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Next-to-Leading Order Perturbative QCD Calculations

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Abstract

The potential of the high luminosity $P\bar{P}$ jet data sets in studying the strong interaction force is discussed. In particular the theoretical aspects of the one-jet inclusive transverse energy distribution will be highlighted. The theoretical uncertainties in the Next-to-Leading order predictions for this particular distribution are discussed in detail.

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1 Introduction

With the increasing integrated luminosity at the TEVATRON, currently over 100 pb^{-1} in run 1A/B for both the CDF and DØ experiments and potentially in excess of 1000 pb^{-1} with the main injector program, QCD studies at hadron colliders enter a phase of high precision tests of the strong interactions. This offers many challenges and opportunities which can be explored in the coming years. For example, we hope to make measurements of the strong coupling constant (α_S) and parton density functions (PDF's) as well as probing the dynamics of strong interactions at very high momentum transfers using the available jet data.

With such a high statistics data set, a large theoretical effort is necessary to make sure the theoretical uncertainties are small enough so that useful information can be extracted from the observables. Already Leading Order (LO) perturbative calculations are insufficient to describe the data and calculations of at least Next-to-Leading Order (NLO) are required. This is most clearly seen in the one-jet inclusive and two-jet inclusive data presented at this workshop. The theoretical aspects of the one-jet inclusive distributions will be discussed here. For a comprehensive discussion of the status of the theoretical issues in the two-jet inclusive distributions see ref. [1].

2 High Precision QCD at $P\bar{P}$-Colliders

The availability of large jet data samples offer unique possibilities for strong interaction studies and tests of QCD at the TEVATRON in the coming decade. However, in order to perform these studies it is necessary to set a clear goal. This should both unify the experimental analysis as well as the theoretical efforts. Up to now strong interaction studies in jet physics at hadron colliders have been rather haphazard. In general one compares a given distribution with the theory, i.e. a LO or NLO perturbative QCD prediction utilizing α_S and PDF's determined mainly from experiments at much lower Q^2. After such a comparison one either reaches the conclusion that the experiment and theory agree within the uncertainties or that for certain regions of phase space there is a discrepancy. If there is agreement one can ask oneself what has been learned from this comparison and the usefulness of the efforts involved to make the comparison. If, on the other hand, there is disagreement, the usual procedure is to adjust the PDF's. However, with the current PDF sets based on a global analysis of many different data sets, it is very difficult to estimate the permissibility of the changes in view of other data. In a way one is indirectly held hostage to other, non-collider, data through the use of the PDF's and α_S.

With the higher statistics data this situation will become more apparent and eventually unmanageable, since we can expect many deviations from LO/NLO QCD. Most of the deviations are either related to uncalculated higher orders or to deficiencies in the input

PDF's. However, in the current situation it is impossible to disentangle these two effects. Indeed, the presence of deviations from QCD might be completely hidden and incorrectly ascribed to the input PDF's. To address this issue and at the same time set a well defined goal for the strong interaction studies for both experimentalists and theorists, one should attempt to use the hadron collider data to determine both α_S and the PDF's without any input from other experiments. In fact the PDF's determined in this way are complementary to the PDF's based on data obtained at HERA and other DIS experiments since the TEVATRON experiments probe the parton densities at moderate x values and much larger momentum transfers, typically $x > 10^{-2}$ and 2,500 GeV$^2 \leq Q^2 \leq$ 250,000 GeV^2. Moreover the PDF's determined at hadron colliders exhibit certain unique opportunities and advantages over other experiments. First of all, the gluon PDF can be directly measured using the di-jet data in the interesting parton fraction range between $10^{-2} < x < 10^{-1}$. The gluon PDF is weakly constrained in this region and mainly inferred from the momentum sum rules. Second, because of the ability of the detectors to perform heavy flavor tagging (i.e. charm and bottom quark tagging) one can combine this with the production of vector bosons to directly probe the flavor content of the sea quarks. Third, one measures the PDF's far above the bottom quark flavor threshold and well within the perturbative domain. This makes the NLO matrix element calculations far more reliable and consequently yielding much smaller theoretical uncertainties than for the low energy predictions used to constrain present day PDF's. The fourth advantage is that one determines the complete set of PDF's using a single detector rather than combining several experiments, each with its own set of systematic uncertainties. This makes the determination of the uncertainties in the extracted PDF's relatively straightforward. Lastly, because at a hadron collider scattering takes place over a wide range of momentum transfers, one can compare the measured evolution of the PDF's with the QCD predictions.

To achieve the goal of determining both α_S and the PDF's at hadron colliders we have to make several intermediate steps in order to gain confidence and become comfortable with the notion that this can actually be done. We also have to identify possible problems in the data/theory and develop the necessary software tools to perform the extractions. There are three distinct phases, each of which will reveal useful information and is a worthwhile goal in itself.

The first phase is to extract α_S for a given set of PDF's. One can either use a parameterization determined with a fixed internal α_S which dictates the evolution strength [2] or use PDF's with varying α_S so that the strong coupling constant can be altered simultaneously in the matrix elements and in the PDF [3]. Many possible observables can be used to determine α_S, each with its own unique features. A few examples are:

- The one-jet inclusive transverse energy distribution measures α_S over a wide range of momentum transfers (from 30 GeV all the way up to 500 GeV) and gives an interesting measurement of the evolution of the strong coupling constant as a function of the momentum transfer in the scattering [4].

- The shape of transverse momentum distribution of W and Z-boson production at high transverse momentum will give a straightforward measurement of α_S. Because one only needs to study the high transverse momentum shape of the distribution the luminosity uncertainty does not enter in the error on α_S. In addition, if one restricts oneself to Z-boson production only the transverse momenta of the charged leptons has to be measured. This gives an α_S measurement which does not require the measurement of the hadron momenta and hence should give a very small experimental error. Also the theoretical uncertainties for vector boson production at transverse energies above 50 GeV at the TEVATRON are very small [5]. With an integrated luminosity of 100 pb^{-1} both CDF and DØ already have of the order of 10,000 Z-bosons each, which should be sufficient to determine $\alpha_S(M_Z)$ with remarkably small experimental errors.

- Cross section ratio's, e.g. the ratio of W-boson plus one jet over W-boson plus zero jets [6] or simply the three jet over two jet cross section ratio for several transverse energy cuts on the jets.

In the second phase we will assume that the charged PDF's for $x > 10^{-2}$ are correctly given by the existing PDF's. We then extract both α_S and the gluon PDF using the triply differential inclusive di-jet data [7]. Finally, we can embark upon the ultimate goal which is the determination of α_S and the PDF's without using any data from other experiments. For this one can pick many combinations of subsets of the data. As an example, we list the following set:

- The transverse momentum distribution of vector bosons and jets will determine α_S.

- The triply differential di-jet data will constrain the gluon, up-quark and down-quark PDF's.

- The longitudinal momentum of the vector boson will constrain the up-quark and down-quark PDF's.

- The W-boson plus charm tagged jets cross section will constrain the strange quark PDF.

- The Z-boson or photon plus charm tagged jet cross section will constrain the charm quark PDF.

- The Z-boson or photon plus bottom-tagged jet cross section will constrain the bottom-quark PDF.

After completion of the above program, we are in a position to test QCD as the theory for strong interactions in a very rigorous manner. To begin with, observed deviations can be understood in their relation to the measured α_S and PDF's. Second, by using the measured

α_S and PDF's in other studies (such as the W-mass determination, the top-quark analysis, multi-jet studies,...) a lot of the common systematic errors will be parameterized in the measured α_S and PDF's and will therefore cancel when comparing data to theory. This makes for an easy identification of deviations from QCD without the escape hatch given by current PDF's. Finally we can compare the parton density functions determined in both deep inelastic scattering and the hadron collider at a common scale. Eventually, this might lead to a unified global fit of the PDF's to *all* hadronic data.

3 The one-jet inclusive transverse energy distribution

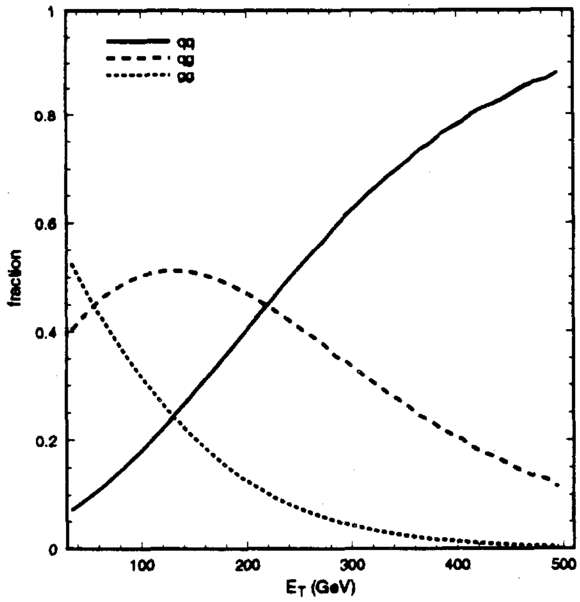

Figure 1: The relative initial state parton contributions divided in quark(antiquark)-quark(antiquark) (qq), quark(antiquark)-gluon (qg) and gluon-gluon (gg) scattering.

Jet production at large transverse energy is the most basic, and by far the most copious, high transverse momentum event at $P\bar{P}$ colliders. The transverse energy is directly related to the impact parameter, or distance scale, in the hard scattering (e.g. a 500 GeV jet corresponds to an impact parameter of order 10^{-2} fm). As such the one-jet inclusive transverse energy distribution probes by far the smallest distance scale of any observable in high energy physics collider experiments. Because of the increasing integrated luminosity at the TEVATRON both the CDF and DØ experiments become sensitive to new physics at the 1 TeV scale. If any new physics is present at this scale we will get the first indication of that through deviations from QCD in this particular distribution.

The new CDF results presented at this workshop on the one-jet inclusive distribution [8], utilizing the run 1A data, show a considerable excess of high transverse momentum jets. In fact this excess is also observed in the di-jet invariant mass (m_{jj}) and summed transverse momentum ($\sum E_T$) distributions. The observed deviations in these distributions can be easily parametrized. First we look at the different initial state contributions for the one-jet inclusive transverse energy distributions. As is shown in fig. 1 the dominant scattering below 200 GeV involves at least one initial state gluon, however above 200 GeV the scattering is dominated by the valence quark-valence antiquark initial state. By multiplying the quark-

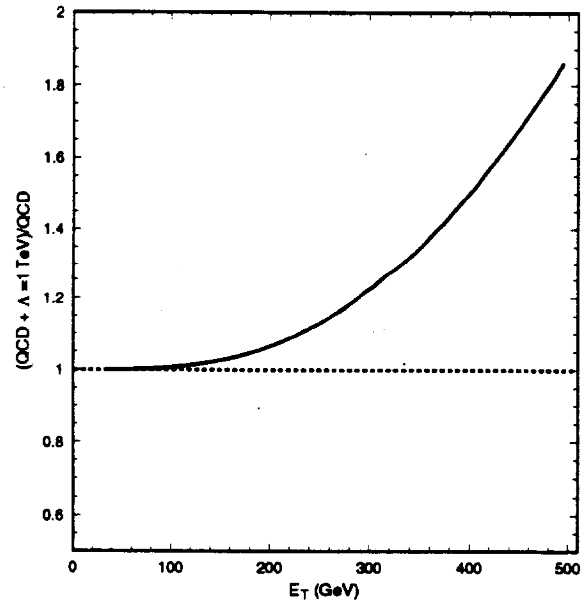

Figure 2: The ratio of QCD plus a formfactor with a scale of 1 TeV over QCD.

antiquark to quark-antiquark contribution with a simple form factor of the form $1 + Q^2/\Lambda^2$, where Q is the invariant mass of the scattering (and approximately $Q \simeq 2E_T \simeq m_{jj} \simeq \sum E_T$) and choosing $\Lambda = 1$ TeV the observed deviations in all the distributions are parametrized quite well. The effect of the form factor on the four quark cross section relative to QCD is shown in fig. 2. As can be seen the deviations from QCD are quite substantial at high E_T and are in good agreement with the observed deviations in the CDF data [8].

At the moment the experimental results are preliminary and some experimental issues still have to be investigated. However, from the theoretical viewpoint one can ask the question how reliable the NLO approximation is and up to what level should we expect the data to agree with the NLO predictions. The first observation is that the $\mathcal{O}(\alpha_S^3)$ parton scattering amplitudes have been calculated in ref. [9] and independently verified in ref. [10]. Using these matrix elements the NLO one-jet and two-jet Monte Carlo programs have been constructed by several independent groups [11]. This means that the NLO QCD predictions for these particular observables are very well known and the possibility of mistakes is very unlikely. The second observation is that the deviations occur at very high transverse momentum of the jets ($E_T > 200$ GeV). This immediately excludes many possible causes of unreliability of the perturbative expansion associated with soft jets and non-perturbative physics. In fact, the perturbative expansion should work better and better as the jet transverse energy increases due to the running of the strong coupling constant. The fact that the data agrees very well with the NLO prediction below 200 GeV, makes it very difficult to ascribe the observed deviations to uncalculated higher order terms. Furthermore the non-perturbative effects are severely suppressed by at least one power in the transverse energy. We will now look more quantitatively to the uncertainties and try to estimate the expected theoretical errors. We will start by separating the uncertainty into two components. First we look at the normalization of the distribution, which is closely related to the value of α_S. Next we will look at the shape of the distribution, which indirectly also depends on α_S.

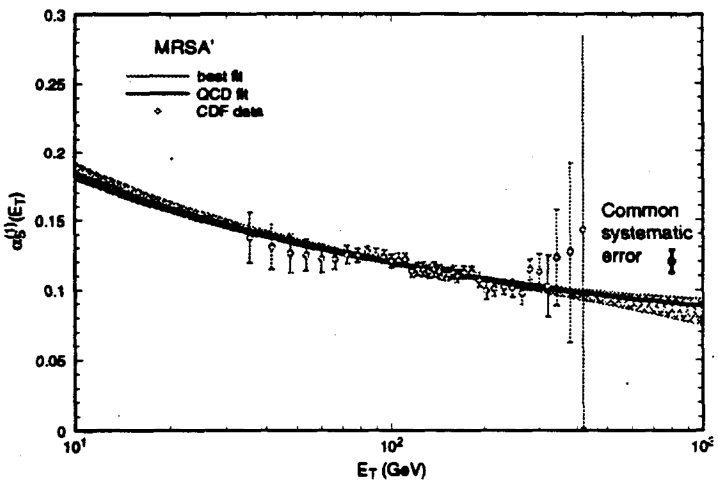

Figure 3: The measured $\alpha_S(E_T)$ from the CDF one-jet inclusive transverse energy measurement using the '88-'89 TEVATRON run with an accumulated luminosity of 4.1 pb^{-1}.

Studying the normalization is equivalent to determining α_S. Because the Born prediction for the one-jet inclusive transverse energy distribution already starts out at order α_S^2, the normalization is very sensitive to the strong coupling constant. The procedure to extract α_S is very simple [4]: we solve the third order polynomial in α_S

$$\frac{d\,\sigma}{d\,E_T} = \alpha_S^2(\mu_R)A(x_T) + \alpha_S^3(\mu_R)B(x_T,\mu_R) \qquad (1)$$

where $x_T = 2E_T/\sqrt{S}$. The LHS is given by the data, while the functions $A(x_T)$ and $B(x_T,\mu_R)$ are calculated using perturbative QCD and the renormalization scale μ_R is chosen to be equal to the momentum transfer in the event, $\mu_R = E_T$. Using the CDF '88-'89 data with an integrated luminosity of 4.1 pb^{-1} [12], measures $\alpha_S(E_T)$ between $30 < E_T < 500$ GeV as shown in fig 3. Subsequently we can evolve α_S from scale E_T to scale M_Z, this should give us a value for $\alpha_S(M_Z)$ independent of the momentum transfer in the event. Any residual dependence on the momentum transfer represents deviations from the expected QCD running of the coupling constant. The results are shown in fig. 4. As can be seen from the figure the average value is $\alpha_S(M_Z) \simeq 0.121 \pm 0.008 \pm 0.003$ where the first error

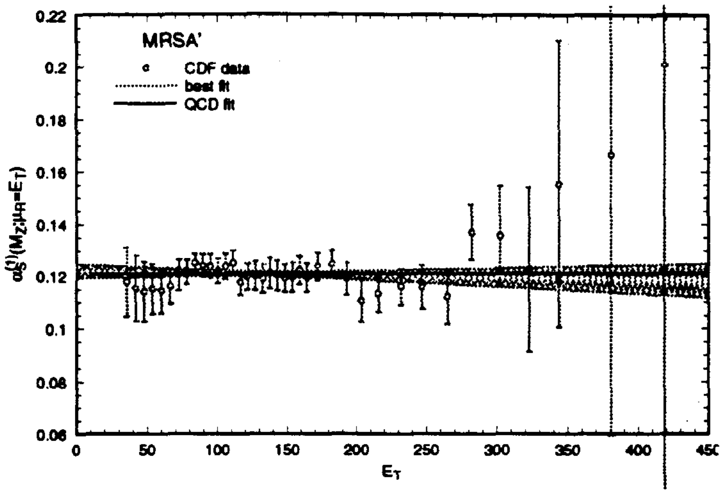

Figure 4: The value of $\alpha_S(M_Z)$ as a function of the momentum transfer at which it was measured.

is the systematic dominated experimental error and the second uncertainty is due to the freedom of renormalization choice in the theoretical predictions. Note that within one-sigma error the running of the strong coupling constant (or in other words the momentum transfer dependence of $\alpha_S(M_Z)$) is independent of the transverse energy of the jet. The measured value of the strong coupling constant is consistent with both the e^+e^--collider values and the DIS-value at which the PDF is evolved. This result was based on the '88-'89 CDF data set. At present both CDF and DØ have in excess of 100 pb^{-1} of integrated luminosity. The factor of twenty five times more data will have a large impact on the experimental error in the extracted α_S there both systematic and statistical error are expected to scale with the square root of the number of events. This would give a projected experimental error from the current run 1A/B of roughly equal or smaller size than the theoretical uncertainty. The future main injector run and eventually the LHC will easily reduce the experimental error further. For an improvement on the theoretical uncertainty the order α_s^4 Next-to-Next-to-Leading Order one-jet inclusive cross section has to be calculated. The final theoretical absolute normalization error in the one-jet inclusive transverse energy distribution is simply related to the theoretical uncertainty in α_S and estimated to be $\Delta\sigma/\sigma \sim 2\Delta\alpha_S/\alpha_S \sim 0.05$.

The reliability of the perturbative QCD prediction for the shape depends on the particular $x_T = 2E_T/\sqrt{S}$ region of the distribution. There are two potential infrared unsafe regions. The first region is the obvious $x_T \to 0$ limit, associated with the emission of soft jets at low momentum transfer. However, another region is present which is the endpoint singularity of the distribution, as $x_T \to 1$. In this limit all soft radiation in addition to the di-jet system is kinematically suppressed and large radiative corrections are expected. It is now important to know at which E_T values the $\log^2(x_T)$ and $\log(1-x_T)$ behaviour become important

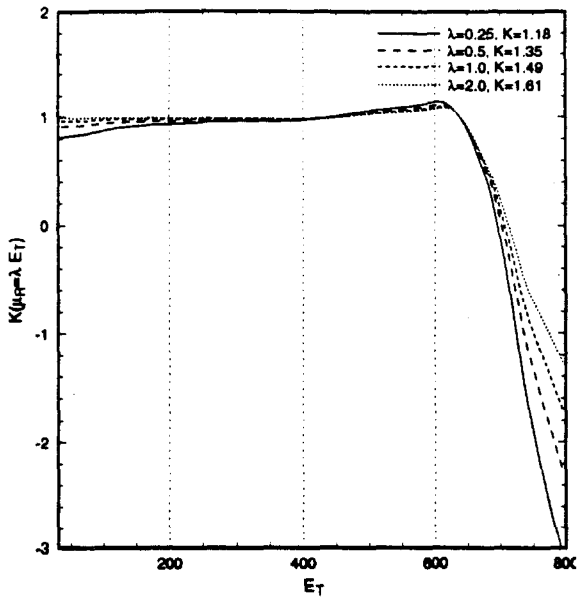

Figure 5: The K-factor normalized to $K(\mu_R = E_T)$ at $E_T = 400$ GeV as a function of the transverse momentum of the jet. (To obtain the unnormalized K-factor multiply with factor indicated in legend.)

and consequently the perturbative expansion breaks down. Because these logarithms are present in the NLO calculation we can quantitatively identify the E_T regions for which the perturbative expansion is valid. In fig. 5 we plot the K factor (that is the ratio of the NLO over the LO differential cross sections) as a function of E_T. We clearly see the onset of the $\log(1 - x_T)$ terms above $E_T \simeq 600$ GeV. The small E_T logarithms are not yet apparent at 30 GeV. One note is that a large part of the K-factor renormalization scale dependence is absorbed in the running coupling constant. This is because $\alpha_S^2 \times K$ is the renormalization scale independent quantity, while both α_S and the K-factor themselves are dependent on the renormalization scale. Indeed taking the running α_S into account the variations due to different renormalization scale choices is less than 5% in the range between 30 and 500 GeV. It is clear that in order to compare the theory with the data for $E_T > 600$ GeV a resummation of the logarithms is in order. However within the region of interest, covered by the experimental data, between 30 and 500 GeV, there are no large logarithms present and the perturbative calculation should be reliable. We estimate the residual higher order effects by varying the renormalization scale between $E_T/4$ and $2E_T$ and find a theoretical uncertainty of less than 5%.

This gives a combined shape and normalization uncertainty due to higher order corrections less than 10% in the E_T region relevant for the experiment. Above E_T of 600 GeV large corrections can be expected. In view of the observed deviations it is clear one cannot explain the discrepancy between NLO QCD and the data by uncalculated higher order terms.

The only remaining source of uncertainty are the input parton density functions. As can

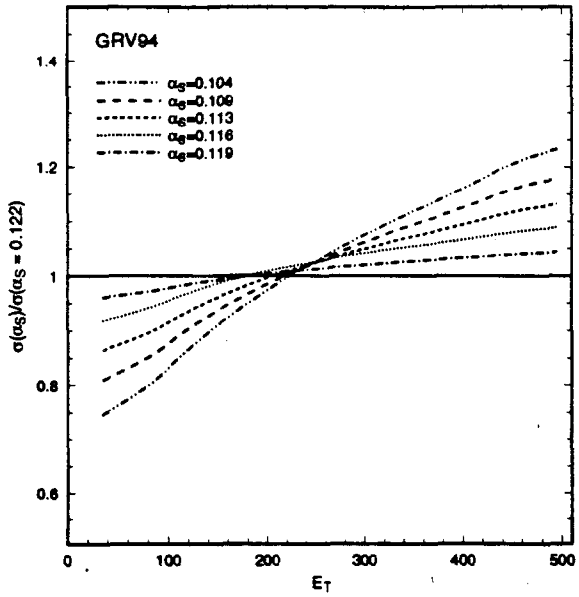

Figure 6: The ratio of the NLO one-jet inclusive transverse energy distribution with several choices of $\alpha_S(M_Z)$ over that obtained with $\alpha_S(M_Z) = 0.122$.

be seen from fig. 1 the dominant scattering mode above momentum transfers of 200 GeV is quark-antiquark annihilation. The associated charged PDF's are mainly determined by low Q^2 DIS experiments and subsequently evolved over a large range to the energy scales of the jets. This means that the charged PDF's at the high E_T energy scales are quite sensitive to the particular value of α_s. In fig. 6 we see that lowering $\alpha_S(M_Z)$ has different effects at low E_T and high E_T. In the gluon initial state dominated low E_T regions lowering the coupling constant has the expected effect, the differential cross section is lowered accordingly. However, at high E_T we see that the lowering of the coupling constant is compensated by the increase of quark density in the proton due to the slower running of the PDF's. In fact the differential cross section is substantially increased by lowering α_S for the highest E_T-bins. We see that the shape of the distribution is rather sensitive to the value of α_S. By determining α_s for $E_T < 200$ GeV where NLO and data agree well we in fact fix the hardness of the tail by fixing the evolution rate of the charged PDF's. To explain the observed deviations by changing the PDF's one has to make rather radical changes in the PDF's. Further investigation of this particular problem is required.

4 Conclusions

The accumulated QCD data at the TEVATRON in run 1A/B with an integrated luminosity well over 100 pb^{-1} for both CDF and DØ can be used to determine both α_S and PDF's without input from other experiments. With this the test of QCD as the theory of strong interactions enters a new phase of precision, many surprises can be expected. The first indications were already presented in this workshop as deviations in the one-jet inclusive transverse energy distribution at high transverse energies. If they are not due to some experimental issue the only two remaining explanations are some unexpected behavior of the PDF's at large parton fractions or the presence of new physics at the 1 TeV scale. Both these possibilities are very exciting. The NLO predictions should, with the current experimental uncertainties, be adequate to describe the data with an uncertainty of at most 10% for each point in the distribution. Apart from the high E_T-tail NLO QCD indeed describes the data very well and can be used to extract $\alpha_S(M_Z)$ for a whole range of momentum transfers. With the rapid increasing integrated luminosity at the TEVATRON the α_S results will quickly become more and more accurate and one will be able to identify the cause of the observed deviations. No matter what the outcome this demonstrates clearly the potential of the TEVATRON program in the main injector phase (delivering in excess of 1 fb^{-1} per detector) as a precision measurement of the strong interaction sector.

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