Fermi National Accelerator Laboratory

FERMILAB-CONF-95-391-T

On Naturalness in the Standard Model

William A. Bardeen
Fermilab, MS 106
P.O. Box 500, Batavia, IL 60510
bardeen@fnal.gov

A talk presented at the 1995 Ontake Summer Institute, Ontake Mountain, Japan
August 27-September 2, 1995

ABSTRACT

The question of the naturality of the Standard Model of the electroweak interactions is discussed. In the context of perturbation theory, the classical scale invariance of the theory implies naturalness condition on the Higgs mass counterterms and a possible explanation of the electroweak scale.

The Standard Model of the electroweak interactions has been very successful in describing the known subatomic world in terms of $SU(3)\otimes SU(2)\otimes U(1)$ gauge dynamics. However, little is directly known about the mechanisms of electroweak symmetry breaking. In the Standard Model an elementary Higgs field is introduced with a negative mass term. The negative mass term induces an instability which causes the Higgs field to condense generating a spontaneous symmetry breaking and masses for the electroweak gauge bosons and the fermions. The scale of this symmetry breaking, and the resulting masses, is determined by the size of the negative Higgs mass term. Quantum corrections can strongly affect the size of the Higgs mass and therefore the scale of electroweak symmetry breaking. If the Standard Model were to represent the correct physics up to a high scale, it is usually assumed that the quantum corrections shift the Higgs mass terms by large amounts due to the quadratic divergences of the loop amplitudes. The fine-tuning required to keep the effective Higgs mass term at the electroweak scale and not the high scale represents a naturalness problem for the Standard Model [1]. In this talk, I will discuss an alternative view of the naturalness problem for the Standard Model.

The Higgs field plays the central role in the spontaneous breaking of the electroweak gauge interactions in the Standard Model. The masses of all known particles are generated through the gauge and Yukawa couplings of the Higgs field. Only the Higgs field has a mass term in the Standard Model. The Higgs Lagrangian is usually written as

$$L_H = (D^\mu \overline{H})(D_\mu H) - \overline{Q}_L G_U U \cdot H - \overline{Q}_L G_D D \cdot \tilde{H} \quad - \quad m_H^2 \overline{H} H \qquad (1)$$

where the quark Yukawa couplings have been explicitly displayed.

The quantum corrections to the Higgs mass terms from gauge boson and fermion loops are usually thought to generate quadratic divergences representing large shifts in the effective Higgs mass. If we use a momentum cutoff to define the loop corrections, the Higgs mass counter terms, at one loop order, are

$$\Delta m_H^2 = -4\sum_f m_f^2 (\Lambda_f^2 / v^2) \; + \; (2m_W^2 + m_Z^2 + m_H^2)(\Lambda_b^2 / v^2) \qquad (2)$$

where Λ_f (Λ_b) is the cutoff used for the fermion (boson) loops and v is the Higgs vacuum expectation value. If the Standard Model is to be valid up to high energy, then the cutoffs are at a high scale and the mass shift of Eq.(2) is expected to be large.

A central puzzle of the Standard Model concerns the mechanism for canceling these large corrections. One possibility, usually rejected, is that the the "bare" Higgs mass term is fine-tuned in each order in perturbation theory to precisely cancel the large corrections of Eq.(2). Another possibility is that the coefficient of the quadratic divergence may vanish due to a relation between the Yukawa couplings and the gauge coupling constants [2]. Implicit in such a relation is the assumption of a common cutoff for the fermion and boson loops which need not be a property of the quantum theory. It is also somewhat problematic that a consistent coupling constant relation could be maintained in higher orders of the perturbation theory or in a precise nonperturbative formulation of the full theory.

The usual statement of the naturalness theorem [3] is that the absence of large corrections can only be maintained through symmetries which protect the Higgs mass term. The symmetry of the theory must increase when the Higgs mass vanishes. An example of this mechanism occurs in supersymmetric models where the quantum corrections do not renormalize the superpotential which contains the Higgs mass terms. The electroweak symmetry breaking scale is generated through soft terms which explicitly break the supersymmetry. If the supersymmetry breaking scale is not large, then there is no fine-tuning problem for the minimal supersymmetric version of the Standard Model. We will consider an alternative solution where scale-invariance plays the role of supersymmetry.

At the classical level, the Standard Model Lagrangian, Eq.(1), is precisely scale invariant except for the Higgs mass term. Therefore, the vanishing of the Higgs mass term would increase the symmetry of the Standard Model by making the scale symmetry exact. As in the supersymmetric case, soft symmetry breaking could generate the Higgs mass and set the scale of electroweak symmetry breaking. This mechanism is completely consistent at tree level. At one loop, there is explicit breaking of the scale symmetry reflected by the logarithmic running of the coupling constants. The divergence of the scale current, the trace of the energy-momentum tensor, is not soft, as at tree level, but contains terms proportional to the beta-functions of the running couplings,

$$\Theta^{\mu}_{\mu} \approx \beta_{\lambda_i}(\{\lambda\}) \cdot O_i + \text{soft terms} \tag{3}$$

where β_{λ} are the beta-functions and {Oi} are dimension four operators.

Since these beta-functions vanish at lowest order in perturbation theory, they can not be responsible for the quadratic divergences seen in the calculation of the Higgs mass at one loop. Therefore, the one-loop quadratic divergences are unrelated to the running of the coupling constants and represent a separate, explicit breaking of the scale symmetry. This explict breaking is generated by the use of a cutoff procedure which violates the scale invariance of the tree level theory. In other cases where the cutoff procedure breaks a global symmetry, counter-terms are normally added to restore the original symmetry structure of the theory. This is not considered to be an issue of fine-tuning but merely an artifact of the particular method used to reqularize the loop calculations. We argue that this is also the case in the Standard Model and that the quadratic divergences are spurious effects of particular regularization procedures. Counter-terms must be added in perturbation theory to cancel the quadratic divergences and preserve structure of the anomalous divergence of the scale current, the trace of the energy-momentum tensor,

$$\begin{aligned} \Theta^{\mu}_{\mu}|_{classical} &= 2m^2 \overline{H} H \\ \Theta^{\mu}_{\mu}|_{one\ loop} &= 2\Delta m^2 \overline{H} H + \sum_i \beta_{\lambda_i} O_i \end{aligned} \tag{4}$$

where $\Delta m^2 \sim m^2 \cdot X$, *not* $\Lambda^2 \cdot X$.

Order by order in perturbation theory, this argument may be used to remove the explicit quadratic divergences without explicitly invoking fine-tuning. The perturbative running of the coupling constants is related to the scale breaking logarithms which occur in each order in perturbation theory. Quadratic divergences represent explicit violations of scale invariance which are not consistent with the scale Ward-Takahashi identities reflected in Eq.(4). The Higgs mass terms reflect a soft breaking of the scale symmetry and are renormalized by soft terms generated in higher orders. We note that the usual dimensional regularization procedure would generate only the logarithmic running of the couplings and not the quadratic divergences if it could be applied consistently to the Standard Model.

In perturbation theory, the scale symmetry would protect the Higgs mass from quadratic divergences and the conventional statement of the fine-tuning problem. Dimensional transmutation would be required to generate quadratic divergences from the logarithmic divergences of the perturbative expansion. This could occur in the infrared through renormalons, confinement dynamics, etc. or in the ultraviolet through Landau singularities, GUT physics, etc. For example, the Standard Model has Landau poles associated with the running of the top quark Yukawa coupling constant which identify a large but finite mass scale in the theory. These effects raise the question of whether it is possible to maintain the soft breaking scenario of the perturbative expansion in the presence of nonperturbative effects associated with these new scales. It may not be possible to answer this question without a better understanding of the nonperturbative definition of the Standard Model.

The fine-tuning issues may reappear in models where the Standard Model is embedded in a more complex theory, visible only at short distance scales. The usual GUT models where the gauge interactions are unified at a high energy scale are examples where the Standard Model is the effective low energy theory and the effective Higgs mass may be a calculable quantity in terms of the GUT physics at the high scale. The sensitivity of the Higgs mass to these new scales will depend on the scale invariance properties of the GUT scale physics. In a normal GUT model, scale invariance is explicitly broken by terms in GUT Higgs potential at the scale of the GUT symmetry breaking. In this case, it may not be possible to use the scale invariance of the effective Standard Model to protect the Higgs mass without additional fine-tuning. Even in cases where the GUT scale arises from a dimensional transmutation of a strong GUT dynamics, it may not be possible to preserve sufficient scale symmetry to protect the Higgs mass from fine-tuning. However, the fine-tuning question is now properly addressed to questions related to the scale invariance and other properties of the embedded GUT model and not on the effective Standard Model.

We have argued that the Standard Model does not, by itself, have a fine tuning problem due to the approximate scale invariance of the perturbative expansion. Fine tuning issues are related to either explicit nonpertubative aspects of the theory, such as the Landau poles, or embeddings of the Standard Model into a more complete dynamics, visible at short distance or high energy. Whether scale invariance can still be used to protect the electroweak scale depends on the structure and dynamics of these more complete formulations. Of course, these arguments do not give an explanation of the observed value of the electroweak scale, as is also true in the supersymmetric model where the supersymmetry breaking scale is adjusted to produce the correct electroweak scale.

Acknowledgements.

I thank the Organizers of the 1995 Ontake Summer Institute for their hospitality and for the stimulating environment for discussions of critical questions in elementary particle physics research. I also thank Chris Hill for discussions on the scale invariance properties of effective field theories for electroweak and other dynamics.

References.

1. M. Veltman, Acta Phys. Polon. B8, 475, 1977; Phys. Lett. 70B, 253(1977); K. Wilson, Phys. Rev. D3, 1818(1971); L. Susskind, Phys. Rev. D20, 2619(1979).

2. M. Veltman, Acta Phys. Polon. B12, 437, 1981; Y. Nambu, in "New Trends in Strong Coupling Gauge Theories", edited by M. Bando, T. Muta and K. Yamawaki (World Scientific, 1989), pg. 3.

3. S. Weinberg, Phys. Lett. 82B, 387(1979); G. 't Hooft, Lectures at the 1979 Cargese Summer Institute (1979), pg. 135.