions is necessary (though not sufficient) in the fermionic construction to realize groups at higher level.

Now we add four more basis sectors to (6.8). These break the level two $SO(16)$, which was embedded in the first 8 left-moving Weyl fermions, down to its maximal subgroup $SU(8)\times U(1)$. $V_9$ then breaks this level two $SU(8)$ down to its maximal subgroup $SU(5)\times SU(3)\times U(1)$. Both the $SU(5)$ and the horizontal flavor group $SU(3)$ are realized at level two.

$$
\begin{aligned}
V_6 &= \big((0^2)(111100)\ (0^41^40^4)\ \|\,(1111111111111111)(0000000000000000)\,(0^{12})\,\big)\\
V_7 &= \big((0^2)(001111)(1^20^21^20^6)\|\,(0000000000000000)(0000000000000000)(1^80^4)\big)\\
V_8 &= \big((0^2)(000000)\quad(0^{12})\quad\|\,(1111000011110000)(0000111100001111)(0^41^8)\big)\\
V_9 &= \big((0^2)(110011)(1^20^41^20^4)\|\,(1111000011110000)(1001011010010110)\,(0^{12})\,\big)
\end{aligned}
\tag{6.9}
$$

For completeness, here is the matrix of $k_{ij}$s which, in the notation and conventions of Kawai, Lewellen, Schwartz, and Tye, determines the phases in the partition function:

$$
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1/2 & -1/2 & -1/2 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & -1/2\\
0 & 0 & 0 & 0 & 0 & -1/2 & 0 & 0 & 0 & 0\\
0 & 0 & 0 & 0 & 0 & -1/4 & -1/2 & -1/2 & -1/2 & 0\\
0 & 0 & 0 & 0 & 0 & +1/4 & -1/2 & -1/2 & 0 & -1/2\\
0 & 0 & 0 & 0 & 0 & +1/4 & 0 & 0 & -1/2 & -1/2\\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1/2 & 0\\
0 & -1/2 & 0 & 0 & 0 & 0 & 0 & 0 & -1/2 & -1/2
\end{pmatrix}
\tag{6.10}
$$

This model has $N$=1 spacetime supersymmetry. The full gauge group is

$$
SU(5)\times\big[SU(3)\times[U(1)]^2\big]_{\text{flavor}}\times\big[SO(5)\times[SU(2)]^4\big]_{\text{hidden}}
\tag{6.11}
$$

There is no anomalous $U(1)$ in this model.

Here is a complete list of the massless chiral superfields in this model:

– an adjoint 24 of $SU(5)$ which has zero charge under the extra $U(1)$s.

– three 10's of $SU(5)$ which are in a triplet of the horizontal $SU(3)$, and which are charged under the extra $U(1)$s.

– three $\bar{5}$'s of $SU(5)$ which are in an antitriplet of the horizontal $SU(3)$, and which are charged under the extra $U(1)$s.

– two vectorlike pairs of $10+\bar{10}$ of $SU(5)$.

– one vectorlike pair of $15+\bar{15}$ of $SU(5)$.

– an $SU(5)$ anomaly-free collection of *chiral* exotics: one $\bar{15}$ of $SU(5)$, three 5's which form an $SU(3)$ triplet, three 5's which form an $SU(3)$ antitriplet, four 5's which form two doublets under one of the $SU(2)$s, and one $\bar{5}$.

– a collection of $SU(5)$ singlets which transform under the horizontal $SU(3)$ (and cancel the $SU(3)$ anomalies when combined with the above): an adjoint 8, a vectorlike $6+\bar{6}$ pair,

a chiral $\bar{6}$, four triplets which form two doublets under another of the $SU(2)$s, a vectorlike $3+\bar{3}$ pair, and two chiral antitriplets.
– two spinors and a vector under the hidden $SO(5)$, five doublet-doublets under various pairs of $SU(2)$s, one singlet which carries only a $U(1)$ charge, and one neutral singlet.

This model demonstrates a number of important facts. First of all, it shows that it is possible to obtain a three generation $SU(5)$ GUT with adjoint Higgs from a perturbative superstring vacuum. Secondly, it shows that it is possible in string theory to obtain a nonabelian horizontal flavor symmetry, with the three generations in three-dimensional irreps of this symmetry. This extra $SU(3)$ symmetry is optional; one can construct a three generation $SU(5)$ model that has only $U(1)$ gauged flavor symmetries.

On the negative side, this model shows that going to Kac-Moody level two can indeed result in the presence of dangerous exotics in larger irreps, in this case a chiral $\bar{15}$ (plus nine associated 5's to cancel its $SU(5)$ anomaly). This makes the present model unrealistic; a more important question, however, is whether obtaining three generations in a string GUT *implies* chiral exotics. We can examine this question by looking at the underlying $SU(8)$ structure. Three generations requires an odd number of 10's of $SU(5)$, which in turn requires an odd number of 28's of $SU(8)$. However, *for the particular fermionic embedding of* $SU(8)$ *used in this model*, we automatically make a massless $\bar{36}$ of $SU(8)$ every time we make a 28. Because the $\bar{36}$ contains a $\bar{15}$ of $SU(5)$, we end up with the unwanted exotic.

So we see that in this model there is a relation between the number of generations and the presence of certain exotics; *however* this relation appears to be an artifact of (1) embedding $SU(5)$ in $SU(8)$, and (2) embedding the root lattice of $SU(8)$ into fermionic charges in a particular way. Thus we strongly suspect that this problem is not generic.

Another problem of this model can be seen by computing the terms in the superpotential involving just the $SU(5)$ adjoint scalars, or terms involving adjoint scalars and $SU(5)$ singlets (which may get vevs). These are the terms which generate the effective self-couplings of the adjoint scalars from the superpotential. The surprising result is that, at least through quintic order, there are no such terms. In other words, the adjoint scalars have a flat potential – they are moduli. Actually this property had been noticed before in orbifold versions of (even generation) stringy GUTs, and a similar problem occurred in flipped $SU(5)$, so perhaps this problem *is* generic.

Let me conclude with the following observation. I have presented two GUT-like models here only because their construction is easier to describe and their massless spectra are simpler. The more generic class of semi-realistic string models break to

$SU(3)_c \times SU(2)_L \times U(1)_Y$ at the string scale. An interesting open question in superstring phenomenology is whether there exists any perturbative string solution whose spectrum and gauge group just below the string scale is exactly that of the MSSM plus hidden fields. The easiest way to answer this question in the affirmative is to construct a model.

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